$$k^2=\frac{(s-s_A)(s-s_T)}{4s}\,,\quad s_T=(M+\mu)^2,\quad s_A=(M-\mu)^2,\quad x=1+\frac{2\mu^2}{k^2}\,,$$

$$b_l(s')=\frac{P_{l+1}'(x')-P_l'(x')}{l+1}\,,\quad h(s')=\frac{q(s')(s'-M^2+\mu^2)^3}{k'\sqrt{s'}}\,, \tag{B3}$$

$$\sin\phi'=\frac{-2M\{t[su-(M^2-\mu^2)^2]\}^{1/2}}{[(s-s_T)(s-s_A)(u-s_T)(u-s_A)]^{1/2}}\,,\quad \hat{a}=\frac{15\pi M\alpha_2^t}{2(M^2-\mu^2)}-\frac{G_\pi^{\ 2}}{\mu^4}=0.177\mu^{-4}\,.$$

The expression for $\hat{a}$ is not quite as simple as that for the corresponding quantity in π-π scattering, owing to the presence of a term originating in the nucleon pole in the Froissart-Gribov representation of the D wave prior to taking the scattering-length limit. The value which we quote for $\hat{a}$ was calculated by Yndurain and Common.[5]

[1]F. J. Yndurain, Phys. Letters 31B, 368 (1970).
[2]A. K. Common, Nuovo Cimento 59A, 115 (1970).
[3]F. J. Yndurain and A. K. Common, Nucl. Phys. B26, 167 (1971).
[4]M. Einhorn and R. Blankenbecler, Ann. Phys. (N.Y.) 67, 480 (1971).
[5]F. J. Yndurain and A. K. Common, Nucl. Phys. B34, 509 (1972).
[6]G. Giacomelli, CERN Report No. CERN-HERA 69-3 (unpublished).
[7]D. Morgan and G. Shaw, Nucl. Phys. B10, 261 (1969).
[8]B. Bonnier and P. Gauron, Nucl. Phys. B36, 11 (1972).
[9]J. C. Le Guillou, A. Morel, and H. Navelet, Nuovo Cimento 5A, 659 (1971).
[10]R. Roskies, Phys. Rev. D 2, 247 (1971).
[11]M. R. Pennington, Nuovo Cimento 5A, 644 (1971).
[12]G. Wanders, in *Springer Tracts in Modern Physics*, Vol. 57, edited by G. Höhler (Springer, Berlin, 1972).
[13]G. Wanders, in Lecture Notes from GIFT Seminar, 1971 (unpublished).
[14]S. M. Roy, Phys. Letters 36B, 353 (1971).

PHYSICAL REVIEW D VOLUME 7, NUMBER 9 1 MAY 1973

Saturation of the Drell-Hearn-Gerasimov Sum Rule*

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The Drell-Hearn-Gerasimov sum rule for the forward spin-flip amplitude of nucleon Compton scattering is decomposed into separate sum rules originating from different isospin components of the electromagnetic current. The resulting sum rules are reexamined using recently available analyses of single-pion photoproduction in the region up to photon laboratory energies of 1.2 GeV. All three sum rules receive important nonresonant as well as resonant contributions. The isovector sum rule whose contributions are known best is found to be nearly saturated, lending support to the assumptions underlying the sum rules. The failure of the isoscalar-isovector sum rule to be saturated is then presumably to be blamed on inadequate data for inelastic contributions.

I. INTRODUCTION

The Drell-Hearn-Gerasimov[1] sum rule for the spin-flip amplitude in forward Compton scattering rests on two assumptions. These are the low-energy theorem[2] for the spin-flip amplitude and the validity of an unsubtracted dispersion relation. Since these are simple and relatively well-accepted assumptions, and are often used together with additional stronger assumptions in deriving other sum rules, it is of interest to look into the validity of the Drell-Hearn-Gerasimov sum rule in the light of the present experimental data.

In their original paper, Drell and Hearn[1] did attempt to investigate the validity of the sum rule for a proton target by using an isobar model of single-pion photoproduction. Their results were generally encouraging, but some important contribution from high energy (greater than 1 GeV) seemed to be likely. Somewhat later, Chau *et al.*[3]

extended the examination of the proton sum rule by using an analysis of single-pion photoproduction through the second resonance region. They found good agreement, without any high-energy contribution. Finally, in the course of an analysis of many sum rules, Fox and Freedman[4] have considered the Drell-Hearn-Gerasimov sum rule, using Walker's partial-wave analysis[5] of pion photoproduction. They found the somewhat surprising result that while the sum rule involving only the isovector part of the electromagnetic current appeared well satisfied, the sum rule involving one isovector and one isoscalar current (equivalent to the difference of proton and neutron sum rules) was badly violated.

Since that time, there has been a considerable improvement in both the pion photoproduction data and their analysis. In particular, relatively good neutron data are becoming available and have been incorporated in the recent analyses of Pfeil and Schwela[6] and Moorhouse and Oberlack.[7]

Given this changed situation, we reexamine in this paper the Drell-Hearn-Gerasimov sum rules for both proton and neutron targets, with particular attention to their difference. In Sec. II we give the relevant definitions and present the contributions to the sum rules using several recent analyses of pion photoproduction. In Sec. III we present some conclusions.

II. ANALYSIS OF CONTRIBUTIONS

The unsubtracted dispersion relation for the forward spin-flip nucleon Compton amplitude f_2,

$$\mathrm{Re}\, f_2(\nu)=\frac{2\nu}{\pi}\int_{\nu_0}^{\infty}\frac{d\nu'}{\nu'^2-\nu^2}\,\mathrm{Im}\, f_2(\nu')\ ,$$

gives the Drell-Hearn-Gerasimov[1] sum rule

$$\frac{2\pi^2\alpha}{M^2}\,\kappa^2=\int_{\nu_0}^{\infty}\frac{\sigma_{3/2}(\nu')-\sigma_{1/2}(\nu')}{\nu'}\,d\nu'\equiv I_p\ , \qquad (1)$$

when the low-energy theorem is applied to $f_2(\nu)$. Here κ is the nucleon's anomalous magnetic moment, M the nucleon's mass, ν_0 the threshold energy for a single-pion photoproduction, which in the laboratory frame is given by

$$\nu_0=\frac{m_\pi^{\ 2}}{2M}+m_\pi \approx 150\ \mathrm{MeV}\ ,$$

and $\sigma_{3/2}$ ($\sigma_{1/2}$) is the total cross section for the process photon + nucleon → hadrons in the net helicity state $\frac{3}{2}$ ($\frac{1}{2}$). These cross sections enter the sum rule through the unitarity equation which relates the imaginary part of the forward scattering amplitude to the total cross section into the intermediate states.

We decompose the left- and right-hand sides of Eq. (1) into components of different isospin character and then relate the corresponding parts. The anomalous magnetic moments of the proton and neutron are defined by

$$\kappa_p=\tfrac{1}{2}\kappa_S+\tfrac{1}{2}\kappa_V\ ,\qquad \kappa_n=\tfrac{1}{2}\kappa_S-\tfrac{1}{2}\kappa_V\ , \qquad (2)$$

where κ_S (κ_V) is the isoscalar (isovector) component.

In this way we obtain the isovector, isoscalar, and the "interference" Drell-Hearn-Gerasimov sum rules:

$$\begin{aligned}
I_{VV}&\equiv\int_{\nu_0}^{\infty}[\sigma_{3/2}^{VV}(\nu')-\sigma_{1/2}^{VV}(\nu')]\frac{d\nu'}{\nu'}=(\tfrac{1}{2}\kappa_V)^2\frac{2\pi^2\alpha}{M^2}=218.5\ \mu\mathrm{b}\ ,\\
I_{SS}&\equiv\int_{\nu_0}^{\infty}[\sigma_{3/2}^{SS}(\nu')-\sigma_{1/2}^{SS}(\nu')]\frac{d\nu'}{\nu'}=(\tfrac{1}{2}\kappa_S)^2\frac{2\pi^2\alpha}{M^2}=0.3\ \mu\mathrm{b}\ ,\\
I_{VS}&\equiv\int_{\nu_0}^{\infty}[\sigma_{3/2}^{VS}(\nu')-\sigma_{1/2}^{VS}(\nu')]\frac{d\nu'}{\nu'}=\tfrac{1}{2}\kappa_V\kappa_S\frac{2\pi^2\alpha}{M^2}=-14.7\ \mu\mathrm{b}\ .
\end{aligned} \qquad (3)$$

Presently the data limit the study of the saturation of these sum rules to the contributions (to the total cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$) of a nucleon and one-pion final hadronic state. Resonance dominance only allows one to estimate a part of the inelastic contributions. Still, one may well hope that the largest contributions come from not too far from threshold, so that the nucleon-plus-one-pion state at least provides an indication of the sum rules' saturation.

Single-pion photoproduction amplitudes have a simple isospin decomposition[5]:

$$M^{+}_{\gamma p\to\pi^+ n}=(\tfrac{1}{3})^{1/2}[M^{(3)}-\sqrt{2}\,(M^{(1)}-M^{(0)})]\,,$$
$$M^{0}_{\gamma p\to\pi^0 p}=(\tfrac{2}{3})^{1/2}[M^{(3)}+(1/\sqrt{2}\,)(M^{(1)}-M^{(0)})]\,,\quad (4)$$
$$M^{-}_{\gamma n\to\pi^- p}=(\tfrac{1}{3})^{1/2}[M^{(3)}-\sqrt{2}\,(M^{(1)}+M^{(0)})]\;.$$

$M^{(3)}$ corresponds to the isospin-$\frac{3}{2}$ state and $M^{(1)}$ to the isospin-$\frac{1}{2}$ state created by the isovector part of the photon current; $M^{(0)}$ describes the interaction of the nucleon with the isoscalar part of this current.

The cross sections of definite isospin character for one-pion photoproduction are then proportional to the following combinations of amplitudes:

$$\sigma^{VV}\propto|M^{(3)}|^2+|M^{(1)}|^2,$$
$$\sigma^{SS}\propto|M^{(0)}|^2,\quad (5)$$
$$\sigma^{VS}\propto-[(M^{(0)})^{*}M^{(1)}+M^{(0)}(M^{(1)})^{*}]\;.$$

Additionally, the cross sections of definite helicity in this process are conveniently expressed in terms of $A_{n\pm},B_{n\pm}$ amplitudes[8,9]:

$$\sigma_{1/2}=\frac{8\pi q}{k}\sum_{n=0}^{\infty}(n+1)\,(|A_{n+}|^2+|A_{(n+1)-}|^2),$$
$$\sigma_{3/2}=\frac{8\pi q}{k}\sum_{n=0}^{\infty}\tfrac{1}{4}[n(n+1)(n+2)]\times(|B_{n+}|^2+|B_{(n+1)-}|^2).$$

The above relations allow us to consider the isovector, isoscalar, and interference sum rules separately, to insert the single-pion-plus-nucleon intermediate states' cross sections into the integrals, and to separate the contributions due to each partial wave.[10] Since we can extract the contributions of definite isospin and of definite total and orbital angular momentum, we are able to separate the contributions due to resonances of known inelasticity and subsequently to evaluate the corresponding part of the inelastic contributions to the integrals.

A. The Isovector Sum Rule

Table I presents the contributions to the isovector, I_{VV}, sum rule. For the photon laboratory energy, ν_{lab}, below 0.45 GeV, we find good agreement[11] between the results obtained with the analyses of Refs. 5, 6, and 7. Since the Pfeil-Schwela fit extends to the lowest energy, we have chosen to list the contributions obtained with their analysis, where possible.[12] For ν_{lab} above 0.45 GeV the values resulting from Refs. 5 and 7 also agree well, and listed are contributions obtained with the very recent Moorhouse-Oberlack fit.

The inelastic part is evaluated as a sum of inelastic contributions of $N(1520)$, $N(1670)$, and $N(1688)$ (D_{13}, D_{15}, and F_{15}, respectively).

The behavior of $\sigma^{VV}_{3/2}-\sigma^{VV}_{1/2}$ as a function of energy is shown in Fig. 1. One can easily recognize on this graph the dominant features of a big negative nonresonant s-wave contribution and a large positive contribution from the $P_{33}(1236)$ and the two other resonances.

TABLE I. The isovector-isovector contributions to the Drell-Hearn-Gerasimov sum rule (Ref. 12).

Partial wave	$0.18\le\nu_{\text{lab}}\le0.45$ GeV (μb)	$0.45\le\nu_{\text{lab}}\le1.2$ GeV (μb)	Total (μb)
0^+	-115^{a}	-51^{b}	-166
1^-	-2^{a}	-9^{b}	-11
1^+ $I=\frac{3}{2}$; includes $P_{33}(1236)$	$+237^{a}$	$+2^{b}$	$+239$
1^+ $I=\frac{1}{2}$	$+12^{a}$	$+17^{b}$	$+29$
2^- $I=\frac{3}{2}$	$+7^{b}$	$+5^{b}$	$+12$
2^- $I=\frac{1}{2}$; includes $D_{13}(1520)$	$+13^{b}$	$+41^{b}$	$+54$
2^+ $I=\frac{3}{2}$	$+1^{b}$	-2^{b}	-1
2^+ $I=\frac{1}{2}$; includes $D_{15}(1670)$	$+2^{b}$	$+1^{b}$	$+3$
3^- $I=\frac{3}{2}$	$+1^{b}$	$+1^{b}$	$+2$
3^- $I=\frac{1}{2}$; includes $F_{15}(1688)$	$+2^{b}$	$+7^{b}$	$+9$
Inelastic	$+12$	$+37^{b}$	$+49$
Total	$+170$	$+49^{b}$	$+219$

[a] Pfeil and Schwela, Ref. 6.
[b] Moorhouse and Oberlack, Ref. 7.

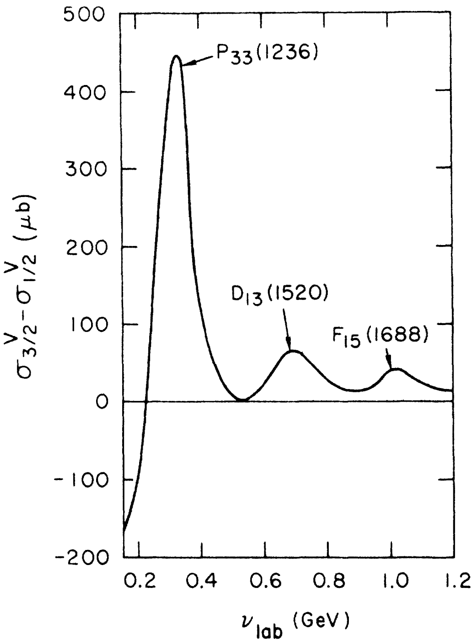

FIG. 1. Single-pion photoproduction contribution to the difference between the photon-nucleon cross sections in the helicity-$\frac{3}{2}$ and helicity-$\frac{1}{2}$ states $\sigma_{3/2} - \sigma_{1/2}$ for the isovector photons.

B. The Isoscalar Sum Rule

Table II presents the results for the isoscalar Drell-Hearn-Gerasimov sum rule. There are serious discrepancies between the results obtained with different fits; the signs of various contributions even disagree between analyses. This difficulty has been previously noted by Fox and Freedman.[4] The isoscalar amplitudes cannot be extracted directly from the data. Instead they are obtained indirectly from the sum and difference of the proton and neutron data, which results in relatively large errors as the isoscalar amplitudes themselves are small compared to the isovector ones.

The behavior of $\sigma^{SS}_{3/2} - \sigma^{SS}_{1/2}$ in Fig. 2 combines the Pfeil-Schwela and the Moorhouse-Oberlack results. There is no systematic trend and further data are necessary, both more accurate and including higher energies, in order to evaluate this sum rule.

C. The Interference Sum Rule

Table III presents the results for the isovector-isoscalar interference sum rule. This part corresponds to $I = 1$ exchange in the t channel, or in other words, it corresponds to the difference

TABLE II. The isoscalar-isoscalar contributions to the Drell-Hearn-Gerasimov sum rule (Ref. 12).

Partial wave	$0.18 \leq \nu_{lab} \leq 0.45$ GeV (μb)	$0.45 \leq \nu_{lab} \leq 1.2$ GeV (μb)	Total (μb)
0^+	−1.07 [a] −0.80 [c] −1.93 [b]	−0.50 [c] −1.06 [b]	−1.30 [c] −2.99 [b]
1^-	−0.33 [a] −0.29 [c] −0.26 [b]	−0.34 [c] −1.61 [b]	−0.63 [c] −1.87 [b]
1^+	+0.30 [a] −0.28 [c] +1.77 [b]	−0.60 [c] +2.62 [b]	−0.88 [c] +4.39 [b]
2^- [includes $D_{13}(1520)$]	+0.30 [c] +1.30 [b]	−0.21 [c] −1.75 [b]	+0.09 [c] −0.45 [b]
2^+ [includes $D_{15}(1670)$]	+0.01 [c] −0.10 [b]	+0.04 [c] +0.25 [b]	+0.05 [c] +0.15 [b]
3^- [includes $F_{15}(1688)$]	+0.25 [c] +0.01 [b]	+1.38 [c] +2.49 [b]	+1.63 [c] +2.50 [b]
Inelastic	+0.41 [c] +0.95 [b]	+0.62 [c] +0.24 [b]	+1.03 [c] +1.19 [b]
Total	−0.40 [c] +1.74 [b]	+0.39 [c] +1.18 [b]	−0.01 [c] +2.92 [b]

[a] Pfeil-Schwela analysis, Ref. 6.
[b] Moorhouse-Oberlack analysis, Ref. 7.
[c] Walker analysis, Ref. 5.

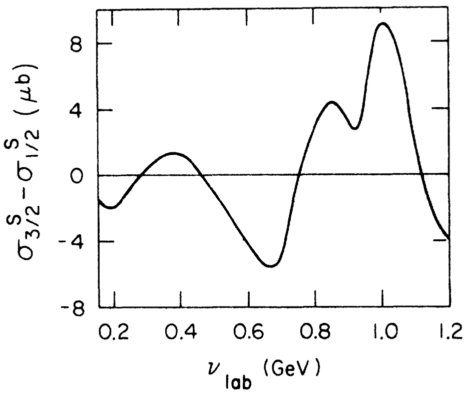

FIG. 2. As in Fig. 1 but for the isoscalar photons.

between the proton and neutron Drell-Hearn-Gerasimov sum rules. It should have a negative value, being proportional to $\kappa_p^2-\kappa_n^2$.

We find agreement between the results obtained with different analyses, where they overlap, and present the values resulting from the Pfeil-Schwela fit for $\nu_{lab} \leq 0.45$ GeV,[12] and from the Moorhouse-Oberlack fit for $\nu_{lab} \geq 0.45$ GeV.

Each of these fits results in a big, nonresonant, s-wave contribution of the wrong, positive sign. This contribution, however, is canceled almost entirely by the nonresonant part of 1^+ partial wave in the Moorhouse-Oberlack analysis. The second and third resonances cause the total result for this sum rule to be of *wrong*, positive sign, which reflects the stronger coupling of these resonances to the proton than to the neutron. Figure 3 shows this behavior in terms of $\sigma^{VS}_{3/2}-\sigma^{VS}_{1/2}$.

III. DISCUSSION AND CONCLUSIONS

The Drell-Hearn-Gerasimov sum rule rests on two assumptions: the low energy theorem and the validity of an unsubtracted dispersion relation for the f_2 (spin-flip) Compton amplitude. The first is quite general and has a very solid theoretical basis. Therefore, the validity of the sum rule is presumably dependent on the second assumption. This one is equivalent to the absence of a fixed pole (with $J^P=1^+$) in the f_2 amplitude.[13]

Strong evidence for the correctness of this hypothesis comes from the evaluation of the contributions to the isovector sum rule, shown in Table I. This sum rule seems to be rather well saturated by the results obtained with the single-pion plus nucleon data, complemented by some estimate of the inelastic contributions, up to $\nu_{lab}=1.2$ GeV. Even more pleasing is that this saturation occurs in a nontrivial way. There are strong cancellations, mainly between the large negative s-wave and the positive $P(1236)$ contributions (see Fig. 1). The large nonresonant s-wave contribution is of interest in itself, however, as it violates two-component duality. The imaginary part of a nondiffractive amplitude like $f_2(\nu)$ should contain *only* s-channel resonances.

Our total numerical result for the isovector sum rule is in general agreement with previous analyses.[3,4] While contributions from still higher values of ν_{lab} need not be small, we expect the contributions listed in Table I to be the largest individual ones, particularly since the sum rule integrands involve the factor $1/\nu_{lab}$ times a difference of total cross sections which is expected

TABLE III. The isovector-isoscalar contributions to the Drell-Hearn-Gerasimov sum rule (Ref. 12).

Partial wave	$0.18 \leq \nu_{lab} \leq 0.45$ GeV (μb)	$0.45 \leq \nu_{lab} \leq 1.2$ GeV (μb)	Total (μb)
0^+	+17 [a]	+5 [b]	+22
1^-	+1 [a]	+2 [b]	+3
1^+	−6 [a]	−17 [b]	−23
2^- [includes $D_{13}(1520)$]	−3 [b]	+16 [b]	+13
2^+ [includes $D_{15}(1670)$]	0	+1 [b]	+1
3^- [includes $F_{15}(1688)$]	0	+8 [b]	+8
Inelastic	−2	+17	+15
Total	+7	+32	+39

[a] Pfeil-Schwela analysis, Ref. 6.
[b] Moorhouse-Oberlack analysis, Ref. 7.

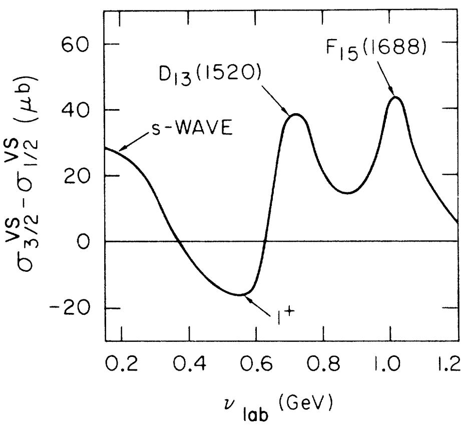

FIG. 3. As in Fig. 1 but for the interference term between the isoscalar and isovector photons.

to vanish at high energies.

The isoscalar sum rule presents some problems. Because of the sensitivity of the isoscalar amplitudes to the relatively small differences between the neutron and proton photoproduction data, it is very difficult to achieve reliable values for the isoscalar contributions. Furthermore, Table II shows that individual, nonresonant, partial waves make (canceling) contributions, each of which is of the order of the expected total value. In such a situation, small shifts in the data or the contributions of inelastic states may easily remove the present disagreement between the total value shown in Table II and that predicted (I_{SS}) by Eq. (3).

In this light, the behavior of the isovector-isoscalar sum rule is puzzling. Since the isovector sum rule is almost saturated, we have every reason to expect the validity of the underlying assumptions for the interference sum rule as well. The total contribution in Table III is of the wrong sign, however. This general difficulty had been previously noted by Fox and Freedman[4] using an earlier analysis[5] of pion photoproduction. We note in particular that there are large nonresonant contributions in the 0^+ and 1^+ partial waves, which tend to cancel. The second and third *resonances* contribute to the sum rule with the wrong sign. This again is of interest in itself as it violates *local* two-component duality. *Global* duality would still seem to be satisfied for this part of $f_2(\nu)$ because the various nonresonant partial waves are canceling in the full amplitude. This leads one to believe that if the sum rule is to work, it may well be the contributions of quite inelastic resonances, many in low partial waves, that saturate the sum rule. Unfortunately, the determination of these contributions would be very difficult experimentally.

In summary, we find no reason to doubt the validity of the Drell-Hearn-Gerasimov sum rule. The isovector sum rule's near-saturation even furnishes some direct evidence of support. There seems little reason to be alarmed at the nonsaturation of the isoscalar and isovector-isoscalar sum rules at the present stage of photoproduction analysis. What is needed is the more direct experimental determination of $\sigma_{1/2}(\nu)$ and $\sigma_{3/2}(\nu)$ using a polarized beam and target, something which is now becoming a real possibility.

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[1]S. D. Drell and A. C. Hearn, Phys. Rev. Letters 16, 908 (1966); S. B. Gerasimov, Yad. Fiz. 2, 598 (1966) [Sov. J. Nucl. Phys. 2, 430 (1966)]; 5, 1263 (1967) [5, 902 (1967)].

[2]M. Gell-Mann and M. L. Goldberger, Phys. Rev. 96, 1423 (1954); F. E. Low, *ibid.* 96, 1428 (1954).

[3]Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. 163, 1632 (1967).

[4]G. C. Fox and D. Z. Freedman, Phys. Rev. 182, 1628 (1969).

[5]R. L. Walker, Phys. Rev. 182, 1729 (1969).

[6]W. Pfeil and D. Schwela, Nucl. Phys. B45, 379 (1972).

[7]R. G. Moorhouse and H. Oberlack, in Proceedings of the Sixteenth International Conference on High Energy Physics, Batavia, 1972 (unpublished), and private communication.

[8]$A_{n\pm}$ and $B_{n\pm}$ correspond to a state with final-pion orbital momentum n, definite parity $P=(-1)^{n+1}$, and total angular momentum $j=n\pm\frac{1}{2}$.

[9]See also, for example, F. J. Gilman, Phys. Reports 4C, 95 (1972).

[10]For example, the single-pion part of $\sigma^{VS}_{3/2}$ is given by

$$\sigma^{VS}_{3/2}=\frac{8\pi q}{k}\sum_{n=0}^{\infty}-\tfrac{1}{4}[n(n+1)(n+2)]$$

$$\times[(B^{(0)}_{n+})^*B^{(1)}_{n+}+B^{(0)}_{n+}(B^{(1)}_{n+})^*$$

$$+(B^{(0)}_{(n+1)^-})^*B^{(1)}_{(n+1)^-}+B^{(0)}_{(n+1)^-}(B^{(1)}_{(n+1)^-})^*].$$

[11] I_{VV} for $\nu_{lab} \leq 0.45$ GeV due to $P_{33}(1236)$ has the following values resulting from different analyses: 260 μb with Walker's fit, 210 μb with the Moorhouse-Oberlack fit, and 237 μb with the Pfeil-Schwela fit.

[12] The Pfeil-Schwela analysis is limited to the first resonance region and to 0^+, 1^-, and 1^+ partial waves.

[13] H. D. I. Abarbanel and M. L. Goldberger, Phys. Rev. 165, 1594 (1968).

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Multiperipheral Cluster Model*

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A multiperipheral cluster model of high-energy inelastic collisions is discussed. A physical justification for the model is given, in that it accounts for "Ericson fluctuations" in the inelastic cross sections. These incoherent resonance contributions are ignored in most multi-Regge models. The major assumption is made that the ratio of the average fluctuation cross sections between different channels is governed by phase space. This allows a description of the cluster decay processes via the statistical bootstrap theory of Hagedorn and Frautschi. A simplified mathematical formulation of the model is studied, and it is shown that, subject to certain resistrictions on the vertex functions for cluster production, Feynman scaling occurs. The relationship with other models of inelastic processes is discussed. It is shown that the model includes both the ordinary multi-Regge model and the diffractive excitation models of Hwa and of Jacob and Slansky, as limiting cases, and that the approach is equivalent to, but is a refinement on, Hagedorn's thermodynamic model. A search for a power-behaved "tail" in the multiplicity distributions at high energies would provide an important test of the model.

I. INTRODUCTION

Two of the most popular models of high-energy inelastic hadron collisions are the multiperipheral or multi-Regge model, in various guises, and the thermodynamic model of Hagedorn (see, for example, the recent review by Frazer *et al.*[1]). These two models are almost orthogonal to each other in approach, and their main results concern quite different areas of interest. It is our object to show how a "multiperipheral cluster model"[2] can combine the essential features of them both, and to give a physical justification for the new model. Similar ideas have been expressed by Ranft and Ranft.[3]

In the multiperipheral cluster model, the total inelastic cross section is taken to be made up of a sum of terms,

$$\sigma_{\text{inel}} = \sum_n \sigma_n , \tag{1}$$

as represented diagrammatically in Fig. 1. Here σ_n represents the cross section for production of n "clusters," via a multiperipheral mechanism (i.e., repeated exchanges of Reggeons). Thus the coherent, or dynamical, part of the cross section in any given channel is assumed to be given by a multi-Regge model. Each "cluster," on the other hand, will be identified with the *incoherent*, or non-Regge, contributions of intermediate resonance states decaying according to the statistical bootstrap model of Hagedorn[4] and Frautschi.[5] The cluster may therefore have a variable mass. At the lower limit of its mass range it will consist of a single stable hadron (e.g., a pion), while at high mass it consists of an unstable resonance which eventually gives rise to a cloud of stable decay products, all moving with limited momenta in the center-of-mass system of the cluster. The multiplicity of decay products is thus proportional to the mass of the cluster (which is also called a "fireball" by Hagedorn[4]). This decay process has recently been studied in detail by Frautschi and the present author.[6,7]

A justification for this model is provided in Sec. II, on the grounds that it accounts for the incoherent resonance contributions to hadron cross sections, commonly called "Ericson fluctuations" in nuclear physics, which were recently discussed by Frautschi.[8] These fluctuation terms are due to random (incoherent) variations in the resonance coupling strengths and spacings: They average to zero in the reaction *amplitudes*, over suitably large energy intervals, but provide important