*Work done in part under the auspices of the U. S. Atomic Energy Commission.

[1]G. A. Rinker, Jr. and L. Wilets, preceding paper, Phys. Rev. D 7, 2629 (1973), and private communications.

[2]B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D 2, 179 (1970); 4, 272(E) (1971); and earlier references therein.

[3]An unsubtracted dispersion relation does not converge without a form factor for (Re*x*). However, there is no *a priori* reason for such a large enhancement factor, nor for the necessary repulsive sign of the potential.

[4]I thank Dr. Rinker for this communication. See Table I in Ref. 1.

[5]S. Barshay, Phys. Letters 37B, 397 (1971). The numerical values of the quantities $y_{3\pi}$, g, and f in Eqs. (10), (13), and (14)–(15), respectively, should be multiplied by four.

[6]Private communication from Dr. Zavattini.

[7]I thank Dr. Rinker for his help here.

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Chiral SU(2)×SU(2) Symmetry in the πK Scattering*†

J. S. Kang

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

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The chiral SU(2)×SU(2) symmetry is studied for the πK scattering in the scheme of the linear realization. Roskies-type relations are used for the crossing symmetry, and the relativistic version of the effective-range approximation is imposed on for unitarity. Two alternatives of the up-down ambiguity for the $I=\frac{1}{2}$, 0^+ resonance are investigated, and our analysis favors the up-solution with $\kappa(870)$. The σ term in the πK scattering is estimated to be about $-1.1\ m_\pi^2$. This is another measure of the chiral SU(2)×SU(2) breaking and is consistent with other features of the symmetry breaking. Finally the threshold effect on the K_{l3} form factor is studied and compared with experiment.

I. INTRODUCTION

Recently improvement has been made in the boson spectroscopy of the reactions $K^-p\to\pi^+K^-n$ (Ref.1) and $K^+p\to\pi K\Delta^{++}$ (Refs. 2 and 3) dominated by the one-pion-exchange mechanism. This enables one to understand the πK elastic scattering at low energies. Since only s and p waves are considered, the resulting $I=\frac{1}{2}$ s-wave phase shift has the up-down ambiguity and thus two choices about the mass of the κ meson. The up solution gives the κ meson at 870 MeV with a width of 30 MeV while for the down solution we have $\kappa(\sim 1150)$ with a broad width of about 400 MeV.

Various attempts have been made to understand these phenomena from different points of view. Pagnamenta and Renner[4] construct a current-algebra model for the $\pi K\kappa$ vertices with the πK intermediate states. The Veneziano model is also used[5] in the study of the πK scattering with an application to the K_{l3} form factors. Carrotte[6] unitarizes a simple parametrization determined by the current-algebra constraints by making use of the relativistic version of the effective-range approximation. Pond[7] applied the hard-pion technique to the πK problems.

The purpose of this paper is to construct the πK scattering amplitudes at low energies which are consistent with unitarity and crossing. We assume that σ, $\vec{\pi}$, K, and κ mesons belong to the simplest representation of the chiral SU(2)×SU(2) symmetry and formulate the most general phenomenological action containing at most two derivatives. The difference between the nonlinear phenomenological Lagrangian method discussed previously[8] and ours is that in the former, one assumes a certain term characterizing the SU(2)×SU(2) breaking (the so-called σ term) to be negligible. This assumption has been challenged in the case of πN scattering, and in our work we shall not make this assumption. This necessitates our treatment of the σ and κ fields as independent degrees of freedom.

Unitarity is imposed on by the method of the relativistic effective-range approximation, which has been successful in the $\pi\pi$ problem,[9] and crossing symmetry is restored by demanding three Roskies-type relations[10] for the s and p waves. Our final solution favors the up solution for the $I=\frac{1}{2}$ s wave. The magnitude of the σ term in the πK scattering is evaluated to be about $-1.12m_\pi^2$ and the chiral SU(2)×SU(2) breaking is measured to be small (~10%), which is consistent with other features of the chiral symmetry breaking. We also applied our phase shifts to the K_{l3} decays. The threshold effect on the scalar form factor $f(t)$ is studied and compared with experiments.

The paper is organized as follows: In Sec. II we review the phenomenological action and formulate it for the πK system in the framework of the chiral SU(2)×SU(2). Here the Feynman rules for the irreducible vertices are derived. In Sec. III the scattering amplitudes are calculated as well as the partial waves. Unitarity and crossing relations are discussed in Sec. IV and the numerical calculation is presented in Sec. V to fix our parameters. We also study the K_{l3} decays in Sec. VI and comparison with the experimental data is made. Finally we conclude in Sec. VII that our construction is a good approximation up to the inelastic threshold.

II. CHIRAL SU(2)×SU(2) SYMMETRY FOR THE πK SYSTEM

A. Review of the Phenomenological Action

Let us briefly review what is meant by the effective action or the phenomenological Lagrangian.[11]

Let $\phi(x) \equiv \{\phi_i(x),\ i=1,2,\ldots\}$ be the renormalized fields corresponding to the particles of a system. When we introduce the c-number classical currents $\eta(x)=\{\eta_i(x),\ i=1,2,\ldots\}$ for these fields we have the Schwinger functional[12]:

$$S[\eta]=-i\ln\left\langle 0\middle| T\left(\exp i\int d^4x \textstyle\sum_i \eta_i(x)\phi_i(x)\right)\middle|0\right\rangle , \tag{2.1}$$

which generates the connected Green functions in the presence of the additional interaction $\mathcal{L}(x) \to \mathcal{L}(x)+\sum_i \gamma_i\phi_i(x)$:

$$\left.\frac{\delta^n S[\eta]}{\delta\eta_{i_1}(x_1)\cdots\delta\eta_{i_n}(x_n)}\right|_{\eta_i(x)=\gamma_i} = i^{n-1}\langle 0|T(\phi_{i_1}(x_1)\cdots\phi_{i_n}(x_n))|0\rangle^c . \tag{2.2}$$

Defining the new c fields $\Phi_i(x)$ by

$$\Phi_i(x)=\frac{\delta S[\eta]}{\delta\eta_i(x)} ,\qquad i=1,2,\ldots \tag{2.3}$$

which may be called the phenomenological fields, and changing variables from $\eta(x)$ to $\Phi(x)$ by the Legendre transformation we obtain a new functional, the Jona-Lasinio functional[13]:

$$A[\Phi]=S[\eta]-\int d^4x \textstyle\sum_i \eta_i(x)\Phi_i(x) , \tag{2.4}$$

with

$$\frac{\delta S[\eta]}{\delta\eta_i(x)}=\Phi_i(x) ,$$

$$\frac{\delta A[\Phi]}{\delta\Phi_i(x)}=-\eta_i(x),\quad i=1,2,\cdots .$$

The new functional $A[\Phi]$ is the generating functional for the one-particle-irreducible vertices (the amputated Green functions which remain connected when any one internal line is removed):

$$\left.\frac{\delta A[\Phi]}{\delta\Phi_i(x)}\right|_{\Phi_i(x)=v_i}=-\gamma_i , \tag{2.5}$$

$$\left.\frac{\delta^2 A[\Phi]}{\delta\Phi_i(x)\delta\Phi_j(y)}\right|_{\Phi_i(x)=v_i;\Phi_j(y)=v_j}=[\Delta^F_{ij}(x,y)]^{-1} , \tag{2.6}$$

$$\left.\frac{\delta^n A[\Phi]}{\delta\Phi_{i_1}(x)\cdots\delta\Phi_{i_n}(x_n)}\right|_{\Phi_i(x)=v_i} = \Gamma^{(n)}_{i_1,\ldots,i_n}(x_1,\ldots,x_n)\quad\text{for}\quad n\geq 3 , \tag{2.7}$$

where

$$v_i=\langle 0|\phi_i(x)|0\rangle,$$
$$i\Delta^F_{ij}(x,y)=\langle 0|T(\phi_i(x)\phi_j(y))|0\rangle .$$

Therefore we get all irreducible vertices by expanding $A[\Phi]$ around the vacuum expectation values of the renormalized fields, $\Phi_i(x)=v_i$ and the v_i's are determined by Eq. (2.5).

We define yet another functional $\Lambda[\Phi;\gamma]$:

$$\Lambda[\Phi;\gamma]\equiv\int d^4x\,\Lambda(x) = A[\Phi]+\textstyle\sum_i\gamma_i\int d^4x\,\Phi_i(x) . \tag{2.8}$$

Then Eqs. (2.5)–(2.7) become

$$\left.\frac{\delta\Lambda[\Phi;\gamma]}{\delta\Phi_i(x)}\right|_{\Phi_i(x)=v_i}=0 , \tag{2.9}$$

$$\left.\frac{\delta^2\Lambda[\Phi;\gamma]}{\delta\Phi_i(x)\delta\Phi_j(y)}\right|_{\Phi_i(x)=v_i;\Phi_j(x)=v_j}=[\Delta^F_{ij}(x,y)]^{-1} , \tag{2.10}$$

$$\left.\frac{\delta^n\Lambda[\Phi;\gamma]}{\delta\Phi_{i_1}(x_1)\cdots\delta\Phi_{i_n}(x_n)}\right|_{\Phi_i(x)=v_i} = \Gamma^{(n)}_{i_1,\ldots,i_n}(x_1,\ldots,x_n)\quad\text{for}\quad n\geq 3 . \tag{2.11}$$

$\Lambda(x)$ is called the phenomenological action when it is expanded in power series of momenta in the momentum space. This is equivalent to expanding $\Lambda(x)$ in terms of derivatives of fields in the coordinate space.

The S-matrix elements have the tree structures when they are expressed in terms of the irreducible vertices. Therefore we have only to calculate the one-particle-irreducible vertices and inverse propagators from $\Lambda(x)$ by Eq. (2.10) and Eq. (2.11), respectively. Then the scattering amplitudes can be obtained from the product of the irreducible vertices and the propagators.

If $\Lambda(x)$ is constructed to contain up to two derivatives, so are the irreducible vertices and the inverse propagators. As long as the internal particles for the tree structure are massive, the propagators are also correct in the second order of momenta. Therefore it is sufficient to construct $\Lambda(x)$ up to two derivatives in order to construct the S-matrix elements to the second order in momenta involved.

B. Construction of the Chiral SU(2)×SU(2) Phenomenological Action for the πK System

Now we construct the most general form of $\Lambda(x)$ containing up to two derivatives. Since we are interested in πK scattering, terms contributing only to the $K\bar{K}\to K\bar{K}$ reaction will not be considered.

The σ and $\vec{\pi}$, and K and κ mesons are regarded as the chiral SU(2)×SU(2) multiplets and are assumed to belong to some irreducible representations. The σ field is responsible for breaking the chiral SU(2)×SU(2) symmetry in the Lagrangian. The validity of PCAC (partially conserved axial-vector current) demands that $\vec{\pi}=\partial_\mu \vec{A}^\mu/f_\pi m_\pi^2$ and σ belong to the same multiplet. Since the σ field must be isoscalar, it follows that $\sigma, \vec{\pi}$ must belong to the $[N/2, N/2]$ representation. Furthermore the fact that K has $I=\frac{1}{2}$ requires K and κ belong to an $[L/2, M/2]$ representation with $|L-M|=1$. We shall assume that $\sigma, \vec{\pi}$, K, and κ belong to the simplest representations compatible with the above observations, i.e., σ and $\vec{\pi}$ belong to the $[\frac{1}{2},\frac{1}{2}]$ representation and K, κ to $[0,\frac{1}{2}]+[\frac{1}{2},0]$. Chiral invariants are formed from the representation tensors[8]

$$\begin{aligned} M^{a}{}_{\dot{b}} &= (\sigma + i\vec{\pi}\cdot\vec{\tau})_{ab} , \\ \chi^a &= (\kappa+K)_a , \\ \chi^{\dot{a}} &= (\kappa-K)_a , \end{aligned} \qquad (2.12)$$

where $a, \dot{a}$ are the tensor indices of $SU(2)_+$ and $SU(2)_-$ of the chiral $SU(2)_+\times SU(2)_-$, respectively.

We assume that the chiral SU(2)×SU(2) is a better symmetry than SU(3). The SU(2)×SU(2) is spontaneously broken and in its symmetric limit the pions alone (and not kaons) become Goldstone bosons.[14] Thus we expand $\Lambda[\Phi]$ in the momentum space about $p_\pi^2=0$, $p_K^2=p_\kappa^2=m_K^2$, because to preserve the SU(2)×SU(2) invariant structure of Λ it is necessary to maintain the symmetry between p_K and p_κ. The mass of the κ meson is identified approximately with the zero of the corresponding inverse propagators, i.e., $\Delta_\kappa^F(m_\kappa^2)^{-1}=0$.

It is now a straightforward matter to formulate $\Lambda(x)$ for the πK system containing up to two derivatives:

$$\begin{aligned} \Lambda(x) = {} & f(\sigma^2+\vec{\pi}^2) + [(\partial\sigma)^2+(\partial\vec{\pi})^2]\, g(\sigma^2+\vec{\pi}^2) + \gamma\sigma \\ & + (\kappa^\dagger\kappa + K^\dagger K)\{A_1(\sigma^2+\vec{\pi}^2) + [(\partial\sigma)^2+(\partial\vec{\pi})^2]A_2(\sigma^2+\vec{\pi}^2) + (\sigma\partial\sigma+\vec{\pi}\cdot\partial\vec{\pi})^2 A_3(\sigma^2+\vec{\pi}^2)\} + (\partial\kappa^\dagger\partial\kappa + \partial K^\dagger\partial K)B(\sigma^2+\vec{\pi}^2) \\ & + (\kappa^\dagger\sigma\kappa - i\kappa^\dagger\vec{\pi}\cdot\vec{\tau}K + iK^\dagger\vec{\pi}\cdot\vec{\tau}\kappa - K^\dagger\sigma K)\{C_1(\sigma^2+\vec{\pi}^2) + [(\partial\sigma)^2+(\partial\vec{\pi})^2]\,C_2(\sigma^2+\vec{\pi}^2) + (\sigma\partial\sigma+\vec{\pi}\cdot\partial\pi)^2 C_3(\sigma^2+\vec{\pi}^2)\} \\ & + (\partial\kappa^\dagger\sigma\partial\kappa - i\partial\kappa^\dagger\vec{\pi}\cdot\vec{\tau}\partial K + i\partial K^\dagger\vec{\pi}\cdot\vec{\tau}\partial\kappa - \partial K^\dagger\sigma\partial K)D(\sigma^2+\vec{\pi}^2) \\ & + (\partial\kappa^\dagger\partial\sigma\kappa - i\partial\kappa^\dagger\partial\vec{\pi}\cdot\vec{\tau}K + i\partial K^\dagger\partial\vec{\pi}\cdot\vec{\tau}K - \partial K^\dagger\partial\sigma K + \kappa^\dagger\partial\sigma\partial\kappa - i\kappa^\dagger\partial\vec{\pi}\cdot\vec{\tau}\partial K + iK^\dagger\partial\vec{\pi}\cdot\vec{\tau}\partial\kappa - K^\dagger\partial\sigma\partial K)E(\sigma^2+\vec{\pi}^2) \\ & + \{\kappa^\dagger[\sigma\partial\sigma+\vec{\pi}\cdot\partial\vec{\pi}+i(\vec{\pi}\times\partial\vec{\pi})\cdot\vec{\tau}]\partial\kappa + \text{H.c.} + K^\dagger[\sigma\partial\sigma+\vec{\pi}\cdot\partial\vec{\pi}+i(\vec{\pi}\times\partial\vec{\pi})\cdot\vec{\tau}]\partial K + \text{H.c.} \\ & + \kappa^\dagger[-i\sigma\partial\vec{\pi}\cdot\vec{\tau} + i\partial\sigma\vec{\pi}\cdot\vec{\tau}]\partial K + \text{H.c.} + K^\dagger[-i\sigma\partial\vec{\pi}\cdot\vec{\tau} + i\partial\sigma\vec{\pi}\cdot\vec{\tau}]\partial\kappa + \text{H.c.}\}F(\sigma^2+\vec{\pi}^2) . \end{aligned} \qquad (2.13)$$

Here space-time dependence of the phenomenological fields σ, $\vec{\pi}$, K, and κ is understood.

The vacuum expectation values of the fields are determined by minimizing $\Lambda[\Phi]$ with respect to these fields as shown in Eq. (2.9). Thus the derivative-independent parts of $\Lambda[\Phi]$ are chosen such that only the σ field has nonvanishing vacuum expectation value. There are 12 functions of $\sigma^2(x)+\vec{\pi}^2(x)$ in $\Lambda(x)$. They are f, g, A_1, A_2, A_3, B, C_1, C_2, C_3, D, E, and F, and are assumed to be expandable as a power series around the vacuum expectation values of the fields. By means of Eqs. (2.10) and (2.11) we can relate the above functions to the irreducible vertices.

From Eqs. (2.9) and (2.10) and the normalization conditions for the propagators

$$\Delta_i^F(q^2)^{-1} \underset{q^2\to m_i^2}{\sim} q^2-m_i^2, \quad i=\sigma, \vec{\pi}, K, \kappa \qquad (2.14)$$

where m_i is the physical mass of the corresponding particle, we obtain[15]

$$
\begin{aligned}
&\gamma = f_\pi m_\pi^{\,2} ,\\
&g = \tfrac{1}{2} ,\\
&f' = -\tfrac{1}{2} m_\pi^{\,2} ,\\
&f'' = -(m_\sigma^2 - m_\pi^{\,2})/4 f_\pi^{\,2} , \qquad (2.15)\\
&B = 1 ,\\
&D = 0,\\
&A_1 = -\tfrac{1}{2}(m_\kappa^{\,2} + m_K^2) ,\\
&C_1 = -(m_\kappa^{\,2} - m_K^2)/2 f_\pi .
\end{aligned}
$$

Here f_π is the pion-decay constant (about 94 MeV).

The one-particle-irreducible vertices can be calculated from Eq. (2.11) and are given in Fig. 1, where

$$
\begin{aligned}
g_{\sigma K K}(q^2) = {} & C_1 - 2 f_\pi (A_1' - f_\pi C_1') \\
& - 2 f_\pi m_K^2 (B' - f_\pi D') \\
& + q^2[f_\pi (B' - f_\pi D) + E] , \qquad (2.16)
\end{aligned}
$$

$$
g_{\pi K \kappa}(q^2) = \frac{1}{2 f_\pi} (m_\kappa^{\,2} - m_K^2)(1 + 2 f_\pi^{\,2} F) - q^2 E .
$$

From the Feynman rules shown in Fig. 1 we realize that there are five parameters to fully describe the πK elastic scattering, i.e., C_1, $A_1' - f_\pi C_1'$, $B' - f_\pi D'$, E, and F.[15] Furthermore the vertex functions are quadratic in momenta because $\Lambda(x)$ contains up to two derivatives of the fields.

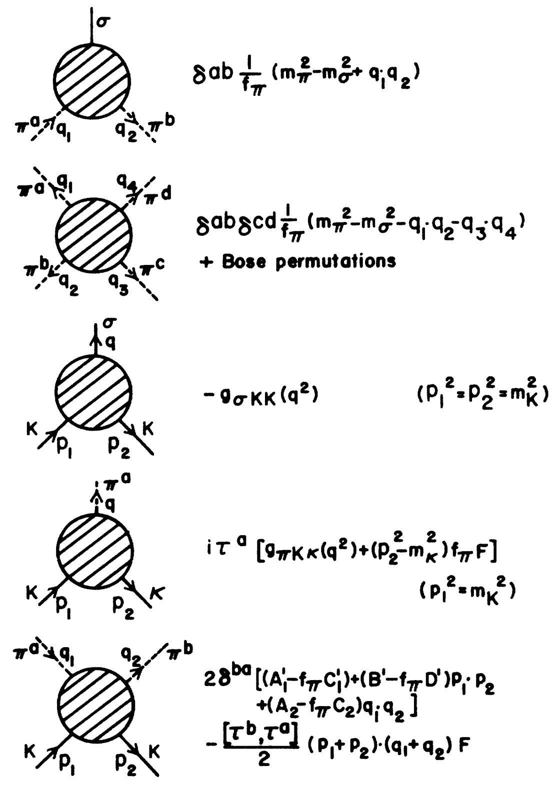

FIG. 1. Feynman rules for the one-particle-irreducible vertices relevant for the πK scattering.

III. SCATTERING AMPLITUDES

Owing to the isospin conservation there are two invariant amplitudes in the πK scattering:

$$
\begin{aligned}
T(s,t,u) = {} & \delta^{ba}\, T^{(+)}(s,t,u) \\
& + \tfrac{1}{2}[\tau^b , \tau^a]\, T^{(-)}(s,t,u), \qquad (3.1)
\end{aligned}
$$

where s, t are the square of the total energy in the c.m. frame and the momentum transfer squared in the $\pi K \to \pi K$ channel, and a, b are the isospin indices of the initial and the final pions, respectively.

As shown in Fig. 2, the scattering amplitudes have the tree structures. With the help of the Feynman rules for the irreducible vertices (Fig. 1), we can construct the off-the-mass-shell amplitudes:

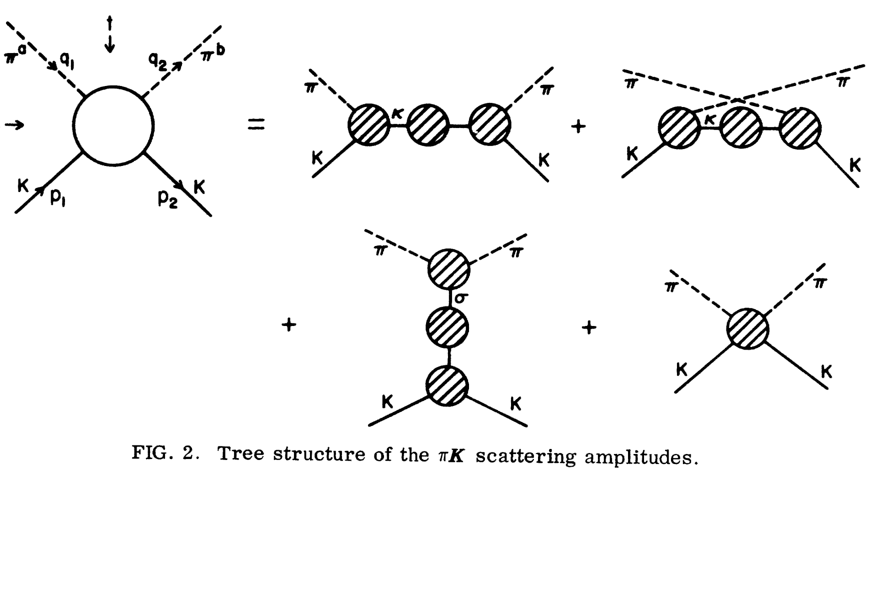

FIG. 2. Tree structure of the πK scattering amplitudes.

$$T^{(+)}(s,t,u)=-g_{\pi K K}(q_2^2)\,g_{\pi K K}(q_1^2)\left(\frac{1}{s-m_\kappa^2}+\frac{1}{u-m_\kappa^2}\right)+\frac{g_{\sigma K K}(t)}{f_\pi}\,\frac{m_\pi^2-m_\sigma^2+q_1\cdot q_2}{t-m_\sigma^2}$$
$$-2f_\pi F[g_{\pi K K}(q_2^2)+g_{\pi K K}(q_1^2)]-f_\pi^2F^2(s+u-2m_\kappa^2)+2(A_1'-f_\pi C_1')$$
$$+2q_1\cdot q_2(A_2-f_\pi C_2)+[2m_\kappa^2-(q_2-q_1)^2](B'-f_\pi D'),$$
$$T^{(-)}(s,t,u)=-g_{\pi K K}(q_2^2)g_{\pi K K}(q_1^2)\left(\frac{1}{s-m_\kappa^2}-\frac{1}{u-m_\kappa^2}\right)-(F+f_\pi^2F^2)(s-u)\,. \tag{3.2}$$

It can be readily checked that Adler's self-consistency condition[16] is satisfied:

$$\lim_{q_1\to 0} T^{(+)}(s,t,u)=0, \tag{3.3}$$

and the two soft-pion theorem is also satisfied:

$$\lim_{q_1,q_2\to 0} T^{(+)}(s,t,u)=-\frac{g_{\sigma K K}(0)}{f_\pi}\,\frac{m_\pi^2}{m_\sigma^2}\equiv\frac{\sigma_{KK}(0)}{f_\pi^2}\,, \tag{3.4}$$

where $\sigma_{KK}(q^2)$ is called the σ term and defined as

$$\delta_{ab}\sigma_{KK}(q^2)=i\langle K(p_2)\,|\,[Q_a^5,[Q_b^5,\Lambda(0)]]\,|\,K(p_1)\rangle$$
$$=-\frac{m_\pi^2}{m_\sigma^2}f_\pi\,g_{\sigma K K}(q^2)\,, \tag{3.5}$$

where $q^2=(p_2-p_1)^2$. In the chiral symmetric limit, σ_{KK} vanishes and thus it is a measure of the chiral $SU(2)\times SU(2)$ breaking in the πK scattering.

In the nonlinear realization of the chiral symmetry the σ and κ fields are considered as the dependent fields on π and K. This is attained from the linear realization by letting m_σ^2 and m_κ^2 go to infinity.[17] In the limit $m_\sigma^2, m_\kappa^2\to\infty$ the scattering amplitudes (3.2) become

$$T^{(+)}(s,t,u)\xrightarrow[m_\sigma^2\to\infty,\ m_\kappa^2\to\infty]{}\frac{1}{4f_\pi^2}(s+u-2m_K^2)+\alpha q_1\cdot q_2\,,$$
$$T^{(-)}(s,t,u)\xrightarrow[m_\sigma^2\to\infty,\ m_\kappa^2\to\infty]{}\frac{1}{4f_\pi^2}(s-u)\,, \tag{3.6}$$

where $\alpha=A_2-f_\pi C_2+2(B'-f_\pi D')+E/f_\pi+F$. Thus it is consistent with the result of Bardeen and Lee[8] on the πK scattering amplitudes in the nonlinear realization.

On the mass-shell the total amplitudes can be rewritten as

$$T^{(+)}(s,t,u)=-g_\pi^2\left(\frac{1}{s-m_\kappa^2}+\frac{1}{u-m_\kappa^2}\right)+\frac{16\pi\sigma}{m_\sigma^2-t}+16\pi(\xi+\eta t)\,,$$
$$T^{(-)}(s,t,u)=-g_\pi^2\left(\frac{1}{s-m_\kappa^2}-\frac{1}{u-m_\kappa^2}\right)+16\pi\zeta(s-u)\,, \tag{3.7}$$

where

$$16\pi\sigma=\frac{3}{2f_\pi}g_{\sigma K K}(m_\sigma^2)(m_\sigma^2-\tfrac{4}{3}m_\pi^2)\,,$$
$$16\pi\xi=-\frac{1}{2f_\pi^2}(m_\kappa^2-m_K^2)-\frac{3}{2f_\pi}g_{\sigma K K}(m_\sigma^2-\tfrac{4}{3}m_\pi^2)+2m_\pi^2(A_2-f_\pi C_2)-4g_\pi f_\pi F-2f_\pi^2F^2(m_K^2+m_\pi^2-m_\kappa^2),$$
$$16\pi\eta=-\tfrac{1}{2}E-\tfrac{3}{2}(B'-f_\pi D')-(A_2-f_\pi C_2)+f_\pi^2F^2\,,$$
$$16\pi\zeta=-F-f_\pi^2F^2\,,$$
$$g_\pi=g_{\pi K K}(m_\pi^2)\,. \tag{3.8}$$

We shall take g_π, σ, ξ, η, and ζ as independent parameters in our work.

We notice immediately that σ is a measure of the magnitude of the σ term:

$$\sigma_{KK}(m_\sigma^2) = -\frac{32\pi}{3}\frac{f_\pi^2 m_\pi^2\sigma}{m_\sigma^2[m_\sigma^2-\frac{4}{3}m_\pi^2]} . \tag{3.9}$$

Partial-wave expansion is convenient at low energies near the threshold because only a few angular momentum states contribute to the total amplitudes and the unitarity condition is particularly of simple form. We expand $T^{(\pm)}(s,t,u)$ as

$$\begin{aligned} T^{(\pm)}(s,t,u) &= 16\pi\sum_{l=0}^{\infty}(2l+1)\,a_l^{(\pm)}(s)\,P_l(z_s) , \\ a_l^{(\pm)}(s) &= \frac{1}{16\pi}\,\frac{1}{2}\int_{-1}^{+1}dz_s\, T^{(\pm)}(s,t,u)\,P_l(z_s), \end{aligned} \tag{3.10}$$

where $z_s = 1+t/2q_s^2$ and q_s^2 is the three-momentum squared in the c.m. frame of the s channel. Then the elastic unitarity condition is of the form

$$\operatorname{Im}\frac{1}{a_l^{(\pm)}(s)} = -\frac{2q_s}{\sqrt{s}} \equiv -\rho(s), \qquad s \geq (m_K+m_\pi)^2. \tag{3.11}$$

On projecting the total amplitudes of Eqs. (3.7) into the partial waves by means of Eqs. (3.10), we find that

$$\begin{aligned} a_0^{(+)}(s) &= -\frac{g_\pi^2}{16\pi}\left[\frac{1}{s-m_K^2}+\frac{1}{2q_s^2}Q_0\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)\right]+\frac{\sigma}{2q_s^2}Q_0\left(1+\frac{m_\sigma^2}{2q_s^2}\right)+\xi-2q_s^2\eta , \\ a_0^{(-)}(s) &= -\frac{g_\pi^2}{16\pi}\left[\frac{1}{s-m_K^2}-\frac{1}{2q_s^2}Q_0\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)\right]+(2s-\Sigma-2q_s^2)\,\zeta , \\ a_1^{(+)}(s) &= -\frac{g_\pi^2}{16\pi}\frac{1}{2q_s^2}Q_1\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)+\frac{\sigma}{2q_s^2}Q_1\left(1+\frac{m_\sigma^2}{2q_s^2}\right)+\tfrac{2}{3}q_s^2\eta , \\ a_1^{(-)}(s) &= +\frac{g_\pi^2}{16\pi}\frac{1}{2q_s^2}Q_1\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)+\tfrac{2}{3}q_s^2\zeta , \end{aligned} \tag{3.12}$$

or, in the channel with definite isospins,

$$\begin{aligned} a_0^{1/2}(s) &= -\frac{g_\pi^2}{16\pi}\left[\frac{3}{s-m_K^2}-\frac{1}{2q_s^2}Q_0\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)\right]+\frac{\sigma}{2q_s^2}Q_0\left(1+\frac{m_\sigma^2}{2q_s^2}\right)+\xi-2q_s^2\eta+2(2s-\Sigma-2q_s^2)\zeta , \\ a_0^{3/2}(s) &= -\frac{g_\pi^2}{16\pi}\frac{2}{2q_s^2}Q_0\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)+\frac{\sigma}{2q_s^2}Q_0\left(1+\frac{m_\sigma^2}{2q_s^2}\right)+\xi-2q_s^2\eta-(2s-\Sigma-2q_s^2)\zeta , \\ a_1^{1/2}(s) &= +\frac{g_\pi^2}{16\pi}\frac{1}{2q_s^2}Q_1\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)+\frac{\sigma}{2q_s^2}Q_1\left(1+\frac{m_\sigma^2}{2q_s^2}\right)+\tfrac{2}{3}q_s^2(\eta+2\zeta) , \\ a_1^{3/2}(s) &= -\frac{g_\pi^2}{16\pi}\frac{2}{2q_s^2}Q_1\left(1+\frac{\Sigma-s-m_K^2}{2q_s^2}\right)+\frac{\sigma}{2q_s^2}Q_1\left(1+\frac{m_\sigma^2}{2q_s^2}\right)+\tfrac{2}{3}q_s^2(\eta-\zeta) , \end{aligned} \tag{3.13}$$

where $\Sigma = 2(m_K^2+m_\pi^2)$, and $Q_l(z)$ is the Legendre polynomial of the second kind.

IV. UNITARITY AND CROSSING

A. Unitarity

The phenomenological action $\Lambda(x)$ defined in Sec. II is expected to have full unitarity when it is expanded as an infinite series of derivatives in the phenomenological fields. Since we truncated the series at two derivatives, the irreducible vertices calculated from Eq. (2.13) are not unitary. At low energies near threshold elastic unitarity is valid and this is most simply expressed in the partial waves as given in Eq. (3.11).

Brown and Goble[9] unitarized the current-algebra amplitudes of $\pi\pi$ scattering derived by Weinberg.[18] They applied the relativistic version of the effective-range approximation and succeeded in explaining the correct width of the ρ meson. In the πK scattering Carrotte[6] used the same method and determined the $I=\frac{1}{2}$ s wave from the mass of the κ meson and the current-algebra constraints, i.e., the scattering length given by current algebra and Adler's self-consistency condition.[16]

In our work we shall use the same effective-range approximation adopted by Brown and Goble[9] for both s and p waves. Denoting $f_l^I(s)$ as the unitarized partial waves of $a_l^I(s)$ given in Eq. (3.13), we have

$$\frac{1}{f_l^I(s)}=\frac{1}{a_l^I(s)}+H_l^I(s)\,, \tag{4.1}$$

where $H_l^I(s)$ is the elastic unitarity function for the unequal masses and has elastic unitary cut from threshold to infinity. It can be readily obtained from the dispersion integral with at least one subtraction. Therefore,

$$\begin{aligned}H_l^I(s)&=C_l^I+\frac{s-s_0}{\pi}\int_{s_0}^{\infty}ds'\,\frac{-\rho(s')}{(s'-s)(s'-s_0)}\\&=C_l^I+h(s)\,,\end{aligned}$$

with

$$\begin{aligned}h(s)=\frac{2}{\pi}\Big[&-\frac{1}{s}\,[(s_0-s)(s_p-s)]^{1/2}\\&\times\ln\frac{(s_0-s)^{1/2}+(s_p-s)^{1/2}}{(s_0s_p)^{1/2}}\\&+\left(\frac{1}{s}-\frac{1}{s_0}\right)(s_0s_p)^{1/2}\ln\sqrt{m_K}\,\Big]\,,\end{aligned} \tag{4.2}$$

where $s_0=(m_K+m_\pi)^2$ and $s_p=(m_K-m_\pi)^2$.

Since $\rho(s)$ is independent of l and I, so is $h(s)$. Subtraction is made at $s=s_0$ because according to PCAC the phenomenological action $\Lambda[\Phi]$ expanded around the soft-pion limit is assumed to be smoothly extrapolated to threshold and we can choose C_l^I to be zero for the s waves. The s-wave scattering lengths will not be changed thereby by unitarization. But for the p waves, owing to the kinematical behavior at threshold, C_l^I's affect their effective ranges which is fourth order in momenta. The phenomenological section $\Lambda[\Phi]$ we constructed is of the second order and thus does not constrain C_l^I's for p waves. These shall be determined in Sec. V by the dynamical constraints on the p waves.

B. Crossing

As a result of the unitarization, crossing symmetry is broken and we have to restore it. Crossing has a simple form for the total amplitudes. But we are dealing with a few partial waves and accordingly the crossing relations have to be expressed in terms of them. For the $\pi\pi$ scattering there are two main types of crossing relations. One is an inequality for partial waves in the unphysical region.[19] The other is an integral equality in the same region.[20] Those methods have been extended to the unequal-mass case.[10,21] The crossing relations of the inequality type in the πK scattering[21] relate partial waves in the s channel to those in the t channel.

Since we unitarize only in the s channel, we shall not use these as constraints. Instead we shall impose the Roskies-type crossing relations[10] on the partial-wave amplitudes $f_l^I(s)$ unitarized through Eq. (4.1). These relations are obtained by integrating any isospin-odd combination of the total amplitudes over the region bounded by $z_s=\pm1$ inside the Mandelstam triangle. They are:

$$\begin{aligned}&\int_{s_p}^{s_0}ds\,q_s^{\,2}[f_0^{1/2}(s)-f_0^{3/2}(s)]=0\,,\\&\int_{s_p}^{s_0}ds\,q_s^{\,4}[f_0^{1/2}(s)-f_0^{3/2}(s)]\\&\qquad=\int_{s_p}^{s_0}ds\,q_s^{\,4}[f_1^{1/2}(s)-f_1^{3/2}(s)]\,,\\&\int_{s_p}^{s_0}ds\,q_s^{\,2}(s-q_s^{\,2}-m_K^{\,2}-1)[f_0^{1/2}(s)+2f_0^{3/2}(s)]\\&\qquad=-\int_{s_p}^{s_0}ds\,q_s^{\,4}[f_1^{1/2}(s)+2f_1^{3/2}(s)]\,,\end{aligned} \tag{4.3}$$

where s_0, s_p are the threshold and the pseudothreshold of the πK scattering, respectively. For the purpose of numerical calculation in Sec. V we can rewrite these relations as follows:

$$\begin{aligned}&\int_{s_p}^{s_0}ds\,q_s^{\,2}\,f_0^{(-)}(s)=0\,,\\&\int_{s_p}^{s_0}ds\,q_s^{\,2}[(s-3q_s^{\,2}-m_K^{\,2}-1)f_0^{1/2}(s)\\&\qquad+2(s-m_K^{\,2}-1)f_0^{3/2}(s)]\\&\qquad\qquad=-3\int_{s_p}^{s_0}ds\,q_s^{\,4}f_1^{1/2}(s)\,,\\&\int_{s_p}^{s_0}ds\,q_s^{\,2}[(s-m_K^{\,2}-1)f_0^{1/2}(s)\\&\qquad+(2s-3q_s^{\,2}-2m_K^{\,2}-2)f_0^{3/2}(s)]\\&\qquad\qquad=-3\int_{s_p}^{s_0}ds\,q_s^{\,4}f_1^{3/2}(s)\,.\end{aligned} \tag{4.4}$$

V. NUMERICAL CALCULATIONS

A. Determination of Parameters

There are 7 parameters altogether to describe the πK scattering: g_π, σ, ξ, η, ζ, and $C_1^{1/2}$, and $C_1^{3/2}$. These shall be determined by imposing dynamical constraints of the s and p waves, and three Roskies-type crossing relations, Eq. (4.3) or (4.4).

Recent improvements in boson spectroscopy provide us with information about the $I=\frac{1}{2}$ s-wave πK phase shift. Since only the s and p waves are taken into account in analyzing experimental data, there is an up-down ambiguity for the $I=\frac{1}{2}$ s-wave phase shift and thus an ambiguity about the mass of the κ meson. One solution gives $m_\kappa = 870$ MeV with the width about 30 MeV and the other solution has resonance at considerably higher energy (about 1150–1350 MeV) (Ref. 2). We shall investigate which alternative,

Case 1: $m_\kappa = 870$ MeV, $\Gamma_\kappa = 30$ MeV,

Case 2: $m_\kappa = 1150$ MeV, $\Gamma_\kappa = 400$ MeV,

is favored by our assumptions on the chiral assignment of the κ meson.

The p waves are better understood experimentally than the s waves. The $I=\frac{1}{2}$ p wave has the K^* meson as resonance and the $I=\frac{3}{2}$ p wave, being exotic, has a small phase shift. These constraints on the p waves can be expressed as

$$\frac{1}{a_1^{1/2}(s)} + \mathrm{Re}\,h(s) + C_1^{1/2} = 0 \quad \text{at} \quad s = m_{K^*}^2, \tag{5.1}$$

$$\frac{1}{a_1^{1/2}(s)} + \mathrm{Re}\,h(s) + C_1^{1/2} = -\mathrm{Im}\,h(s) \quad \text{at} \quad s = (m_{K^*} - \tfrac{1}{2}\Gamma_{K^*})^2, \tag{5.2}$$

$$\delta_1^{3/2} = 0 \ \text{ at, say, } 2 \text{ GeV}. \tag{5.3}$$

We can now determine the 7 parameters one by one. First g_π is fixed by the decay-width formula of $\kappa \to K\pi$;

$$\Gamma_\kappa = \frac{g_\pi^2}{8\pi}\frac{3}{4m_\kappa^3}\{[m_\kappa^2 - (m_K + m_\pi)^2][m_\kappa^2 - (m_K - m_\pi)^2]\}^{1/2}. \tag{5.4}$$

We have

Case 1: $\dfrac{g_\pi^2}{16\pi} = 1.42 m_\pi^2$,

Case 2: $\dfrac{g_\pi^2}{16\pi} = 19.7 m_\pi^2$.

After this step the first of the crossing relations (4.4) contains only one parameter ζ. We solve this nonlinear equation in ζ numerically. Since F is real, Eq. (3.8) gives us the bound $\zeta < 1.10\times10^{-2}$. A solution for the two cases is

Case 1: $\zeta = -0.220\times10^{-2}$,

Case 2: $\zeta = -0.662\times10^{-2}$.

The p waves contain σ, η, $C_1^{1/2}$, and $C_1^{3/2}$. g_π^2 and ζ are already fixed. We determine σ, η, and $C_1^{1/2}$ by making use of Eqs. (5.1)–(5.3). $C_1^{3/2}$ shall be fixed later. When σ is given, η is fixed by Eq. (5.3). On eliminating $C_1^{1/2}$ in Eq. (5.2) by means of Eq. (5.1) we have one equation as a function of σ. Assuming that the $I=\frac{1}{2}$ p-wave scattering length is positive we find

Case 1: $\sigma > 3m_\pi^2$,

Case 2: $\sigma > 8m_\pi^2$.

Solving for σ with this limitation we have, within the range $0<\sigma<250m_\pi^2$ (Ref. 22),

Case 1: Sol. A $\sigma = 15.94 m_\pi^2$, $\eta = -0.401\times10^{-2}$,
Sol. B $\sigma = 43.86 m_\pi^2$, $\eta = -0.762\times10^{-2}$;

Case 2: No solution.

Therefore one finds that the up solution ($m_\kappa = 870$) is favorable unless σ_{KK} is abnormally large.

Now the $I=\frac{1}{2}$ p wave is completely specified. If we plot the phase shift it reaches 90° at the energy of the K^* mass and decreases thereafter in the case of solution A, while in the case of the solution B the phase shift keeps increasing beyond the resonance region. Thus we shall discard solution A.

The second of the crossing relations (4.4) contains ξ only by now. Assuming that the $I=\frac{1}{2}$ s-wave scattering length is positive it has a bound

$$\xi > -1.89 m_\pi^2 .$$

A numerical solution for this nonlinear equation is

$$\xi = -1.76 m_\pi^2 .$$

Finally we fix $C_1^{3/2}$ by means of the third equation in the crossing relations [Eq. (4.4)]. For the positive value of $C_1^{3/2}$ we find many solutions:

$$C_1^{3/2} = 20.97, \sim23.9, \sim28.8, \sim46.0, \ldots .$$

In general $\delta_1^{3/2}$ becomes smaller as $C_1^{3/2}$ increases. Therefore it is sufficient to look into $C_1^{3/2} = 20.97$ to check if the final $I=\frac{3}{2}$ p wave is consistent with the ansatz we made at the beginning of this section.

The s-wave phase shifts are compared with the experimental data in Fig. 3. The $I=\frac{1}{2}$ phase shift gives a rather good approximation up to the inelastic threshold and the $I=\frac{3}{2}$ phase shift is small as expected.[23] Besides, $\delta_1^{3/2}$ is small (less than 1.5°) up to 1 GeV in the case of the smallest solution, $C_1^{3/2} = 20.97$, and it becomes even smaller in the case of other solutions.

The s-wave scattering lengths, defined by

$$(a^I)^{-1} = \lim_{s\to s_0} q_s \cot\delta_0^I(s),$$

are $a^{1/2} = 0.055 m_\pi^{-1}$, $a^{3/2} = 0.049 m_\pi^{-1}$.

Our analysis, however, does not entirely ex-

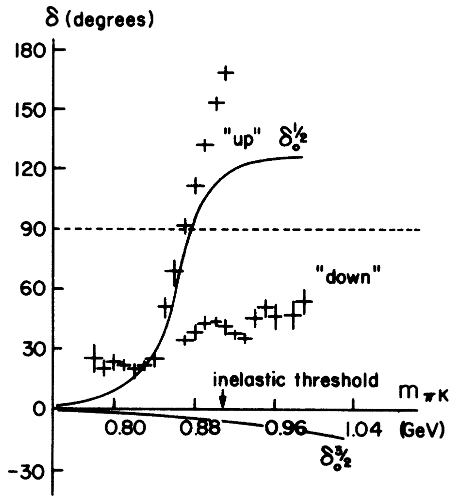

FIG. 3. The s-wave phase shifts. The experimental data are taken from Ref. 3.

clude the possibility of finding a down solution. In solving a system of nonlinear equations numerically, we put constraints on the parameter space by the physical considerations. But this gives us only the upper and the lower bound for ζ, ξ, respectively. Numerical calculations have been carried out within a reasonable range under these constraints and we do not find any down solution. Therefore we can say our analysis favors the up solution with $\kappa(870)$.

B. σ Term

As mentioned in Sec. II B, we have the chiral SU(2)×SU(2) as an approximate symmetry of strong interactions where the pions become massless Goldstone bosons in the symmetric limit. The degree of the symmetry breaking can be measured in various ways. For example, if the chiral SU(2)×SU(2) is exact, then we expect zero-mass pions, vanishing σ terms, and no correction to the Goldberger-Treiman relation. We have, however,

$$\frac{m_\pi^2}{m_K^2}=0.075,$$

$$\frac{\sigma_{NN}}{m_N}=0.04\text{–}0.12 \quad \text{(see Refs. 24 and 25)}, \qquad (5.5)$$

$$1-\frac{m_N G_A}{f_\pi g_{\pi NN}}=0.08\pm 0.02\,.$$

Therefore the chiral SU(2)×SU(2) breaking is about 10%.

The magnitude of the σ term in the πK scattering can be readily evaluated from Eq. (3.5):

$$\sigma_{KK}(m_\sigma^2)=-1.12\,m_\pi^2 \quad \text{(see Ref. 26)}$$

or

$$\frac{|\sigma_{KK}|}{m_K^2}=0.08\,. \qquad (5.6)$$

This is another measure of the chiral SU(2)×SU(2) breaking and it is consistent with the results of Eq. (5.5). There is, however, a discrepancy between the scattering lengths we predict and the current-algebra results. In the latter, one retains only leading terms in the soft-pion limit and discards the σ term which contains the matrix elements of the commutator of the axial charge with the divergence of the axial-vector currents. This is based on the observation that the σ term is of first order in the symmetry breaking and can be neglected compared with other terms in the first approximation. As a result current algebra predicts that the isospin-even s-wave scattering length vanishes,[18,27] i.e.,

$$a^{(+)}=\tfrac{1}{3}(a^{1/2}+2a^{3/2}) =0\,. \qquad (5.7)$$

For a long time the extreme smallness of $a^{1/2}+2a^{3/2}$ in the πN scattering has been attributed to the smallness of σ_{NN}. But a recent estimation by Cheng and Dashen[24] shows that σ_{NN} is about 110 MeV, although it could be smaller by a factor of two or three.[25] This will change the isospin-even scattering length to be as big as $a^{1/2}(\pi N)$, which is inconsistent with the experimental data. They argue that the Born term of the axial-vector scattering amplitude which vanishes in the soft-pion limit compensates the σ-term contribution on the mass shell. But this is not the case for the πK scattering. The Born term of the axial-vector scattering with the kaons is identically zero by parity conservation and the σ term has a noncompensating contribution to $a^{1/2}+2a^{3/2}$. In our parametrization

$$\tfrac{1}{3}(a^{1/2}+2a^{3/2})=0.05\,m_\pi^{-1}\,. \qquad (5.8)$$

This is also as big as $a^{1/2}$ and $a^{3/2}$ of the πK scattering. At the present stage no data is available for the isospin-even πK scattering length and a direct measurement of this quantity is desirable.

VI. APPLICATION TO THE K_{l3} DECAYS

We can also apply our results to the study of the K_{l3} decays.[28] The K_{l3} form factors are defined as

$$\langle \pi^0(q)|V^\mu_{\bar K}(0)|K^+(k)\rangle=\tfrac{1}{2}[f_+(t)(k+q)^\mu+f_-(t)(k-q)^\mu],$$

$$t=(k-q)^2\,. \qquad (6.1)$$

Instead of $f_\pm(t)$ we shall consider their combination, the scalar form factor $f(t)$, in the following:

$$f(t)=f_+(t)+\frac{t}{m_K{}^2-m_\pi{}^2}f_-(t)\,. \qquad (6.2)$$

As emphasized by many authors, the form factor $f(t)$, which is proportional to the divergence of the matrix element (6.1), is the quantity which arises naturally in the current-algebra analysis. It is also the quantity that is directly measured by the Dalitz-plot density in $K_{\mu 3}$ decay experiments.

Theorems on those form factors are based on the chiral SU(3)×SU(3) as an approximate symmetry of the strong interactions, where its symmetric limit is realized as having a massless pseudoscalar octet of Goldstone bosons. According to the Ademollo-Gatto theorem,[29] $f(0)$ is not renormalized in the first order of the symmetry breaking,

$$f(0)=1+0(\lambda^2\ln\lambda)\,. \qquad (6.3)$$

The Callan-Treiman relation[30] can be written as

$$f_+(m_K{}^2)+f_-(m_K{}^2)=\frac{f_K}{f_\pi}\,, \qquad (6.4)$$

which becomes in terms of $f(t)$

$$f(m_K{}^2-m_\pi{}^2)\approx\frac{f_K}{f_\pi}-f_+'(0)m_\pi{}^2\,. \qquad (6.5)$$

Here f_K is the leptonic decay constant of the kaons.

The slope of $f(t)$ is given by Dashen, Li, Pagels, and Weinstein[31] from the chiral perturbation theory. Their theorem states that

$$\left.\frac{d}{dt}f(t)\right|_{t=m_K{}^2+m_\pi{}^2}=\tfrac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)\frac{1}{m_K{}^2-m_\pi{}^2}+X+O(\lambda,\ln\lambda,t)\,. \qquad (6.6)$$

The first term on the right-hand side of Eq. (6.6) comes from the pole terms of the local current commutators and the axial-vector currents. It is of order[32] $\ln\lambda$ and model-independent. The second term, X, is due to the πK intermediate state and is of the order of one. This term arises from differentiating terms of order $\ln(\lambda-t)$ with respect to t. Diagrammatically we can express the scalar form factor $f(t)$ as shown in Fig. 4.

In the approximation of retaining only the πK intermediate state, the scalar form factor $f(t)$ has the structure[33]

$$f(t)=P(t)\exp\left[\frac{t}{\pi}\int_{(m_K+m_\pi)^2}^\infty dt'\frac{\delta_0^{1/2}(t')}{t'(t'-t)}\right]\,, \qquad (6.7)$$

where $P(t)$ is an arbitrary polynomial in t except that it is normalized at $t=0$ with

$$P(0)=f(0)=1+O(\lambda^2\ln\lambda)\,. \qquad (6.8)$$

Since $P(t)$ is regular at the threshold the term X must come entirely from the Omnès exponential integral. Near the threshold $\delta_0^{1/2}(t)$ behaves like $\rho(t)f_0^{1/2}(t)$ and the exponent in Eq. (6.7) is $O(\lambda\ln(\lambda-t))$. Therefore we can take the first two terms in the exponential expansion to the order $O(\lambda\ln\lambda)$. Furthermore, $P'(m_K{}^2+m_\pi{}^2)$ is fixed by the model-independent term of $f'(t)$ in Eq. (6.6). As a result we can write

$$f(t)=1+\tfrac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)\frac{t}{m_K{}^2-m_\pi{}^2}-\frac{f_0^{1/2}(t_0)}{\pi}\ln\frac{t_0-t}{t_0}+O(t^2,\lambda^2,\lambda^2\ln\lambda)\,. \qquad (6.9)$$

The scalar form factor, Eq. (6.9), is shown in Fig. 5 with the experimental value of $f_K/f_\pi=1.28\pm0.06$. It is also compared with the recent SLAC experimental data.[34] The Callan-Treiman relation is satisfied and it is consistent with the data at high t in the physical region. For small t, however, it does not seem to be consistent with the experiment. But in this region there is an uncertainty among the experimental data.[28,34] Therefore the behavior of $f(t)$ is not clear at the moment for small t in the physical region.

VII. CONCLUSION

In conclusion we have constructed the chiral SU(2)×SU(2) symmetric scattering amplitudes for

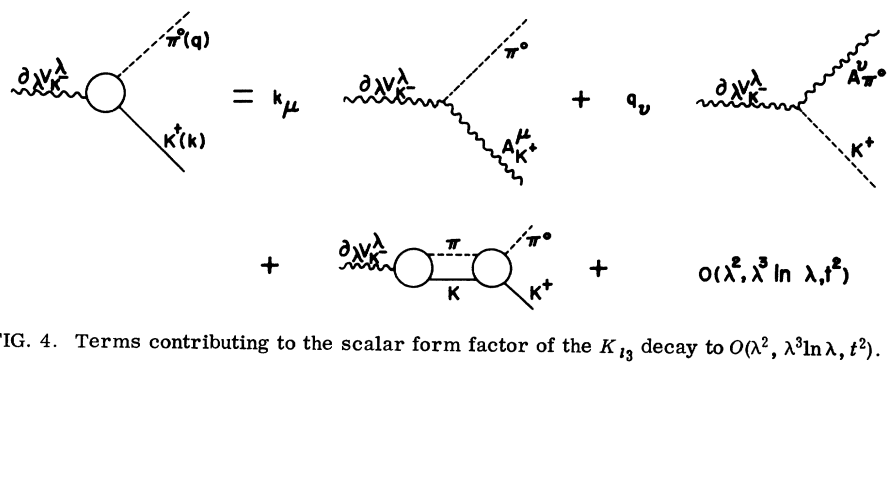

FIG. 4. Terms contributing to the scalar form factor of the K_{l3} decay to $O(\lambda^2,\lambda^3\ln\lambda,t^2)$.

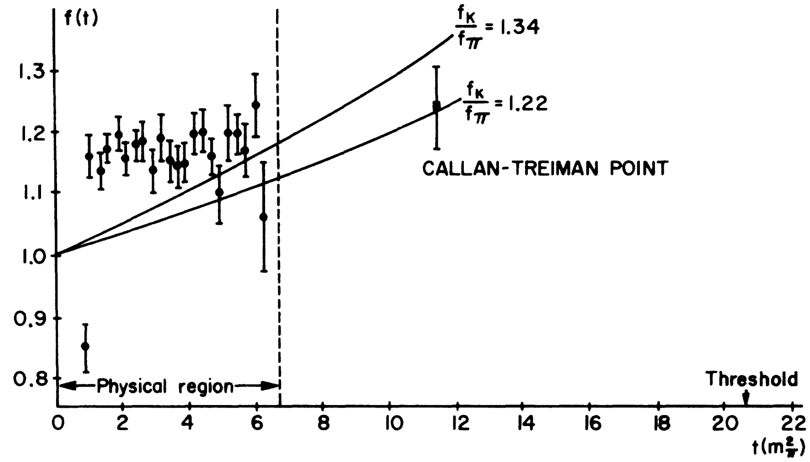

FIG. 5. The behavior of the scalar form factor, $f(t)$. The experimental data are taken from Ref. 34.

the πK reactions which are consistent with unitarity and crossing. As for the up-down ambiguity in the experimental data, our analysis favors the solution with the smaller mass of the κ meson (870 MeV). The magnitude of the σ term is estimated and the chiral symmetry breaking is measured as about 8%, which is consistent with other results. Finally the $I=\frac{1}{2}$ s-wave phase shift has been applied to the K_{l3} decays.

The amplitudes so constructed seem to be a good approximation up to the inelastic threshold.

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[22]By Eq. (3.9) σ measures the magnitude of the σ term in the πK scattering, and $\sigma=250\,m_\pi^{\,2}$ corresponds to $\sigma_{KK}\approx 6.4\,m_\pi^{\,2}$.

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than 1°), and soon it becomes negative.

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Multiplicity Distributions Resulting from Multiperipheral Plus Diffractive Production*

William R. Frazer†
Department of Physics, University of California, San Diego, La Jolla, California 92037

Roberto D. Peccei‡
Department of Physics, Stanford University, Stanford, California 94305

Stephen S. Pinsky§
Department of Physics, University of California, Riverside, Riverside, California 92502

Chung-I Tan‖
Department of Physics, Brown University, Providence, Rhode Island 02919

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We discuss the phenomenology of multiplicity distributions in high-energy hadronic collisions in terms of the short-range correlation picture, modified to include diffractive processes leading to low-multiplicity final states. We formulate two models, both of which give reasonable fits to present data on high-energy partial cross sections. Model-independent methods for separating diffractive and multiperipheral production are discussed.

I. INTRODUCTION

One of the most interesting open questions concerning multiparticle hadronic reactions is the behavior of the partial cross sections $\sigma_n^i(s)$ for producing n particles of a given type i (plus any number of other particles), as a function of both n and the square of the c.m. energy, s. Current efforts to organize and understand data on multiparticle reactions are dominated by two competing points of view, the diffractive picture and the short-range correlation (multiperipheral) picture. The *diffractive picture* assumes that for large s the partial cross sections approach nonzero constant values, $\sigma_n(s) \to \sigma_n$. In order to reproduce the apparent linear increase of the average multiplicity $\langle n \rangle$ with $\ln s$, proponents of the diffractive picture usually assume that $\sigma_n \sim c/n^2$ for sufficiently large n. Specific diffractive models[1] have been constructed which, at the cost of additional assumptions, exhibit concrete predictions to compare with data.

The *short-range-correlation* picture assumes that all correlations vanish between particles whose rapidities y are separated by a distance large compared to a correlation length L. This picture implies, without further *ad hoc* assumptions, that the average multiplicity rises linearly with $\ln s$. Since it reproduces general features of multiperipheral models, this picture is also called the *multiperipheral picture*. Again, specific simple multiperipheral models[2] have been constructed and their predictions are testable experimentally.

The diffractive and short-range-correlation pictures represent, in some sense, extreme points of view. It is not hard to imagine that the real world contains both diffractive and short-range-correlation elements. Recent high-energy data from the National Accelerator Laboratory (NAL) and CERN Intersecting Storage Rings (ISR) are,