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Extrapolation of the pp Forward Scattering Amplitude

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The pp forward spin-nonflip amplitude is analytically continued by Ciulli-Cutkosky-Deo method. The resultant amplitude has qualitatively the same characteristic features as that observed by Sugawara and the present author in their iterative method. In particular, it is quite conclusive that the spin flip cannot be neglected in the high-energy region up to about 30 GeV/c.

Recently Sugawara and the present author[1] have calculated the pp forward spin-nonflip amplitude by their newly proposed iterative method, which consists of an iterative evaluation of the usual dispersion relation and the inverted one. The remarkable feature of the result is that the imaginary part has a large dip just above the $\bar{p}p$ threshold, which makes its high-energy real part nearly twice that determined experimentally ignoring spin-flip.[2] It would be, however, worthwhile to confirm the above result by an entirely different way of continuation, since in many cases the analytic continuation turns out to be drastically model- and/or method-dependent.[3-5] In this note we evaluate the amplitude by applying the method introduced by Ciulli, Cutkosky, and Deo,[6] and developed by Chao and Pietarinen[7] in their KN analysis. The method is known to be the best, as long as we do the continuation in terms of a series expansion of a certain function. The main conclusions are summarized at the end of this note.

First we conformally map the lab-energy ω plane onto the inside of a unifocal ellipse in the z plane as shown in Fig. 1. It is assumed the relevant total cross section σ_T is known above D and C, while the data on the real part are also available in the region $[A, B]$. We assume the same experimental input as in Ref. 1, which includes the 450-MeV phase-shift analysis.[8] We choose D, A, and B at 16-, 70- and 350-MeV kinetic energies, respectively. C is fixed at the $\bar{p}p$ threshold. z_∞ corresponds to $\omega=\infty$. The sum R of semimajor and semiminor axes becomes 9.79, which gives a reasonable asymptotic convergence rate.

Our discrepancy function $D(z)$ is constructed in the following way. Let $A(z)$ be the spin-averaged spin-nonflip forward amplitude normalized by $\mathrm{Im}A=(\omega^2-m^2)^{1/2}\sigma_T$, where m is the nucleon mass. We introduce $T(z)$ by

$$T(z)=[A(z)-V(\omega)]/(\omega-m)-f_\pi/(\omega-\omega_\pi), \qquad (1)$$

where

$$V(\omega)=\frac{1}{\pi}\int_m^\infty \frac{\mathrm{Im}\,V(\omega')}{\omega'-\omega}\,d\omega',$$

with $\mathrm{Im}\,V(\omega)=4\pi k/(a^{-2}+k^2)$ and $k=[\frac{1}{2}m(\omega-m)]^{1/2}$. Here a is the S-wave scattering length; therefore the threshold behavior is guaranteed automatically. f_π is so chosen that $T(z)$ has no pion pole. Then, $D(z)$ is defined by

$$D(z)=[H(z)-H(z_\infty)]/(z-z_\infty), \qquad (2)$$

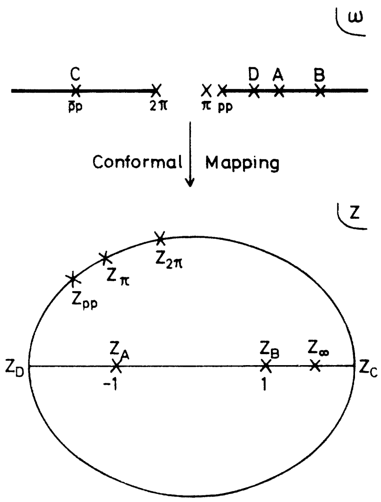

FIG. 1. Conformal mapping from the lab-energy ω plane to the z plane. Above D and below C, the total cross-section data for pp or $\bar{p}p$ are available. Between A and B the reliable real part of pp forward spin-nonflip amplitude is available. z_∞ corresponds to $\omega=\infty$.

where

$$H(z)= T(z) - \frac{1}{\pi}\int_{z_D}^{z_C} \frac{\operatorname{Im} T(z')}{z'-z}\,dz' . \tag{3}$$

$D(z)$ thus defined is analytic inside the whole ellipse and real on the real axis, while it has weak logarithmic singularites at $z=z_C$ and z_D. The singularities, however, cause no practical trouble. We assume the Pomeranchuk theorem to make the integral in (3) convergent. The real-imaginary ratio will vanish at $z=z_\infty$.

The discrepancy function $D(z)$ is now expanded in terms of polynomials in z, and the coefficients $d_1, d_2, \ldots$ can be determined by utilizing the experimental information in the region $[z_A, z_B]$. The polynomials are orthonormalized with respect to the experimental errors in the least-squares sense. We assume, for simplicity, a uniform error $\Delta \operatorname{Re} A$ in the range 1–4 GeV^{-1}. The truncation problem can be solved by introducing the same probability density for the coefficients c_1, $c_2, \ldots$ of truncating series as that devised in Ref. 7. The most probable values for $c_1, c_2, \ldots$ can

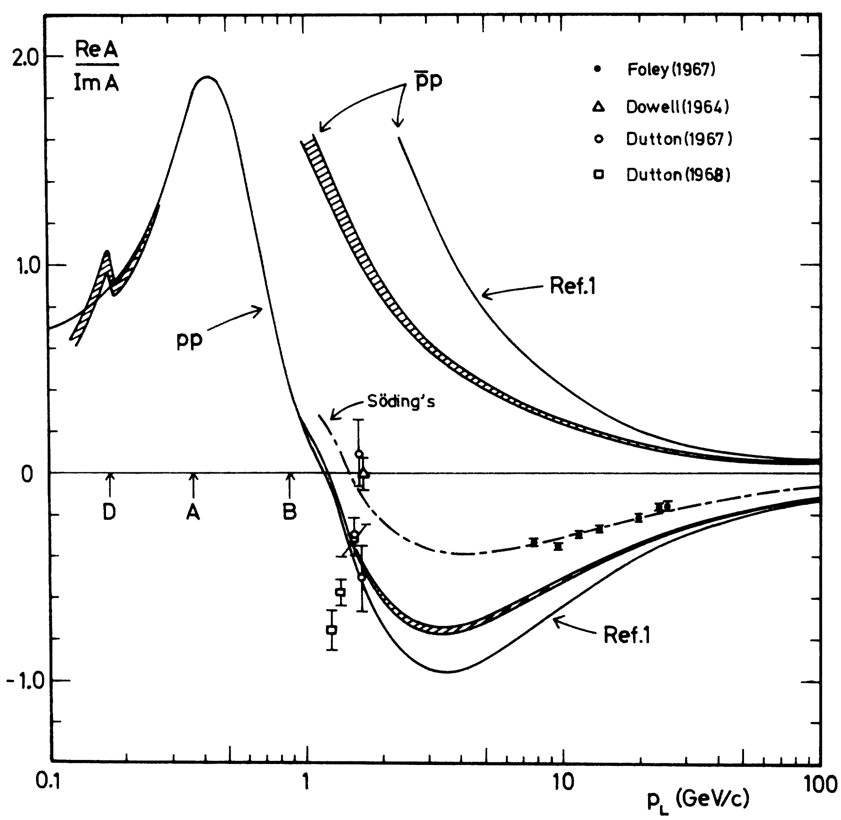

FIG. 2. Real-imaginary ratios for pp and $\bar{p}p$ are shown, when $\Delta \operatorname{Re} A = 2.0\ \mathrm{GeV}^{-1}$ and $a=-7.8$ fm. The result obtained by the iterative method (Ref. 1) is also shown for comparison. The high-energy data (Ref. 2) determined ignoring spin flip are definitely above our prediction, though they are consistent with the solution (Ref. 4) without the dip. Near the GeV region there is a confusion among the experimental data (Ref. 9).

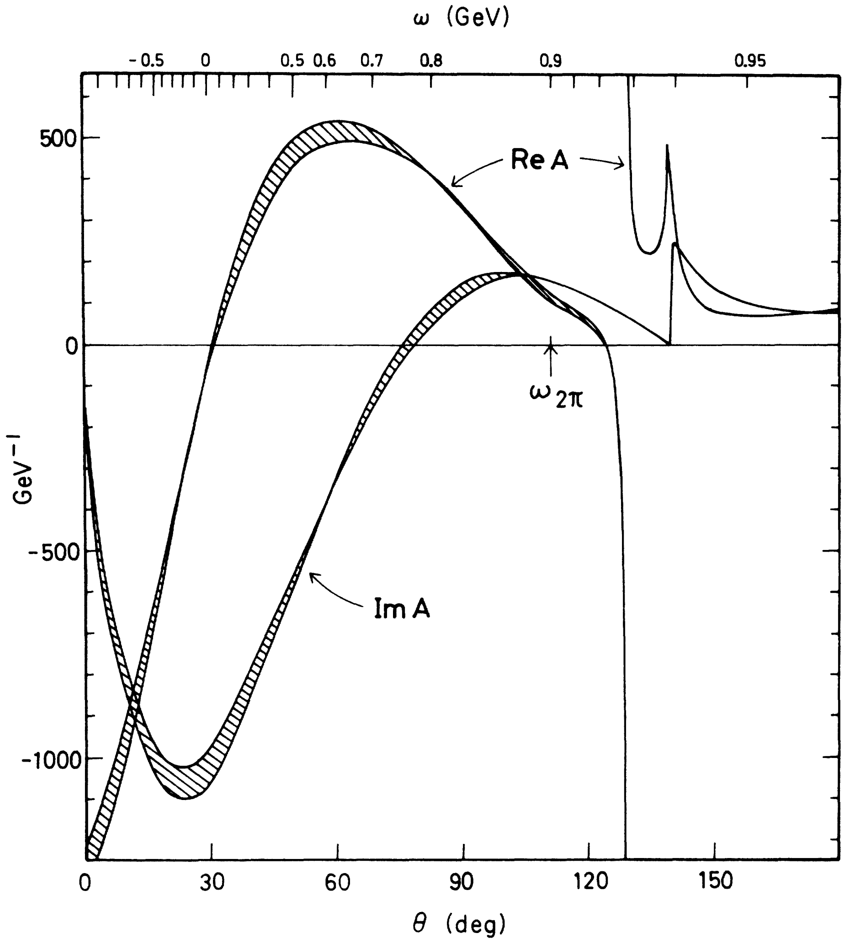

FIG. 3. ReA (z) and Im$A(z)$ on the ellipse are shown. θ is the phase angle of $\zeta = z + (z^2 - 1)^{1/2}$, which maps the ellipse on a circle with radius R. ΔReA and a, the S-wave scattering length, are the same as in Fig. 2.

be determined by maximizing the density with respect to $c_1, c_2, \ldots$ and the scale factors[7] λ where the coefficients $d_1, d_2, \ldots$ are considered as statistical data. The error thus estimated involves both a truncation error and the statistical one.

Now we discuss our numerical results. The calculated real-imaginary ratio in case of ΔReA $= 2.0$ GeV^{-1} and $a = -7.8$ fm is shown in Fig. 2 with the one obtained in Ref. 1. The variation of ΔReA in the assumed range does not change the result significantly, and so for the variation of a from -7.8 to -20 fm. The ratio for pp runs slightly above the previous one[1]; however, it is still definitely below the experimental data[2] in the range 7–26 GeV/c, which are determined ignoring the spin-flip amplitude. The ratio is also consistent with the analysis of Ref. 5, where a dispersion integral over the unphysical cut is simply expanded in terms of Chebyshev polynomials. Near the GeV region our result will help to dissolve the confusion of mutually inconsistent experimental data.[9] Our ratio for $\bar{p}p$ is nearly two times smaller than that of Ref. 1. The predicted amplitude on the ellipse is shown in Fig. 3. Its imaginary part on the unphysical cut is qualitatively similar to that of Ref. 1. That is, it has a large broad dip on the side of the $\bar{p}p$ threshold and a peak near the 2π threshold, though the area above the dip is only one third of the previous prediction,[1] and we do not observe any abrupt change just above the $\bar{p}p$ threshold. These would be expected, however, since the probability density used in truncation makes the amplitude on the ellipse small and smooth as possible. The variations of the positions for D, A, and B within the reasonable range do not affect all of the results mentioned above.

We summarize our main conclusions:

(i) The Ciulli-Cutkosky-Deo method works quite stably in determining the pp forward amplitude. The qualitative features of the predicted amplitude are consistent with that of Ref. 1, e.g., the imaginary part has a broad dip above the $\bar{p}p$ threshold and a peak near 2π threshold.

(ii) In particular, the predicted Re/Im ratio for pp is again large enough for us to conclude that the spin-flip amplitude cannot be neglected even in the forward direction in the high-energy region below 30 GeV/c. The ratio is also definite enough to dissolve the experimental confusion in the near-GeV region.

(iii) One does, however, not observe any abrupt change in the imaginary part at the $\bar{p}p$ threshold, and the depth of the dip is only one fifth that of Ref. 1, which would be expected in the present method of continuation.

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New Limits on the Difference Between the Muon- and Electron-Nuclear Interactions*

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Muonic-atom and electron-nucleus scattering measurements of nuclear radii have been examined in light nuclei in order to test their relative consistency. The electron data were selected to provide information at the "equivalent momentum transfer," which is approximately 0.3 fm^{-1}. All corrections known to us were applied or considered. Scattering experiments normalized to the Darmstadt ^{12}C cross-section work are found to exhibit a systematic difference from the muon results. When these experiments are renormalized to the recent Amsterdam ^{12}C work, the discrepancy disappears. In the latter case, no muon/electron measurements disagree by more than 0.02 fm. We estimate the standard deviation to be 0.014 fm about a mean of -0.002 fm. The possibility of anomalous lepton-nucleon interactions is discussed. The limits found here still admit observable anomalous effects in, for example, muonic H and He Lamb shifts and muon-nucleon scattering experiments.

The muon and electron are generally assumed to be point Dirac particles which differ from each other only in mass. Colliding-electron-beam experiments have shown no violation of quantum electrodynamics (QED) and the assumption of a point electron up to momentum transfers of 8 fm^{-1}. These experiments may be interpreted as placing a limit on the electron "size" of $\langle r^2\rangle_e < 0.006\ fm^2$. The agreement of the muon $g-2$ experiment with QED places a limit on the muon "size" of $\langle r^2\rangle_\mu < 0.004\ fm^2$ within 2 standard deviations. Comparisons between high-energy (12 GeV) e-p and μ-p scattering experiments indicate $\langle r^2\rangle_\mu - \langle r^2\rangle_e < 0.014\ fm^2$ with 97.7% confidence.[1]

In the present work we have examined existing μ-atom and e-nucleus scattering data in order to test their relative consistency. A persistent, otherwise unaccountable discrepancy might in-