smaller $-O((m^2/s)^{2-|\epsilon_1+\epsilon_2|})$ – than the $O((1/\ln s)^N)$ error terms not explicitly written, it has been included to show how the familiar longitudinal structure found in (19) enters here.

[31]When the effects of isospin conservation are included, f_2 for a particular charge state can acquire a term proportional to lns. This is still consistent with the effective short range of kinematic correlations.

[32]W. R. Frazer *et al.*, Rev. Mod. Phys. 44, 284 (1972).
[33]M. LeBellac, Phys. Letters 37B, 413 (1971).

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Multiplicity Distribution in High-Energy *pp* Collisions*

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An analysis is made of the fluctuation observed in the multiplicity distribution at 205 GeV/c. A compound Poisson distribution is proposed to describe the distribution of multiparticle production. Applications are given to currently available pp data.

A recent NAL-ANL experiment[1] on the multiplicity of charged particles produced by 205-GeV/c pp collisions indicates that the prong distribution deviates from the Poisson law,

$$P_n(m) = e^{-m}\frac{m^n}{n!}, \qquad (1)$$

where $m=\langle n\rangle$ designates the average number of prongs. The authors have used the *width parameter*

$$f_2 = \langle n(n-1)\rangle - \langle n\rangle^2 \qquad (2)$$

to test the validity of the Poisson distribution (1) and found a very large fluctuation; for negative prongs $f_2^- = 0.95 \pm 0.21$ instead of zero as expected from (1). Thus the Poisson distribution is not valid for their data. The purpose of this paper is to investigate the fluctuation observed in the NAL-ANL and other pp experiments, and to propose a compound Poisson process to describe the multiparticle distribution.

First, let us investigate how the NAL-ANL data[1] deviate from the Poisson distribution (1), according to which the cross section for observing events of n prongs is given by

$$\sigma_n = \sigma_{\text{inel}} P_n(\langle n\rangle), \qquad (3)$$

where $\sigma_{\text{inel}} = \sum \sigma_n$ is the total inelastic cross section. For this purpose we use the following consistency test.[2] Knowing the experimental cross sections σ_n^{exp}, we compute

$$X_n = \ln(n!\,\sigma_n^{\text{exp}}) \qquad (4)$$

and plot X_n vs n. Clearly, if the Poisson distribution holds, the points should lie on a straight line, its slope being equal to $\ln\langle n\rangle$ and its intercept at $n=0$ equal to $\ln(\sigma_{\text{inel}}/e^{\langle n\rangle})$. Figure 1 shows such a plot for the negative prongs $n=n_-$. The solid line is a least-squares fit in the range $n_- \le 5$. We obtain a reasonable fit, the χ^2 being equal to 2.01 for 3 degrees of freedom. From the slope and the intercept we estimate

$$\langle n_-\rangle = 3.08 \pm 0.36$$

and

$$\sigma_{\text{inel}} = 29.4 \pm 5.8 \text{ mb},$$

which are consistent with 2.82 ± 0.08 and 32.7 ± 1.2 of the experimental values of Ref. 1.

If we extrapolate the fitted straight line beyond the fitting range, namely the dashed line in Fig. 1, then we notice that the points outside the fitting range deviate systematically further and further from the extrapolated straight line. We can estimate the excess of events for $n_- \ge 6$ by comparing with the fitted Poisson distribution in the range from $n_-=1$ to 5. In this way we find the amount of cross sections corresponding to these events in excess and obtain ~0.6 mb to compare with 6.52 mb of the measured cross sections. Therefore, although these events amount to only ~2% of the total, they give rather a large contribution to the fluctuation. This is because these events occur in the part of distribution with large number of prongs.

With these remarks we proceed to consider the modification to be made for the Poisson distribution (1) in order to take into account the fluctuations observed in various multiplicity distributions. First we recall that the basic assumption to be made to derive the Poisson distribution is that the particles are produced independently with-

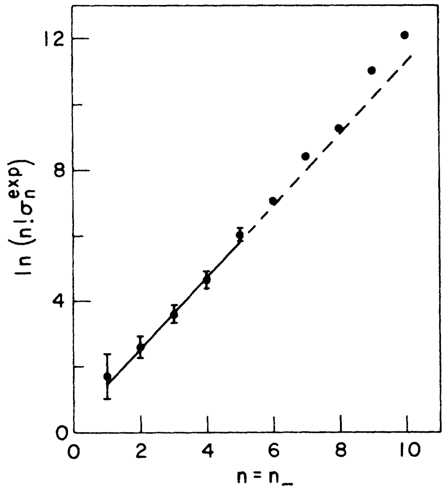

FIG. 1. Consistency test of the Poisson distribution with NAL-ANL data. The solid straight line is the least-squares fit. Systematic deviations from the Poisson distribution are noticeable by comparing the points of $n_- \geq 6$ with the extrapolated straight line shown by the dashed line.

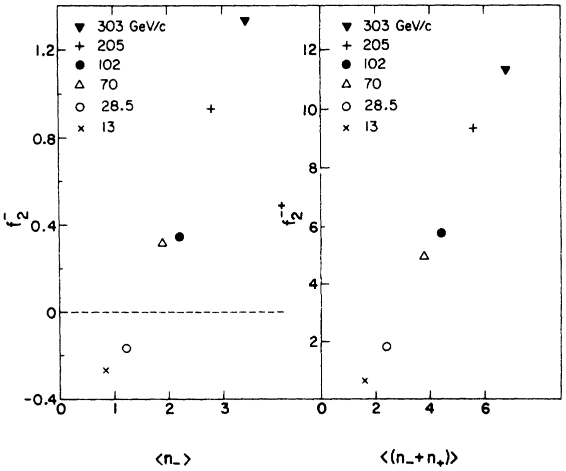

FIG. 2. Correlations between the width parameter $f_2 = \langle n(n-1)\rangle - \langle n \rangle^2$ and the average multiplicity $\langle n \rangle$. (a) $n = n_-$. Values of $\langle n_- \rangle$ and f_2^- together with their errors are reproduced in Table I. (b) $n = n_- + n_+$. Values of $\langle (n_- + n_+) \rangle$ and f_2^{-+} are deduced from (a) by assuming $n_+ = n_-$.

out interaction. This assumption is unjustified because of strong interactions among particles. As pointed out by Landau,[3] the number of particles is not constant during the stage of expansion of the system constituted by particles produced in the primary act of pp collision. The number of particles observed in an event is not the number of particles produced in the first act of pp collision, but rather the net sum of particles which are produced in the primary act of pp collision and those created as well as annihilated during the state of expansion. The mutual interaction among particles ends when all particles are far apart and behave like free particles.

Next we note that in the independent-emission model, the parameter m of the Poisson distribution (1) represents the probability of emission of a single particle and that the probability for emitting n particles is equal to m^n. Thus this parameter needs to be modified to account for the strong interaction of particles produced in the primary pp collision. We propose the following compound Poisson distribution with two parameters α and β:

$$p_n(\alpha,\beta) = e^{-\alpha}\frac{(\alpha+\alpha\beta n)^n}{n!} \tag{5}$$

and write the cross section for n particles as

$$\sigma_n = c p_n(\alpha,\beta), \tag{6}$$

c being a normalization constant. Note that the series $\sum p_n(\alpha,\beta)$ is absolutely convergent if $\alpha|\beta| < 1$, but its sum is different from 1 as in the case $\beta = 0$.

It should be mentioned that our choice of a linear term $(\alpha + \alpha\beta n)$ to replace m in (1) is rather empirical. If instead of this linear approximation we choose, for instance, $(\alpha + \alpha\beta n^\gamma)$, then from the data analyzed in the present work we find $\gamma = 0.91 \pm 0.11$. Therefore, for simplicity, we set $\gamma = 1$.

The meaning of the parameters α and β will become apparent, if we express (5) in terms of (1):

$$p_n(\alpha,\beta) = P_n(\alpha)(1+\beta n)^n . \tag{7}$$

Thus, α is the analog of the average multiplicity and is positive definite, whereas β describes the deviation from the Poisson distribution through the factor $(1+\beta n)^n$. It is easily seen from (7) that the distribution under consideration $p_n(\alpha,\beta)$ is broader than the Poisson distribution $P_n(\alpha)$ if $\beta > 0$, and narrower if $\beta < 0$. Therefore β is directly related to the width parameter (2) and its sign is determined by that of f_2. We shall see later that the magnitude of β is rather small; consequently the factor $(1+\beta n)^n$ is always positive even if β is negative. In this regard, it should be mentioned that the sign of f_2 depends essentially on the convention of choosing the variable n. Thus, its physical meaning is not clear. Indeed, consider, e.g., the negative particles $n = n_-$; then we find that f_2^- changes sign when the incident momentum increases from 13 to 205 GeV/c, as has been noted by Charlton *et al.*[1] On the other hand, if we choose both positive and negative mesons $n = n_- + n_+$ and assume $n_+ = n_-$, then we have

$$f_2^{-+} = 4 f_2^- + 2\langle n_- \rangle , \tag{8}$$

which is always positive for the same set of data.

TABLE I. Experimental data and fits for $n = n_-$.

Experiments		Data $\langle n_-\rangle$	f_2^-	α	β	χ^2/point	$\langle n_-\rangle$	f_2^-
NAL-UCLA	303 GeV/c	3.43 ± 0.08	1.36 ± 0.25	2.23 ± 0.28	0.065 ± 0.004	2.21	3.42	1.50
NAL-ANL	205 GeV/c	2.82 ± 0.08	0.95 ± 0.21	2.18 ± 0.23	0.048 ± 0.002	1.52	2.92	0.74
Mich.-Roch.	102 GeV/c	2.19 ± 0.07	0.34 ± 0.10	1.79 ± 0.2	0.044 ± 0.003	0.50	2.22	0.36
IPHE [a]	70 GeV/c	1.90 ± 0.04	0.32 ± 0.13	1.59 ± 0.38	0.035 ± 0.003	1.94	1.85	0.19
LBL	28.5 GeV/c	1.25 ± 0.03	−0.16 ± 0.09	1.53 ± 0.45	−0.057 ± 0.007	0.79	1.24	−0.17
LBL	13 GeV/c	0.82 ± 0.04	−0.26 ± 0.12	b				

[a] See Ref. 4.
[b] See Ref. 8.

This is illustrated in Figs. 2(a) and 2(b). We have used besides NAL-ANL data[1] also those of Serpukhov at 70 GeV/c,[4] LBL at 28.5 and 13 GeV/c,[5] NAL-UCLA at 303 GeV/c,[6] and Michigan-Rochester at 102 GeV/c.[7] From these plots, we observe the following characteristic feature: f_2 increasees with $\langle n\rangle$ irrespective of the choice of n, whether $n = n_-$ or $n = n_- + n_+$. This indicates that the derivation from the Poisson distribution increases with the multiplicity.

In an attempt to estimate parameters α and β, we proceed to perform least-squares fits using currently available pp data. For the sake of simplicity, we have arbitrarily chosen for variable $n = n_-$, in spite of the fact that it would be more realistic if we could make use of all mesons, charged as well as neutral ones. The results of fits thus made together with experimental data taken from Refs. 1, 4, 5, and 6 are presented in Table I.[8] Some fitted curves are shown in Fig. 3.

Generally speaking, these fits are more satisfactory than those with a Poisson distribution, i.e., $\beta = 0$.[9] As a further check of our fits, we have computed for each experiment the average $\langle n_-\rangle$ and the width parameter f_2^- using the fitted parameters α and β. The values thus obtained are listed in the last two columns of Table I. They are consistent with experimental values listed in the same table.

It is noteworthy that the parameter β we have introduced to account for deviations from the Poisson distribution is found to be rather small. Thus, we have strong feelings that the main feature of the multiplicity distribution follows Poisson's law, and that the term in β of the compound Poisson distribution may be regarded as a correction term.

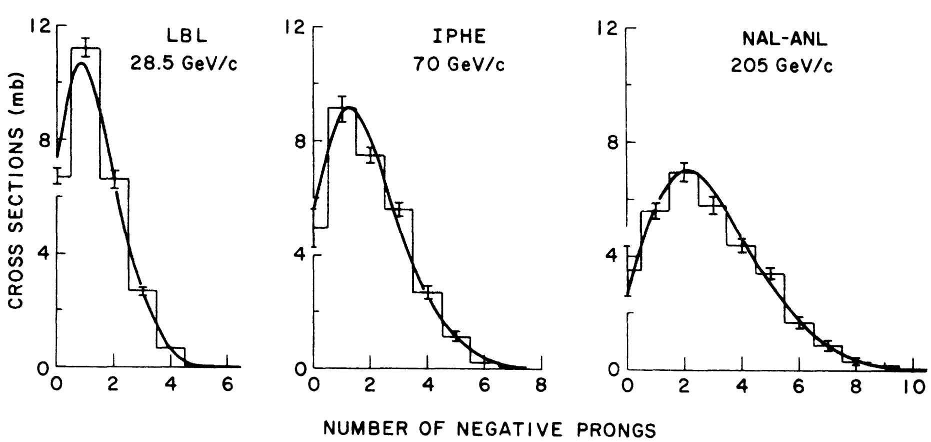

FIG. 3. Fits with compound Poisson distribution. Parameters of fit and χ^2 are listed in Table I.

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]G. Charlton, Y. Cho, M. Derrick, R. Engelmann, T. Fields, L. Hyman, K. Jaeger, U. Mehtani, B. Musgrave, Y. Oren, D. Rhines, P. Schreiner, H. Yuta, L. Voyvodic, R. Walker, J. Whitmore, H. B. Crawley, Z. Ming Ma, and R. G. Glasser, Phys. Rev. Letters 29, 515 (1972).

[2]We refer to a previous paper for a detailed description of the test with other experimental data: T. F. Hoang, Nuovo Cimento 2A, 467 (1971).

[3]L. D. Landau, Izv. Akad. Nauk. SSR Ser. Fiz. 17, 51 (1953); Usp. Fiz. Nauk 56, 309 (1955). See also S. Z. Belen'kij and L. D. Landau, Suppl. Nuovo Cimento 3, 15 (1956).

[4]Soviet-French collaboration, Phys. Letters 42B, 519 (1972). The data here used are taken from the paper by E. Berger, Ref. 9.

[5]D. B. Smith, R. J. Sprafka, and J. A. Anderson, Phys. Rev. Letters 23, 1064 (1969).

[6]F. T. Dao, D. Gordon, J. Lach, E. Malamud, T. Meyer, R. Poster, and W. Slater, Phys. Rev. Letters 29, 1627 (1972).

[7]J. W. Chapman, N. Green, B. P. Roe, A. A. Seidl, D. Sinclair, J. C. Vander Velde, C. M. Bromberg, D. Cohen, T. Ferbel, P. Slattery, S. Stone, and B. Werner, Phys. Rev. Letters 29, 1686 (1972).

[8]We have left aside 13-GeV/c data because it is not possible to attempt a reasonable fit with only four points to estimate three known constants of the fitting distribution.

[9]For comparison we mention that the data here analyzed have been fitted by E. L. Berger with a Poisson distribution [see Phys. Rev. Letters 29, 887 (1972)] and that LBL 28.5-GeV/c data have been tested with the Poisson distribution for various choices of n in a previous paper (Ref. 2).

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Errata

Covariant Phase-Space Calculations of n-Body Decay and Production Processes. II. Rajendra Kumar [Phys. Rev. D 2, 1902 (1970)]. 1. In Eq. (3.5), $\zeta_k = -C(s; M_k^2, s_k'; \bar{s}_{k-1}, s_k; \bar{s}_k)$ should read

$$\zeta_k = \overline{C}(s; M_k^2, s_k'; \bar{s}_{k-1}, s_k; \bar{s}_k),$$

where

$$\overline{C}(s; a, b; c, d; t) \equiv [(s+a-b)(s+c-d)+2s(a+c-t)] \times [\lambda(s, a, b)\lambda(s, c, d)]^{-1/2}.$$

2. In the expressions for Z_k and $\overline{Z}_k$, Eqs. (3.5) and (3.9), m_k^2 should read M_k^2. 3. In Eq. (4.38), τ should read τ_2. 4. In the second line following Eq. (4.39), $s_2-(t_1')$ should read $s_{2\text{-}}(t_1')$.

Relativistic Treatment of Low-Energy Nucleon-Nucleon Scattering. R. A. Bryan and A. Gersten [Phys. Rev. D 6, 341 (1972)]. In computing the one-meson-exchange potentials of the wide mesons (the ρ and the ϵ) in the two-pole approximation described in the Appendix, we did not use the value of the pion mass listed in Table I (138.7 MeV); rather, we used the value 135.0 MeV. Furthermore, we rounded off the value of $\gamma = \Gamma m/(m^2 - 4m_\pi^2)^{1/2}$ to 139 MeV in the case of the ρ, and to 430 MeV in the case of the ϵ. Thus, in the notation of the Appendix, we obtain the following parameters:

	A	m_1	m_2
ρ	0.65064	993.19 MeV	605.12 MeV
ϵ	0.72900	1173.00 MeV	504.62 MeV

The one-pion-exchange potential was computed using the parameters listed in Table I; thus, the pion mass was set to 138.7 MeV in that case.

All the above remarks apply to our calculations for all four models described in the paper (Fits A, B, C, and D).

One of us (R.A.B.) would like to thank Mr. Tom Connor for reminding us of the numerical choices described in this Erratum.