der of 10%) to account for this discrepancy. But this only shows that the experimental errors cannot concur in producing a violation of the condition (7), which therefore imposes bounds on the magnitude and relative sign of such errors.

As an example of another possible use of conditions (7), consider the distribution indicated in Ref. 1:

$$G(x)=\lambda(1+3x^2)^{1/2},$$

whose analytic form has been determined on the basis of a model to represent π^+p elastic scattering at 1236 MeV. In this case, only the coefficient λ must be determined from experimental data. As one can easily check, $c_0=1.3801\lambda$, which makes the value $\lambda=0.988$ indicated in Ref. 1 unacceptable.

On the other hand, for a lower value like $\lambda=\sqrt{3}/2$, which is still compatible with experimental data, $c_0=1.195$, but this is hardly surprising since for this value of λ, Eq. (1) is known[6] to have an exact solution. Note also that $c_n=0$ for odd n, while $c_n>0$ for n even. Since $[\sigma/2(2n+1)]^{1/2}=1/\sqrt{2}$ for $\lambda=\sqrt{3}/2$ and $n=1$, it follows that out of all conditions only the one for $n=0$ provides an effective bound.

From a theoretical viewpoint, one might express the hope that the bounds (7), (8), and (10) could provide an indication as to the direction and range of an extension of the Martin-Newton condition covering all cases of physical interest.

I am deeply indebted to Professor A. Martin for suggesting the proof which leads to the lower bound (10), and for correcting a number of errors in an earlier version of the manuscript.

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Coupled Angular Momentum Channels and Axial-Vector Resonance in $\pi\omega$ Scattering*

Pu Shen and Kyungsik Kang
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 21 August 1972)

The possibility of binding an axial-vector meson in the $\pi\omega$ system is considered. A substantial admixture of s and d waves is required to bootstrap this particle, and the decay angular distribution calculated is in good agreement with the recent experimental information.

In this note, we would like to report on a simple S-matrix dynamical calculation which gives a resonant state in the $J^P=1^+$ state of the $\pi\omega$ channel. In this state, the $\pi\omega$ system can take two angular momentum values $l=0$ and 2, and the calculation is carried out by taking into account these two angular momentum channels.

All previous attempts[1,2] based on the self-consistency idea have preferred a bound state in the 2^- state. They are usually carried out either in the static approximations or in the relativistic dispersion methods. But none of them have seriously considered the possible admixture of the two angular momentum channels in the unnatural-spin-parity state. In fact, the recent experiments[3,4] not only favor 1^+ over 2^- for this $\pi\omega$ resonance but also give the angular distributions in detail, indicating the presence of both s and d waves.

We can explain these experiments from a *two-channel N/D calculation,* which assumes the driving forces coming from the u-channel exchanges only. The matrix N/D equations are solved algebraically by approximating the phase-space integrals at energies below threshold by a few poles, and the parameters are determined by demanding the self-consistency of the resonance in addition to the conditions coming from the ρ meson coupled to the $\pi\omega$ system. The results show that the mass and width of this resonance agree well with those of the B meson, and the decay angular correlations deduced from our coupling constants are in good agreement with the recent experiments.

The driving force coming from the $\rho(1^-)$ and $B(1^+)$ exchanges in the u channel are calculated by taking the effective interactions at the $\pi\omega\rho$ and $\pi\omega B$ vertices (Fig. 1) as

$$\begin{aligned} L_{\pi\omega\rho} &= (g_{\pi\omega\rho}/m_\rho)e_{\mu\nu\lambda\sigma}e_\mu(\omega)q_\nu(\omega)e_\lambda(\rho)q_\sigma(\rho)\,, \\ L_{\pi\omega B} &= g_s m_B e_\mu(B)e_\mu(\omega) \\ &\quad + (g_d/m_B)e_\mu(B)q_\mu(\omega)e_\nu(\omega)q_\nu(B)\,. \end{aligned} \tag{1}$$

By following the same scheme used in I to remove the kinematic singularities from the transition amplitudes $T^J(L \to L')$ between the states of definite angular momenta, we find that (1) is equivalent to taking the imaginary part of $T^J(L\to L')$ as

$$\begin{aligned} \mathrm{Im}T^1(1\to 1) &= q_1^2 g_{\pi\omega\rho}^2 \pi\delta(s-m_\rho^2)\,, \\ \mathrm{Im}T^1(0\to 0) &= q_0^2 g_s^2 \pi\delta(s-m_B^2)\,, \\ \mathrm{Im}T^1(0\to 2) &= q_0 q_2 g_s g_d \pi\delta(s-m_B^2)\,, \\ \mathrm{Im}T^1(2\to 2) &= q_2^2 g_d^2 \pi\delta(s-m_B^2)\,, \end{aligned} \tag{2}$$

with

$$\begin{aligned} q_0 &= \tfrac{1}{3}\left[\left(\frac{s}{s_0}\frac{W}{M}+2\right)m_B - \frac{sk^2}{\sqrt{s_0}Mm_B}\frac{g_d}{g_s}\right], \\ q_1 &= \left(\frac{2sk^2}{3m_\rho^2}\right)^{1/2}, \\ q_2 &= \tfrac{1}{3}\sqrt{2}\left[\left(\frac{s}{s_0}\frac{W}{M}-1\right)m_B\frac{g_s}{g_d} - \frac{sk^2}{\sqrt{s_0}Mm_B}\right], \end{aligned} \tag{3}$$

where $W=(M^2+k^2)^{1/2}$, $E=(m^2+k^2)^{1/2}$ (k being the barycentric momentum); $s_0=(M+m)^2$; and M (m) is the mass of ω (π). Notice that the transition amplitudes exhibit the correct threshold behavior [i.e., $T^1(L\to L')\sim(sk^2)^{(L+L')/2}$] through q_L, which gets the mixed contributions from the s and d terms in (1) for the unnatural-spin-parity state.

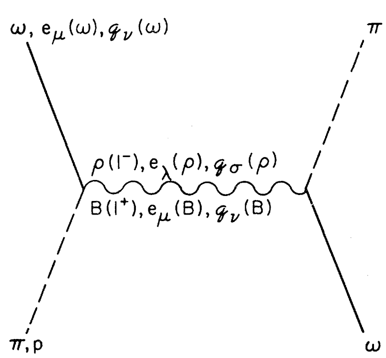

FIG. 1. The ρ- and B-meson exchanges in the $\pi\omega$ interactions.

Furthermore, q_0 and q_2 (also q_1) are polynomials in s, which makes the calculation of the phase-space integral much simpler. With these q_L factors we define the kinematic-singularity-free transition amplitudes by

$$\overline{T}^{J=1}(L\to L') = (q_L q_{L'})^{-1}T^{J=1}(L\to L') = \overline{T}_{LL'}(s)\,, \tag{4}$$

which we will work with hereafter.

As was noticed in I and II, the ρ exchange in the u channel gives rise to an attractive force to the 1^+ state. It was also concluded there and in other previous works[2] that the 1^+ s-wave B exchange produces a repulsive force to the 1^- state from the threshold behavior of the force thereby unable to reciprocally bootstrap ρ and 1^+ B. However, we find that although the 1^+ s-wave exchange does give a weak repulsive force to the 1^- state near threshold, it is effectively attractive around the ρ-meson pole position. Furthermore, the inclusion of the d-wave interaction gives an even stronger attractive force to the 1^- state, if the admixture of d wave is comparable to s wave.

For these reasons, we proceed to reexamine the possibility of a reciprocal bootstrap mechanism between the 1^- ρ and 1^+ B mesons by keeping both s- and d-wave interactions. Since there is no simple method of estimating the t-channel contribution, and since we want to limit the number of arbitrary parameters, we neglect the t-channel exchanges in the present calculation, as in all of the previous works.[1,2]

The reduced amplitudes defined in (4) satisfy the dispersion relation

$$\overline{T}_{LL'}(s) = \overline{B}_{LL'}(s) + \frac{1}{\pi}\int_R \frac{\sum_n \overline{T}^\dagger_{Ln}(s')\rho_n(s')\overline{T}_{nL'}(s')}{s-s'}ds'\,, \tag{5}$$

where $\overline{B}_{LL'}(s)$ is the corresponding Born term, and $\rho(s)$ is given for the $J^P=1^+$ state by

$$\rho(s) = \frac{k}{8\pi\sqrt{s}}\begin{pmatrix} q_0^2 & 0 \\ 0 & q_2^2 \end{pmatrix} \tag{6a}$$

and for $J^P=1^-$ by

$$\rho(s) = \frac{k}{8\pi\sqrt{s}}q_1^2\,. \tag{6b}$$

We solve (5) by the N/D method but within the approximation scheme suggested by Pagels[5] in order to avoid numerical integration.

The scheme consists essentially of approximating the phase-space integrals at energies below threshold by some poles. The phase-space integrals take the form

$$F_n(s)=\frac{s}{\pi}\int_R \frac{\rho_n(x)}{x^2(x-s)}dx=f_n(s)G(s)+g_n(s)\,, \qquad (7)$$

where

$$G(s)=\int_R \frac{\{[x-(M+m)^2][x-(M-m)^2]\}^{1/2}}{x^2(x-s)}dx \qquad (8)$$

and $f_n(s)$ and $g_n(s)$ are some polynomials in s due to q_L ($L=0,1,2$) being polynomials in s. We approximate $G(s)$ for $s<s_0$ by some pole terms which are adjusted to reproduce $G(s)$ as closely as possible. We find that the one-pole approximation, i.e. (using pion mass units), $G(s)=c/(s-a)=-3.024/(s-124.5)$, is rather good (see Fig. 2) in reproducing the exact $G(s)$ within 10% accuracy over a large range.

In terms of the N and D matrices, the solution of (5) is given by

$$\overline{T}_{LL'}(s)=[N(s)D(s)^{-1}]_{LL'}\,, \qquad (9)$$

$$D_{LL'}(s)=\delta_{LL'}-sf_L(s)\left[G(s)-\frac{c}{s-a}\right]N_{LL'}(s) - \frac{cs}{s-a}f_L(a)N_{LL'}(a)\,, \qquad (10)$$

$$N_{LL'}(s)=\overline{B}_{LL'}(s) -\sum_n \frac{c}{s-a}[s\overline{B}_{Ln}(s)-a\overline{B}_{Ln}(a)]f_n(a)N_{nL'}(a)\,. \qquad (11)$$

This solution has the correct discontinuity on the right-hand cut, $\mathrm{Im}\overline{T}^{-1}(s)=-\rho(s)$, and on the left-hand cut, $\mathrm{Im}\overline{T}(s)=\mathrm{Im}\overline{B}(s)$. Hence we may expect that many of the features of the exact solution are preserved by this approximated solution.

The solution of the N/D equation usually needs a cutoff for the driving forces coming from the vector exchanges. However, in Pagels's approximation with the assumption of the nearby-singularity dominance, the matrix N/D equation is reduced to algebra without an explicit cutoff parameter in the result, although it is still implied by the nature of approximation in that the logarithmic factor in $G(s)$ is effectively replaced by a constant. The cutoff introduced in $g_n(s)$ does not add a new parameter to the problem, since $g_n(s)$ has no effect on the final result so long as it is a polynomial in s, which is the case in the present calculation. The approximation scheme depends on the kinematic factor of the particular partial wave, whereas the Born term, which governs essentially the low-energy dynamics, remains unchanged, in contrast to other

TABLE I. Inputs used are $m_\pi=1.00$ (139.6 MeV), $m_\rho=5.48$ (765 MeV), and $m_\omega=5.61$ (784 MeV).

m_B ($m_\pi=1$)	$g_{\rho\pi\omega}$	g_s	g_d	g_d/g_s	Γ_B [Eq. (15)]	Γ_B [Eq. (16)]
8.24	9.25	2.55	−5.50	−2.16	0.969	0.969
	9.25	2.70	−6.00	−2.22	0.953	1.08
	9.30	2.55	−5.50	−2.16	0.966	0.969
	9.30	2.70	−6.00	−2.22	0.951	1.08
8.59	9.25	2.40	−5.50	−2.29	1.11	0.950
	9.25	2.65	−6.50	−2.45	1.08	1.15
	9.30	2.50	−6.00	−2.40	1.09	1.02
	9.30	2.65	−6.50	−2.45	1.07	1.15
8.95	9.25	2.50	−6.50	−2.60	1.21	1.10
	9.25	2.60	−7.00	−2.69	1.19	1.18
	9.30	2.60	−7.00	−2.69	1.19	1.18
	9.30	2.70	−7.50	−2.78	1.17	1.27
9.31	9.25	2.55	−7.50	−2.94	1.29	1.20
	9.25	2.65	−8.00	−3.02	1.27	1.29
	9.30	2.55	−7.50	−2.94	1.26	1.20
	9.30	2.65	−8.00	−3.02	1.26	1.29

schemes that involve the approximation of the driving forces. The self-consistency in the present approximation thus implies certain restrictions for the acceptable Born terms to allow the bootstrap particles. Other advantages as well as justification of this approximation scheme can be found in the original work[5] of Pagels.

Let us suppose that the driving forces coming from the ρ and B exchanges in the u channel are attractive enough to produce a bound state with $0<m_\rho^2<(M+m)^2$ in the 1^- amplitude and a resonant state with $m_B^2>(M+m)^2$ in the 1^+ amplitude. The condition that $T^{1^-}(s)$ have a simple pole at $s=m_\rho^2$ implies $D_{11}(m_\rho^2)=0$, which gives

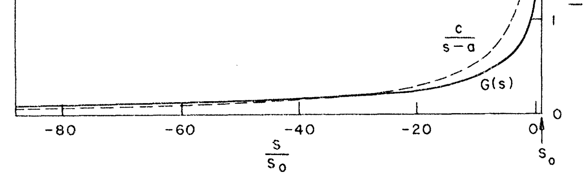

FIG. 2. The exact $G(s)$ and the pole approximation $-3.024/(s-124.5)$, with s and s_0 in units of pion mass squared. The parameters are determined by the least-squares fit of the exact $G(s)$ up to $s=-80s_0$ after matching at $s=s_0$.

TABLE II. Comparison of our predictions with other theoretical and experimental results.

Ref. No.	m_B (MeV)	Γ_B (MeV)	$g_{\pi\omega\rho}$	g_s	g_d/g_s	$\lvert D/S\rvert^2$	$W(\theta)\sim a\sin^2\theta+b\cos^2\theta$ a	b
3	···	···	···	···	···	0.03–3	0.45 ± 0.04	0.10 ± 0.08
4	···	···	···	···	···	0.04–0.36	0.48 ± 0.05	0.06 ± 0.10
7	···	···	13.3	2.4	−2.9	0.04	0.39	0.22
9	1233 ± 10	100 ± 20	···	···	···	···	···	···
Present results	1225	158	9.25	2.60	−2.56	0.033	0.38	0.24

$$1-\frac{caf_1(a)}{m_\rho^{\,2}-a}[\bar{B}_{11}(a)-(m_\rho^{\,2}-a)\bar{B}'_{11}(a)]=0\,. \tag{12}$$

Since this bound state must be consistent with the ρ particle that is exchanged in the crossed channel, we have the condition $1/g_{\pi\omega\rho}{}^2=-D'_{11}(m_\rho{}^2)/N_{11}(m_\rho{}^2)$, which can be rewritten as

$$\bar{B}_{11}(a)+\frac{g_{\pi\omega\rho}{}^2}{m_\rho{}^2-a}=0\,. \tag{13}$$

The condition that $T^{1^+}(s)$ have a simple pole at $s=m_B{}^2$ must be determined from the determinant

$$\mathrm{Det}\{D_{LL'}(s)\}=\Delta_R(s)+i\Delta_I(s)\quad (L,L'=0,2)$$

by demanding that

$$\Delta_R(m_B{}^2)=0\,, \tag{14}$$

$$\Gamma_B=-\Delta_I(m_B{}^2)/m_B\Delta_R{}'(m_B{}^2)\,, \tag{15}$$

where Γ_B is the decay width of the B meson calculated directly from the effective interaction (1),

$$\Gamma_B=\frac{Q}{6\pi}\left[g_s{}^2+\frac{E_0}{2M}\left(g_s+\frac{Q^2}{m_BE_0}g_d\right)^2\right], \tag{16}$$

where (E_0, Q) is the energy momentum of the ω particle in the rest frame of the B meson.

With the experimental masses of m_π, m_ρ, and m_ω as input we calculate $g_{\pi\omega\rho}$, g_s, g_d, m_B, and Γ_B directly from (12)–(15). A computer search for these parameters is made over a wide range of values by taking $m_\rho=5.48$ and $m_\omega=5.61$ in the $m_\pi=1$ unit. In particular we tried to satisfy (12)–(14) to within 1% while both Γ_B from (15) and (16) have values less than 200 MeV.

For the case of $g_d/g_s>0$, we find that there is no good solution even for a wide range of values of the parameters, i.e., $5\leq g_{\pi\omega\rho}\leq 20$, $0\leq g_s\leq 30$, $0\leq g_d\leq 60$, and $0.9\leq m_B\leq 2$ GeV. Thus we conclude that the reciprocal bootstrap between 1^- and 1^+ B is not possible if $g_d/g_s>0$. This conclusion is in agreement with experiments[6] in that the zero-helicity amplitude F_0 is small, as one can easily see from the angular distribution

$$W(\theta)\sim\sin^2\theta+\frac{m_B}{M}\left(\frac{E_0}{m_B}+\frac{Q^2}{m_B{}^2}\frac{g_d}{g_s}\right)^2\cos^2\theta\,. \tag{17}$$

A positive ratio of g_d/g_s can, even if it is very small, give a rather large $\cos^2\theta$ or F_0 contribution to the angular distribution. However, for the case of $g_d/g_s<0$, we find that there exists a set of parameters satisfying (12)–(15) which are very reasonable and stable, i.e., the solutions appear in the region of $9.2\leq g_{\pi\omega\rho}\leq 9.3$, $1.15\leq m_B\leq 1.3$ GeV for an approximately constant ratio of g_d/g_s around -2.5. The results are summarized in Table I.

In general, our results are in good agreement with experiments[3,4] as well as with other theoretically predicted results,[7,8] as we see from Table II. In Table II, the number $|D/S|^2$ is the ratio of d-wave and s-wave contributions. $W(\theta)$ is the angular distribution for the B decay with a normalization $2a+b=1$. The data in the last row are the averaged values taken from Table I.

In conclusion, it is possible to explain the axial-vector $\pi\omega$ resonance with mass around 1.2 GeV from the N/D dynamics. The ratio of the coupling constants $g_d/g_s=-2.56$ implies a substantial d-wave contribution. This is in agreement with the recent experiments in which only a small amount of ω helicity-zero amplitude appeared in the angular distribution of the B decay.

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πN Total Cross Section, Neutrino Reactions, and Electroproduction in the Regge Asymptotic Region*

Paul Langacker† and Mahiko Suzuki‡

Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

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It is shown from the Deser-Gilbert-Sudarshan representation that the Pomeranchukon residue of the W_2 function for the axial-vector current is independent of Q^2. This justifies the relation $\sigma_{\text{tot}}^{\pi p}(\infty) = \frac{1}{2}\pi f_\pi^{-2} F_2^{(A)}(\infty)$ previously derived from generalized scaling.

In the formal framework of the Deser-Gilbert-Sudarshan (DGS) representation,[1] we study the Q^2 dependence of the Regge residues of the function $W_2^{(A)}(\nu, Q^2)$ for the axial-vector current.[2,3] We find that besides the scaling of $\nu W_2^{(A)}(\nu, Q^2)/m_N^2$ and the Regge behavior in ω of $F_2^{(A)}(\omega)$, *the Pomeranchukon residue is independent of Q^2 all the way from $Q^2 = 0$ to ∞.* The high-energy limit of the πN total cross section is thus related unambiguously to $F_2^{(A)}(\infty)$, which provides us with a test of the free-quark-algebra result[4] $F_2^{(A)}(\omega) = F_2^{(V)}(\omega)$ at least as $\omega (= 2\nu/Q^2) \to \infty$. Generalized scaling[5,6] is made plausible for secondary Regge trajectories.

The axial-vector structure function $W_2^{(A)\pm}(\nu, Q^2)$ is defined as

$$\overline{\sum_{\text{spin}}} \int \frac{d^4x}{4\pi} e^{iqx} \langle p | [A_\mu^\mp(x), A_\nu^\pm(0)] | p \rangle = \frac{1}{m_N^2} \tilde{p}_\mu \tilde{p}_\nu W_2^{(A)\pm}(\nu, Q^2) + \text{other terms}, \tag{1}$$

where $\tilde{p} = p - q(p \cdot q)/q^2$. Through PCAC (partially conserved axial-vector current) $W_2^{(A)\pm}(\nu, 0)$ is related to πN scattering by[6]

$$\sigma_{\text{tot}}^{\pi^\pm p}(\nu) = \tfrac{1}{2}\pi f_\pi^{-2} \nu W_2^{(A)\pm}(\nu, 0)/m_N^2, \tag{2}$$

where $\sigma_{\text{tot}}^{\pi^\pm p}(\nu)$ is the $\pi^\pm p$ total cross section for a massless external pion at laboratory energy ν/m_N, and $\sqrt{2} f_\pi = 0.96\, m_\pi$ is the pion decay constant.

We now write a DGS representation[1] for $W_2^{(A)\pm}(\nu, Q^2)$; this representation incorporates causality and the mass spectral conditions, and it is useful for relating different kinematical regions in Q^2. It is written as

$$W_2^{(A)\pm}(\nu, Q^2) = \int_0^\infty d\mu \int_{-1}^1 db\, \sigma^\pm(\mu, b) \times \delta(-Q^2 + 2b\nu - \mu)\epsilon(\nu + b m_N^2). \tag{3}$$

The support of $\sigma^\pm(\mu, b)$ is determined by the mass spectral conditions.[1] For positive Q^2 and ν, we obtain, by integrating Eq. (3) over b,

$$\frac{\nu W_2^{(A)\pm}(\nu, Q^2)}{m_N^2} = \frac{1}{2m_N^2} \int_0^\infty d\mu\, \sigma^\pm\left(\mu, \frac{Q^2 + \mu}{2\nu}\right). \tag{4}$$

As was pointed out by many people, the right-hand side of Eq. (4) becomes independent of Q^2 in the Bjorken scaling limit ($Q^2 \to \infty$ with $\omega = 2\nu/Q^2$ fixed) if $\sigma^\pm(\mu, b)$ decreases fast enough in μ to permit interchange of the order of $Q^2 \to \infty$ and $\int d\mu$. If $\sigma^\pm(\mu, b)$ is truncated effectively at $\mu = \mu_{\max}$, scaling sets in for $Q^2 \gg \mu_{\max}$.

In the Regge limit ($\nu \to \infty$ with Q^2 fixed)

$$\nu W_2^{(A)\pm}(\nu, Q^2)/m_N^2 \underset{\text{Regge}}{\sim} \sum_i \beta_i^\pm(Q^2) \nu^{\alpha_i - 1}. \tag{5}$$

This asymptotic behavior is realized if $\sigma^\pm(\mu, b)$ behaves near $b = 0$ as

$$\sigma^\pm(\mu, b) \underset{b \to 0}{\sim} \sum_i \gamma_i^\pm(\mu) b^{1 - \alpha_i}. \tag{6}$$

Combining (4), (5), and (6), we obtain a relation

$$\beta_i^\pm(Q^2) = \frac{1}{2m_N^2} \int_0^\infty d\mu\, \gamma_i^\pm(\mu) \left(\frac{2}{Q^2 + \mu}\right)^{\alpha_i - 1}, \tag{7}$$

which leads to the well-known Q^2 dependence of the Regge residues $\beta_i^\pm(Q^2) \sim (Q^2)^{1 - \alpha_i}$ as $Q^2 \to \infty$.[7]