PHYSICAL REVIEW D VOLUME 7, NUMBER 9 1 MAY 1973

Isobar Analysis of $\gamma p \to \eta p$

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(Received 25 September 1972)

An analysis of the available data below 2.24-GeV c.m. energy for the process $\gamma p \to \eta p$ yields values of $G \equiv \Gamma(R \to \gamma p)\Gamma(R \to \eta p)$ for several s-channel resonances, R. By varying the details of the direct-channel isobar model it is found that the values of G are reasonably model-independent. When these values are combined with information from other reactions it is possible to calculate electromagnetic widths for the important resonance states in this process. These include essential contributions from S_{11} and P_{11} states; in addition, admixtures of P_{13}, D_{13}, F_{17}, and G_{17} resonances are also needed in order to obtain a good fit to the data. An excellent fit is found by including some recently conjectured stray baryonic states which do not couple significantly to the πN channel.

I. INTRODUCTION

We have analyzed the older data,[1-7] the corrected data[8] from Ref. 9, and the new data[8] for the process

$$\gamma p \to \eta p \qquad (1)$$

by use of a direct-channel resonance model which includes a number of features not previously applied to this reaction. (The new and corrected data which we have used are given in Table I.) Moreover, in the course of extensive searching for good fits, features of the model, described in Sec. II, were varied in order to isolate those parts of our results which are relatively model-independent.

The analysis has been conducted in two parts. The first part includes an energy-dependent analysis of all data below laboratory photon energy $E_\gamma = 1.5$ GeV, and the second part includes all data below $E_\gamma = 2.2$ GeV. The reason for having two cutoff energies is due to the sparseness of data at the higher energies. As might be expected, this second part of the analysis is similar to the first part at low energy and shows significant deviation only in the high-energy region.

Our conclusions regarding which resonances are present differ somewhat from the conclusions of recent analyses and reviews.[10-14] In addition, we find that there are certain partial-decay rates which we can calculate at an improved level of confidence, since they do not seem to be strongly dependent upon the variations in features of the parametrization.[15]

It has been nine years since Nishimura[16] first proposed applying a resonance model to reaction (1). Meson exchanges were included as well as s-channel resonances, although S_{11} and P_{11} resonances were not considered. During the same year Minami and Moss[17] studied the same reaction and pointed out the possibility of a strong D_{13} contribution near threshold. Later, Minami[18] further studied η photoproduction and was able to estimate the radiative width of the $S_{11}(1550)$ resonance. More work by Minami[19] indicated the need for both D_{13} and S_{11} states near threshold. However, using a K-matrix approach, Logan and Uchiyama-Campbell[20] were able to explain the enhancement of the $\gamma p \to \eta p$ total cross section near threshold in terms of only the S_{11} resonance, and Heusch[21] reported that either a P_{11} or S_{11} resonance could be used to match the data near threshold with preference given to a P_{11} state at about 1480 MeV.

The first elaborate fitting of differential-cross-section data with various possible combinations of pole and resonance contributions was performed by Deans and Holladay[10,11] using 44 experimental points. The major resonance contributions were found to come from the $S_{11}(1550)$ and the $D_{13}(1520)$ with a small but important additional contribution from higher partial waves which they attributed to the $F_{15}(1688)$. Little need was found for a P_{11} state; however, they included it with a rather low mass of 1400 MeV. In 1969 there were 130 data points available, and Bajpai and Donnachie[13] applied a generalized interference model to analyze these data. It was concluded that a good representation of the data was given by Reggeized ω and B exchange plus the $S_{11}(1550)$ and $P_{11}(1780)$ resonance states. Recently, in the course of analyzing deuteron reactions, Hicks[12] reanalyzed the data for (1) using 85 data points below photon lab energy $E_\gamma = 1.0$ GeV, and concluded that the only clearly

TABLE I. Data for the process $\gamma p \to \eta p$. We have used that portion of the new and corrected data for η photoproduction, kindly supplied to us by J. R. Holt (Ref. 8), below $E_\gamma = 2.2$ GeV. The points used in this analysis are tabulated below. They are normalized in accordance with $R_{\gamma\gamma}$ as given in Eq. (19).

E_γ (GeV)	$\theta_{\text{c.m.}}$ (deg)	$\frac{d\sigma}{d\Omega}$ (μb/sr)	$\Delta\frac{d\sigma}{d\Omega}$ (μb/sr)
1.24	12	0.342	0.140
1.24	18	0.287	0.047
1.24	26	0.358	0.062
1.25	32	0.549	0.322
1.53	7	0.315	0.075
1.52	14	0.250	0.028
1.52	22	0.321	0.046
1.51	32	0.275	0.042
1.52	39	0.233	0.032
1.54	46	0.236	0.059
1.84	7	0.205	0.084
1.83	15	0.273	0.035
1.83	19	0.363	0.068
1.84	22	0.276	0.062
1.83	26	0.319	0.039
1.84	32	0.227	0.041
1.84	33	0.345	0.068
1.86	39	0.192	0.025
1.87	46	0.192	0.067
1.97	28	0.374	0.050
1.97	35	0.309	0.033
1.97	42	0.250	0.051
2.04	28	0.325	0.046
2.03	35	0.266	0.037
2.03	42	0.221	0.049
2.11	28	0.293	0.045
2.10	35	0.265	0.033
2.10	42	0.231	0.046
2.19	28	0.284	0.047
2.16	35	0.231	0.034
2.16	42	0.247	0.044

observable resonance in the region was the $S_{11}(1550)$.

The data below $E_\gamma = 1.0$ GeV are very nearly isotropic, and the enhancement in this region is now generally agreed to be mostly due to the $S_{11}(1550)$ resonance. Above 1.0 GeV the data are more sparse and the angular dependence may be more complicated. Recent polarization measurements[9] indicate that the polarization is positive from 0.83 to 1.1 GeV. This result gives strong support for the presence of the P_{11} state used by Bajpai and Donnachie[13] since this polarization is compatible with a P- and S-wave interference. In a preliminary analysis of the data[1-6] including the polarization data,[7] Deans[22] found that a small admixture of higher partial waves was needed in addition to the dominant $S_{11}(1550)$ and $P_{11}(1750)$ states. That such an admixture of additional states could not be excluded was also pointed out by Donnachie at the 1971 Cornell Conference.[14]

Unfortunately there are not yet enough data at any energy to do a model-independent partial-wave analysis. When such an analysis does become feasible, the application of Cutkosky's ACE[23] (accelerated-convergence expansion) method of analysis (using the analyticity properties to maximize the convergence of the partial-wave series) will provide the most efficient use of sparse data. Thus, as in all previous analyses, we are forced to adopt a model. We use a resonance mode, similar in part to that of Deans and Holladay,[10] the details of which are described in Sec. II. Basically, each resonance appears as a Breit-Wigner form, but several details of the parametrization have been allowed to vary. In view of duality,[24,25] one must take special care in regard to exchange contributions. Our calculations with exchanges included have shown that they are not required by the data, and we have excluded them from the model. Thus, there are no double-counting problems present in this analysis. The remaining major part of the model includes nonresonant background terms, in place of the nucleon Born terms; these are included using the method of Orito[26] and Schorsch *et al.*[27]

II. MODEL

A. Pole Terms

We considered three types of terms: pole terms, resonance terms, and background terms. The pole terms (proton pole and ω exchange) were calculated as in Ref. 10. These terms did not contribute significantly to our better solutions and with the current level of data can be replaced by nonresonant background terms discussed below. Since we have not used pole terms in any of the solutions reported in this paper we do not reproduce the parametrizations of these terms here.

B. Resonance Terms

For the resonance contributions to the electric and magnetic multipole amplitudes we assume

$$E_{l\pm} = \frac{-ie^{i\phi}(\Gamma^E_{\gamma p}\Gamma_{\eta p})^{1/2}}{2[qkj(j+1)]^{1/2}(W_r - W - \frac{1}{2}i\Gamma)}, \tag{2}$$

and similarly for $M_{l\pm}$, where k and q are c.m. momenta of the γ and η, respectively, l is the final-state orbital angular momentum, $J = l \pm \frac{1}{2}$ is the total angular momentum, W is the total c.m. energy, W_r is the resonance mass, and $j = l \pm 1$ for $E_{l\pm}$ and $j = l$ for $M_{l\pm}$. The total energy-dependent width Γ is a sum of partial widths for the given resonance state, and it is approximated by

$$\Gamma = \frac{p v_l(Rp)}{p_r v_l(Rp_r)} \Gamma_r, \tag{3}$$

where R is an interaction radius (chosen to be 1 F in the quoted results), the v_l's are barrier factors,[28] and the index r means evaluated at resonance. Consequently, Γ reduces to Γ_r at $W=W_r$. The p is a characteristic momentum for the process; in the quoted results it is taken to be the momentum a pion would have in formation or decay of the state, $\pi N \to N^*$. For the important $S_{11}(1550)$ we have refined prescription (3) by writing

$$\Gamma = \Gamma_{\eta N} + \Gamma_{\pi N} + \Gamma_{\text{other}} \tag{4}$$

$$= \frac{\alpha q_{\eta N}}{q^r_{\eta N}}\Gamma_r + \frac{\beta q_{\pi N}}{q^r_{\pi N}}\Gamma_r + \Gamma_{\text{other}}, \tag{5}$$

such that

$$\Gamma_{\text{other}} = (1-\alpha-\beta)\Gamma_r . \tag{6}$$

In the results here we use the simple choice $\alpha = 0.6, \beta = 0.4$, and $\Gamma_{\text{other}} = 0$. The main contribution to Γ_{other} would probably be the three-body state $N\pi\pi$. We have done calculations in which α and β were allowed to vary with $\alpha + \beta = 1$, and the results are rather insensitive to these variations.

We also experimented as in Ref. 29 with exponentially damping the resonance expression (2) outside the region $W = W_r \pm \frac{1}{2}\Gamma_r$, and we tried omitting the energy dependence altogether, $\Gamma(W) = \Gamma_r$. Both of these modifications cause a noticeable effect, but no over-all change in final results and conclusions.

The product of the partial widths in (2) is given by

$$(\Gamma^E_{\gamma p}\Gamma_{\eta p})^{1/2} = [2kRv_n(kR)]^{1/2}[2qRv_l(qR)]^{1/2}\gamma^E , \tag{7}$$

and similarly for $E \to M$, where $n=l$ except for E_{l-} when $n = l-2$. The real numbers and γ^E γ^M are considered free parameters (unless they are known from other studies) to be varied in obtaining a fit to the data. Thus for each resonance (except S_{11} for which $\gamma^M=0$ and P_{11} for which $\gamma^E=0$) there are at least three real parameters to be varied, γ^E, γ^M, and the phase angle ϕ from (2).

The inclusion of a phase ϕ on the coupling is one aspect of our attempt to develop a more flexible phenomenological formulation.[30,31] In an inelastic channel, unitarity does not constrain the phases of the amplitudes. In fact, measurements of the unpolarized differential cross section and the polarization do not determine the over-all phase.

We are aware of the fact that the resonance mass W_r and width Γ_r as quoted in the compilations[32] may be model-dependent. We have therefore permitted some variation in these quantities. Carried to extremes this can be a dangerous procedure, so we have been rather cautious, and the masses and widths finally used are all consistent with currently quoted values.[32]

C. Background Terms

The third type of term used is the background term. Along with the introduction of the phase angle ϕ, the slowly varying background allows each multipole to have a somewhat more general form. Even if the inclusion of a background is a productive idea, its precise form is at the moment rather obscure. We have used one term from the prescription used by Orito[26] and by Schorsch *et al.*[27] for S_{11} and P_{11} states, which incorporates a desirable threshold behavior. Specifically,

$$E^b_{0^+} = \frac{(a+ib)R^2k}{[2(1+Rq)(1+R^3k^3)]^{1/2}} \tag{8}$$

and

$$M^b_{1^-} = \frac{(a'+ib')R^3qk}{[2(1+R^3q^3)(1+R^3k^3)]^{1/2}}, \tag{9}$$

where a, b, a', and b' are real parameters.

D. Cross Section and Polarization

The multipole amplitudes contribute to the Chew-Goldberger-Low-Nambu (CGLN) amplitudes[33] $\mathfrak{F}_i$ as indicated in Table II. In terms of these $\mathfrak{F}_i$ amplitudes the unpolarized differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}\{|\mathfrak{F}_1|^2 + |\mathfrak{F}_2|^2 - 2x\,\text{Re}(\mathfrak{F}_1^*\mathfrak{F}_2) + \tfrac{1}{2}(1-x^2)[|\mathfrak{F}_3|^2 + |\mathfrak{F}_4|^2 + 2\,\text{Re}(\mathfrak{F}_1^*\mathfrak{F}_4) + 2\,\text{Re}(\mathfrak{F}_2^*\mathfrak{F}_3) + 2x\,\text{Re}(\mathfrak{F}_3^*\mathfrak{F}_4)]\}, \tag{10}$$

where $x = \cos\theta_{\text{c.m.}}$, and the polarization in the direction $\hat{k}\times\hat{q}$ is given by

$$\mathcal{P}\frac{d\sigma}{d\Omega} = \frac{q}{k}(1-x^2)^{1/2}\,\text{Im}[-2\mathfrak{F}_1^*\mathfrak{F}_2 - \mathfrak{F}_1^*\mathfrak{F}_3 + \mathfrak{F}_2^*\mathfrak{F}_4 + (1-x^2)\mathfrak{F}_3^*\mathfrak{F}_4 + x(\mathfrak{F}_2^*\mathfrak{F}_3 - \mathfrak{F}_1^*\mathfrak{F}_4)]. \tag{11}$$

E. Comments About the Model

Our model is thus made up of direct-channel resonances with S- and P-wave background terms. Several features of the model may be modified with relatively small effect upon the product of the partial widths (7) determined in fitting the data. These features include the characteristic momentum p, the interaction radius R, the barrier factors v_l, and the energy dependence of the total width Γ. In particular, we find it is the value of the sum

$$G = G_E + G_M , \tag{12}$$

TABLE II. Contributions to the $\mathcal{F}_i$ amplitudes.

	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$	$\mathcal{F}_4$
S_{11}	E_{0+}			
P_{11}		M_{1-}		
P_{13}	$3x(M_{1+}+E_{1+})$	$2M_{1+}$	$3(E_{1+}-M_{1+})$	
D_{13}	$3M_{2-}+E_{2-}$	$6xM_{2-}$		$-3(M_{2-}+E_{2-})$
D_{15}	$\frac{3}{2}(5x^2-1)(2M_{2+}+E_{2+})$	$9xM_{2+}$	$15x(E_{2+}-M_{2+})$	$3(M_{2+}-E_{2+})$
F_{15}	$3x(4M_{3-}+E_{3-})$	$\frac{9}{2}(5x^2-1)M_{3-}$	$3(M_{3-}+E_{3-})$	$-15x(M_{3-}+E_{3-})$
F_{17}	$\frac{5}{2}(7x^3-3x)(3M_{3+}+E_{3+})$	$6(5x^2-1)M_{3+}$	$\frac{15}{2}(7x^2-1)(E_{3+}-M_{3+})$	$15x(M_{3+}-E_{3+})$
G_{17}	$\frac{1}{2}(15x^2-3)(5M_{4-}+E_{4-})$	$(70x^3-30x)M_{4-}$	$15x(M_{4-}+E_{4-})$	$-\frac{1}{2}(105x^2-15)(M_{4-}+E_{4-})$

where

$$G_E=(\Gamma^E_{\gamma p}\Gamma_{\eta p})_r \tag{13a}$$

and

$$G_M=(\Gamma^M_{\gamma p}\Gamma_{\eta p})_r\,, \tag{13b}$$

which is important and reasonably invariant under various reasonable parametrizations.

Even in view of the relative stability of G there are centainly objections which can be made in regard to the parametrization. Several are given in Ref. 34, where this same model is used to study the process $\gamma p\to K^+\Lambda$. The difficulty with more general parametrizations and more sophisticated models is that so many more unknown parameters are introduced that it becomes virtually impossible to obtain stable solutions. Until the level of the data improves we feel justified in using a rather simple and perhaps somewhat crude model to study these reactions.

III. MULTIPLE MINIMA AND RESTRICTIONS

At the current time there are 186 differential-cross-section and polarization data points below $W=2.238$ GeV c.m. energy ($E_\gamma=2.2$ GeV), and 147 points below $W=1.922$ GeV ($E_\gamma=1.5$ GeV). The model outlined in Sec. II could have over $5N$ adjustable parameters just from γ^E, γ^M, ϕ, W_r, Γ_r, and the background if only the N well-established isospin-$\frac{1}{2}$ nucleon resonances below approximately 2 GeV are included. By most standards N is a number greater than 8, and any attempt to fit these data points with such a large number of free parameters would lead to such a serious multiple-minima problem that very little could be learned by fitting the data. It is therefore necessary to look for as many constraints as possible for the various parameters.

The most obvious first constraint is to fix the values of W_r and Γ_r from studies of strong interactions. For this, we simply use the compilation

TABLE III. Maximum parameter values.

State	Amplitude	$10^6(x_\eta x_{E,M})_{max}$	$\gamma^{E,M}_{max}$ (MeV)	$(\sigma_r)_{max}$ (μb)	$(\sigma_{exp})_{max}$ (μb)
$S_{11}(1550)$	E_{0+}	2500	2.1	25.2	15.0
$S_{11}(1700)$	E_{0+}	150	0.6	1.02	1.7
$P_{11}(1750)$	M_{1-}	1000	1.9	6.3	1.8
$P_{13}(1860)$	E_{1+}		0.65 [a]		1.5
	M_{1+}		0.65 [a]		
$D_{13}(1520)$	E_{2-}	22	0.50	0.49	15.0
	M_{2-}	7.5	0.33	0.17	
$D_{15}(1670)$	E_{2+}	2	0.06	0.045	2.0
	M_{2+}	4	0.09	0.090	
$F_{15}(1688)$	E_{3-}	5	0.11	0.11	1.7
	M_{3-}	5	0.13	0.11	

[a] Estimated from maximum value of σ_{exp}.

of Ref. 32. Second, studies of pion photoproduction[14,35–37] and η production[29,38,39] through the reaction $\pi^- p \to \eta n$ may be used to obtain restrictions on G_E and G_M for some states. Thus, the values of γ^E and γ^M may be determined within limits for certain resonance states. In Table III we give generous approximate ranges for various parameters associated with those states which have been studied in sufficient detail to have reasonably well-known partial widths.

The maximum reasonable values for the dimensionless products $x_E x_\eta$ and $x_M x_\eta$ in Table III, where

$$x_E = \left(\frac{\Gamma^E_{\gamma p}}{\Gamma}\right)_r , \tag{14}$$

$$x_M = \left(\frac{\Gamma^M_{\gamma p}}{\Gamma}\right)_r , \tag{15}$$

and

$$x_\eta = \left(\frac{\Gamma_{\eta p}}{\Gamma}\right)_r , \tag{16}$$

were deduced from Refs. 14, 29, 31, 35–39. In cases of uncertainty or disagreement among different determinations, we have been rather generous in our upper limit estimate. These maximum values may be used to find approximate upper limits on the magnitudes of the γ^E and γ^M parameters which are also given in Table III. It is instructive to tabulate maximum contributions to the total cross section made by each of the resonance states, since in this way one can immediately see whether or not a given state is likely to be observed in the reaction.

Evaluated at resonance, $W = W_r$, the total cross section σ_r in process (1) from a single (noninterfering) E or M multipole is given by

$$\sigma_r = \frac{\pi(2J+1)}{k_r^{\,2}} x_\nu x_\eta , \tag{17}$$

where ν is either M or E. For comparison in Table III we also list a generous upper limit on the experimental value of the total cross section $\sigma_{\rm exp}$.

Upon examining these results in Table III we see that it is quite unlikely one would find strong evidence for either the $D_{15}(1670)$ or the $F_{15}(1688)$ in η photoproduction. In addition, we have realistic information on likely ranges for some of the variable parameters which appear in the model. It will be noted that for two states, $S_{11}(1550)$ and $P_{11}(1750)$, the value of σ_r is greater than $\sigma_{\rm exp}$. Thus, we do not expect to find values of γ^E and γ^M as large as the upper limits given for these two states.

There is a third constraint which may be used to eliminate multiple mimima. We require that the polarization at $\theta_{\rm c.m.} = 90°$ between $E_\gamma = 0.8$ and 1.1 GeV lie between 0.1 and 0.8 in accord with the results of Ref. 7.

A fourth constraint is to require that acceptable solutions agree with the recent results of Heusch *et al.*[40] in regard to the backward-forward asymmetry in the differential cross section from threshold to $E_\gamma = 1.1$ GeV.

Solutions with all four of the above restrictions imposed are called type A solutions. Those in which the first restriction is relaxed, i.e., for some states the parameters Γ_r and W_r are allowed to vary within restricted ranges, are called type B solutions. It is also interesting to consider solutions (type C) which include the last three restrictions, but allow for stray baryonic states as conjectured by Donnachie[41] for reactions $\gamma p \to \eta p$ and $\gamma p \to K^+\Lambda$. The relationship of these stray states to the latter process has been studied recently by Deans *et al.*[34,42]

TABLE IV. Parameters for type-D solutions.[a]

State	Parameters	Solutions $D1$	$D2$	$D3$	$D4$
S_{11}	W_r	1540	1544	1552	1550
	Γ_r	97	123	145	145
	γ^E	1.10	1.68	1.82	1.88
	ϕ	90	90	90	90
S_{11}	W_r			1712	1703
	Γ_r			174	139
	γ^E			0.50	0.38
	ϕ			28	21
P_{11}	W_r				1456
	Γ_r				300
	γ^M				−0.20
	ϕ				7
P_{11}	W_r	1800	1761	1800	1782
	Γ_r	400	145	372	365
	γ^M	0.69	−0.10	0.15	0.26
	ϕ	34	148	154	151
Background	a		2.60	−1.88	−3.51
	b		−13.12	−4.22	−5.56
	a'		2.56	4.51	6.35
	b'		9.31	1.87	2.78
	χ^2/N	5.93	2.71	1.60	1.57

[a] All entries are in MeV except ϕ, which is in degrees, and the background which is dimensionless and is multiplied by 10^3.

IV. METHOD

Our procedure is to minimize

$$\chi^2 = \sum_{\text{data}} \left[\frac{\frac{d\sigma}{d\Omega}(\text{theory}) - \frac{d\sigma}{d\Omega}(\text{experiment})}{\text{experimental error}} \right]^2 \tag{18}$$

by permitting certain parameters to vary. The possible variable parameters include γ^E, γ^M, ϕ, Γ_r, W_r, and the background terms. Whether or not a given parameter is in fact varied depends upon whether the solution is a type A, B, or C solution.

We use two independently written programs which ensures against errors. These programs are used in conjunction with the CERN minimizer MINUIT,[43] which has been modified slightly. For many of our minimizations we used random starting values. Specifically, for each parameter to be varied, the initial value is randomly chosen from a specified interval. For the couplings and phases this interval is generous, for the masses and widths the interval is rather small. After a minimum is found the procedure is restarted with new random starting values. This approach reduces considerably the bias due to the choice of starting values and gives a broader picture of the locations of local minima, which in turn gives a measure of the confidence which can be placed in the uniqueness of the global minimum. For each combination of parameters to be varied, we have done at least ten minimizations, and the total number of minimizations comes to well over a thousand. Thus, we have tested extensively the relative importance of each parameter.

A complication is introduced by the fact that some of the experiments involved measuring the $\gamma\gamma$ decay mode of the η. Thus, to obtain the cross section for reaction (1) requires a multiplication of the data by the ratio

$$R_{\gamma\gamma} = \frac{\Gamma(\eta \to \text{all modes})}{\Gamma(\eta \to \gamma\gamma)} \simeq \frac{1}{0.38}. \tag{19}$$

The published experimental values of this ratio

TABLE V. Values of parameters for solutions below $E_\gamma = 1.5$ GeV.[a]

State	Parameters	$A1$		$B1$		$C1$	
S_{11}	W_r	1550	1770	1538	1662	1551	1709
	Γ_r	135	245	150	100	137	195
	γ^E	1.43	0.58	1.70	0.26	1.33	0.57
	ϕ	90	−13	90	−49	90	−53
P_{11}	W_r	1470	1750	1469	1800	1575	1740
	Γ_r	240	300	243	167	176	251
	γ^M	−0.04	0.22	0.83	0.19	1.28	−0.48
	ϕ	78	50	67	63	−18	19
P_{13}	W_r	1860		1830		1830	1586
	Γ_r	300		252		330	79
	γ^E	0.25		0.19		0.0	0.40
	γ^M	0.03		0.01		0.22	0.0
	ϕ	160		128		151	−103
D_{13}	W_r	1520		1520		1530	
	Γ_r	120		120		110	
	γ^E	0.24		0.46		0.30	
	γ^M	−0.33		−0.33		−0.40	
	ϕ	−14		6		132	
F_{15}	W_r	1688		1688		1626	
	Γ_r	127		127		75	
	γ^E	0.11		−0.03		0.03	
	γ^M	−0.09		−0.11		0.54	
	ϕ	−157		24		88	
Background	a	0.09		−5.78		19.07	
	b	9.07		3.48		−1.19	
	a'	1.69		1.47		−7.46	
	b'	5.61		−2.72		−7.58	
χ^2/N		1.63		1.22		0.98	

[a] All entries are in MeV except ϕ which is in degrees, and the background which is dimensionless and is multiplied by 10^3.

have varied considerably during the time that all the experiments were performed. Consequently, we adjusted the data so that all the experiments would reflect a constant value for $R_{\gamma\gamma}$. Unfortunately, some of the experimental accounts are less than specific concerning the value of $R_{\gamma\gamma}$ used. Because of this and because of the relative experimental normalization differences, we made some computer runs in which the data from each experiment were automatically renormalized to minimize χ^2. To prevent unrealistic renormalizations, for each floating data set a term is added to χ^2 of the form

$$\frac{1-\epsilon^{-1}}{\Delta\epsilon},$$

where ϵ is the factor by which the data set is multiplied and $\Delta\epsilon$ is a constant (0.05–0.10) which constrains the amount of renormalization. These renormalized points were used in obtaining all quoted results in this paper – both tabular and graphical.

TABLE VI. Values of parameters for solutions below $E_\gamma = 2.2$ GeV.[a]

State	Parameters	$A2$		$B2$		$C2$	
S_{11}	W_r	1550	1700	1538	1662	1551	1699
	Γ_r	135	245	150	101	134	195
	γ^E	1.43	0.60	1.70	0.26	1.24	0.53
	ϕ	90	−12	90	−50	90	−48
P_{11}	W_r	1470	1750	1469	1799	1580	1728
	Γ_r	240	300	245	160	168	203
	γ^E	−0.05	0.22	0.82	0.19	1.09	−0.38
	ϕ	80	47	67	70	−21	−7
P_{13}	W_r	1860		1830		1833	1585
	Γ_r	300		254		250	82
	γ^E	0.25		0.20		0.0	0.39
	γ^M	0.01		0.03		−0.02	0.0
	ϕ	161		127		118	−112
D_{13}	W_r	1520	2040	1520	2023	1528	2090
	Γ_r	120	250	120	300	110	224
	γ^E	0.39	0.13	0.38	0.10	0.03	0.12
	γ^M	−0.33	0.01	−0.33	−0.11	−0.28	−0.11
	ϕ	−18	85	3	137	152	−61
F_{15}	W_r	1688		1688		1613	
	Γ_r	127		90		90	
	γ^E	0.11		−0.03		0.17	
	γ^M	−0.11		−0.07		0.53	
	ϕ	−155		−36		42	
F_{17}	W_r	1990		1979		1970	
	Γ_r	240		276		300	
	γ^E	0.33		−0.05		0.04	
	γ^M	0.02		0.25		0.40	
	ϕ	122		115		31	
G_{17}	W_r	2190		2206		2208	
	Γ_r	250		197		193	
	γ^E	0.36		0.34		0.22	
	γ^M	0.51		0.20		0.11	
	ϕ	−26		−176		89	
Background	a	−0.18		−5.63		19.23	
	b	9.09		3.64		−0.57	
	a'	1.66		1.36		−4.50	
	b'	5.62		−2.65		−10.30	
χ^2/N		1.68		1.47		1.30	

[a] All entries are in MeV except ϕ which is in degrees, and the background which is dimensionless and is multiplied by 10^3.

V. RESULTS

We first examine a solution (type D) which includes only S and P waves over the energy region from threshold to $E_\gamma = 1.5$ GeV. It has been pointed out in several studies[13-15,40] that these were the major contributors, and we wish to see how they do in a quantitative study. Table IV shows how well various contributions are able to fit the data. We start with S_{11} and P_{11} resonance states ($D1$), then a background is added in these partial waves ($D2$). The importance of an admixture of the $S_{11}(1700)$ may be seen by solution $D3$, and finally the $P_{11}(1470)$ in $D4$. Certainly, these type D solutions are basic; yet, it is also clear that other contributions are required as can be seen by examining χ^2/N. (Here N is the number of data points minus the number of variable parameters.)

In Table V we present our best results for type A, B, and C solutions below $E_\gamma = 1.5$ GeV. In computer runs not reported here we found additional solutions in which the parameters were outside the range specified in Table III. The improvement was not significant and we favor the reported solutions which are in agreement with the ranges in Table III.

The best values for χ^2/N in type A and B solutions are not as good as one might desire. For

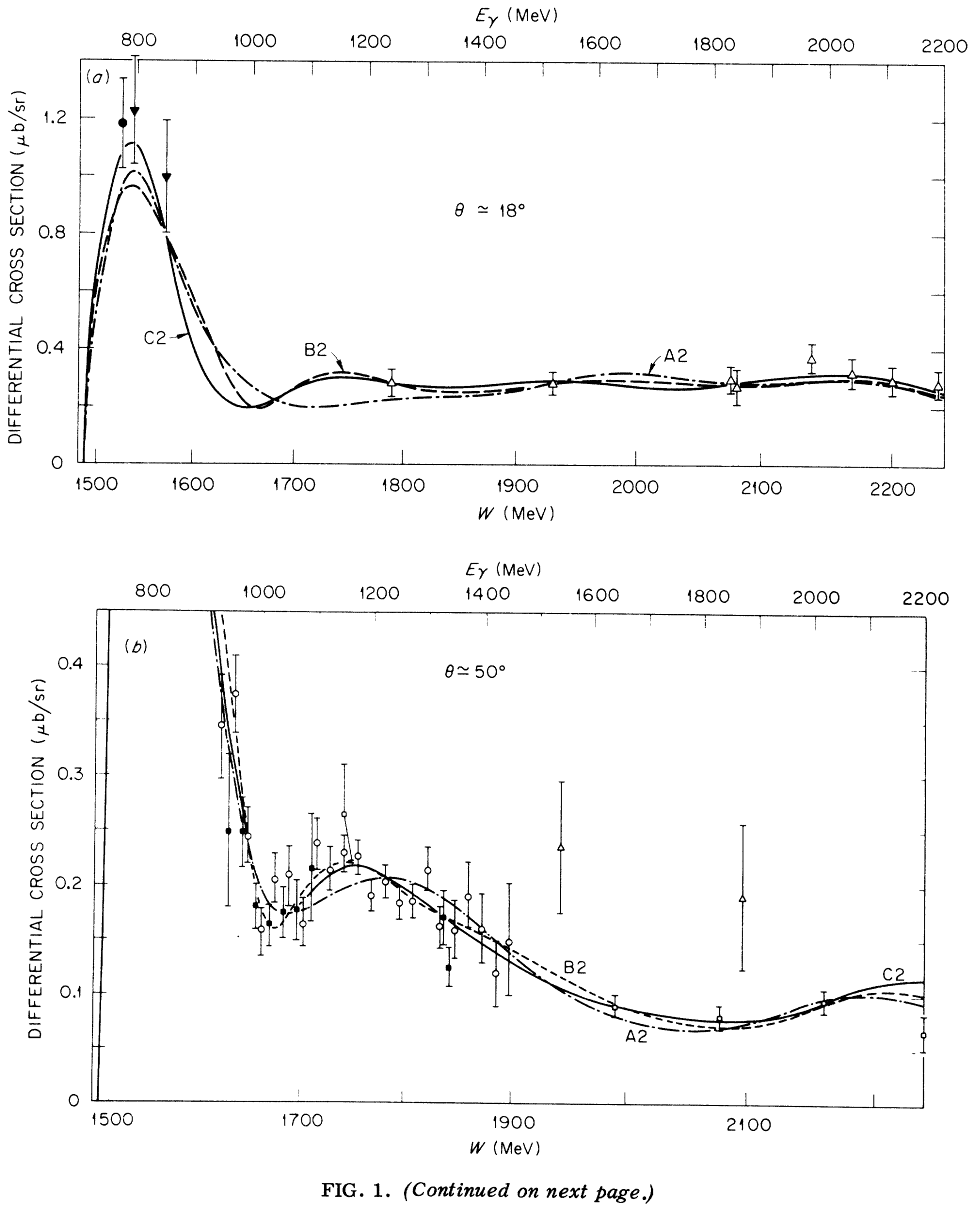
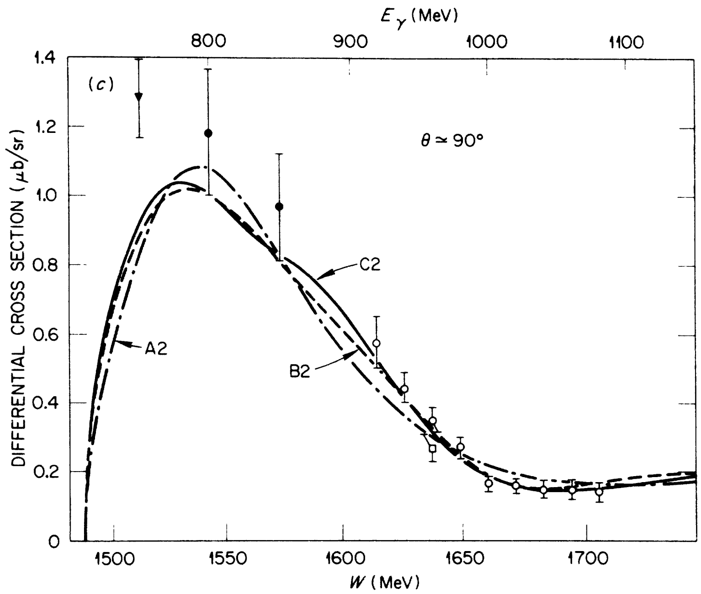
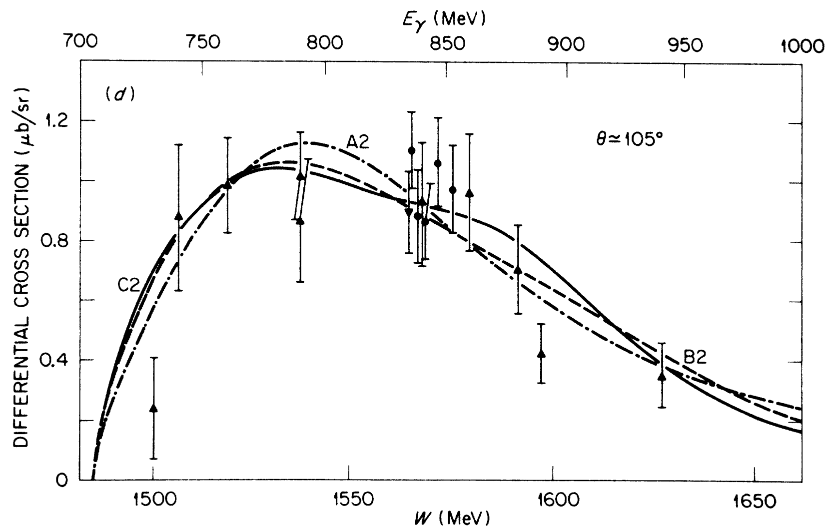


FIG. 1. *(Continued on next page.)*





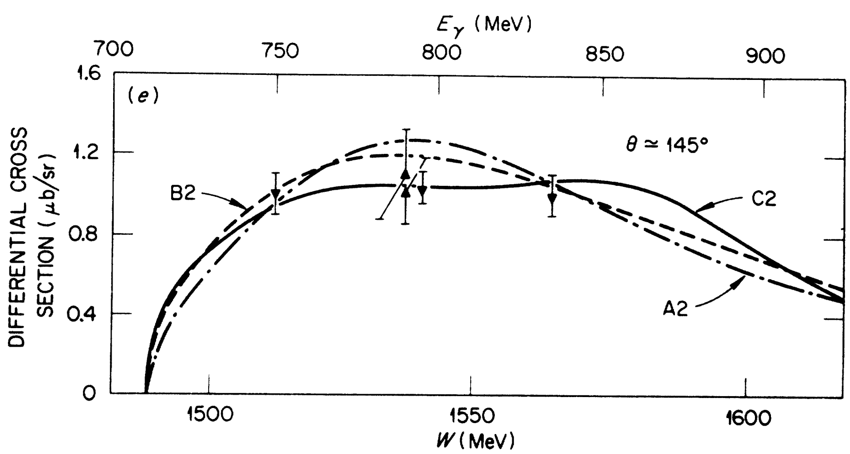


FIG. 1. Differential cross section as a function of energy at fixed angle. The identification of data for all figures follows: ■ Ref. 1, ● Ref. 2, ▲ Ref. 3, ○ Ref. 4, ▼ Ref. 5, □ Ref. 6, ▽ Ref. 7, △ Ref. 8.

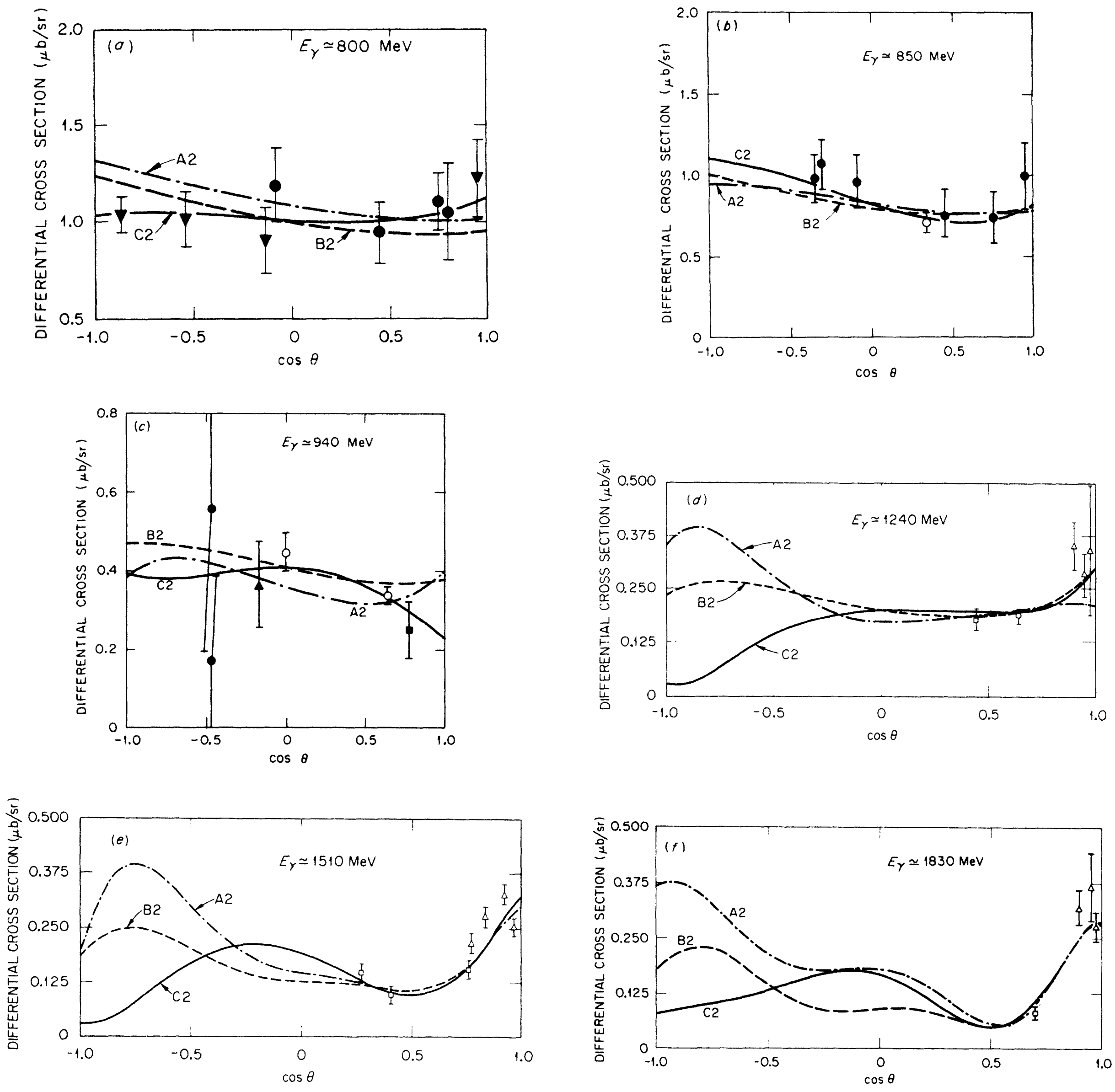


FIG. 2. Differential cross section as a function of angle at fixed energy.

this reason, we have tried using the stray states suggested by Donnachie,[41] and the $D_{13}(1670)$ found to be important in $\gamma p \to K^+\Lambda$.[34,42] These states are supposed to couple insignificantly to the πN channel and consequently they would not be expected to show up in studies of πN interactions. The resulting value of χ^2/N is very good (0.98); but, there are a rather large number of parameters involved, and of course the additional P_{11} and P_{13} states are only speculative. We find no need for the $D_{13}(1670)$ in this study.

From Tables IV and V we note that the value of χ^2/N is not as good for $A1$ as for either $D3$ or $D4$. The major reason for this is that in $D3$ and $D4$ we have allowed the mass W_r and the width Γ_r to vary. This serves to emphasize the importance of the $S_{11}(1550)$, $S_{11}(1700)$, and $P_{11}(1750)$ in this reaction provided liberal ranges are allowed for W_r and Γ_r. From $B1$ we see that the $P_{11}(1470)$ and $D_{13}(1520)$ states may make important additional contributions. The value $\chi^2/N = 1.22$ is certainly a big improvement over that obtained for the type D solutions.

In Table VI we show the solutions obtained when the higher energy data are included. These results are similar to those in Table V for the low-energy parameters. The only additions are the $F_{17}(1990)$, $G_{17}(2190)$, and $D_{13}(2040)$ states. The latter of these seems to be unimportant and could be omitted without causing a noticeable change in the fit to the data. A graphical comparison with the data is shown in Figs. 1–3 for the solutions which appear in Table VI. It will be noted that the cross section shows a peak in the forward direction at higher energy. This may very well be evidence of Regge behavior. We have chosen not to include Regge exchanges in our parametrization however, since a double counting problem would then be present, and we wish to see how well a pure direct-channel resonance model works.

A close examination of Tables IV, V, and VI shows that certain parameters have a degree of stability throughout the various solutions. Basically, these same parameters have also shown the same type of stability in computer runs not presented here, runs in which various features of the model were varied as discussed in Sec. II. In Table VII we give the quantity G from (12) which shows a rather high degree of stability, and an estimated value of the radiative width Γ_γ, where

$$\Gamma_\gamma = (\Gamma^E_{\gamma p} + \Gamma^M_{\gamma p})_r \qquad (20)$$

for the decay $N^* \to N + \gamma$. We do not quote any errors, but estimates of 20% for G and 40% for Γ_γ may not be unrealistic, and for the states with masses above 1800 MeV these errors may be substantially larger.

VI. DISCUSSION AND CONCLUSIONS

We have used an isobar model to determine which resonance terms are needed to explain the data for η photoproduction. It is clear that an admixture of states in addition to the predominant $S_{11}(1550)$, $S_{11}(1700)$, and $P_{11}(1750)$ is required. A fair fit is obtained assuming these additional contributions come from the $P_{13}(1860)$, $D_{13}(1520)$, $F_{17}(1990)$, and $G_{17}(2190)$ states. An excellent fit is obtained when the conjectured[41] states $P_{11}(1575)$ and $P_{13}(1585)$ are also included along with a stray $F_{15}(1625)$. At the present time we do not believe this is strong evidence for these states, however.

Our solutions are in general agreement with all available data, including the backward-forward asymmetry results of Ref. 40, and the polarization results from Ref. 7 (Fig. 3). There is a noticeable lack of differential-cross-section data in the backward direction and our solutions are quite different in this region, especially at the higher energies, as can be seen from Fig. 2. More data in the backward direction would therefore be most welcome. Additional polarization data would also be very helpful in selecting unique solutions, which would, of course, yield better values for the partial widths of the important resonance states. In spite of these difficulties with the current level of the data our solutions are sufficiently stable to enable us to calculate, approximately, some of the radiative decay widths. In general, these results in Table VII are in good agreement with other determinations where applicable,[11,34–36] and the new values seem reasonable.

There is some evidence here as in other recent work[34,44] which suggests that the mass and width of the second S_{11} resonance state might be lower than previously supposed. Our result in B1 and B2 is $W_r = 1662$ MeV and $\Gamma_r = 100$ MeV. Clearly, this is not strong evidence for such values, since

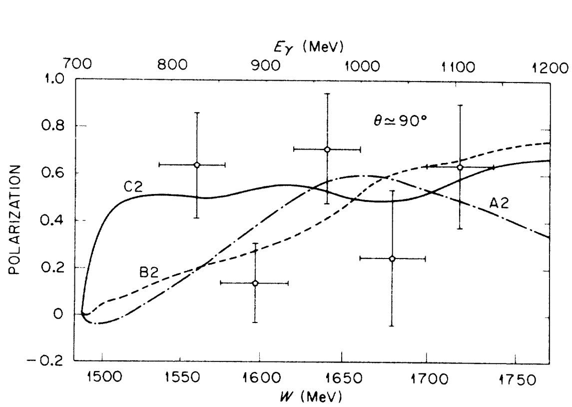


FIG. 3. Polarization in direction $\hat{k} \times \hat{q}$ as a function of energy at $\theta_{c.m.} \simeq 90°$.

TABLE VII. Couplings and radiative widths.

State	G (MeV^2)[a]	$\Gamma_{\eta N}$ (MeV)[b]	Γ_γ (MeV)
$S_{11}(1550)$	24	81	0.30
$S_{11}(1700)$	3.9	13	0.30
$P_{11}(1750)$	2.3	40	0.06
$P_{13}(1860)$	1.7	3	0.57
$F_{17}(1990)$	1.8	6	0.30
$D_{13}(2040)$	0.7	5	0.14
$G_{17}(2190)$	3.3	3	1.10

[a] From Tables V and VI.
[b] Assumed.

the other solutions have this state at the more traditional[32] higher values.

In connection with the values obtained for the parameters in Tables IV–VI a word of caution is in order for those who attempt to fit the data for $\gamma p \to \eta p$ just through the first peak in the cross section (below $E_\gamma \simeq 1.0$ GeV) without regard to how their solutions join the solutions at slightly higher energy. It is very easy to miss an admixture of states, which is important in addition to the dominant S_{11} and P_{11} isobars. This may cause a greater uncertainty in the values obtained for the partial widths of the dominant contributions.

In view of the fact that the results for radiative widths obtained in Table VII are in general agreement with the findings in Ref. 34, the remarks made there about quark model multiplet assignments also apply to our results. Clearly, much more work must be done before definite assignments can be made for most states which are involved. About all we can say is that our results are not inconsistent with the assignments:

$$S_{11}(1550) \in {}^2\{8\}_{1/2}\,[\underline{70}, 1^-]_1 ,$$

$$S_{11}(1700) \in {}^4\{8\}_{1/2}\,[\underline{70}, 1^-]_1 ,$$

$$P_{11}(1750) \in {}^2\{8\}_{1/2}\,[\underline{70}, 0^+]_2 \text{ or } {}^2\{8\}_{1/2}\,[\underline{56}, 0^+]_2 ,$$

$$P_{13}(1860) \in {}^2\{8\}_{3/2}\,[\underline{56}, 2^+]_2 ,$$

$$D_{13}(1520) \in {}^2\{8\}_{3/2}\,[\underline{70}, 1^-]_1 .$$

Finally, in this work, unlike in the study of $K\Lambda$ photoproduction,[34,42] there is no clear improvement obtained by including a $D_{13}(1670)$ state. In all solutions where this state was used it had a very small coupling and could be omitted with essentially no change in the final result.

ACKNOWLEDGMENTS

We wish to thank Professor J. R. Holt for kindly calling our attention to corrections in the data of Ref. 9, and for supplying us with some relevant new data. One of us (H.R.H.) is indebted to R. L. Schult for many helful discussions during the early stages of this work.

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*Work supported in part by the U. S. Atomic Energy Commission. Present address: Oak Ridge National Laboratory, Oak Ridge, Tennessee.

†Operated by Union Carbide Corporation, Nuclear Division for the U. S. Atomic Energy Commission.

‡Work supported in part by the U. S. F. Research Council.

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PHYSICAL REVIEW D VOLUME 7, NUMBER 9 1 MAY 1973

Extrapolation of the pp Forward Scattering Amplitude

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(Received 6 November 1972)

The pp forward spin-nonflip amplitude is analytically continued by Ciulli-Cutkosky-Deo method. The resultant amplitude has qualitatively the same characteristic features as that observed by Sugawara and the present author in their iterative method. In particular, it is quite conclusive that the spin flip cannot be neglected in the high-energy region up to about 30 GeV/c.

Recently Sugawara and the present author[1] have calculated the pp forward spin-nonflip amplitude by their newly proposed iterative method, which consists of an iterative evaluation of the usual dispersion relation and the inverted one. The remarkable feature of the result is that the imaginary part has a large dip just above the $\bar{p}p$ threshold, which makes its high-energy real part nearly twice that determined experimentally ignoring spin-flip.[2] It would be, however, worthwhile to confirm the above result by an entirely different way of continuation, since in many cases the analytic continuation turns out to be drastically model-and/or method-dependent.[3-5] In this note we evaluate the amplitude by applying the method introduced by Ciulli, Cutkosky, and Deo,[6] and developed by Chao and Pietarinen[7] in their KN analysis. The method is known to be the best, as long as we do the continuation in terms of a series expansion of a certain function. The main conclusions are summarized at the end of this note.

First we conformally map the lab-energy ω plane onto the inside of a unifocal ellipse in the z plane as shown in Fig. 1. It is assumed the relevant total cross section σ_T is known above D and C, while the data on the real part are also available in the region $[A, B]$. We assume the same experimental input as in Ref. 1, which includes the 450-MeV phase-shift analysis.[8] We choose D, A, and B at 16-, 70- and 350-MeV kinetic energies, respectively. C is fixed at the $\bar{p}p$ threshold. z_∞ corresponds to $\omega=\infty$. The sum R of semimajor and semiminor axes becomes 9.79, which gives a reasonable asymptotic convergence rate.

Our discrepancy function $D(z)$ is constructed in the following way. Let $A(z)$ be the spin-averaged spin-nonflip forward amplitude normalized by $\mathrm{Im}A=(\omega^2-m^2)^{1/2}\sigma_T$, where m is the nucleon mass. We introduce $T(z)$ by

$$T(z)=[A(z)-V(\omega)]/(\omega-m)-f_\pi/(\omega-\omega_\pi), \quad (1)$$

where

$$V(\omega)=\frac{1}{\pi}\int_m^\infty \frac{\mathrm{Im}\,V(\omega')}{\omega'-\omega}\,d\omega',$$

with $\mathrm{Im}\,V(\omega)=4\pi k/(a^{-2}+k^2)$ and $k=[\frac{1}{2}m(\omega-m)]^{1/2}$. Here a is the S-wave scattering length; therefore the threshold behavior is guaranteed automatically. f_π is so chosen that $T(z)$ has no pion pole. Then, $D(z)$ is defined by

$$D(z)=[H(z)-H(z_\infty)]/(z-z_\infty), \quad (2)$$