Lepton Energy and Angle Dependence Theorems for Radiative Neutrino Scattering

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(Received 25 September 1972)

Explicit expressions for dependence of cross sections for neutrino processes on lepton energy and angle variables have been obtained as implications of lepton pair locality by Pais and Treiman. We show here that for neutrino processes accompanied by emission of a photon, a modified version of their theorem can be obtained giving dependence of the cross sections on the lepton-system variables.

Pais and Treiman[1,2] showed some time ago that dependence of the cross sections for neutrino processes on suitably chosen lepton-system variables can be given explicitly if a local current-current form is assumed for the semileptonic part of the weak-interaction Lagrangian. "Nonlocal" violation of these theorems would be expected due to electromagnetic interactions even when the local form for weak interactions is valid. We show here that for processes involving emission of a single photon, which involve the weak and electromagnetic processes in the lowest order, explicit expressions for dependence of the cross section on the lepton energy and angle variables can still be obtained as a consequence of the current-current form for the weak-interaction Lagrangian. Expressions of Pais and Treiman for dependence on these variables are modified in a definite form.

One of the main points of interest in obtaining the implications of the current-current Lagrangian – well established in low-energy weak interactions – is that deviations from these may provide clues towards a more complete theory. For example, models in which scalar bosons mediate weak interactions give rise to a current-current form when energies are small, but develop nonlocal effects at sufficiently large energies. Such violations have already been calculated in specific models.[3,4]

Our kinematics is similar to that of Ref. 1. Consider the process

$$\bar{\nu}_l(q_1)+p(p_1)\to l^+(q_2)+\alpha(p)+\gamma(k),$$

where $\alpha(p)$ is the final-state hadronic system with total momentum p. We work throughout in the laboratory frame ($\vec{p}_1=0$) and take mass of the lepton equal to zero. We define $L=q_2-q_1$ and $N=q_2+q_1$. Let us choose the z direction along $\vec{L}$, the y axis along $\vec{L}\times\vec{k}$ and x axis along $(\vec{L}\times\vec{k})\times\vec{L}$. Further, let θ be the angle between $\vec{k}$ and the z axis and let ϕ be the angle between the x axis and the projection of $\vec{N}$ on the xy plane. If there are n particles in the system α, we will need in all $3n+2$ variables to describe the scattering process. Some of them are chosen to be

(1) E, the laboratory energy of the incident neutrino;

(2) the angle ϕ as defined above;

(3) $t=(q_2-q_1)^2$, the square of the momentum transfer from the leptons;

(4) $w=p^2$, the square of the invariant mass of the final-state hadronic complex;

(5) $s=(k+p)^2$; and

(6) k_0, the energy of the emitted photon.

The remaining $3n-4$ variables are internal to the system α and do not concern us here.

The amplitudes for the process are

$$T_{fi}^{(1)}=-\frac{eG}{\sqrt{2}}\,\epsilon^\mu\bar{v}(q_1)\gamma^\sigma(1-\gamma_5)\frac{1}{\gamma\cdot(k+q_2)+m_l}\times\gamma_\mu v(q_2)\langle\alpha|J_\sigma^w|p\rangle$$

and

$$T_{fi}^{(2)}=\frac{eG}{\sqrt{2}}\,\epsilon^\mu\bar{v}(q_1)\gamma^\sigma(1-\gamma_5)v(q_2)M_{\mu\sigma},$$

where

$$M_{\mu\sigma}=i\int d^4x\,e^{ik\cdot x}\langle\alpha|T^*\{J_\mu^{\rm em}(x)J_\sigma^w(0)\}|p\rangle.$$

Gauge invariance of the sum of these can be checked. On replacing ϵ^μ by k^μ, $T_{fi}^{(1)}$ becomes

$$-\frac{eG}{\sqrt{2}}\,\bar{v}(q_1)\gamma^\sigma(1-\gamma_5)v(q_2)\langle\alpha|J_\sigma^w|p\rangle$$

and $T_{fi}^{(2)}$ becomes

$$\frac{eG}{\sqrt{2}}\,\bar{v}(q_1)\gamma^\sigma(1-\gamma_5)v(q_2)\langle\alpha|J_\sigma^w|p\rangle$$

since

$$k^\mu M_{\mu\sigma} = -\left\langle\alpha\left|\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}[J_0^{\rm em}(\vec{x},0),\, J_\sigma^w(0)]\right| p\right\rangle + \text{divergence of seagull term}$$
$$= \langle\alpha|J_\sigma^w(0)|p\rangle\,.$$

Squaring the amplitude and summing over spins, we have

$$\sum |T|^2 = [\tau_1^{\lambda\sigma}T^1_{\sigma\lambda} + \tau_2^{\lambda\sigma}T^2_{\sigma\lambda} + (\tau^{\lambda\sigma\nu}T_{\nu\sigma\lambda} + \text{H.c.})]\frac{G^2e^2}{m_l q_{10}}\,, \tag{1}$$

where

$$\tau_{1\lambda\sigma} = \frac{1}{k\cdot q_2}(q_{1\lambda}k_\sigma + k_\sigma q_{1\lambda} - k\cdot q_1 g_{\lambda\sigma} + i\epsilon_{\alpha\beta\lambda\sigma}k^\alpha q_1^\beta)\,,$$

$$\tau_{2\lambda\sigma} = -(q_{1\lambda}q_{2\sigma} + q_{2\lambda}q_{1\sigma} - q_1\cdot q_2 g_{\lambda\sigma} + i\epsilon_{\alpha\beta\lambda\sigma}q_2^\alpha q_1^\beta)\,,$$

$$\begin{aligned}\tau_{\lambda\sigma\nu} = \frac{1}{k\cdot q_2}[&2q_{2\nu}(q_{1\lambda}q_{2\sigma} + q_{2\sigma}q_{1\lambda} - q_1\cdot q_2 g_{\lambda\sigma} + i\epsilon_{\alpha\beta\lambda\sigma}q_2^\alpha q_1^\beta)\\ &+ q_{1\lambda}(k_\sigma q_{2\nu} - k\cdot q_2 g_{\nu\sigma} + k_\nu q_{2\sigma}) - k\cdot q_1(g_{\nu\lambda}q_{2\sigma} - q_{2\lambda}g_{\nu\sigma} + q_{2\nu}g_{\lambda\sigma})\\ &+ q_{1\nu}(k_\lambda q_{2\sigma} + k\cdot q_2 g_{\lambda\sigma} - q_{2\lambda}k_\sigma) - q_1\cdot q_2(k_\lambda g_{\nu\sigma} - k_\sigma g_{\nu\lambda} + k_\nu g_{\sigma\lambda})\\ &+ q_{1\sigma}(k_\lambda q_{2\nu} - k\cdot q_2 g_{\nu\lambda} + k_\nu q_{2\lambda}) + i(q_{2\sigma}\epsilon_{\alpha\beta\nu\lambda}q_1^\alpha k^\beta + \epsilon_{\alpha\beta\gamma\lambda}q_1^\alpha q_2^\beta k^\gamma g_{\nu\sigma}\\ &\qquad + q_{2\nu}\epsilon_{\alpha\beta\lambda\sigma}k^\alpha q_1^\beta + k_\sigma\epsilon_{\alpha\beta\lambda\nu}q_1^\alpha q_2^\beta + k\cdot q_2\epsilon_{\alpha\nu\lambda\sigma}q_1^\alpha + k_\nu\epsilon_{\alpha\beta\sigma\lambda}q_1^\alpha q_2^\beta\\ &\qquad + g_{\lambda\sigma}\epsilon_{\alpha\beta\gamma\nu}q_1^\alpha k^\beta q_2^\gamma + q_{2\lambda}\epsilon_{\alpha\beta\nu\sigma}q_1^\alpha k^\beta + g_{\nu\lambda}\epsilon_{\alpha\beta\gamma\sigma}q_1^\alpha q_2^\beta k^\gamma\\ &\qquad + k_\lambda\epsilon_{\alpha\beta\nu\sigma}q_2^\alpha q_1^\beta + q_1\cdot q_2\epsilon_{\alpha\lambda\nu\sigma}k^\alpha + q_{1\sigma}\epsilon_{\alpha\beta\nu\lambda}k^\alpha q_2^\beta + q_{1\nu}\epsilon_{\alpha\beta\lambda\sigma}k^\alpha q_2^\beta\\ &\qquad + k\cdot q_1\epsilon_{\alpha\nu\lambda\sigma}q_2^\alpha + q_{1\lambda}\epsilon_{\alpha\beta\nu\sigma}q_2^\alpha k^\beta)]\,,\end{aligned}$$

$$T^1_{\lambda\sigma} = \langle\alpha|J_\lambda^w(0)|p\rangle\langle\alpha|J_\sigma^w(0)|p\rangle^\dagger\,,\qquad T^2_{\lambda\sigma} = M_{\mu\lambda}(M_{\mu'\sigma})^\dagger g^{\mu\mu'}\,,$$

and

$$T_{\nu\sigma\lambda} = (M_{\nu\sigma})^\dagger\langle\alpha|J_\lambda^w(0)|p\rangle\,.$$

Now, $\langle\alpha|J_\sigma^w(0)|p\rangle$ and $M_{\mu\sigma}$ are functions of the momenta of the photon and the hadrons in α and depend on the lepton momenta only through the constraints imposed by four-momentum conservation. These, therefore, are functions of L but not of N, which implies they are independent of two variables since N has two constraints $N^2=-t$ and $N\cdot L=0$. These we choose to be E and ϕ. Therefore, dependence on E and ϕ comes only through the lepton tensors $\tau^{1,2}_{\lambda\sigma}$ and $\tau_{\lambda\sigma\nu}$ in the cross section.

Integrating (1) over the variables of the hadronic system and also ϕ, the differential cross section is given as a function of E by

$$\frac{d\sigma}{ds\,dt\,dw\,dk_0} = \frac{1}{E[(E-E')^2-t]^{1/2}}\left(AE^2 + BE + C + \frac{A_1E^3 + B_1E^2 + C_1E + D_1}{(PE^2 + QE + R)^{1/2}}\right). \tag{2}$$

P, Q, and R are given by

$$P = m_n{}^2 t\sin^2\theta + (X - Y\cos\theta)^2\,,$$

$$Q = \frac{1}{m_n}\{m_n{}^2 t\sin^2\theta\, Y + (X - Y\cos\theta)[2XY - (X^2+Y^2)\cos\theta]\}\,,$$

$$R = \frac{1}{4m_n{}^2}\{m_n{}^4 t^2\sin^2\theta + [2XY - (X^2+Y^2)\cos\theta]^2\}\,,$$

where $Y = \frac{1}{2}(m_n{}^2 + t - s)$ and $X = (Y^2 - m_n{}^2 t)^{1/2}$. If integrations are performed only over the variables internal to the hadronic system α, we have the ϕ-dependence theorem

$$\begin{aligned}\frac{d\sigma}{ds\,dt\,dw\,dk_0\,d\phi} = \frac{1}{E[(E-E')^2-t]^{1/2}}\Big[&a\cos2\phi + b\sin2\phi + c\cos\phi + d\sin\phi + e\\ &+ \frac{a_1\cos3\phi + b_1\sin3\phi + c_1\cos2\phi + d_1\sin2\phi + e_1\cos\phi + f_1\sin\phi + g_1}{q + r\cos\phi}\Big]\,,\end{aligned} \tag{3}$$

where a, b, a_1, b_1, etc., are functions of s, t, w, k_0, and E. q and r are given by

$$q = 2E(X - Y\cos\theta) + \frac{1}{m_n}[2XY - (X^2 + Y^2)\cos\theta],$$
$$r = \sin\theta\,[(-tm_n)(4m_nE^2 + 4YE + m_nt)]^{1/2}.$$

Expressions (2) and (3) have been obtained assuming a local current-current form for the weak-interaction Lagrangian. These results also hold in vector-boson-mediated theories of weak interactions, but are changed in some other models. In scalar-boson mediated theories, for example, higher powers of E or $\cos\phi$ and $\sin\phi$ also occur, but reduce to the above expressions, as expected, in the limit of a very large mass for the intermediate bosons. A point of special interest in this case is that since the photon can couple to the charged-lepton and intermediate-boson lines, deviations from the locality theorems can be calculated independent of the details of strong interactions, in contrast to the deviations from the Pais and Treiman theorems[4] which needed additional assumptions regarding the hadronic currents participating in weak interactions. Details of such a calculation will be published elsewhere.

One of us (S.K.) would like to thank A. K. Kapoor for useful discussions.

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PHYSICAL REVIEW D VOLUME 7, NUMBER 9 1 MAY 1973

Meson Charge-Exchange Reactions in a Regge-Cut Van Hove Model*

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(Received 9 March 1972)

We present an analysis of four meson charge-exchange reactions ($\pi^- p \to \pi^0 n$, $\pi^- p \to \eta^0 n$, $K^- p \to \bar{K}^0 n$, and $K^+ n \to K^0 p$) within the context of our Regge-pole-cut model based on the Durand–Van Hove model. Quite good fits to all differential cross sections and polarizations are obtained. Predictions for the KN charge-exchange polarizations are also presented.

I. INTRODUCTION

In the past few years it has become abundantly clear, on both theoretical and phenomenological grounds, that the j-plane structure of hadron scattering amplitudes must include cuts.[1] The discontinuity structure of these cuts being unknown, a number of models have been proposed for generating cut corrections to simple Regge exchange[2] or otherwise approximating the effects of j-plane cuts.[3]

The Durand–Van Hove model[4] has previously been successful as a means for studying the complicated structure of daughter poles[5] in the j plane. It has been modified to introduce Regge cuts in backward πN scattering so as to resolve the parity-doubling problem.[2(d)] However, the strength of the cut in the last model is not related in any natural way to the properties of the particles lying on the trajectories and is completely adjustable to fit the data. As formulated, the model of Ref. 2(d) cannot be applied to forward scattering.

Motivated by the observation that the Durand–Van Hove model is a zero-width particle exchange model generating pure Regge poles (and daughters), and by the expectation that nonzero total widths for the exchanged particles and Regge cuts in the j plane will arise somehow from unitarity effects, we constructed[6] a model in which the Regge-cut contributions arise from the introduction of j-dependent widths for all particles in the exchanged resonant tower; that is, we modified the t-channel spin-j propagator to a simple Breit-Wigner form with resonance poles on the second sheet of the t