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Exotic SU(3) Representations in Inclusive Reactions*

H. B. Mathis and Richard H. Capps
Physics Department, Purdue University, Lafayette, Indiana 47907
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The general inclusive reaction $a+b\to c+X$ is considered in the b fragmentation region. Several well-known hypotheses concerning early scaling in cases of $(b\bar{c})$ or $(ab\bar{c})$ states of exotic Y (hypercharge) and I_z are extended to states of nonexotic Y and I_z and exotic SU(3) representation. Several testable predictions are made. These may be useful in studying both the SU(3) symmetry of various Regge couplings and the effects of SU(3) mass differences.

I. INTRODUCTION AND GENERAL PROCEDURE

The optical-theorem analysis of Mueller has been a useful tool for analyzing the energy-dependent terms in the invariant cross sections for inclusive reactions of the type $a+b\to c+X$, where a, b, and c are hadrons, and X is any set of particles.[1] We are concerned with the fragmentation region of particle b, which may be either the projectile or target. The Mueller analysis relates the invariant cross section F in this region to the imaginary part of the $a+(b\bar{c})$ scattering amplitude in an unphysical region, as shown in Fig. 1. The energy-dependent terms are those for which the exchanged Reggeon (α in Fig. 1) is not a Pomeranchukon. Measurements of the leading energy-dependent terms provide information concerning the couplings of the vector- and tensor-meson trajectories to the particle a, and to the state $(b\bar{c})$.

Many papers have been written concerning the effects expected or observed when either of the states $(b\bar{c})$ or $(ab\bar{c})$ is of exotic Y (hypercharge) and I_z.[2-4] However, if SU(2) or SU(3) symmetry is approximately valid, exotic $(b\bar{c})$ and $(ab\bar{c})$ states exist that are of nonexotic Y and I_z. An example is the state $(2)^{-1/2}\,(p\pi^+ + \Sigma^+K^+)$, which belongs to the exotic SU(3) representation $\underline{27}$. In this paper, we pay particular attention to such states.

The invariant cross section for the process j is denoted by F_j and given by $F_j = E d\sigma/d^3p$, where E and p are the energy and momentum of the produced particle b. In the fragmentation region of b at high energy, the dominating Regge exchanges in the Mueller analysis are those of Fig. 1, with α identified with the Pomeranchukon and the V and T (vector and tensor) meson trajectories. In this region, F_j is given by[1,2]

$$F_j(s, p_{\parallel}^b, p_{\perp}^2) = A_j(p_{\parallel}^b, p_{\perp}^2) + s^{\alpha(VT)-1} B_j(p_{\parallel}^b, p_{\perp}^2)\,, \quad (1)$$

where s is the square of the total energy in the center-of-mass system, $p_{\parallel}^b$ is the momentum of c in the direction of a in the b rest system, $p_{\perp}$ is the transverse momentum of c, and $\alpha(VT)$ is the intercept of the Regge trajectories of the $Y = I_z = 0$, V, and T mesons.

We define nonexotic states to be meson states that can be formed from quark-antiquark pairs, and baryon states that can be formed from three quarks. In a recent paper by one of the authors (R.C.), it was shown from generally accepted assumptions that there must be a region of small $p_{\parallel}^b$ such that the energy-dependent terms [B terms in Eq. (1)] are small for exotic $(b\bar{c})$, compared to their average size when $(b\bar{c})$ is not exotic.[3] The extent of this "extreme fragmentation region" may be appreciable in $p_{\parallel}^b$ and $p_{\perp}^2$, and is to be determined by experiment. For example, if b and c are pseudoscalar mesons, there is experimental evidence that the extreme fragmentation region is at least as large as $p_{\parallel}^b < 1.0$ GeV/c.[5]

Chan *et al.* have hypothesized that the B terms in Eq. (1) are small when $(ab\bar{c})$ is exotic.[2] This "exotic $ab\bar{c}$ hypothesis" cannot be true outside the extreme fragmentation region, but is supported by many experiments within the extreme fragmentation region.[3]

In this paper some results of Refs. 2 and 3 are extended to interactions such that $(b\bar{c})$ and $(ab\bar{c})$ are of nonexotic Y and I_z, but one or both of them corresponds to an exotic SU(2) or SU(3) representation. SU(3) symmetry of the Regge couplings is assumed, and relations between the B terms of different processes are obtained. We pay special attention to relations that are expected to remain nearly true despite the large mass differences within SU(3) multiplets. It is assumed that the vector and tensor trajectories are nonexotic, i.e., are $Y = I_z = 0$ states of SU(3) singlets and octets.

Our results are given in Sec. II. We consider only processes for which particle b is a nucleon. Sections II A, II B, and II C are concerned with processes for which particle c belongs, respectively, to the meson octet, baryon resonance decuplet, and baryon octet.

II. RESULTS

A. States of Baryon Number One

We consider first the case when particle b is a nucleon and c is a meson, so that the baryon number of the state $(b\bar{c})$ is one. In the extreme fragmentation region, the state $(b\bar{c})$ behaves as a linear combination of a singlet, octet, and decuplet. The least ambiguity in our predictions will occur when the I_z and Y of $(b\bar{c})$ correspond to a decuplet state but not to an octet state. Of such possibilities, that which is most accesible experimentally is the $p\pi^+$ possibility, so we consider only the processes $a+p \to \pi^- + X$ in the p fragmentation region, where a is a pseudoscalar meson.

The $p\pi^+$ state can be written as $2^{-1/2}(\psi_{27}+\psi_{10})$, where ψ_i is a state of the SU(3) representation i. If the ψ_{27} part of the $(b\bar{c})$ state does not interact with the V and T-meson trajectories, SU(3) symmetry of the couplings leads to the simple result

$$B(p:\pi^-|\pi^-) = B(p:\pi^-|K^-), \qquad (2)$$

where the notation $(b:c|a)$ refers to the process $a+b \to c+X$ in the b fragmentation region. Thus, Eq. (2) should be valid in the extreme fragmentation region. This result is independent of the SU(3) representations of the contributing trajectories; it may be obtained simply by summing over the s-channel SU(3) representations of the $a-(b\bar{c})$ scattering process.[6]

Although both the processes $\pi^- p \to \pi^- X$ and $K^- p \to \pi^- X$ have been studied experimentally, there are not yet enough data in the literature to make an estimate of the energy-dependent term of the $K^- p \to \pi^- X$ invariant cross section. When $B(p:\pi^-|K^-)$ is measured, Eq. (2) will be useful as a direct test of the SU(3) prediction of the coupling ratios of the V and T meson trajectories to π and K mesons. Such a direct comparison cannot be made by measuring the energy dependence of πN and KN elastic scattering, because the $\pi N/KN$ ratio depends on the F/D values of the couplings of the octet V and T trajectories to nucleons.

It is pointed out in Ref. 3 that for small $p_\parallel^b$, one expects both the A and B terms of the invariant cross section to be small when $(b\bar{c})$ is exotic. Thus, for small $p_\parallel^{\text{lab}}$, the assumption of an SU(3) singlet Pomeranchukon leads to the prediction $A(p:\pi^-|P_1) = A(p:\pi^-|P_2)$, where P_1 and P_2 are any two members of the P-meson octet. Perhaps the most useful choices for P_1 and P_2 are the π^+ and K^+, since the processes are of exotic $(ab\bar{c})$ in these cases and are expected to contain very small B terms in the extreme fragmentation region.

The result $A(p:\pi^-|\pi^+) = A(p:\pi^-|K^+)$ follows even if the ψ_{27} part of the $p\pi^+$ $(b\bar{c})$ state contributes.

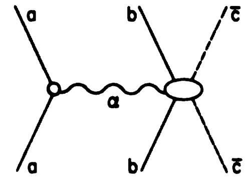

FIG. 1. Mueller-Regge diagram for b fragmentation region.

This is one of many similar Pomeranchukon-exchange predictions, and is well known.[7] However, it seems to have been overlooked in the literature that this comparison may be very useful as a check of the usual Regge-pole model of high-energy πN and KN scattering. It appears that the high-energy limit of the KN total cross section is about 20% lower than that of the πN cross section.[8] In the usual Regge model, this implies that the Pomeranchukon coupling of the K is less than that of the π, in violation of the identification of the Pomeranchukon as an SU(3) singlet. If this model is correct, the A term of the $K^+p \to \pi^- X$ invariant cross section should be about 20% smaller than that of the $\pi^+ p \to \pi^- X$ cross section.

Experimental evidence suggests that for $p_\parallel^{\text{lab}} < 0.4$ GeV/c, $F(p:\pi^-|\pi^+)$ is nearly energy-independent in the range of incident π momenta 6–22 GeV/c.[5,9] The comparison of $F(p:\pi^-|\pi^+)$ at 7 GeV/c with $F(p:\pi^-|K^+)$ at 12.7 GeV/c of Chen *et al.* is consistent with the hypothesis that for small $p_\parallel^{\text{lab}}$, $F(\pi^+)$ is slightly larger than $F(K^+)$.[10] However, the manner in which the data are presented in various references makes it difficult to estimate the size of the effect and to determine whether or not it is statistically significant. It would be worthwhile to compare the data for these processes at small $p_\parallel^{\text{lab}}$ with the specific aim of finding the experimental value of the ratio of F's and the error of this ratio.

B. Octet-Antidecuplet States of Zero Baryon Number

We next consider the case when the particles a, b, and c are members of the pseudoscalar-meson octet, nucleon octet, and baryon-resonance decuplet, respectively. The $(b\bar{c})$ state is a linear combination of the SU(3) representations $\underline{8}$, $\underline{10}^*$, $\underline{27}$, and $\underline{35}^*$. Since this state is of baryon number zero, only the octet part is nonexotic. Thus, only the octet part contributes in the extreme fragmentation region, so that useful predictions concerning the ratios of energy-dependent terms of invariant cross sections can be made.

In this case (unlike the case considered in Sec. IIA) the condition that the vector- and tensor-meson trajectories belong to either singlets or octets increases the number of useful results that may be obtained. In order to use this condition, we will expand the elastic $a+b+\bar{c}$ amplitude in terms of t-channel representations. The t-channel amplitude, shown in Fig. 2, is $a+\bar{a}\rightarrow(\bar{b}+c)+(b+\bar{c})$. The general expression for F, in terms of SU(3) Clebsch-Gordan coefficients and t-channel amplitudes, is[11]

$$F(s,p_\parallel^b,p_\perp^2)=\sum_{\mu\nu\gamma}\sum_{\mu'\nu'\gamma'}\sum_{\rho\lambda\alpha\beta}\begin{pmatrix}\mu_b & \mu_c^* & \mu & \gamma\\ \nu_b & -\nu_c & \nu & \end{pmatrix}\begin{pmatrix}\mu_b^* & \mu_c & \mu' & \gamma'\\ -\nu_b & \nu_c & \nu' & \end{pmatrix}\begin{pmatrix}\mu & \mu' & \rho & \alpha\\ \nu & \nu' & \lambda & \end{pmatrix}\begin{pmatrix}\mu_a & \mu_a^* & \rho & \beta\\ \nu_a & -\nu_a & \lambda & \end{pmatrix}$$

$$\times(-1)^{\bar{\nu}_a+\bar{\nu}_b+\bar{\nu}_c}F_{\rho\alpha\beta}^{\mu\gamma,\mu'\gamma'}(s,p_\parallel^b,p_\perp^2)\,, \tag{3}$$

where μ_i is the SU(3) representation of the i state, ν_i is the (Y,I,I_z) label of the i state, and $\bar{\nu}=I_3+\frac{1}{2}Y$. The labels μ and ν refer to the $(b\bar{c})$ state, while μ' and ν' refer to the $(\bar{b}c)$ state. The labels γ, γ', α, and β are the "coupling types" at the various vertices; in our application these labels are necessary only in the case of octet-octet-octet coupling. Each $F_{\rho\alpha\beta}^{\mu\gamma,\mu'\gamma'}$ may be written as a sum of an energy-independent term (A term) and energy-dependent term (B term), as in Eq. (1).

We now return to the special case in which particles a, b, and c belong to the meson octet, baryon octet, and baryon-resonance decuplet. In this case the coupling indices γ and γ' are unnecessary. We follow the procedure discussed in Sec. I, considering the energy-dependent terms of the F_j in the extreme fragmentation region. Limitation of $(b\bar{c})$ and $(\bar{b}c)$ to nonexotic states implies that the indices μ and μ' of the $B_{\rho\alpha\beta}^{\mu,\mu'}$ must refer to octets. Since the t-channel representation ρ must be a singlet or octet, there are only five allowed B amplitudes, $B_{8ss}^{8,8}$, $B_{8sa}^{8,8}$, $B_{8as}^{8,8}$, $B_{8aa}^{8,8}$, and $B_1^{8,8}$. Here, the second and third subscripts refer to the symmetric and antisymmetric octet-octet-octet couplings at the $(b\bar{c})(\bar{b}c)\rho$ and $(a\bar{a}\rho)$ vertices, respectively, where ρ is the intermediate representation shown in Fig. 2.

The contributions of these five B terms to a large number of measurable inclusive cross sections are listed in columns 2–6 of Table I. The superscripts on the B all refer to octets, and so are omitted. In the processes listed, the meson a is either a π^+, K^+, or K^-. The B terms for π^-, K^0, and $\bar{K}^0$ processes may be obtained by reflecting all the particles about the I_z axis; for example, $B(p:\Delta^+|K^0)=B(n:\Delta^0|K^-)$.

As mentioned in Sec. I, there is much experimental evidence for the validity of the exotic $ab\bar{c}$ hypothesis in the extreme fragmentation region.[2,3] This hypothesis implies that the B of Table I are zero if the $(ab\bar{c})$ representation is $\underline{27}$, $\underline{10}$, or $\underline{10}^*$. The following equalities then follow from the octet-octet crossing matrix[12]:

$$B_{8sa}=B_{8as}\,,$$
$$B_1=\tfrac{16}{5}B_{8ss}\,, \tag{4}$$
$$B_{8aa}=\tfrac{9}{5}B_{8ss}\,.$$

These equations imply that the five B's may be written in terms of two amplitudes, which we take

TABLE I. Allowed B terms in the extreme fragmentation region for the processes meson + nucleon → decuplet + X. Equality signs connect amplitudes with the same expansion in this region.

1 Reaction	2 $\frac{B_{8ss}}{25}$	3 $\frac{B_{8sa}}{125^{1/2}}$	4 $\frac{B_{8as}}{125^{1/2}}$	5 $\frac{B_{8aa}}{15}$	6 $\frac{B_1}{20}$	7 C	8 D
$\frac{3}{2}(p:\Delta^+\|\pi^+)=\frac{3}{2}(n:\Delta^0\|\pi^+)=\frac{3}{4}[(p:\Delta^0\|\pi^+)+(n:\Delta^+\|\pi^+)]$	2	0	0	0	1	2	0
$\frac{3}{2}(p:\Delta^+\|K^-)=\frac{3}{2}(n:\Delta^0\|K^-)=\frac{3}{2}[(n:\Delta^+\|K^-)+(p:\Delta^0\|K^-)]$	−1	1	0	0	1	1	1
$\frac{3}{2}(p:\Delta^+\|K^+)=\frac{3}{2}(n:\Delta^0\|K^+)=\frac{3}{2}[(p:\Delta^0\|K^+)+(n:\Delta^+\|K^+)]$	−1	−1	0	0	1	1	−1
$3(n:\Sigma^{*-}\|K^-)=6(p:\Sigma^{*0}\|K^-)=(p:\Delta^{++}\|\pi^+)=3(n:\Delta^+\|\pi^+)$	2	0	0	2	1	4	0
$3(n:\Sigma^{*-}\|K^+)=6(p:\Sigma^{*0}\|K^+)=(n:\Delta^-\|\pi^+)=3(p:\Delta^0\|\pi^+)$	2	0	0	−2	1	0	0
$3(p:\Sigma^{*+}\|\pi^+)=6(n:\Sigma^{*0}\|\pi^+)=(n:\Delta^-\|K^-)=3(p:\Delta^0\|K^-)$	−1	1	1	1	1	2	2
$3(p:\Sigma^{*+}\|K^+)=6(n:\Sigma^{*0}\|K^+)=(p:\Delta^{++}\|K^-)=3(n:\Delta^+\|K^-)$	−1	1	−1	−1	1	0	0
$3(n:\Sigma^{*-}\|\pi^+)=6(p:\Sigma^{*0}\|\pi^+)=(n:\Delta^-\|K^+)=3(p:\Delta^0\|K^+)$	−1	−1	1	−1	1	0	0
$3(p:\Sigma^{*+}\|K^-)=6(n:\Sigma^{*0}\|K^-)=(p:\Delta^{++}\|K^+)=3(n:\Delta^+\|K^+)$	−1	−1	−1	1	1	2	−2

to be $C=\frac{3}{25}B_{8ss}$ and $D=(\frac{1}{125})^{1/2}B_{8sa}$. Columns 7 and 8 of Table I show the coefficients in the expansion of the B's in C and D that follows if the exotic $ab\bar{c}$ hypothesis is valid.

None of the equalities of the table are of the type found in Sec. II A, involving particles c of the same isospin multiplet, and particles a of different multiplets. There are two types of relations in the table. The first involves the isospin group only; corresponding particles of the related B's are in the same isospin multiplet. In the second type, the particles c of the related B's are in different multiplets. One does not expect this latter type of prediction to be accurate, as the dependence of F on the mass of the produced particle c is believed to be strong. Our predictions may be used to study this dependence.

At present, there are not many data available on inclusive cross sections for which the particle c is a member of the resonance decuplet. It is expected that such data will become available in the next few years.

C. Octet-Octet States of Zero Baryon Number

In this subsection we consider the case when the particles a, b, and c are members of the pseudoscalar meson octet, baryon octet, and baryon octet, respectively. The procedure is the same as that used in Sec. II B. However, in this case there are three possibilities for nonexotic $(b\bar{c})$ or $(\bar{b}c)$ states, a singlet, symmetrically coupled octet, or antisymmetrically coupled octet. Because of this, there are still twenty independent amplitudes if one neglects exotic $(b\bar{c})$ and $(\bar{b}c)$ states. Clearly, there are fewer predicted relations. If we limit b to a proton or neutron, c to a p, n, Λ, or Σ, and a to a K, $\bar{K}$, or charged π, the following relations are obtained:

$$
\begin{aligned}
&(p:\Sigma^0|P)=\tfrac{1}{2}(n:\Sigma^-|P)\,,\\
&(n:\Sigma^0|P)=\tfrac{1}{2}(p:\Sigma^+|P)\,,\\
&(p:\Sigma^+|K^-)+(n:\Sigma^-|K^+)+(p:\Sigma^+|\pi^+)\\
&\qquad=(n:\Sigma^-|K^-)+(p:\Sigma^+|K^+)+(n:\Sigma^-|\pi^+)\,,\\
&(n:\Lambda|K^-)+(p:\Lambda|K^+)+(n:\Lambda|\pi^+)\\
&\qquad=(p:\Lambda|K^-)+(n:\Lambda|K^+)+(p:\Lambda|\pi^+)\,,\\
&(n:p|K^+)+(p:n|K^-)+(p:n|\pi^+)\\
&\qquad=(p:n|K^+)+(n:p|K^-)+(n:p|\pi^+)\,,\\
&(p:p|K^+)+(n:n|K^-)+(n:n|\pi^+)\\
&\qquad=(n:n|K^+)+(p:p|K^-)+(p:p|\pi^+)\,,
\end{aligned}
\tag{5}
$$

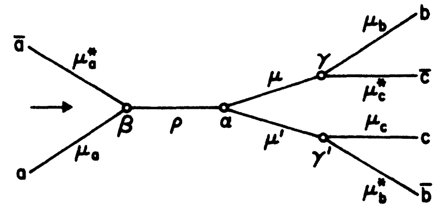

FIG. 2. The t-channel coupling scheme for the $(a+b+\bar{c})$ scattering amplitude.

where $(b:c|a)$ is a shortened notation for $B(b:c|a)$. The meson in these relations is either a π^+, K^+, or K^-; B terms involving a π^-, K^0, or $\bar{K}^0$ may be obtained by reflecting around the I_z axis.

If we impose further the condition that $B=0$ when $(ab\bar{c})$ is exotic, some of the B's of Eq. (5) are zero. The following simpler conditions may then be obtained from Eq. (5):

$$
\begin{aligned}
&(n:\Sigma^-|K^-)=(p:\Sigma^+|K^-)+(p:\Sigma^+|\pi^+)\,,\\
&(p:\Lambda|K^-)=(n:\Lambda|K^-)+(n:\Lambda|\pi^+)\,,\\
&(n:p|\pi^+)=(n:p|K^+)+(p:n|K^-)\,.
\end{aligned}
\tag{6}
$$

These relations are of the same general type as that of Eq. (2), and may be useful in checking the SU(3) symmetry of the couplings of the V and T meson trajectories to the π and K and to the various $(b\bar{c})$ states.

III. CONCLUDING REMARKS

If one wishes to determine the size of the extreme fragmentation region, it is more convenient to consider a process for which $(b\bar{c})$ is of exotic (Y,I_z) than it is to consider one of the processes discussed here. Similarly, if one wishes to test the exotic $ab\bar{c}$ hypothesis, it is most convenient to consider a reaction for which $ab\bar{c}$ is of exotic (Y,I_z). The relations given here involving (c) particles of the same isospin multiplet and (a) particles of different multiplets are useful for testing the SU(3) invariance of the couplings of V and T trajectories. Comparison of reactions involving c particles of different multiplets are useful for determining the effect of the c mass on F. Furthermore, as pointed out in Sec. II A, comparison of $\pi^+p\to\pi^-X$ and $K^+p\to\pi^-X$ cross sections can provide a test of the usual interpretation of the apparent difference between the high-energy limits of the KN and πN total cross sections.

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Electromagnetic Contribution to the Lepton-Pair Decay of the K_1^0 Meson*

J. Smith and Z. E. S. Uy†

The Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

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If we assume that there is no anomalously large *CP*-violating $K_S^0\to\mu^+\mu^-$ decay mode, then a possible conflict with a unitarity bound could arise similar to the present situation with $K_L^0\to\mu^+\mu^-$. It is therefore important to have a reliable estimate of the electromagnetic contribution for $K_S^0\to\mu^+\mu^-$. In this paper, we assume CP invariance (so $K_S^0\equiv K_1^0$) and give the results of a detailed calculation of both the real and imaginary parts of the $K_1^0\to\mu^+\mu^-$ amplitude based on contributions from 2γ, $\pi^+\pi^-$, and $\pi^+\pi^-\gamma$ states.

I. INTRODUCTION

The experiment of Clark *et al.*[1] gave an upper bound for the decay rate $K_L^0\to\mu^+\mu^-$ which was inconsistent with the theoretical lower bound for this decay.[2] One possible way to resolve this puzzle was given by Christ and Lee,[3] who proposed a CP violation. In order to obtain agreement between theory and experiment for $K_L^0\to\mu^+\mu^-$ decay, the decay rate for $K_S^0\to\mu^+\mu^-$ must be anomalously large. By this we mean that the decay rate must be several orders of magnitude larger than that expected on the basis of quantum electrodynamics. Experimental searches for the decay $K_S^0\to\mu^+\mu^-$ have so far been inconclusive. However, there is now considerable interest in carrying out experiments to substantially decrease the present upper limits on the rates for $K_S^0\to\mu^+\mu^-$ and $K_S^0\to 2\gamma$. If the Christ-Lee mechanism is invalid, then it is probable that the $K_S^0\to\mu^+\mu^-$ decay is of electromagnetic origin, and it is important to know the expected decay rate.

From the theoretical point of view it is possible to give very plausible estimates of the electromagnetic rates for the *CP*-conserving processes $K_1^0\to\mu^+\mu^-$ and $K_1^0\to 2\gamma$. The latter decay is usually assumed to proceed via a $\pi^+\pi^-$ intermediate state. The imaginary part of the amplitude is then given by the $2\gamma\to\pi^+\pi^-$ CP-even amplitude for which the Born approximation is taken.[4] Due to gauge invariance the real part of the $K_1^0\to 2\gamma$ amplitude is finite in this model and we expect the perturbation-theory estimate to be a reasonable guide for experimenters. The correction term due to the inclusion of the latest values for the low-energy S-wave $\pi^+\pi^-$ phase shift is small.[5] Also vector-meson exchanges in the $2\gamma\to\pi^+\pi^-$ amplitude have been calculated and are small.[6] However, it would be satisfying to have some experimental check of the $K_1^0\to 2\gamma$ rate because the same model (pion-loop) is inherently assumed to give the electromagnetic decay amplitude for $K_1^0\to\mu^+\mu^-$. The calculation by Martin, de Rafael, and Smith[5] of the imaginary part of the $K_1^0\to\mu^+\mu^-$ amplitude assumed that it was dominated by the 2γ, $\pi^+\pi^-$, and $\pi^+\pi^-\gamma$ intermediate states. Sehgal[7] made an estimate of the imaginary part of the $K_1^0\to\mu^+\mu^-$ amplitude based on the 2γ state alone. However, this is not a good approximation because the $\pi^+\pi^-$ state gives a comparable contribution. Once the $\pi^+\pi^-$ state is included then the $\pi^+\pi^-\gamma$ state must also be included to remove a spurious infrared