where a, b, a_1, b_1, etc., are functions of s, t, w, k_0, and E. q and r are given by

$$q = 2E(X - Y\cos\theta) + \frac{1}{m_n}\left[2XY - (X^2+Y^2)\cos\theta\right],$$

$$r = \sin\theta\left[(-tm_n)(4m_nE^2 + 4YE + m_nt)\right]^{1/2}.$$

Expressions (2) and (3) have been obtained assuming a local current-current form for the weak-interaction Lagrangian. These results also hold in vector-boson-mediated theories of weak interactions, but are changed in some other models. In scalar-boson mediated theories, for example, higher powers of E or $\cos\phi$ and $\sin\phi$ also occur, but reduce to the above expressions, as expected, in the limit of a very large mass for the intermediate bosons. A point of special interest in this case is that since the photon can couple to the charged-lepton and intermediate-boson lines, deviations from the locality theorems can be calculated independent of the details of strong interactions, in contrast to the deviations from the Pais and Treiman theorems[4] which needed additional assumptions regarding the hadronic currents participating in weak interactions. Details of such a calculation will be published elsewhere.

One of us (S.K.) would like to thank A. K. Kapoor for useful discussions.

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PHYSICAL REVIEW D VOLUME 7, NUMBER 9 1 MAY 1973

Meson Charge-Exchange Reactions in a Regge-Cut Van Hove Model*

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(Received 9 March 1972)

We present an analysis of four meson charge-exchange reactions ($\pi^- p \to \pi^0 n$, $\pi^- p \to \eta^0 n$, $K^- p \to \bar{K}^0 n$, and $K^+ n \to K^0 p$) within the context of our Regge-pole-cut model based on the Durand–Van Hove model. Quite good fits to all differential cross sections and polarizations are obtained. Predictions for the KN charge-exchange polarizations are also presented.

I. INTRODUCTION

In the past few years it has become abundantly clear, on both theoretical and phenomenological grounds, that the j-plane structure of hadron scattering amplitudes must include cuts.[1] The discontinuity structure of these cuts being unknown, a number of models have been proposed for generating cut corrections to simple Regge exchange[2] or otherwise approximating the effects of j-plane cuts.[3]

The Durand–Van Hove model[4] has previously been successful as a means for studying the complicated structure of daughter poles[5] in the j plane. It has been modified to introduce Regge cuts in backward πN scattering so as to resolve the parity-doubling problem.[2(d)] However, the strength of the cut in the last model is not related in any natural way to the properties of the particles lying on the trajectories and is completely adjustable to fit the data. As formulated, the model of Ref. 2(d) cannot be applied to forward scattering.

Motivated by the observation that the Durand–Van Hove model is a zero-width particle exchange model generating pure Regge poles (and daughters), and by the expectation that nonzero total widths for the exchanged particles and Regge cuts in the j plane will arise somehow from unitarity effects, we constructed[6] a model in which the Regge-cut contributions arise from the introduction of j-dependent widths for all particles in the exchanged resonant tower; that is, we modified the t-channel spin-j propagator to a simple Breit-Wigner form with resonance poles on the second sheet of the t

plane. As simple model for the j dependence of the total widths, we chose a square-root j dependence with branch point moving linearly in t: $\Gamma_j \sim\gamma[j-\alpha_c(t)]^{1/2}$, $\alpha_c(t)=\alpha_0+\alpha_c' t$. This has the double advantage of allowing analytic inversion of the propagator to obtain $j=\alpha(t)$ and of determining the cut strength in terms of a single parameter γ. This parametrization leads to approximately constant widths for the particles on the trajectory; this is not inconsistent with the experimentally observed widths.[7] The parameter γ is determined by the total widths of the particles on the Regge trajectories (see Sec. II).

In Ref. 6 we applied this model to pion-nucleon charge-exchange (CEX) scattering, with good results. In particular we predicted that, on the basis of our model calculations, the polarization in πN CEX scattering for $-t \gtrsim 0.4$ $(\text{GeV}/c)^2$ would be large and positive, falling to zero rapidly for $-t \gtrsim 0.6$ $(\text{GeV}/c)^2$ and remaining near zero for larger t values. This was contrasted with the strong-cut Reggeized absorption model (SCRAM),[2(b)] which gave (at that time) essentially the opposite prediction. No data were then available in the large-t region. Our previous results are in substantial agreement with the new measurements from CERN.[8,9]

In order to determine whether or not our model can successfully confront a large body of data for reactions involving trajectories other than that of the ρ meson, we have studied in this note four meson-nucleon charge-exchange reactions ($\pi^- p\to\pi^0 n$, $\pi^- p\to\eta^0 n$, $K^- p\to\overline{K}^0 n$, and $K^+ n\to K^0 p$). The forward $\pi^- p$ CEX reaction is reanalyzed here to remove some unnecessary approximations and to include a t-dependent factor in the residue function that was previously omitted. The model is then extended to fit the other three CEX reactions. Quite good fits to the presently available data are obtained. Predictions for KN CEX polarization are also given.

II. THE MODEL

We refer to Ref. 6 for the details of the construction of the model, contenting ourselves here with a brief sketch. In the Durand–Van Hove model, the signatured A' and B amplitudes for meson-nucleon scattering[10] are given, to leading order in $x_t=\cos\theta_t$, by

$$-A'^{(\tau)}(s,t)=\tfrac{1}{2}\sum_j \frac{(2j+1)g^2(j)}{m^2(j)-t}\left[b(j)+\frac{m^2}{p^2}a(j)\right] \times(-pq)^j[P_j(-x_t)+\tau P_j(x_t)], \quad (1)$$

$$-B^{(\tau)}(s,t)=\tfrac{1}{2}m\mu\sum_j \frac{(2j+1)g^2(j)}{m^2(j)-t}\tilde{a}(j)(-pq)^j \times[P_j'(-x_t)-\tau P_j'(x_t)], \quad (2)$$

where m = nucleon mass, μ = pseudoscalar meson mass, and p and q are the t-channel center-of-mass momenta of the nucleons and mesons, respectively. The function $g^2(j)$ is the spin-j coupling constant $[g^2(j)=g_{jN\overline{N}}g_{j\pi\pi}$ for the coupling of a spin-j object to the $N\overline{N}$ and $\pi\pi$ vertices$]$. The functions $a(j)$ and $b(j)$ measure the strength of the two possible couplings at the $N\overline{N}$ vertex:

$$(N\overline{N}\text{ coupling})_j=b(j)P_{\nu_1}P_{\nu_2}\cdots P_{\nu_{j-1}}P_{\nu_j} +ma(j)P_{\nu_1}P_{\nu_2}\cdots P_{\nu_{j-1}}i\gamma_{\nu_j}, \quad (3)$$

where there are j factors of $P_\mu=\frac{1}{2}(p_1-p_2)_\mu$ in the first term. Also, $\tilde{a}(j)=a(j)/j$, where $\tilde{a}(j)$ is regular near $j=0$.[6] The mass of the exchanged spin-j object is taken as

$$\alpha_p' m^2(j)=j-\alpha_0-i\gamma[j-\alpha_c(t)]^{1/2}, \quad (4)$$

where α_0, α_p', and γ are constants which correspond to the trajectory's intercept, slope, and cut strength, and where $\alpha_p(t)$ and $\alpha_c(t)$ are the trajectory functions of the "bare" Regge pole and branch point. The input trajectories for the pole $\alpha_p(t)$ and the branch point $\alpha_c(t)$ are taken as linear functions of t:

$$\alpha_p(t)=\alpha_0+\alpha_p' t,$$

$$\alpha_c(t)=\alpha_0+\alpha_c' t=\alpha_p(t)-a^2 t,$$

where

$$a^2=\alpha_p'-\alpha_c'.$$

This leads to effective Regge trajectories of the form

$$j=\alpha_\pm(t)=\alpha_p(t)-\tfrac{1}{2}\gamma^2\pm i\tfrac{1}{2}\gamma(4a^2t-\gamma^2)^{1/2}. \quad (5)$$

As in Ref. 6 we require that $\alpha_+(t)$ develop an imaginary part for $t>4\mu^2$ which we relate to the width of the resonances on the trajectory. Thus for $t<4\mu^2$, $\text{Im}\alpha_+(t)=0$, and for $t>4\mu^2$, $\text{Re}\alpha_+(m_{\text{res}}^2)=J_{\text{res}}$ and $\text{Im}\alpha_+(m_{\text{res}}^2)=\alpha_p' m_{\text{res}}\Gamma_{\text{res}}$. For the two Regge poles (ρ and A_2) which contribute to the four CEX reactions under investigation, $J_{\text{res}}=1, 2$ for $m_{\text{res}}^2=0.585$, 1.72 $(\text{GeV}/c^2)^2$ and $\Gamma_{\text{res}}=0.150$, 0.085 GeV/$c^2$. These constraints are used to fix α_0 and γ, leaving one free parameter for each trajectory function, which we take to be its slope α_p'. Because of the above constraints we do not require exchange degeneracy of the ρ-A_2 trajectory [which is only approximately verified even in pure pole models – see Refs. 11 and 18(a)]. Our fits show, however, that within the context of our model there is a universal slope $[\alpha_p' = 0.9\ (\text{GeV}/c)^{-2}]$ for the ρ and A_2 trajectories.

The sums over j in Eqs. (1) and (2) are evaluated by utilizing the Sommerfeld-Watson transformation. To evaluate the cut contribution, we assume that $g^2(j)\Gamma(j+\frac{3}{2})/[\sin(\pi j)\Gamma(j)]$ is a smooth function of j for $\text{Re}j < 0$. The resulting amplitudes for the various reactions are given in the Appendix.

III. APPLICATIONS

We have applied the above model to a number of meson charge-exchange reactions, in particular to $\pi^- p \to \pi^0 n$,[12] $\pi^- p \to \eta^0 n (\eta^0 \to 2\gamma)$,[13] $K^- p \to \bar{K}^0 n$,[14] and $K^+ n \to K^0 p$.[15] The slope parameters for the ρ and A_2 trajectories were fixed in the analysis of $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta^0 n$, respectively. These were then used without variation in the fits to the KN reactions.

A. $\pi^- p \to \pi^0 n$

This reaction was previously analyzed in Ref. 6. Because of an unnecessary approximation in that evaluation of the cut contributions (essentially that $\ln s \gg 1$, which is not valid at present experimental energies), and because of an error in the t-dependent part of the residues for the pole contributions (for $t < 0$, these are quite smooth, slowly varying functions of t and introduce only small corrections), we have reanalyzed the data[12] in our fit. The results for the forward differential cross sections and polarizations are presented in Figs. 1 and 2. The over-all quality of the fits is fairly good. As shown in Fig. 1, the forward differential cross section falls below the data at small $|t|$ for the higher-energy data. This can be traced to a number of effects, mainly to our constraint that the ρ trajectory pass exactly through spin one at the ρ mass. If we float the intercept, then the high-energy data can be fitted as well, at the expense of introducing another parameter into the model (the intercept α_0) and passing only *near* spin one at $t = m_\rho^2$.

We should point out that, unlike conventional Regge models, the strength of the helicity-flip and -nonflip amplitudes in our model are strongly correlated through the structure built in via the Durand–Van Hove model. The multiple constraints imposed here, that the trajectories pass through the correct spin values with approximately the correct widths, coupled with the fits to the differential cross sections, polarizations, and $\pi^\pm p$ total cross-section differences, leave very little freedom in the choice of parameters.

More complicated t dependences for the various residues, and floating the trajectory intercepts,

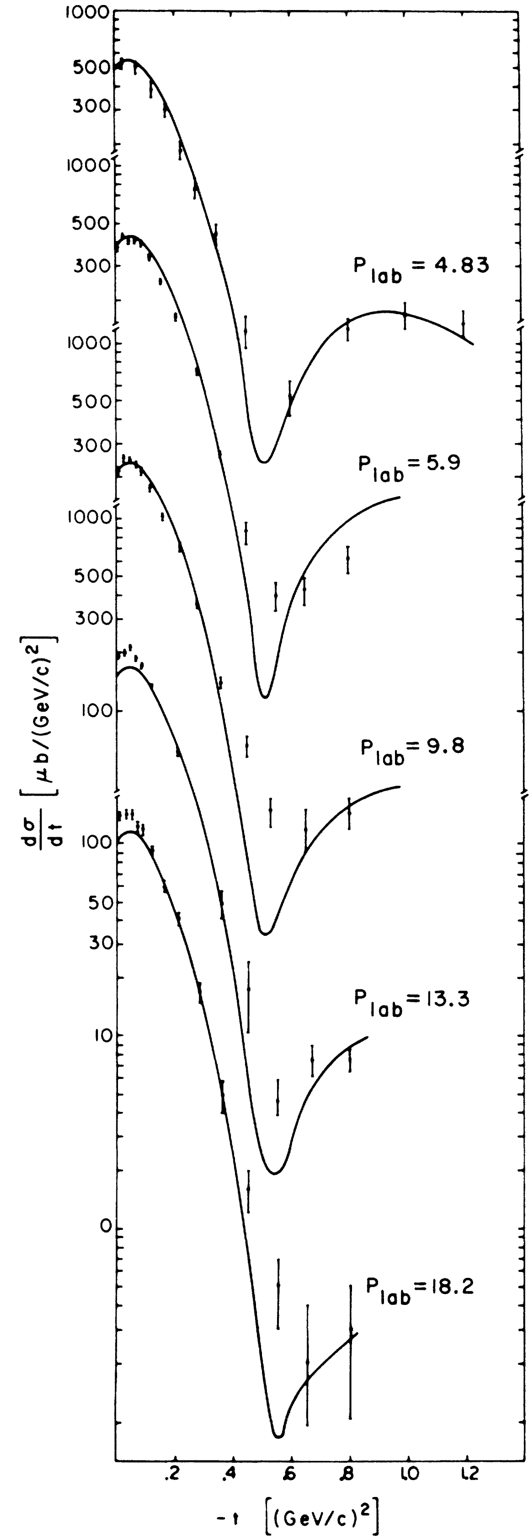

FIG. 1. πN CEX differential cross sections. The data are taken from Refs. 12(a) ($p_{\text{lab}} = 4.83$ GeV/c) and 12(b) ($p_{\text{lab}} = 5.9$, 9.8, 13.3, 18.2 GeV/c). The curves are calculated using the parameters in Table I.

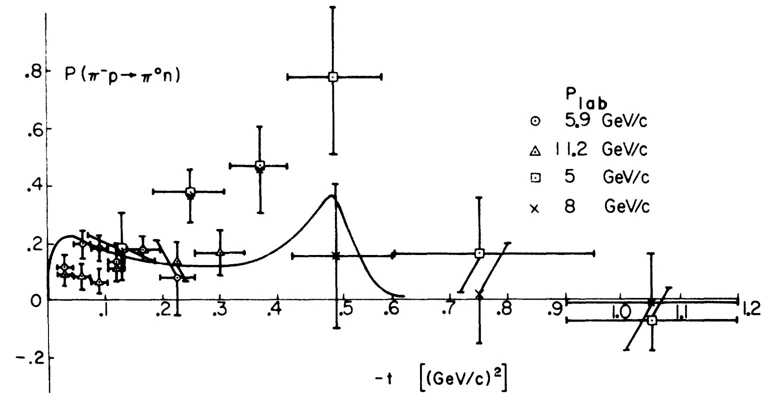

FIG. 2. πN CEX polarization. The data are taken from Refs. 12(c) (p_{lab} = 5.9 and 11.2 GeV/c) and 12(d) (p_{lab} = 5 and 8 GeV/c). The curve is calculated at p_{lab} = 6.0 GeV/c but shows little s dependence. For $-t \gtrsim 0.6$ (GeV/c)^2, the polarization is approximately zero.

would give much better over-all fits to the data.

The fit to the πN CEX polarization shown in Fig. 2 is also acceptable, although the magnitude of the CEX polarization in the dip region $[-t \approx 0.5$ (GeV/c)$^2]$ is not as large as the new CERN large-angle data[12d] would indicate.[16] However, we wish to emphasize that the relatively large, positive bump in the polarization at $-t \approx 0.5$ (GeV/c)^2, followed by a rapid falloff for $-t > 0.5$ (GeV/c)^2, is a *firm* prediction of our model and is in agreement with the new data. For $-t > 0.6$ (GeV/c)^2, our polarizations are very small, also in agreement with the CERN large-angle data. This is in contrast with the earlier predictions of the strong-cut Reggeized absorption model[2b,9] and earlier fits in the complex Regge-pole model.[3] Our fits overestimate the total cross-section difference $\Delta\sigma = \sigma_t(\pi^- p) - \sigma_t(\pi^+ p)$ by approximately 0.3 mb for $p_{lab} < 10$ GeV/c, but agree with experiment[17] for larger values of p_{lab}.

B. $\pi^- p \to \eta^0 n$ ($\eta^0 \to 2\gamma$)

For this reaction only the A_2 meson has the appropriate quantum numbers to couple at both the $\pi\eta$ and $N\bar{N}$ vertices. For the purposes of this note, we have ignored all questions of A_2 splitting, taking the A_2 as the lowest-lying resonance on a Regge trajectory passing through $J = 2$ at $m_A = 1.31$ GeV/c^2 with a width $\Gamma_A = 0.085$ GeV/c^2.[7] We have assumed an extra factor of j in the coupling constants to remove the spin-zero ghost pole in the A_2 amplitude (see Appendix). The results of our fits to the differential cross sections[13] and polarization[12c,12d,13] are presented in Figs. 3 and 4. Adequate fits are obtained without assuming any t dependence for the residues; this yields a three-parameter fit to the data (see Table I).

For lower energies, the forward dip in $\pi^- p \to \eta^0 n$ for $-t < 0.1$ (GeV/c)^2 arises from a dip in the $A'^{(+)}$ amplitude, which is much larger than the $B^{(+)}$ amplitude. This is in contrast to the forward dip in πN CEX, where only Re$B^{(-)}$ is small, while Im$B^{(-)}$ is comparable to both Re$A'^{(-)}$ and Im$A'^{(-)}$ near $t = 0$; the large helicity-flip amplitude gives rise to the forward dip in πN CEX. The smallness of the $B^{(+)}$ amplitudes in $\pi^- p \to \eta^0 n$ is due primarily to

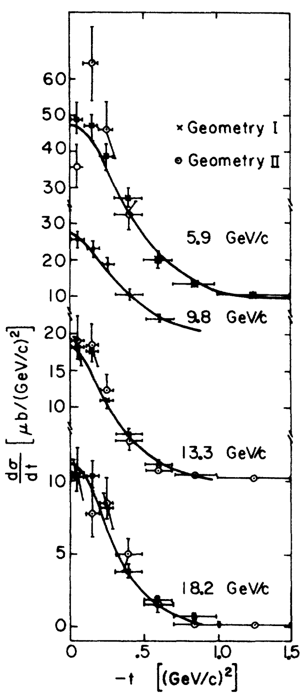

FIG. 3. $\pi^- p \to \eta^0 n$ ($\eta^0 \to 2\gamma$) differential cross sections. The data are taken from Ref. 13. Two geometries with different angular acceptances were used in this experiment; Geometry II covers a larger t range at all energies.

TABLE I. The values of the parameters that were obtained in fitting the CEX reactions. The parameters are defined in Eq. (A7); the trajectory or trajectories are given in parentheses after each reaction.

	k	a_0	b_0	b_1	α_p'
$\pi^- p \to \pi^0 n$ (ρ)	−0.11	2.56	18.2	5.65	0.90
$\pi^- p \to \eta^0 n$ (A_2)	···	3.74	3.43	···	0.89
$K^- p \to \bar{K}^0 n$ (ρ)	2.93	15.8	8.33	11.1	0.90
$K^- p \to \bar{K}^0 n$ (A_2)	···	2.24	6.53	···	0.89
$K^+ n \to K^0 p$ (ρ)	0.49	16.9	6.41	25.8	0.90
$K^+ n \to K^0 p$ (A_2)	···	8.38	6.25	···	0.89

cancellations between the pole and cut contributions to $B^{(+)}$; this accounts for the model's prediction of a small, positive polarization, which is in reasonable agreement with the data of Refs. 12(c), 12(d), and 13.

C. $K^- p \to \bar{K}^0 n$

Both the ρ and A_2 mesons have the appropriate quantum numbers to couple to the $K\bar{K}$ and $N\bar{N}$ vertices. We have taken the slope of the ρ trajectory from our fits to $\pi^- p$ CEX [$\alpha_\rho' = 0.90$ (GeV/c)$^{-2}$] and that of the A_2 trajectory from $\pi^- p \to \eta^0 n$ [$\alpha_\rho' = 0.89$ (GeV/c)$^{-2}$]. Only the strength and t dependence of the coupling constants are varied. The differential cross sections[14] and predicted polarization are presented in Figs. 5 and 6. For $-t < 1.0$ (GeV/c)^2 the agreement with the data is reasonable. As before, we found no need for any t dependence in the coupling constants of the A_2, while such t dependences were required in the ρ couplings (see Table I). We predict a fairly large, negative polarization for $-t < 0.3$ (GeV/c)^2; unfortunately, the $K^- p$ CEX polarization is as yet unmeasured. Previous theoretical calculations[18] of this polarization generally predict a large negative polarization for $-t < 0.4$ (GeV/c)^2, although there is considerable disagreement on its magnitude. The results of Ref. 18(c) in particular are based on SU(3) octet symmetry (see Sec. III D).

D. $K^+ n \to K^0 p$

This is the line-reversed reaction to $K^- p \to \bar{K}^0 n$ discussed in Sec. III C. Again, the ρ and A_2 are

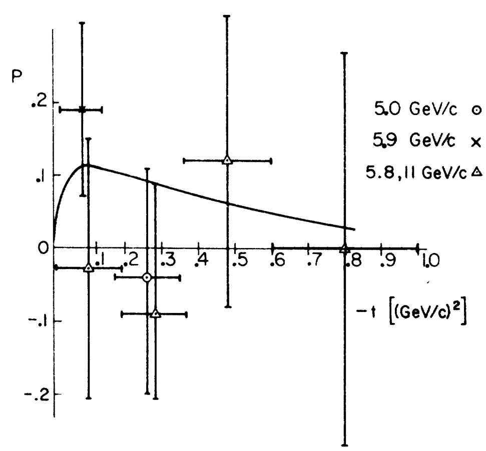

FIG. 4. $\pi^- p \to \eta^0 n$ polarization. The data are taken from Refs. 12(e) (p_{lab} = 5.0 GeV/c), 12(c) (p_{lab} = 5.9 GeV/c), and 12(d) (p_{lab} = 5.8 and 11 GeV/c data added to improve statistics).

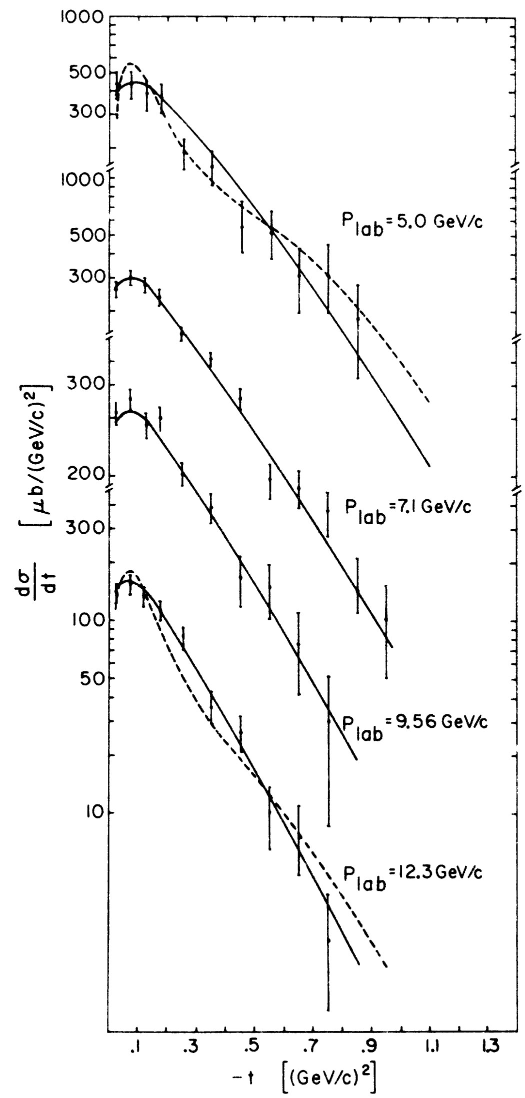

FIG. 5. $K^- p$ CEX differential cross sections. The data are taken from Ref. 14; the fits are made with the parameters of Table I. The dashed curves show the SU(3)-symmetric fits obtained by using the $K^+ n \to K^0 p$ parameters as explained in the text.

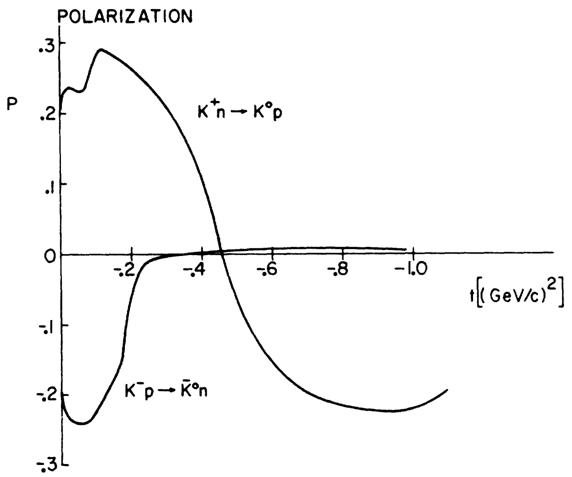

FIG. 6. *KN* polarization predictions.

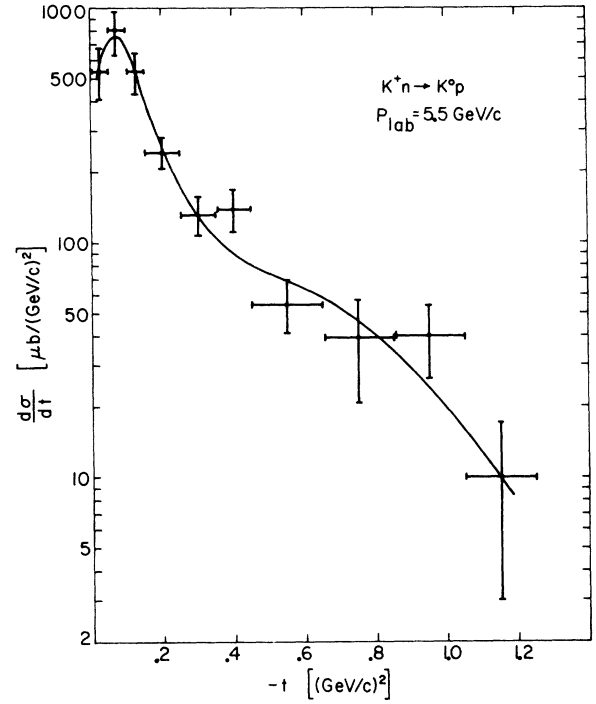

FIG. 7. $K^+n \to K^0p$ differential cross section at $p_{\text{lab}} = 5.5$ GeV/*c*. The data are taken from Ref. 15.

the dominant contributions. If SU(3) symmetry is good, the amplitudes of the four reactions A–D are related for all s and t by[11]

$$\langle \pi^- p | \pi^0 n\rangle = -\sqrt{2}F(s,t),$$
$$\langle K^- p | \bar{K}^0 n\rangle = F(s,t) + D(s,t),$$
$$\langle K^+ n | K^0 p\rangle = -F(s,t) + D(s,t),$$
$$\langle \pi^- p | \eta^0 n\rangle = (\tfrac{2}{3})^{1/2} D(s,t).$$

Use of the parameters for $K^-p \to \bar{K}^0n$ and the above relations does not yield acceptable fits to the $K^+n \to K^0p$ data.[15] However, fits to the $K^+n \to K^0p$ data, coupled with the SU(3) relations above, produce reasonable fits to the $K^-p \to \bar{K}^0n$ differential cross sections (the dashed curve of Fig. 5). Our fits agree with the relative signs of the various A' and B amplitudes predicted by the SU(3) relations, but quantitatively they fail to satisfy the relations by roughly a factor of two at small t.

We have, therefore, fitted the $K^+n \to K^0p$ reaction independently of the $K^-p \to \bar{K}^0n$ reaction. The differential cross section[15] and predicted polarization are shown in Figs. 7 and 6, respectively. The sign of the $K^+n \to K^0p$ polarization is opposite that of $K^-p \to \bar{K}^0n$, as expected for line-reversed reactions, but the t-dependent structure is quite different, especially in the large-t region. The polarization is in rough agreement with the SU(3) prediction of Ref. 18(c) for $-t < 0.3$ (GeV/c)^2, but differs markedly for larger t values.

IV. CONCLUSIONS

We have presented a simple model for meson charge-exchange reactions. The model provides quite adequate fits to the differential cross sections and polarization (where known). A clear advantage is that the amplitudes are determined solely by the highest-lying Regge trajectories, with their associated Regge cuts, and that all but one of the trajectory parameters are fixed by requiring the trajectory to satisfy t-channel constraints. A trajectory's one free parameter (essentially its "slope") is the most sensitive parameter in the model and is consistently determined to be $\alpha_p' \approx 0.9$, in good agreement with the effective trajectory slopes expected from "universality" ($\alpha_p' \approx 0.5/m_\rho^2$). This model is one of the few Regge-cut models which agree with the new πN CEX polarization data. While the complex Regge-pole model also agrees with the new data,[3] we feel that the economy in the number of parameters (5 for our model vs 11 for the complex-pole model) provides a significant simplification. Also our model incorporates, in a natural way, the physical widths of the particles on a Regge trajectory. Whether or not this model can adequately fit the elastic reactions, with their more complicated problems of Pomeranchukon exchange, remains unanswered. We hope to return to this problem in a future note.

ACKNOWLEDGMENT

We wish to thank Mr. David Bridges for providing the minimization routine utilized in these calculations, and for a number of helpful comments during the course of these calculations.

APPENDIX

In this appendix we present the equations which are used in the analysis of the CEX reactions. By contracting in the external factors of p_ν and q_ν and using the coupling at the nucleon vertex given by Eq. (3), we obtain the signatured t-channel helicity amplitudes[6]:

$$-f_{++}^{t\tau}(s,t)=2p\sum_j \frac{(2j+1)g^2(j)}{m^2(j)-t}\left[b(j)+\frac{m^2}{p^2}a(j)\right](pq)^j[P_j(-x_t)+\tau P_j(x_t)], \tag{A1}$$

$$-f_{+-}^{t\tau}(s,t)=\frac{\sqrt{\varphi}}{2p}\sum_j \frac{(2j+1)g^2(j)}{m^2(j)-t}m^2\tilde{a}(j)(pq)^{j-1}[P_j'(-x_t)-\tau P_j'(x_t)], \tag{A2}$$

where

$$p^2=\tfrac{1}{4}t-m^2,\quad q^2=\tfrac{1}{4}t-\mu^2,\quad \sqrt{\varphi}=2pq\sqrt{t}\sin\theta_t,\quad x_t=\cos\theta_t,\ \text{and}\ \tau=\pm1$$

(for $\eta\pi$ these definitions must be changed slightly).

These can be related to the standard analytic amplitudes A' and B defined by Singh.[10] After performing the Sommerfeld-Watson transformation, taking the leading terms for large s and negative t, crossing over to the s channel, and assuming that

$$K(j)=\frac{(2j+1)\Gamma(j+\frac{1}{2})\alpha_p'}{\sqrt{\pi}\,2^j},$$

$\tilde{a}(j)$ and $b(j)$ are smooth and fairly slowly varying functions of j, so that they can be taken outside the cut integral, we obtain for the unsignatured amplitudes

$$\begin{aligned}A'(s,t)=&K(t)\left[\frac{1}{\alpha_+(t)}b(t)+\frac{m^2}{p^2}\tilde{a}(t)\right]\left(1+\frac{2\mu}{Q}\right)\Gamma(1-\alpha_+)\left(\frac{s}{s_0}\right)^{\alpha_+}\\ &-\tfrac{1}{2}K(t)\Gamma(1-\alpha_c)\left(\frac{s}{s_0}\right)^{\alpha_c}\\ &\times\left\{\left[\frac{1}{\alpha_+}b(t)+\frac{m^2}{p^2}\tilde{a}(t)\right]\left(1+\frac{2\mu}{Q}\right)w(x_+)-\left[\frac{1}{\alpha_-}b(t)+\frac{m^2}{p^2}\tilde{a}(t)\right]\left(1-\frac{2\mu}{Q}\right)w(x_-)+\frac{2\gamma\sqrt{\alpha_c}}{\alpha_+\alpha_-}w(x_c)\right\},\end{aligned} \tag{A3}$$

$$B(s,t)=K(t)\frac{2m\tilde{a}(t)}{s}\left\{-\left(1+\frac{2\mu}{Q}\right)\Gamma(1-\alpha_+)\left(\frac{s}{s_0}\right)^{\alpha_+}+\tfrac{1}{2}\Gamma(1-\alpha_c)\left(\frac{s}{s_0}\right)^{\alpha_c}\left[\left(1+\frac{2\mu}{Q}\right)w(x_+)-\left(1-\frac{2\mu}{Q}\right)w(x_-)\right]\right\}, \tag{A4}$$

where $\alpha_\pm=\alpha_\pm(t)$ are given by Eq. (5); $Q=-2iq=(4\mu^2-t)^{1/2}$; $x_\pm=a(\ln s)^{1/2}(Q\pm2\mu)$; $w(x)=e^{-x^2}[1-\mathrm{erf}(-ix)]$; and $\mathrm{erf}(x)$ is the error function. The scale mass s_0 has been taken to be 1 GeV^2. For $\alpha_c<0$, $\sqrt{\alpha_c}=-i\sqrt{-\alpha_c}$. Including signature, the amplitudes are given by

$$A'^{(\tau)}(s,t)=A'(-s,t)+\tau A'(s,t), \tag{A5}$$

$$B^{(\tau)}(s,t)=B(-s,t)+\tau B(s,t), \tag{A6}$$

where τ is the signature of the Regge trajectory (−1 for the ρ and +1 for the A_2). The quantities $K(t)$, $b(t)$, and $\tilde{a}(t)$ were parametrized in our fits as

$$K(t)=e^{kt},\qquad b(t)=b_0e^{b_1t},\qquad \tilde{a}(t)=a_0. \tag{A7}$$

For the ρ fits five parameters were used (the four above plus the trajectory slope α_p'), while for the A_2 contributions k and b_1 are not needed and can be taken to be zero. To remove the ghost state at $j=0$ in the trajectories with positive signature, an extra factor of $j=\alpha(t)$ is included in the coupling constants $\tilde{a}(t)$ and $b(t)$. The unsignatured amplitudes $A'(s,t)$ and $B(s,t)$ are modified slightly:

$$\begin{aligned}A'(s,t)=&K(t)\left[b+\frac{m^2}{p^2}\alpha_+\tilde{a}\right]\left(1+\frac{2\mu}{Q}\right)\left(\frac{s}{s_0}\right)^{\alpha_+}\\ &-iK(t)\left(\frac{s}{s_0}\right)^{\alpha_c}\left\{\tfrac{1}{2}\left[b+\frac{m^2}{p^2}\alpha_+\tilde{a}\right]\left(1+\frac{2\mu}{Q}\right)w(x_+)-\tfrac{1}{2}\left[b+\frac{m^2}{p^2}\alpha_-\tilde{a}\right]\left(1-\frac{2\mu}{Q}\right)w(x_-)+\frac{i\gamma m^2\tilde{a}}{(\pi\ln s)^{1/2}}\frac{1}{p^2}\right.,\end{aligned} \tag{A8}$$

$$B(s,t)=K(t)\frac{2m\tilde{a}}{s}\left\{-\left(1+\frac{2\mu}{Q}\right)\alpha_+\left(\frac{s}{s_0}\right)^{\alpha_+}+\tfrac{1}{2}i\left(\frac{s}{s_0}\right)^{\alpha_c}\left[\alpha_+\left(1+\frac{2\mu}{Q}\right)w(x_+)-\alpha_-\left(1-\frac{2\mu}{Q}\right)w(x_-)-\frac{2i\gamma}{(\pi\ln s)^{1/2}}\right]\right\}. \quad \text{(A9)}$$

Equations (A3) and (A4) were used for the ρ trajectory's contributions to the scattering amplitude, and Eqs. (A8) and (A9) were used for the A_2 contributions. The formulas for the differential cross section, total cross section, and polarization are given by[10]

$$\frac{d\sigma}{dt}=\frac{1}{64\pi m^2 p_{\text{lab}}^{\;2}}\left[(4m^2-t)|A'|^2+\frac{\varphi}{4m^2-t}|B|^2\right],$$

$$\sigma_t=\frac{1}{p_{\text{lab}}}\text{Im}\,A'(s,t=0), \quad \text{(A10)}$$

$$P(t)=-\frac{\sin\theta_s}{16\pi\sqrt{s}}\frac{\text{Im}(A'B^*)}{d\sigma/dt}.$$

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. UO-ITS-AT (45-1)-2230.

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