stably in determining the pp forward amplitude. The qualitative features of the predicted amplitude are consistent with that of Ref. 1, e.g., the imaginary part has a broad dip above the $\bar{p}p$ threshold and a peak near 2π threshold.

(ii) In particular, the predicted Re/Im ratio for pp is again large enough for us to conclude that the spin-flip amplitude cannot be neglected even in the forward direction in the high-energy region below 30 GeV/c. The ratio is also definite enough to dissolve the experimental confusion in the near-GeV region.

(iii) One does, however, not observe any abrupt change in the imaginary part at the $\bar{p}p$ threshold, and the depth of the dip is only one fifth that of Ref. 1, which would be expected in the present method of continuation.

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New Limits on the Difference Between the Muon- and Electron-Nuclear Interactions*

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Muonic-atom and electron-nucleus scattering measurements of nuclear radii have been examined in light nuclei in order to test their relative consistency. The electron data were selected to provide information at the "equivalent momentum transfer," which is approximately 0.3 fm^{-1}. All corrections known to us were applied or considered. Scattering experiments normalized to the Darmstadt ^{12}C cross-section work are found to exhibit a systematic difference from the muon results. When these experiments are renormalized to the recent Amsterdam ^{12}C work, the discrepancy disappears. In the latter case, no muon/electron measurements disagree by more than 0.02 fm. We estimate the standard deviation to be 0.014 fm about a mean of −0.002 fm. The possibility of anomalous lepton-nucleon interactions is discussed. The limits found here still admit observable anomalous effects in, for example, muonic H and He Lamb shifts and muon-nucleon scattering experiments.

The muon and electron are generally assumed to be point Dirac particles which differ from each other only in mass. Colliding-electron-beam experiments have shown no violation of quantum electrodynamics (QED) and the assumption of a point electron up to momentum transfers of 8 fm^{-1}. These experiments may be interpreted as placing a limit on the electron "size" of $\langle r^2\rangle_e < 0.006$ fm^2. The agreement of the muon $g-2$ experiment with QED places a limit on the muon "size" of $\langle r^2\rangle_\mu < 0.004$ fm^2 within 2 standard deviations. Comparisons between high-energy (12 GeV) e-p and μ-p scattering experiments indicate $\langle r^2\rangle_\mu - \langle r^2\rangle_e < 0.014$ fm^2 with 97.7% confidence.[1]

In the present work we have examined existing μ-atom and e-nucleus scattering data in order to test their relative consistency. A persistent, otherwise unaccountable discrepancy might indicate a difference between the muonic and electronic interactions *with nuclei* at intermediate energy.

It should be noted that, contrary to some recently published remarks, muon-nucleus experiments alone cannot determine the muonic form factor since such can always be attributed to the nuclear form factor: One measures only a folded nuclear-muonic charge distribution. This, of course, implies that to the extent that an anomalous interaction can be represented by a reasonable form factor, it is not observable in such experiments. The same is true for electron-nucleus experiments.

Electron-nucleus scattering experiments are usually analyzed by adjusting the parameters of an assumed charge distribution to yield a fit to the observed cross section. The information which can be obtained depends on the momentum transfers in the experiment. The Born approximation (which is *not* used in any of the analyses utilized here) gives a scattering amplitude proportional to the form factor

$$F(q) = 1 - \frac{1}{3!} q^2 \langle r^2 \rangle_N + \frac{1}{5!} q^4 \langle r^4 \rangle_N + \cdots .$$

Thus as $q \to 0$, for example, the rms radius is measured, while at large q, information about the higher moments (shape) of the charge distribution is obtained. All of the experiments utilized here were analyzed in terms of the two-parameter Fermi distribution.

In muonic-atom experiments, each transition measures some one radial moment of the nuclear charge distribution. "Which moment" has been a subject of recent interest.[2-5] In fact, the moment is determined by the difference in electrostatic potential generated by the muon in the states involved in the transition. We may approximate the potential as[2-5]

$$V_\mu(r) = V_0 + V_2 r^2 + V_4 r^4 + \cdots \simeq V_0' + B r^k$$

(for the $2p \to 1s$ transitions, for example, $k = 2 - Z/69.5$). A given transition thus measures $\langle r^k \rangle_N$.

Engfer has suggested[6] that a test of the consistency of muon and electron experiments would be to analyze the electron data at an "equivalent momentum transfer," defined to be such that the relative weight of the $\langle r^2 \rangle_N$ and $\langle r^4 \rangle_N$ terms in the Born amplitude approximate that in $\langle V_\mu \rangle$. We here set

$$1 - \frac{1}{3!} q^2 r^2 + \frac{1}{5!} q^4 r^4 \approx \text{const} \times (V_0' + B r^k),$$

and determine $q = q_{eq}$ by equating the ratio of the first and second derivatives of the two expressions at a point R chosen to be near the maximum of the muon energy integrand $\rho(r) r^{2+k}$. (V_0' and B are irrelevant.) This gives

$$(q_{eq} R)^2 = 10(2-k)/(4-k).$$

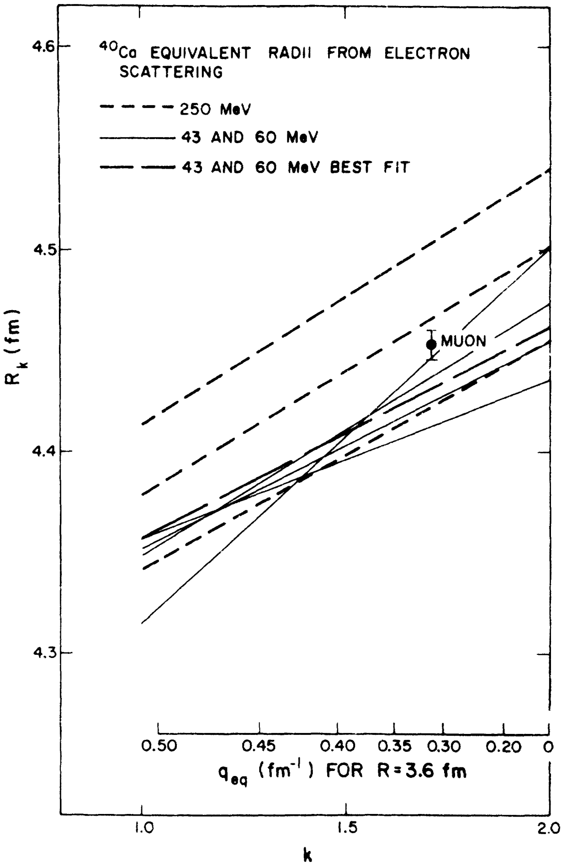

FIG. 1. Calcium-40 equivalent radii, including nuclear polarization and dispersion corrections for the muon and low-energy electron scattering measurements. The low-energy electron curves are normalized to the ^{12}C cross section from Ref. 17.

We consider in some detail the case of ^{40}Ca, for which $k = 1.71$, $q_{eq} \approx 0.31$ fm^{-1}. In Fig. 1 we have plotted the equivalent radius function R_k for charge distributions obtained in electron scattering experiments. [See Eq. (1) below for the definition of R_k.] The solid and long-dashed curves reproduce the low-energy scattering data[7] to within one standard deviation. R_k is best determined in the vicinity of $k \sim 1.3$, corresponding to a representative experimental value for q of 0.45 fm^{-1}. It would appear that the relevant equivalent radius, $R_{1.71}$, could be determined better by a factor of ~5 if the experiment were concentrated on lower momentum transfers.

The short-dashed curves are equivalent radii for three charge distributions obtained by fitting 250-MeV data[8] in different ways. It is clear that no moment in the range $1<k<2$ is well determined by those data. This further emphasizes the fact that high-energy electron scattering experiments are not especially useful for direct comparison with muonic atom data. The information thus obtained is complementary rather than comparable. One of the complementary aspects is that the slope of the equivalent radius curve in this range is well determined by scattering at high energy but not at low energy, nor by measurement of a single muonic-atom transition energy. Knowledge of the slope may be used to make a reliable comparison between independent measurements of R_k when k is somewhat different for the different measurements.

We note that q_{eq} is a slowly varying function of Z over the range $10 \leq Z \leq 90$ because the Z or A dependence of R nearly cancels that of the factor involving k. A representative value is $q_{eq} \sim 0.3$ fm^{-1}.

The present approach is to use existing full two-parameter analyses of electron scattering data[7,9,10] whenever such data are meaningfully influenced by values of $q \approx q_{eq}$. We use the charge distribution quoted in the report of those experiments to calculate (predict) the $2p \rightarrow 1s$ muonic transition energy. Second-order vacuum polarization and Lamb-shift corrections are included. The vacuum-polarization potential is added to the electrostatic potential produced by the nucleus before solution of the radial Dirac equations. We evaluate the Lamb shift through second order in the electric field according to the prescription of Barrett *et al.*[11] Uncertainty in the Lamb shift is due primarily to the average muon excitation energy, which we take to be the muon binding energy to within a factor of 2. This generous error estimate[11] certainly provides an upper bound on the effect for the muon $1s$ state. We have limited consideration to light nuclei in order to reduce the nuclear polarization and dispersion uncertainties, which are discussed further below. Other known corrections (fourth-order vacuum polarization, electron screening, relativistic nuclear recoil) are negligible. A comparison between electron-predicted and measured muonic transition energies is shown in Table I for the six cases where existing data permit such an analysis. For each element, the first line contains results using the electron scattering charge distributions as reported. The second line contains results from charge distributions renormalized by a factor computed in Born approximation to a more recent measurement of the ^{12}C cross section (discussed further below).

The errors which we give for the electron-pre-

TABLE I. Comparison of muonic-atom and electron scattering results for selected nuclei. $E_{\text{electron}}^{(\text{calc})}$ is the muonic transition energy calculated from charge distributions reported in Refs. 7, 9, and 10, as indicated. The second line for each element is the same quantity calculated from charge distributions renormalized to more recent carbon cross-section measurements (Ref. 17). Uncertainties in the renormalized quantities have been adjusted to reflect the smaller quoted errors in the carbon cross section. The dominant uncertainties in this column are due to the uncertainties in the electron-deduced form factors. In ^{40}Ca, for example, the Lamb shift (cf. Ref. 11) contributes 0.335 keV to the transition energy, with an uncertainty of 20% or (007).

Z	A	$E_{\text{electron}}^{(\text{calc})}$ (keV)	$E_{\text{muon}}^{(\text{exp})}$ (keV)[a]	k	C_Z (fm/MeV)	δR_k (fm) Without pol. or disp.	δR_k (fm) With pol. and disp.
13	27	347.08(33)[b]	346.85(14)	1.81	196.0	0.045(71)	0.035(71)
		346.80(20)				−0.010(48)	−0.020(48)
14	28.086	400.60(42)[b]	400.20(11)	1.80	152.0	0.061(66)	0.051(66)
		400.23(25)				0.005(42)	−0.005(42)
20	40	785.05(200)[c]	783.73(14)	1.71	44.0	0.058(88)	0.048(88)
		784.38(150)				0.029(66)	0.019(66)
22	47.90	933.13(200)[d]	931.57(50)	1.68	32.0	0.050(66)	0.042(66)
		931.66(160)				0.003(54)	−0.005(54)
28	58.71	1430.31(800)[d]	1425.60(40)	1.60	14.6	0.069(110)	0.062(110)
		1427.44(700)				0.027(100)	0.020(100)
29	63.546	1514.77(600)[d]	1510.59(33)	1.58	13.0	0.054(78)	0.049(78)
		1511.70(500)				0.014(65)	0.009(65)

[a] Compiled from Ref. 4.
[b] Reference 9.
[c] Reference 7.
[d] Reference 10.

dicted energies are $\Delta E = \Delta R_2 / C_Z$ [see Eq. (2) below], where ΔR_2 is the uncertainty assigned in the experimental papers and includes both statistical and systematic errors. Better estimates for the individual errors in R_k could be obtained directly in the process of fitting the experimental cross sections. We obtain an estimate of the total error from the weighted variance of the six points, a result dependent only upon the relative magnitudes of the quoted errors.

The energy differences evident in Table I could be attributed to specific muon- or electron-nuclear interactions or to differences in their intrinsic charge distributions. We consider the latter for conceptual purposes. The conversion from an energy difference to a size difference is facilitated by utilizing the Ford and Wills equivalent radius parameter[2]

$$R_k = \left[\frac{k+3}{3}\langle r^k\rangle_N\right]^{1/k}. \tag{1}$$

The sensitivity of the transition energy to R_k is given by

$$\delta R_k = -C_Z \delta E\,, \tag{2}$$

where C_Z has been tabulated.[2,4] The derived values of δR_k are also given in Table I and Fig. 2 with the sign convention such that $\delta R_k > 0$ corresponds to $\langle r^2\rangle_{N\mu} > \langle r^2\rangle_{Ne}$. Before speculating on these as a fundamental difference or limit, we consider other possible effects.

1. *Nuclear polarization/dispersion.* The former is the term used for muonic atoms; the latter is used for the same effect in electron scattering. For the light nuclei considered here, we estimate the muonic nuclear polarization energy to be proportional to $A^{11/3}$. (This is the A dependence of the dominant dipole contribution for light nuclei, assuming a giant resonance frequency proportional to $A^{-1/3}$.) We have assumed this dependence and normalized to Chen's calculation[12] for the total effect in ^{40}Ca ($\Delta E_{NP} = 0.19$ keV for the $2p \to 1s$ transitions, which corresponds to an increase in δR_k of about 0.008 fm). We include dispersion effects according to the Bottino and Ciocchetti[13] formula,

$$\frac{\delta_{\rm disp} R_k}{R_k} \simeq \frac{\delta_{\rm disp} R_2}{R_2} \simeq -\frac{N}{2Z(A-1)}\,\frac{1}{(1+1/Z\alpha\pi)}\,.$$

This is regarded as an upper bound on the main dipole contribution, which is the lowest-order term in powers of the initial electron momentum. The magnitudes of the next terms are not known. One might consider contributions from enhanced collective modes, but these would be expected to show considerable variation with nuclear species.[14]

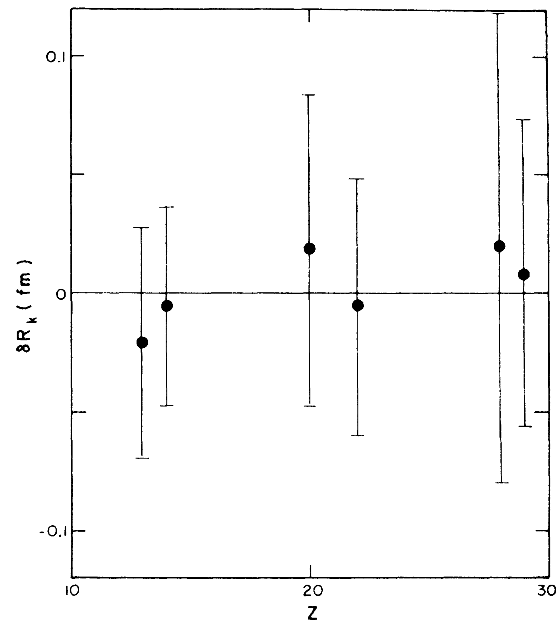

FIG. 2. $R_k^{(\mu)} - R_k^{(e)}$ including nuclear polarization and dispersion, with $R_k^{(e)}$ normalized to the ^{12}C cross section from Ref. 17.

Because the polarization and dispersion effects contribute with the same sign (i.e., tend to cancel in the comparison) and are subject to similar uncertainties, we have presented the final comparison in Table I first ignoring both effects and then including both effects.

2. *Is $\langle r^k\rangle$ the proper moment?* The Ford-Wills moment analysis is used explicitly only in two ways. The first is to select the region of momentum transfers in electron scattering which is appropriate for comparison with muonic atoms. While selection is important, we are only interested in an inclusive *range* of $q \sim q_{eq}$. The second use is for expressing the comparison in terms of δR_k. The direct comparison is already available from E(predicted) $-E$(observed). δR_k is proportional to this difference and permits a physical interpretation.

3. *Is the two-parameter Fermi distribution adequate?* Provided the radial-moment analysis is valid, the relevant model dependence of the assumed nuclear charge distribution is contained in the slope of the equivalent radius curve. We have found negligible difference in this respect among various charge distribution forms constrained to fit high-energy scattering data. Independent determinations of the slope may be made in heavy elements by measuring several muonic-atom transition energies. These determinations agree very well[5] with those made by high-energy elec-

tron scattering. It appears that to within the necessary accuracy, we can ignore the possible influence of a muon-electron difference on the determination of the slope of the equivalent radius curve. Similarly, we have found little difference in the appropriate radial moment among charge distributions which reproduce a specific muon transition energy or a specific value of the Born $F(q_{eq})$.

4. *Normalization of the electron scattering data.* All of the scattering data utilized here have relied upon a comparison with scattering on ^{12}C. At the relatively low momentum transfers considered, the data fall well before the first minimum so that an absolute cross section determination is essential. The absolute ^{12}C comparison cross section is determined by calculation using an independently fit charge distribution. One need not be concerned about the validity or model dependence of the ^{12}C charge distribution, but only whether it can accurately reproduce the absolute cross section at the energies and momentum transfers where comparisons are required. We emphasize this point because recent experiments and analyses by Sick and McCarthy[15] at $375 < E < 745$ MeV and $1 < q < 4$ fm^{-1} lead to a somewhat larger radius ($\langle r^2\rangle_C^{1/2} = 2.46 \pm 0.025$ fm) than those used in the quoted analyses (2.395 ± 0.028 fm in all cases except ^{40}Ca, where 2.42 ± 0.04 fm was used). We do not regard the Sick and McCarthy work to be relevant here *unless* it casts doubt on the experiments (not the analysis) of Bentz *et al.*[16] A more serious disagreement now exists between recent low-energy measurements at Amsterdam[17] ($\langle r^2\rangle_C^{1/2} = 2.453 \pm 0.008$ fm) and the earlier ones at Darmstadt.[16] The Amsterdam measurements agree very well with the Stanford results,[15] which seems reassuring although not mandatory. In addition, the agreement between low[7] and high[8] energy measurements of ^{40}Ca is much improved if the Amsterdam carbon measurements are used for normalization of the low-energy data.

We estimate one standard deviation in the combined results in the following way. We combine quadratically a typical error due to the quoted uncertainty in the carbon radius, 100% of the difference between the calculated dispersion and nuclear polarization corrections, 20% of the calculated Lamb shift, and the standard deviation (scatter) of the six points in Fig. 2. This last contribution assumes that the six experiments contain no further common systematic error. This yields

$$\delta R_k = 0.046 \pm 0.023 \text{ fm} \quad \text{(Darmstadt normalization)}$$

$$= -0.002 \pm 0.014 \text{ fm} \quad \text{(Amsterdam normalization)}.$$

Three standard deviations (in either case) would encompass all of the points plus all of the above systematic errors added linearly. We interpret this difference in terms of a difference between $\langle r^2\rangle_\mu$ and $\langle r^2\rangle_e$. It is the mean square radii which are additive; in self-evident notation

$$\langle r^2\rangle_{N\mu} = \langle r^2\rangle_N + \langle r^2\rangle_\mu ,$$

$$\langle r^2\rangle_{Ne} = \langle r^2\rangle_N + \langle r^2\rangle_e ,$$

$$\tfrac{3}{5}\{[R_2(N\mu)]^2 - [R_2(Ne)]^2\} = \langle r^2\rangle_\mu - \langle r^2\rangle_e = \tfrac{6}{5}R_2\delta R_2 .$$

We assume

$$\delta R_k \simeq \delta R_2 ,$$

which leads to

$$\delta_{\mu-e}\langle r^2\rangle \simeq \tfrac{6}{5}R_2\delta R_k .$$

We thus obtain

$$\delta_{\mu-e}\langle r^2\rangle = 0.24 \pm 0.12 \text{ fm}^2 \quad \text{(Darmstadt)}$$

$$= -0.01 \pm 0.07 \text{ fm}^2 \quad \text{(Amsterdam)}.$$

This form-factor interpretation is compatible with muon $g-2$ experiments and high-energy e-p and μ-p scattering experiments only for the Amsterdam normalization.

Within the framework of μ-e "universality," differences between μ-N and e-N effective interactions are to be expected, as has been pointed out and estimated by Barshay.[18] Leptons do couple (order α^2) to spin-zero mesons via a two-photon intermediate state, and the mesons in turn couple strongly to the nucleon. The amplitude for this process contains a factor of the lepton mass. In the case of the π^0, all quantities are known experimentally when the π^0 and the two photons are real, and this puts a lower bound on the process. Pseudoscalar mesons (π^0) are not of direct interest here because they lead to an effective $\vec{\sigma}_l\cdot\vec{\sigma}_N$ interaction which is nuclear-spin dependent and vanishes in lowest order for even-even nuclei. For the relatively weak interactions considered here, such an effect is further reduced from that produced by an equally strong scalar interaction by a factor of order $(m_\pi^2/4m_N m_l)(Z\alpha m_l/m_\pi)$, where m_l and m_N are the lepton and nucleon masses, respectively. Coupling via scalar mesons (such as the ϵ) is capable of producing an observable effect, and drops off inversely as the energy at high energy. At low energies, the interaction can be approximated by a Yukawa form

$$V(r) = (G/4\pi)e^{-m_\epsilon r}/r .$$

In both electron scattering and μ-atom experiments, the inclusion of such a potential changes the deduced nuclear size (to lowest order in

m_l/m_ϵ) by

$$\delta\langle r^2\rangle_N = -\frac{6A}{Z}\left(\frac{G}{4\pi\alpha}\right)\frac{1}{m_\epsilon{}^2}.$$

With $m_\epsilon = 750$ MeV, a value $G \lesssim 0.01$ would be allowed by the present data, assuming that the Amsterdam normalization and the nuclear polarization/dispersion calculations are correct to within the errors which we assign. We have examined other tests of QED and found no experiment which would rule out a coupling as large as $G = 0.01$.

Other authors have considered direct coupling of scalar mesons to leptons.[18-20] Such coupling is phenomenologically equivalent to the above description, although the linear dependence on the lepton mass is not manifest. We should note that since none of the nuclei considered here has a large neutron excess, the present results do not set a very severe limit on the size of an interaction which couples to protons and neutrons with opposite sign. Such an interaction an order of magnitude larger ($G = 0.1$) would be easily allowed.

We close with an appeal for further electron scattering experiments in the range $q \sim q_{eq} \simeq 0.3$ fm^{-1}, including experiments on neutron-rich nuclei and a redetermination of the ^{12}C absolute cross section. By further reducing the uncertainties in the comparison, we have the possibility of either clarifying nuclear polarization/dispersion effects or of obtaining a quantitative measure of anomalous lepton-nuclear interactions. We should also look to future experiments involving muons and nucleons, such as muonic hydrogen and muon-proton (muon-nucleus) scattering at medium energy. A coupling $G = 0.01$ and $m_\epsilon = 750$ MeV would produce a 0.1% change in the $2s_{1/2} - 2p_{1/2}$ energy in muonic hydrogen, and 0.7% in muonic helium. According to estimates,[18,19] the same coupling could produce a 1–2% effect in μ-p scattering at 200 MeV.

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