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Multiplicity Distributions Resulting from Multiperipheral Plus Diffractive Production*

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We discuss the phenomenology of multiplicity distributions in high-energy hadronic collisions in terms of the short-range correlation picture, modified to include diffractive processes leading to low-multiplicity final states. We formulate two models, both of which give reasonable fits to present data on high-energy partial cross sections. Model-independent methods for separating diffractive and multiperipheral production are discussed.

I. INTRODUCTION

One of the most interesting open questions concerning multiparticle hadronic reactions is the behavior of the partial cross sections $\sigma_n^i(s)$ for producing n particles of a given type i (plus any number of other particles), as a function of both n and the square of the c.m. energy, s. Current efforts to organize and understand data on multiparticle reactions are dominated by two competing points of view, the diffractive picture and the short-range correlation (multiperipheral) picture. The *diffractive picture* assumes that for large s the partial cross sections approach nonzero constant values, $\sigma_n(s) \to \sigma_n$. In order to reproduce the apparent linear increase of the average multiplicity $\langle n\rangle$ with $\ln s$, proponents of the diffractive picture usually assume that $\sigma_n \sim c/n^2$ for sufficiently large n. Specific diffractive models[1] have been constructed which, at the cost of additional assumptions, exhibit concrete predictions to compare with data.

The *short-range-correlation* picture assumes that all correlations vanish between particles whose rapidities y are separated by a distance large compared to a correlation length L. This picture implies, without further *ad hoc* assumptions, that the average multiplicity rises linearly with $\ln s$. Since it reproduces general features of multiperipheral models, this picture is also called the *multiperipheral picture*. Again, specific simple multiperipheral models[2] have been constructed and their predictions are testable experimentally.

The diffractive and short-range-correlation pictures represent, in some sense, extreme points of view. It is not hard to imagine that the real world contains both diffractive and short-range-correlation elements. Recent high-energy data from the National Accelerator Laboratory (NAL) and CERN Intersecting Storage Rings (ISR) are, as we shall show, consistent with the predictions of the short-range-correlation picture for inelastic multiparticle production. On the other hand, we know that diffractive processes are present – certainly in elastic scattering, and in some fraction of the inelastic production. This fraction is quite uncertain at present, and we shall use two models in the phenomenological analyses of Sec. IV – one which assumes that inelastic events all fit the short-range-correlation picture, and one which (under cetain assumptions) determines the fraction of diffractive processes from the data.

In order to study the behavior of the partial cross sections $\sigma_n(s)$ from a general point of view which does not tie us to an extreme multiperipheral or extreme diffractive picture, we start in Sec. II with a general discussion of the implications of Mueller-Regge analysis.[3] In Sec. III we construct specific models for partial cross sections by using Mueller-Regge analysis plus a model for correlation functions. The remainder of the paper is devoted to comparison of these models with the data, and to some further theoretical remarks.

II. MUELLER-REGGE ANALYSIS OF PARTIAL CROSS SECTIONS

A. General Considerations

It is convenient to define a generating function $I^i(z, Y)$ for the partial cross sections[4]:

$$I^i(z, Y) = \sum_{n=0}^{\infty} p_n^i(Y) z^n , \tag{1}$$

where

$$p_n^i(Y) = \frac{\sigma_n^i(Y)}{\sigma(Y)}$$

and

$$Y = \ln(s/s_0) ,$$

where s_0 is usually chosen to be $s_0 = m_a m_b$, so that Y is the rapidity separation between the incoming particles a and b. The normalization is sometimes chosen to be $\sigma = \sigma_{\text{tot}}$, sometimes $\sigma = \sigma_{\text{inel}}$. This point is discussed in detail in Sec. II C.

This generating function can be expressed in terms of integrals, f_n^i, over the n-particle inclusive correlation functions, C_n^i:

$$I^i(z, Y) = \exp\left[\sum_{n=1}^{\infty} \frac{f_n^i}{n!} (z-1)^n\right] , \tag{2}$$

where

$$\begin{aligned} f_1^i &= \int_0^Y dy_1 C_1(y_1) \\ &= \langle n^i \rangle , \\ f_2^i &= \int_0^Y \int_0^Y dy_1 dy_2 C_2(y_1, y_2) \\ &= \langle n^i(n^i - 1)\rangle - \langle n^i \rangle^2 , \end{aligned} \tag{3}$$

etc.

See, for example, Ref. 4 for a detailed definition of the C_n. These formulas are valid independently of the theoretical picture and provide the link between the partial cross sections σ_n and the inclusive average correlation functions f_n.

The behavior with energy of the f_n's can be inferred from Mueller-Regge analysis of inclusive reactions. One finds[5] that

$$f_n(Y) \leq O(Y^n) . \tag{4}$$

This bound follows from the fact that $f_n(Y)$ is an n-dimensional integral over a region of volume Y^n of a nonsingular limiting distribution.

B. Short-Range-Correlation (SRC) Picture

In the short-range-correlation picture all correlation functions $C_n(y_1, y_2, \ldots, y_n)$ vanish whenever the rapidity separation $|y_j - y_i|$ of any pair of particles becomes large compared to a correlation length L. The effective region of integration in the n-dimensional integral which defines $f_n(Y)$ is then reduced from Y^n to YL^{n-1}, and the bound in Eq. (4) is strengthened to

$$f_n(Y) \leq O(Y)$$

or $\quad (5)$

$$f_n(Y) = a_n Y + b_n .$$

Such behavior is obtained from Mueller-Regge analysis only with an additional assumption: that the leading J-plane singularity is factorizable.

From Eq. (5) one infers a generating function $I_M(z, y)$ for the short-range-correlation (or multiperipheral) picture (we now drop the superscript denoting particle type, except where necessary for clarity):

$$\begin{aligned} I_M^i(z, Y) &= \exp\left[Y \sum \frac{a_n}{n!}(z-1)^n + \sum \frac{b_n}{n!}(z-1)^n\right] \\ &= \exp[YA(z) + B(z)] . \end{aligned} \tag{6}$$

The partial probabilities $p_n(y)$ are given by

$$p_n(Y) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} I_M(z, Y)\Big|_{z=0} . \tag{7}$$

It follows from the above that in this picture[6]

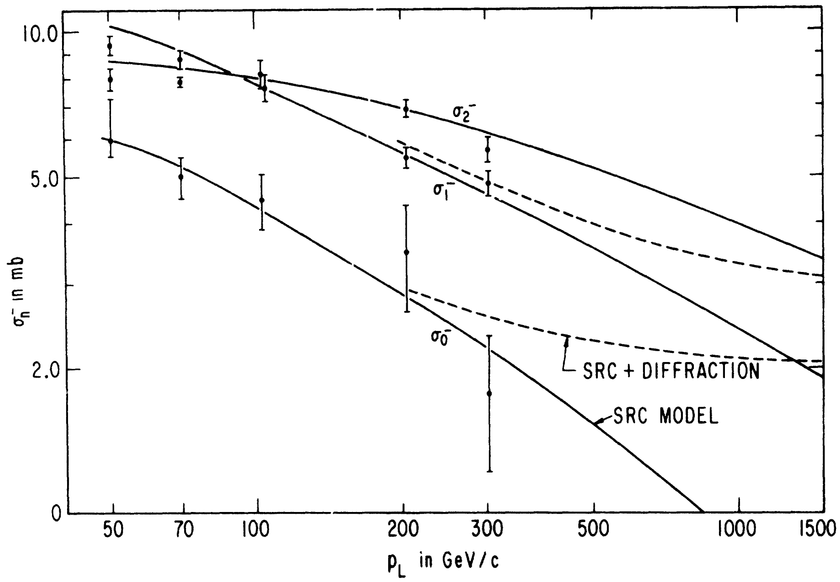

FIG. 1. Behavior of the low-multiplicity inelastic partial cross sections as a function of energy. Curves are fits using models described in Sec. IV.

$$p_n(Y)=e^{A(0)Y}h_n(Y)\ , \tag{8}$$

where $h_n(Y)$ is an nth-order polynomial in Y.

The crucial feature of Eq. (8) is that for sufficiently large Y *all* $p_n(y)$ must fall with the same power of the energy. Clearly, however, the polynomial factor $h_n(y)$ may overcome the exponential damping factor at smaller values of Y. In Fig. 1 we display the experimental inelastic partial cross sections σ_0^-, σ_1^-, and σ_2^-, respectively, for producing 0, 1, and 2 negative particles in proton-proton collisions, as a function of energy. The data come from the recent Serpukhov[7] and NAL experiments.[8-10] Although it is hard to draw any definitive conclusions from Fig. 1, it is clear that as far as these inelastic cross sections go, the short-range-correlation prediction is in reasonable accord with the data with $A(0)\sim-\frac{1}{2}$. We should note, however, that in this energy range the elastic cross section is approximately constant, within errors.

Thus we are forced to face the dilemma that a phenomenology based on a short-range correlation model must exclude at least the elastic cross section, plus any diffractive production which may be present in the inelastic cross section. We are not, of course, the first to recognize this dilemma. It has been discussed recently in the Mueller-Regge context by Le Bellac.[11] It is present in all simple multiperipheral models; for example, the Chew-Pignotti model,[2] which yields an elastic cross section falling with the same power as the inelastic partial cross sections. It is also present in a somewhat different form in the original work of Amati, Fubini, and Stanghellini,[12] which suggested the necessity of a branch point in the J plane in the elastic cross section. Presumably our present difficulties also arise from our neglect of branch points (in inclusive Mueller amplitudes), which is necessary to derive the short-range-correlation picture from Mueller-Regge analysis. The present controversial state of the theory of J-plane branch points prevents us from dealing with our dilemma from a fundamental theoretical approach; we concentrate instead on the development of a consistent phenomenology.

We speculate that if the elastic contribution (which is clearly diffractive) and all inelastic diffractive processes are excluded, then the remainder of the partial cross sections can be generated from a short-range-correlation picture. Such a separation of events into diffractive and multiperipheral has been advocated by Wilson.[13]

C. Modified Short-Range-Correlation Picture: Inclusion of Diffraction

Before outlining a method for adding diffractive processes in a Mueller-Regge analysis, we must distinguish between two types of diffractive pictures, which we shall call high- and low-multiplicity diffractive processes. By *high-multiplicity* diffractive processes we signify mechanisms like the $\sigma_n\sim1/n^2$ tail of the multiplicity distribution hypothesized by Hwa, Jacob, and Slansky, and by Quigg, Wang, and Yang,[1] to account for growth of average multiplicities like $\ln s$. Such a high-multiplicity tail implies moments $f_n(s)$ which grow

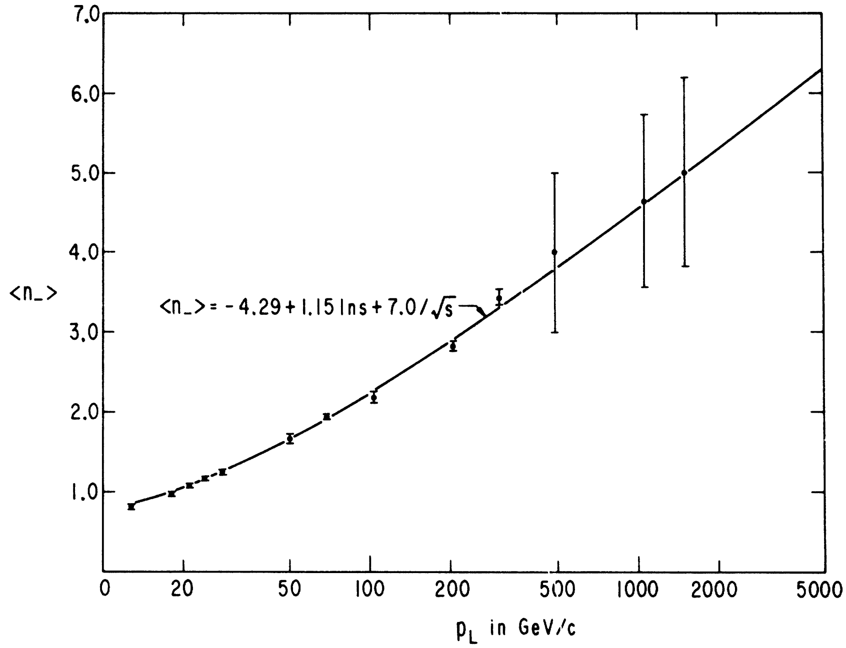

FIG. 2. Average multiplicity of negative particles as a function of energy. Data are from Refs. 7–10, 14, 15. The fit is of the form proposed by Tow in Ref. 16. Addition of a term in $(\ln s)/\sqrt{s}$, required by theory (Cahn, Ref. 17), does not improve the fit, which has $\chi^2 = 6$ for 10 degrees of freedom.

like $(\sqrt{s})^{n-1}$, violating the bound in Eq. (4) which follows from the basic assumptions of Mueller-Regge analysis. There is no need for this tail in models which include a short-range-correlation component, since this component naturally produces the $\ln s$ growth of multiplicities. Moreover, existing data are compatible with the bound given by Eq. (4), and even with the more severe bound in Eq. (5) as can be seen from Figs. 2[7-10,14-17] and 3.[7-10] For these reasons, we shall not consider high-multiplicity diffraction further, although it does offer an alternative mechanism which is not ruled out by existing data.

Low-multiplicity diffractive processes contribute significantly to low-multiplicity cross sections only, and therefore give contributions to the average multiplicity (and to all other moments of the multiplicity distribution) which are asymptotically constant as a function of energy. We now discuss the addition of such processes to the formalism developed in previous sections.

Let us denote by σ_n^D and by σ_n^M the partial cross sections produced by diffractive and short-range-correlation mechanisms, respectively. Then

$$\sigma = \sum(\sigma_n^D + \sigma_n^M)$$
$$= \sigma_D + \sigma_M \; . \qquad (9)$$

We define a generating function $I_M(z, Y)$ by

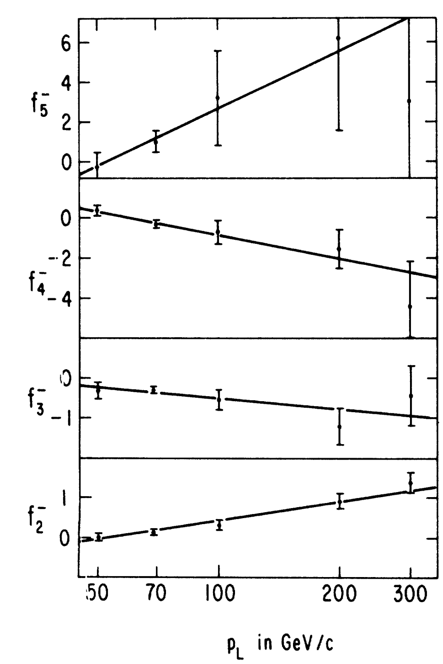

FIG. 3. Correlation parameters as a function of energy. Linear fits, required by the short-range correlation picture, are shown. Fits have $\chi^2 = 2.0$, 1.6, 0.94, and 0.22 for f_2^-, f_3^-, f_4^-, and f_5^-, respectively. Data are from Refs. 7, 8, 9, and 10.

$$I_M(z,Y)=\sum_{n=0}^{\infty}\frac{\sigma_n^M}{\sigma_M}z^n\ . \tag{10}$$

This generating function can, of course, be expressed in terms of the "short-range" correlation parameters f_n^M:

$$I_n(z,Y)=\exp\left[\sum_{n=1}^{\infty}\frac{f_n^M(Y)}{n!}(z-1)^n\right]\ . \tag{11}$$

These "short-range" correlation parameters are defined with respect to the SRC component of the cross sections. That is,

$$f_1^M=\langle n_M\rangle=\frac{\sum n\sigma_n^M}{\sigma_M}\ ,$$

$$f_2^M=\frac{1}{\sigma_M}\sum n(n-1)\sigma_n^M-\left(\frac{1}{\sigma_M}\sum n\sigma_n^M\right)^2\ , \tag{12}$$

etc.

By assumption, these correlation parameters grow at most like Y,

$$f_n^M(Y)=a_nY+b_n\ . \tag{13}$$

In order to relate these short-range-correlation parameters to the usual f_n's defined in Eq. (3), we first seek a relation between the generating functions. We define a generating function for diffractive production in analogy to Eq. (10) for SRC production,

$$I_D(z,Y)=\sum_{n=0}^{\infty}\frac{\sigma_n^D}{\sigma^D}z^n$$

$$=\exp\left[\sum_{n=1}^{\infty}\frac{f_n^D}{n!}(z-1)^n\right]\ , \tag{14}$$

where these sums cut off at some low value of n, according to our assumption of low-multiplicity diffraction. Recall the definition of the full generating function,

$$I(z,Y)=\sum_{n=0}^{\infty}\frac{\sigma_n}{\sigma}z^n$$

$$=\exp\left[\sum_{n=1}^{\infty}\frac{f_n}{n!}(z-1)^n\right]\ ; \tag{15}$$

the normalization to $\sigma=\sigma_{\text{inel}}$ is now conventional, but normalization to $\sigma=\sigma_{\text{tot}}$ is advocated in many theoretical papers. We shall return to the question of normalization at the end of this section.

These generating functions are related by

$$I(z,Y)=\lambda I_M(z,Y)+(1-\lambda)I_D(z,Y)\ , \tag{16}$$

where

$$\lambda=\frac{\sigma_M}{\sigma}\ . \tag{17}$$

The ratio $\lambda(Y)$ is independent of Y if σ_D and σ_M are strictly constant, but may have weak energy dependence (logarithmic or perhaps a very small power) in some models.

One can now use Eq. (16) to calculate the relations among the f's, the first two of which read

$$f_1=\lambda f_1^M+(1-\lambda)f_1^D\ , \tag{18a}$$

$$f_2=\lambda(1-\lambda)(f_1^M)^2+\lambda f_2^M-2\lambda(1-\lambda)f_1^Mf_1^D+(1-\lambda)f_2^D+\lambda(1-\lambda)(f_1^D)^2\ . \tag{18b}$$

These of course reduce to $f_n=f_n^M$ for the case $\lambda=1$. Note, however, that for $\lambda\neq 1$ the first term in Eq. (18b) behaves like Y^2 (by assumption, all f_n^D are constant). In general, f_n goes like Y^n, as is to be expected when the SRC hypothesis is violated.[18] Expressions for the higher f_n's are quite complicated, but it may be useful to write down the term which behaves like Y^n:

$$f_1\sim\lambda f_1^M\ , \tag{19a}$$

$$f_2\sim\lambda(1-\lambda)(f_1^M)^2\ , \tag{19b}$$

$$f_3\sim\lambda(1-\lambda)(1-2\lambda)(f_1^M)^3\ , \tag{19c}$$

$$f_4\sim\lambda(1-\lambda)(1-6\lambda+6\lambda^2)(f_1^M)^4\ , \tag{19d}$$

$$f_5\sim\lambda(1-\lambda)(1-14\lambda+36\lambda^2-24\lambda^3)(f_1^M)^5\ . \tag{19e}$$

Note that the leading term for every f_n is specified in terms of only two parameters, f_1^M and λ. Thus very accurate high-energy multiplicity data will provide the opportunity to test for the presence of a term $f_n\sim Y^n$, and, by determining its magnitude, to measure the fraction of the cross section coming from SRC and from diffractive processes.

We conclude this section with a discussion of the question of normalization: What cross section σ should be used in Eq. (15); total, inelastic, or yet another choice? Although theorists have usually normalized to $\sigma=\sigma_{\text{tot}}$, experimenters and phenomenologists have found the dissimilar behavior of elastic and inelastic reactions suggestive that elastic events be excluded and hence that distributions be normalized to $\sigma=\sigma_{\text{inel}}$. In terms of the foregoing formalism, they are recognizing that σ_{el} belongs to σ_D. It still remains possible that $\sigma_M=\sigma_{\text{inel}}$; that is, inelastic production comes entirely from SRC processes. In that case, $\lambda=1$ with $\sigma=\sigma_{\text{inel}}$ and all the above formulas simplify; for example, Eqs. (18) and (19) become just $f_n=f_n^M$. It also remains possible, however, that a significant fraction of σ_{inel} is of the diffractive type; in that case, the normalization $\sigma=\sigma_{\text{inel}}$ will not provide great simplicity and we shall face the task of separating σ_D and σ_M. We shall discuss methods of

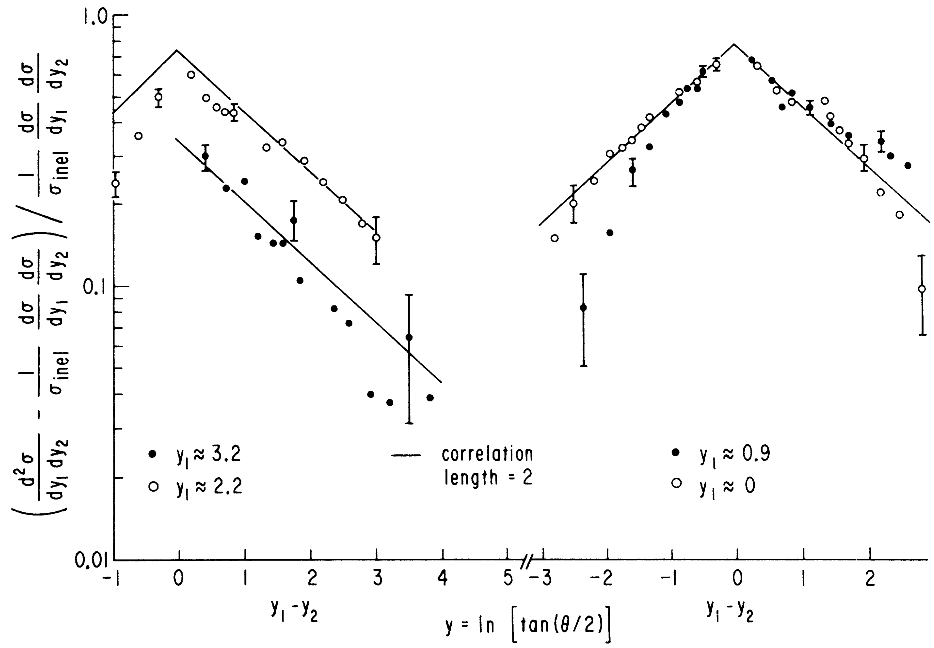

FIG. 4. Correlation functions in central region, from preliminary data of Pisa-Stony Brook group at the ISR (Ref. 14).

separation in Secs. IV and V C.

Consistent normalization is necessary if one is to achieve consistently simpler results. That is, if normalization to σ_{inel} seems required to obtain f_n's which satisfy the SRC bound, Eq. (5), then normalization to σ_{inel} will also be necessary in two-particle correlation functions if one is to see SRC behavior $C_2 \sim \exp[-|y_2 - y_1|/L]$. It is interesting that the preliminary data of the Pisa-Stony Brook group[19] shown in Fig. 4, which do exhibit SRC behavior, are indeed normalized to σ_{inel}.

There remains the theoretical question of the consistency of any normalization other than σ_{tot}. In the Mueller-Regge formalism one naturally normalizes to σ_{tot} in order to divide out the factorizable Pomeranchukon couplings. If, however, our speculations at the end of the last section are correct, diffractive production σ_D may be associated with branch-point contributions. To obtain a factorizable amplitude, which is necessary to derive the SRC picture, one should normalize to the Pomeranchukon *pole* contribution, which is perhaps just σ_M. We realize that this is not a tight argument, but we believe it merits further study, both experimental and theoretical.

III. MUELLER-REGGE MODEL FOR THE CORRELATION FUNCTIONS

We have remarked earlier that asymptotic short-range-correlation functions C_n are obtainable in a Mueller-Regge model when the leading singularity is factorizable. It is instructive to consider a simple model, along these lines, for the f_n's. We consider a world with two trajectories,[20] a leading trajectory with intercept $\alpha_P(0) \approx 1$ and a secondary trajectory with $\alpha_M(0) \approx \frac{1}{2}$. Assuming factorizable singularities, we find that the Mueller diagrams that contribute to the asymptotic limit of f_n are those shown in Fig. 5. If we now assume superduality and use the asymptotic form over the entire range of integration,[21] we find the results $f_n^M = a_n Y + b_n$, where $a_1 = g_0$,

$$a_{n\geq 2} = n!\,L^{n-1}(g_1)^2(g_2)^{n-2}\,, \tag{20}$$

where $g_{PP} = g_0$, $g_{PM} = g_1$, $g_{MM} = g_2$, and $L^{-1} = \alpha_P(0)$

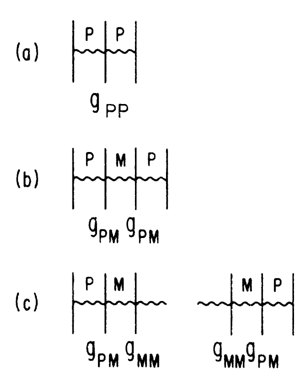

FIG. 5. Mueller diagrams contributing to asymptotic form of (a) single-particle spectrum, (b) two-particle correlation function, and (c) many-particle correlation functions.

$-\alpha_M(0) \approx \frac{1}{2}$. The b_n coefficients get contributions from diagrams besides those of Fig. 5, and hence cannot easily be calculated, even approximately, in closed form. Henceforth we shall make the asymptotic approximation of neglecting the b_n. We should remark that in the above we have not distinguished among particle types, for simplicity, but we shall shortly return to this point.

Using the above results one finds the generating function

$$I_M(z, Y) = \exp\left\{Y\left[g_0(z-1) + \frac{g_1^2(z-1)^2 L}{1 - Lg_2(z-1)}\right]\right\}. \tag{21}$$

The partial cross sections can be determined from the generating function of the model. For this purpose it is convenient to rewrite Eq. (21) as

$$I(z, Y) = \exp\left(-\tau Y + \lambda z + \frac{\gamma \rho z}{1 - \rho z}\right), \tag{22}$$

where

$$\begin{aligned} \tau &= g_0 - \frac{g_1^2 L}{1 + g_2 L}, \\ \lambda &= \left(g_0 - \frac{g_1^2}{g_2}\right) Y, \\ \gamma &= \frac{g_1^2 Y}{L g_2^2 (1 + L g_2)}, \\ \rho &= \frac{g_2 L}{1 + g_2 L}. \end{aligned} \tag{23}$$

Expanding the generating function in this form one finds for the partial cross sections

$$\frac{\sigma_n^M(Y)}{\sigma_M} = e^{-\tau Y} \sum_{j=0}^{n} \frac{\lambda^j}{j!} \rho^{n-j} [L_{n-j}(-\gamma) - L_{n-j-1}(-\gamma)], \tag{24}$$

where L_j are Laguerre polynomials. This is a three-parameter formula for the partial cross sections for all n and for sufficiently large Y. If the Regge-Regge coupling constant g_2 vanishes, then the resulting generating function is just the one considered by Mueller,[4] and the partial cross sections are expressible in terms of Hermite polynomials of imaginary argument.

Before we discuss the phenomenological content of this simple model we would like to comment on the quantum-number generalization of our results. Up to now we have considered partial cross sections for producing n particles of a given type, plus anything else. The generalization to several particle types is straightforward, and we shall just write down the result for three types as an example. Let $p_{\alpha\beta\gamma}$ be the probability of production of α particles of the first type, β of the second, and γ of the third. Then define the generating function

$$I(x, y, z, Y) = \sum_{\alpha\beta\gamma=0}^{\infty} p_{\alpha\beta\gamma}(Y) x^\alpha y^\beta z^\gamma. \tag{25}$$

One finds that

$$I(x, y, z, Y) = \exp\left[\sum_{\alpha\beta\gamma=0}^{\infty} \frac{f_{\alpha\beta\gamma}}{\alpha!\beta!\gamma!}(x-1)^\alpha (y-1)^\beta (z-1)^\gamma\right], \tag{26}$$

where

$$\begin{aligned} &f_{000} = 0, \quad f_{100} = \langle\alpha\rangle, \quad f_{010} = \langle\beta\rangle; \quad f_{001} = \langle\gamma\rangle, \\ &f_{n00} = f_n^\alpha, \quad f_{110} = \langle\alpha\beta\rangle - \langle\alpha\rangle\langle\beta\rangle, \text{ etc.} \end{aligned} \tag{27}$$

It is of more immediate interest to consider in detail a particular type of particle in a given process. An interesting example is provided by π^-'s produced in proton-proton collisions. The asymptotic behavior of the relevant correlation functions, f_2^{-M}, f_3^{-M}, etc., is given by the diagrams of Fig. 6. We recall that, since we are dealing with correlation functions, the leading (Pomeranchuk) singularity is specifically excluded from the internal blob in Fig. 6. This suggests that perhaps one may be able to apply the concept of exchange degeneracy for these diagrams. In particular one can speculate that, since the processes only involve π^-'s and hence are exotic, it may well be that f_2^{-n}, f_3^{-n}, etc. all vanish; that is,

$$f_{n\geq 2}^{-M} \approx 0. \tag{28}$$

Such a behavior would imply that the short-range correlation generating function for π^-'s would be Poissonian.

IV. PHENOMENOLOGICAL IMPLICATIONS

In this section we compare with experiment various of the models discussed. We consider only negative-particle production in pp collisions, since

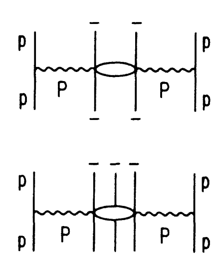

FIG. 6. Mueller diagrams contributing to asymptotic correlation functions among negative particles.

high-energy data exist only for these processes. Charged-particle production is of course trivially related to these processes by charge conservation, and we exhibit the relationship in detail in Sec. V B. Before we can begin we must decide how much of the inelastic events is diffractive, for our SRC models only apply to the nondiffractive part. We shall consider two different alternatives:

(a) All inelastic events arise from a short-range-correlation mechanism.

(b) Some of the low-n σ_n cross sections contain some diffraction.

In case (a), since it is well known that a pure Poisson distribution does not give a good fit to the data, we cannot impose our exchange-degeneracy condition. We shall, however, make use of this condition in case (b), since here it is not ruled out by the data. As a possible model for case (a) we can take the one given by the generating function of Eq. (21). However, we have found that a reasonable fit to the existing data is also obtainable by entirely neglecting the Regge-Regge coupling g_2. For this simplified model, which as we have mentioned is the one discussed by Mueller,[4] one has

$$I^-(z, Y) = \exp[\langle n^-\rangle(z-1) + \tfrac{1}{2}f_2^-(z-1)^2], \tag{29}$$

which yields a simple recursion formula for σ_n^-,

$$\sigma_n^- = \frac{1}{n}[(\langle n\rangle^- - f_2^-)\sigma_{n-1}^- + f_2^-\sigma_{n-2}^-], \tag{30}$$

and

$$\sigma_0^- = e^{-\lambda}\sigma_{\text{incl}}, \quad \lambda = \langle n^-\rangle - \tfrac{1}{2}f_2^-$$
$$\sigma_1^- = (\langle n^-\rangle - f_2^-)\sigma_0^- . \tag{31}$$

We have determined the parameters $\langle n^-\rangle$ and f_2^- from the fits displayed in Figs. 2 and 3. The resultant partial cross section distributions are shown for various energies in Figs. 1, 7, and 8. The total χ^2 for the fits at 50, 70, 100, 200, and 300 GeV is $\chi^2 = 66$ for 50 degrees of freedom. The worst fits are at 70 and 300 GeV. The 300-GeV fit, shown in Fig. 8, has $\chi^2 = 20$ for 13 degrees of freedom.

For case (b) the model is even simpler. Since existing data do not permit an unambiguous separation of diffractive from SRC production, we adopt the further assumption that the SRC production obeys exchange degeneracy (see Sec. III); that is, all correlations among negative particles vanish, $f_n^- = 0$ for $n > 1$. This implies that the SRC part follows a Poisson distribution, and we have

$$\sigma_n^- = \sigma_n^D + \sigma_M \frac{(\langle n^-\rangle_M)^n}{n!}\exp(-\langle n^-\rangle_M),$$

where (32)

$$\langle n^-\rangle_M = c_M Y + d_M .$$

The parameters of the Poisson distribution are determined by fitting to the higher-multiplicity events at each energy. We have done this by fitting $\ln(n!\sigma_n)$ to a straight line for $n_- > 2$. The diffractive contributions σ_0^D, σ_1^D, and σ_2^D are then determined by subtraction. The fit at 200 GeV is shown in Fig. 9. A good fit is also obtained at 100 GeV, and a fair fit at 300 GeV. The model breaks down at lower energies; at 50 GeV one finds $\sigma_D < 0$. We have therefore attempted such fits only at $E_L \geq 100$ GeV. The parameters we obtain by averaging data

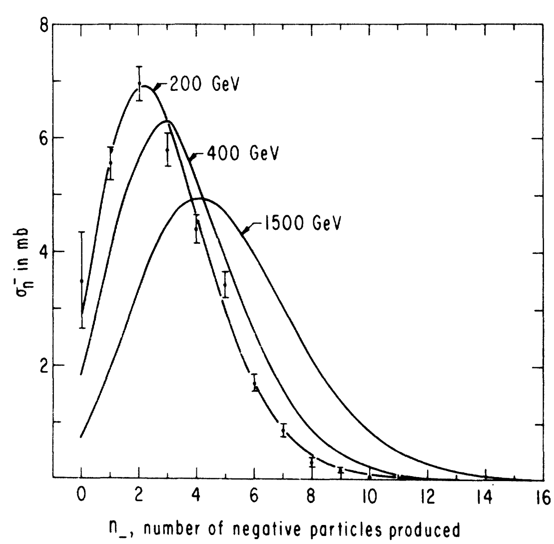

FIG. 7. Fit of short-range correlation model, model (a) of Sec. IV, to 200-GeV data, showing extrapolation of model to 400 and 1500 GeV.

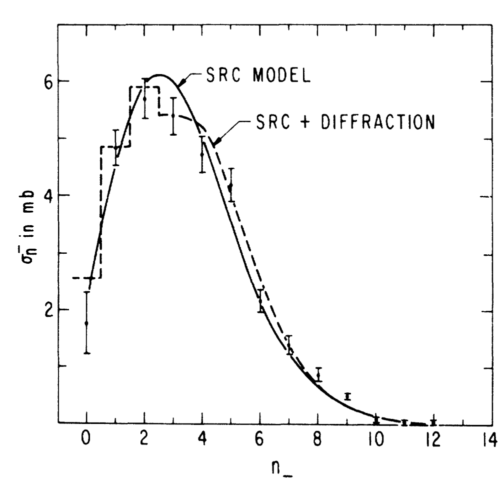

FIG. 8. Fits of models described in Sec. IV to 300-GeV data.

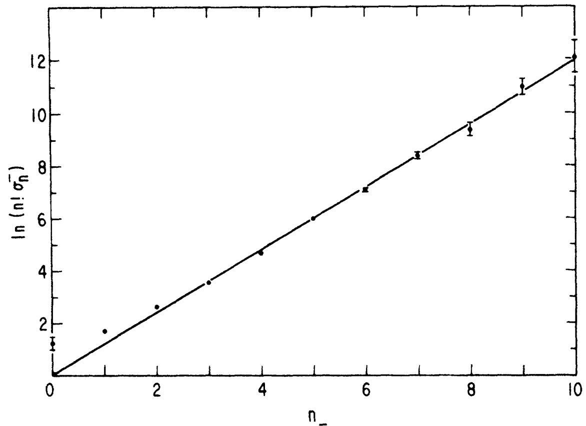

FIG. 9. Method of separation of multiperipheral and diffractive production, with assumption of exchange degeneracy (Poisson distribution) at 200 GeV. Straight-line fit for $n_- \geq 3$ is good ($\chi^2 = 5.3$ for 6 d.f.). Deviation at low multiplicity is ascribed to diffractive production.

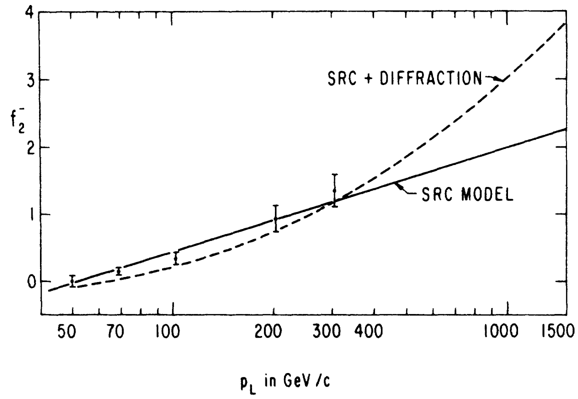

FIG. 11. Comparison of extrapolation of f_2^- in SRC and SRC+diffractive production models. The SRC+diffraction curve was not determined by fitting these data directly (see Sec. IV); better fits could be obtained by tuning the parameters a bit.

at 100, 200, and 300 GeV are $\sigma_0^D = 2.0$ mb, $\sigma_1^D = 2.7$ mb, $\sigma_2^D = 1.7$ mb, $\langle n^-\rangle_M = -4.56 + 1.33 \ln s$, $\sigma_M = 26.8$ mb, $\sigma_D = 6.4$ mb, and $\lambda = 0.81$.[22] Again the fits at 100 and 200 GeV are good; the fit at 300 GeV is still poor ($\chi^2 = 18$ for 13 points), although slightly better than in the pure SRC model (see Fig. 8).

Although both models give fairly good representations of present data, their extrapolations to higher energies are significantly different. The dotted curves in Fig. 1 show the characteristic diffractive behavior of σ_0 and σ_1: leveling off to constant asymptotic values. Figure 10 compares the extrapolated multiplicity distributions at the highest ISR energy. The two-peak structure conjectured by Wilson[13] begins to appear. The correlation parameters f_n^-, if normalized to σ_{inel}, behave like Y^n in the presence of diffraction, as

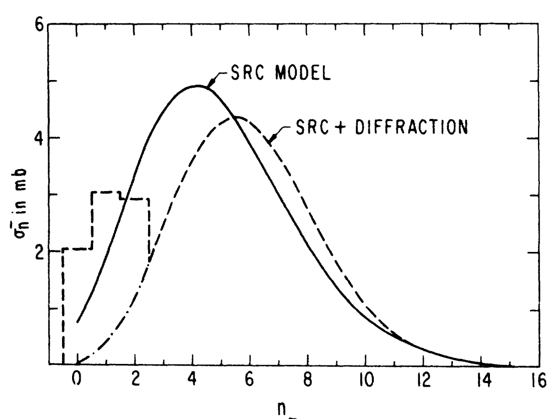

FIG. 10. Comparison of pure SRC model and model with SRC plus diffraction, when extrapolated to 1500 GeV.

discussed in Sec. II C. The behavior of f_2^- in our two models is compared in Fig. 11. It seems that accurate experiments at ISR energies will tell us whether there is a significant diffractive component in σ_{inel}.

V. REMARKS

A. Charge-Conservation Constraints

Throughout this paper we have consistently considered multiplicity distributions and related parameters as a function of n^-, the number of negative particles produced. We find this variable more convenient for the following two reasons. (1) Distributions as a function of this variable are free from constraints resulting from charge conservation. A trivial example is the fact that distributions in the other popular variable, n_{ch}, must vanish when n_{ch} is odd. Hence, a multiplicity distribution in n_{ch} can never be a Poisson distribution. We shall demonstrate a less obvious manifestation of such correlations in the f_n's below. (2) The hypothesis of exchange degeneracy takes a simple form in terms of n^-.

Since the multiplicity distribution in terms of n^- contains the same information as the distribution as a function of n_{ch}, it is obvious that one can develop rules for translating from one language to the other. Such work has been carried out by several authors.[23] We review here the formalism developed by Webber, in order to derive a formula which translates the correlation parameters f_n from one language to another.

Webber finds the following relation for generating function in terms of n_{ch}:

$$I_{\rm ch}(h)=\exp\left(\sum_{n=1}^{\infty} f_n^{\rm ch} h^n/n!\right)$$
$$=(1+h)^Q \exp\left[\sum_{n=1}^{\infty} f_n^-(2h+h^2)^n/n!\right], \qquad (33)$$

where Q is the charge of the initial state; $Q=2$ in pp collisions. After a little algebra one can deduce from this the following explicit relation:

$$\frac{f_n^{\rm ch}}{n!}=(-1)^{n+1}\frac{Q}{n}+\sum_{k=[n/2]}^{n} f_k^- \frac{2^{2k-n}}{(n-k)!(2k-n)!}, \qquad (34)$$

where $[n/2]$ denotes the smallest integer greater than or equal to $n/2$. Explicit formulas for the first few f's are

$$\langle n^{\rm ch}\rangle = Q+2\langle n^-\rangle, \qquad (35a)$$
$$f_2^{\rm ch}=-Q+2f_1^-+4f_2^-, \qquad (35b)$$
$$f_3^{\rm ch}=2Q+12f_2^-+8f_3^-, \qquad (35c)$$
$$f_4^{\rm ch}=6Q+12f_2^-+48f_3^-+16f_4^-, \qquad (35d)$$
$$f_5^{\rm ch}=24Q+120f_3^-+160f_4^-+32f_5^-. \qquad (35e)$$

Note that even in the absence of any correlations among the negative particles, the $f_n^{\rm ch}$ are nonzero as a result of charge conservation alone.

B. Compound-Poisson-Distribution Interpretation

The short-range-correlation generating function defined in Eq. (10) is amenable to an appealingly simple interpretation: It can be viewed as the generating function of a compound Poisson distribution.

For notational simplicity let us consider the case where the type i means *any* particle and let us drop both the superscripts i and M. Now if one supposes that particle production occurs in clusters of particles with each cluster produced independently, then the probability $p_n(Y)$ of producing n particles is given by

$$p_n(Y)=\sum_{j=0}^{n} p_j^c(Y)\lambda_{j,n}, \qquad (36)$$

where $p_j^c(Y)$ is the probability of producing j clusters, and where $\lambda_{j,n}$ is the probability that these j clusters decay into a total of n particles. Since by assumption the clusters are produced independently, the probability $p_j^c(Y)$ follows a Poisson distribution

$$p_j^c(Y)=e^{-n_c(Y)}\frac{[n_c(Y)]^j}{j!}, \qquad (37)$$

where $n_c(Y)$ is the average number of clusters produced.

Let w_k be the probability that a given cluster decays into exactly k particles. Then construct the generating function

$$g(z)=\sum_{k=0}^{\infty} w_k z^k. \qquad (38)$$

It is not hard to show that the probabilities $\lambda_{j,n}$ are generated by[24]

$$[g(z)]^j=\sum_{n=0}^{\infty}\lambda_{j,n} z^n. \qquad (39)$$

Thus one can write the generating function $I(z,Y)$ in the form

$$I(z,Y)=\sum_{n=0}^{\infty} p_n z^n$$
$$=\sum_{n=0}^{\infty} z^n \sum_{j=0}^{n}\lambda_{j,n} p_j^c(Y)$$
$$=\sum_{j=0}^{\infty} p_j^c(Y)[g(z)]^j. \qquad (40)$$

On using (37), this yields

$$I(z,Y)=\exp\{n^c(Y)[g(z)-1]\}, \qquad (41)$$

which is the generating function of a compound Poisson distribution.[24] It is clear that the short-range-correlation generating function has this form provided that we neglect the nonasymptotic terms involving the b_n's. Thus we have the identification

$$g(z)=1+\frac{Y}{n_c}\sum_{k=1}^{\infty}\frac{a_k}{k!}(z-1)^k, \qquad (42)$$

where a_k is defined in Eq. (5), $f_n=a_nY+b_n$. The average number of clusters $n_c(Y)$ can be determined by requiring that $g(0)=w_0=0$; that is, that the probability of a cluster decaying into zero particles vanishes. This yields the result

$$n_c=-Y\sum_{k=1}^{\infty}\frac{a_k}{k!}(-1)^k. \qquad (43)$$

Note that $n_c(Y)\propto Y$, and therefore $g(z)$ is independent of Y. The probabilities w_k for a cluster to decay into k particles are therefore energy-independent. These features resemble those found in multiperipheral models.

The asymptotic correlation coefficients a_n have a particularly simple interpretation in the picture we have described. Differentiation of Eq. (42) with evaluation of the derivatives at $z=1$ yields

$$g^{(r)}(1)=\frac{a_rY}{n_c}. \qquad (44)$$

Comparison with Eq. (38) shows that these derivatives yield moments of the cluster decay distribution w_k,

$$a_rY=n_c\langle k(k-1)\cdots(k-r+1)\rangle. \qquad (45)$$

For $r=1$, this reads

$$n_c\langle k\rangle = a_1 Y \sim \langle n\rangle\,; \tag{46}$$

that is, the asymptotic average multiplicity is the product of the average number of clusters produced times the average number $\langle k\rangle$ of decay particles per cluster. Then, for $r=2$,

$$\frac{f_2}{f_1}\sim\frac{a_2}{a_1}=\frac{\langle k(k-1)\rangle}{\langle k\rangle}\,. \tag{47}$$

The quantity $\langle k(k-1)\rangle$ is just twice the average number of *pairs* of particles in a cluster decay. In general, f_r is proportional to $r\,!$ times the average number of r-tuples produced in a cluster decay.

For this interpretation to be consistent it is necessary that the coefficients w_k calculated by expanding Eq. (23) be positive.[6] Since, however, this is a sufficient but not necessary condition for the σ_n to be positive, the positivity of the w_k is not guaranteed in general and must be examined experimentally.

The compound Poisson interpretation we have formulated is useful for developing some intuition about the magnitude of the parameters f_n. To interpret Eq. (47) most readily, consider a simple model in which clusters decay into a definite number of particles k. Then Eq. (47) reduces to $f_2/f_1 \sim k-1$. Hence one expects $f_2\sim f_1$ if clusters decay into pairs of particles. The value one finds from the parametrizations in Figs. 2 and 3 is $f_2^-/f_1^- \sim 0.6$, corresponding to 1.6 negative particles per cluster.

C. Model-Independent Separation of Multiperipheral and Diffractive Production

In Sec. IV, model (b), we were able to separate SRC (multiperipheral) and diffractive contributions to σ_n only by making the assumption of exchange degeneracy in the SRC portion. In this section we list model-independent separation methods; all of them have been discussed previously, either in this paper or elsewhere. (See especially Ref. 13.) Even though none of them provide clear results with present data, they should be useful as the data are refined, especially at ISR energies.

The definitions of SRC and diffractive production have been given in Secs. II B and II C. We remind the reader, however, that we consider only *low-multiplicity* diffractive processes (see Sec. II C) in which the growth of average multiplicity arises only from SRC production. With this assumption, the following separation methods follow without further assumptions:

(1) Measure the correlation parameters $f_n(Y)$, excluding as usual elastic events, as a function of energy. If they are well fitted by $a_n Y + b_n$, there is no evidence for any diffractive part. This is the case with present data, but we need greater accuracy and a greater energy range. If not, try to fit with a polynomial of nth order in Y, and identify the coefficient of Y^n. These coefficients determine σ_M and σ_D according to Eq. (19).[25]

(2) Measure the energy dependence of the partial cross sections σ_n. If an asymptotically constant part is seen, this is σ_n^D. Figure 1 shows that there is no constant part evident at present energies, but indicates on the basis of model (b) in Sec. III that we might see such flattening at ISR energies if there is indeed a significant diffractive part of σ_{inel}.

(3) Look at the multiplicity distribution at the highest possible energy, in hope of seeing a clear separation between low-multiplicity diffractive processes and higher-multiplicity SRC processes, as indicated in Fig. 10 from our model.

(4) Measure the correlation function

$$C_2(y_1,y_2)=\frac{d^2\sigma}{dy_1dy_2}-\frac{1}{\sigma}\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2} \tag{48}$$

in the central region of the rapidity plot, where presumably SRC events dominate. Vary σ until a value is found for which $C_2(y_1,y_2)$ vanishes exponentially for $|y_1-y_2|$ large, $C_2\sim\exp(-|y_1-y_2|/L)$. This value is $\sigma=\sigma_M$. The preliminary Pisa-Stony Brook data appear to have this property for $\sigma=\sigma_{\text{inel}}$, which would indicate that there is no significant diffractive component in σ_{inel}.

VI. CONCLUSION

We have discussed the phenomenology of multiplicity distributions in high-energy hadronic collisions in terms of the short-range-correlation picture, modified to include diffractive processes leading to low-multiplicity final states. We have formulated two models: one in which diffractive processes are present in elastic scattering only, and one in which diffractive processes contribute also to low-multiplicity inelastic reactions. Both give reasonable agreement with the present data on partial cross sections. As more accurate data become available and are extended to higher energies, these models will be tested more severely and it will become possible to discriminate between them. Even more interesting is the prospect of separating diffractive and short-range-correlation processes without resorting to specific models.

Finally, we hope that our work, if it has accomplished nothing else, has emphasized that a Poisson distribution is by no means the unique "prediction" of the multiperipheral or short-range-correlation picture. The models we have constructed, although still oversimplified models and *not* definitive predictions of the multiperipheral picture, should be much more realistic and useful for phenomenology.

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[2]See, for example, G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1961); C. DeTar, Phys. Rev. D 3, 128 (1971).

[3]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[4]A. H. Mueller, Phys. Rev. D 4, 150 (1971).

[5]See, for example, R. D. Peccei, in *Proceedings of the Seventh Rencontre de Moriond*, edited by J. Tran Thanh Van (CNRS, Paris, 1972). This bound follows from the application of Regge asymptotics to the Mueller discontinuity formula. One need assume only that no J-plane singularities lie higher than $J=1$ (one need not, of course, assume factorization, which leads to the stronger bound in Eq. 5). One then obtains the scaling property of inclusive distributions, from which Eq. (4) follows, as a "naive" Regge bound. It would be very useful to obtain rigorous bounds, extensions of the Froissart bound. Work in this direction has been done by A. Patrascioiu [MIT Report No. CTP-305 (unpublished)], but his results are not immediately applicable to our problem. Naive bounds may miss a few powers of $\ln s$; e.g., total cross section can increase as fast as $\ln^2 s$, whereas the naive Regge bound indicates constant behavior. Such uncertainties do not, however, affect the utility of Eq. (4) in ruling out power-law behavior found in some models.

[6]For these statements to be rigorous it is necessary that the Mueller expansion of the generating function in Eq. (2) converge. We so assume, and the model discussed in Sec. III is reassuring in this regard.

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[18]The fact that the bound in Eq. (4) is saturated when diffractive processes are present has been shown by J. Ellis, J. Finkelstein, and R. D. Peccei, Nuovo Cimento 12A, 763 (1972).

[19]G. Bellettini, in Proceedings of the Sixteenth International Conference on High-Energy Physics, Batavia, Ill., 1972 (unpublished).

[20]It is not difficult to generalize these considerations to an arbitrary number of trajectories, at the cost of introducing a large number of new parameters.

[21]This is a drastic assumption – that the correlation functions have the same form at small rapidity separations as theory tells us they should have at large separations. We are encouraged, however, by the preliminary Pisa-Stony Brook data in Fig. 4, which show just this behavior. One might also question whether such an assumption is consistent with energy-conservation sum rules. In fact, such sum rules do not constrain the a_n but only the b_n, because it is the region at the ends of the rapidity plot which carries most of the energy, whereas the $\ln s$ behavior comes from the central region.

[22]If our discussion at the end of Sec. II C is correct, the ratio $\sigma_D/\sigma_{\text{tot}}$ is a measure of the fraction of the Pomeranchuk singularity which is nonfactorizable. From our phenomenological analysis one finds a value ≈ 0.3. It is amusing to note that this is consistent with recent estimates of the ratio of Pomeranchuk cut to pole strength [see F. E. Low's talk, in Proceedings of the Sixteenth International Conference on High-Energy Physics, Batavia, Ill., 1972 (unpublished)].

[23]L. S. Brown, Phys. Rev. D 5, 748 (1972); K. Biebl and J. Wolf, Nucl. Phys. B44, 301 (1972); B. R. Webber, *ibid.* B43, 541 (1972).

[24]See, for example, W. Feller, *Introduction to Probability Theory and Its Applications* (Wiley, New York, 1968), Vol. I.

[25]Essentially the same proposal has been made by K. Fiałkowski [Phys. Letters 41B, 379 (1972)].