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Possible Difference Between the Muon- and Electron-Nuclear Interactions*

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The possible existence of an order α^2, scalar, mass-dependent, lepton-hadron interaction is noted in connection with the recently investigated possible discrepancies in a number of $2p \to 1s$ transition energies in muonic atoms. Some relevant further experiments are suggested for the Los Alamos Meson Physics Facility.

Very recently Rinker and Wilets have investigated the possible existence of discrepancies in a number of $2p \to 1s$ transition energies in muonic atoms.[1] The effective nuclear radii may be slightly larger than those deduced from the low-energy electron scattering experiments. There is a physical effect which contains the possibility of a distinction between muon and electron in these experiments. This is the interaction of virtual spin-zero mesons, such as the ϵ^0 or π^0, with the leptons via a pair of virtual photons. These mesons can decay into two real photons with a rate parametrized by

$$R_{2\gamma} = |e^2 x|^2 \left(\frac{m^3}{64\pi}\right) \qquad (e^2 \equiv 4\pi\alpha), \tag{1}$$

where m denotes the meson mass and x (in GeV^{-1}) is a parameter, real for π^0, but in general, complex for ϵ^0. Consider the scalar, isoscalar dipion ϵ^0. The amplitude (or effective coupling constant) connecting it to a lepton pair via two *real* photons is[2]

$$|\mathrm{Im}g_{\epsilon l}| = \left(\frac{e^2}{4\pi}\right)|e^2(\mathrm{Re}x)|\left(\frac{m_l}{4v_l}\right)\ln\left|\frac{1+v_l}{1-v_l}\right| \cong 7\times 10^{-5}, \tag{2}$$

where $v_l = (1 - 4m_l^2/m_\epsilon^2)^{1/2}$ and m_l is the lepton mass. The number is evaluated for $m_l = m_\mu$ and $m_\epsilon \cong 750$ MeV, with $|\mathrm{Re}x| \approx 1$, which from Eq. (1) implies a reasonable rate, $R_{\epsilon\to 2\gamma} \geq 14\ \mathrm{keV} \approx \alpha^2\Gamma_\epsilon$ ($\Gamma_\epsilon \cong 300$ MeV). If we consider a ($\mathrm{Re}g_{\epsilon\mu}$) one order of magnitude larger than this number,[3] then we find that a repulsive potential

$$V(r) = \frac{g_{\epsilon N}(\mathrm{Re}g_{\epsilon\mu})}{4\pi}\frac{e^{-m_\epsilon r}}{r} \qquad \left(\frac{g_{\epsilon N}}{4\pi} \approx \frac{g_{\pi N}}{4\pi} \approx 1\right) \tag{3}$$

produces a $(\Delta E)_{\mathrm{muon}}$ of about 1 keV for $Z = 29$, $A = 64$. Four times the potential in Eq. (3) (for example, $|\mathrm{Re}x| \approx 4 \Rightarrow R_{\epsilon\to 2\gamma} \approx 16\alpha^2\Gamma_\epsilon$) then yields virtually an exact fit to the systematic discrepancy in $(\Delta E)_{\mathrm{muon}}$ for all six atoms[4] (observed when the ^{12}C radius measured at Darmstadt is used[1]). The effective interaction for electrons is down by $(\frac{1}{200})$ because of the lepton mass factor in Eq. (2).

A similar discussion for the π^0 [with $|x| = |\mathrm{Re}x| \cong 0.265$ from $(R_{\pi^0\to 2\gamma})_{\mathrm{exp}}$] gives $|\mathrm{Im}g_{\pi^0\mu}| \cong 10^{-5}$ from Eq. (2) (with m_π considered $\to 2m_\mu$ and thus v_μ taken $\to 0$). The pseudoscalar nature of the π^0 implies a $\vec{\sigma}_\mu \cdot \vec{\sigma}_N$ interaction and the above number causes a completely negligible hyperfine splitting in ^{209}Bi (a $\mathrm{Re}g_{\pi^0\mu} \cong 10^{-3}$ could give a splitting of about 0.2 keV).[4]

When use[1] is made of the ^{12}C radius measured at Amsterdam the discrepancy disappears, but an effect at the level of a two-photon process *without* the above factor of 40 enhancement in $\mathrm{Re}g_{\epsilon\mu}$ is possible. It should also be noted that effective coupling parameters of the same order of magnitude as the above enhanced coupling have been discussed[5] in connection with a possible "anomalous" interaction of systems of pions directly with muons.

Moderate energy experiments, which are suitable for LAMPF (Los Alamos Meson Physics Facility), on one- and two-pion production near threshold in muon-proton collisions would be interesting in order to explore the possibility that muons can emit pionic systems with coupling parameters near the level of α. Precision measurements of elastic scattering at moderate energies from the proton and nuclei would also be very valuable. With the above coupling parameter ($\mathrm{Re}g_{\epsilon\mu} \approx 10^{-3}$) the interference between the hadron exchange amplitude and the single-photon exchange amplitude could lead to a few percent anomaly for 200-MeV muons on protons.[5]

The Lamb shift in muonic helium, an experiment to be performed at CERN,[6] would be altered by about (-12.6 eV) ($\mathrm{Re}g_{\epsilon\mu}$), or by less than about 3%.[7]

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[1]G. A. Rinker, Jr. and L. Wilets, preceding paper, Phys. Rev. D 7, 2629 (1973), and private communications.

[2]B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D 2, 179 (1970); 4, 272(E) (1971); and earlier references therein.

[3]An unsubtracted dispersion relation does not converge without a form factor for (Rex). However, there is no *a priori* reason for such a large enhancement factor, nor for the necessary repulsive sign of the potential.

[4]I thank Dr. Rinker for this communication. See Table I in Ref. 1.

[5]S. Barshay, Phys. Letters 37B, 397 (1971). The numerical values of the quantities $y_{3\pi}$, g, and f in Eqs. (10), (13), and (14)–(15), respectively, should be multiplied by four.

[6]Private communication from Dr. Zavattini.

[7]I thank Dr. Rinker for his help here.

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Chiral SU(2)×SU(2) Symmetry in the πK Scattering*†

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The chiral SU(2)×SU(2) symmetry is studied for the πK scattering in the scheme of the linear realization. Roskies-type relations are used for the crossing symmetry, and the relativistic version of the effective-range approximation is imposed on for unitarity. Two alternatives of the up-down ambiguity for the $I=\frac{1}{2}$, 0^+ resonance are investigated, and our analysis favors the up-solution with $\kappa(870)$. The σ term in the πK scattering is estimated to be about $-1.1\ m_\pi^{\ 2}$. This is another measure of the chiral SU(2)×SU(2) breaking and is consistent with other features of the symmetry breaking. Finally the threshold effect on the K_{l3} form factor is studied and compared with experiment.

I. INTRODUCTION

Recently improvement has been made in the boson spectroscopy of the reactions $K^-p \to \pi^+K^-n$ (Ref.1) and $K^+p \to \pi K\Delta^{++}$ (Refs. 2 and 3) dominated by the one-pion-exchange mechanism. This enables one to understand the πK elastic scattering at low energies. Since only s and p waves are considered, the resulting $I=\frac{1}{2}$ s-wave phase shift has the up-down ambiguity and thus two choices about the mass of the κ meson. The up solution gives the κ meson at 870 MeV with a width of 30 MeV while for the down solution we have $\kappa(\sim1150)$ with a broad width of about 400 MeV.

Various attempts have been made to understand these phenomena from different points of view. Pagnamenta and Renner[4] construct a current-algebra model for the $\pi K\kappa$ vertices with the πK intermediate states. The Veneziano model is also used[5] in the study of the πK scattering with an application to the K_{l3} form factors. Carrotte[6] unitarizes a simple parametrization determined by the current-algebra constraints by making use of the relativistic version of the effective-range approximation. Pond[7] applied the hard-pion technique to the πK problems.

The purpose of this paper is to construct the πK scattering amplitudes at low energies which are consistent with unitarity and crossing. We assume that σ, $\vec{\pi}$, K, and κ mesons belong to the simplest representation of the chiral SU(2)×SU(2) symmetry and formulate the most general phenomenological action containing at most two derivatives. The difference between the nonlinear phenomenological Lagrangian method discussed previously[8] and ours is that in the former, one assumes a certain term characterizing the SU(2)×SU(2) breaking (the so-called σ term) to be negligible. This assumption has been challenged in the case of πN scattering, and in our work we shall not make this assumption. This necessitates our treatment of the σ and κ fields as independent degrees of freedom.

Unitarity is imposed on by the method of the relativistic effective-range approximation, which has been successful in the $\pi\pi$ problem,[9] and crossing symmetry is restored by demanding three Roskies-type relations[10] for the s and p waves. Our final solution favors the up solution for the $I=\frac{1}{2}$ s wave. The magnitude of the σ term in the πK scattering is evaluated to be about $-1.12m_\pi^{\ 2}$ and the chiral SU(2)×SU(2) breaking is measured to be small (~10%), which is consistent with other features of the chiral symmetry breaking. We also applied our phase shifts to the K_{l3} decays. The threshold effect on the scalar form factor $f(t)$ is studied and compared with experiments.