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Power-Law Energy Spectrum Deduced from the Angular Distribution of Nuclear Interactions of 20 to ~10^3 GeV*†

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The $\log_{10}F$ plots of the angular distribution for the forward-cone secondaries, observed in high-energy nuclear interactions produced in nuclear emulsion, are found to fit well to a linear function of $\eta(\theta)$ [$\cong 0.46 - \ln\tan\theta$], i.e., $\log_{10}F = \alpha\eta(\theta)+\gamma$. This fact is used to argue for the power-law differential energy spectrum in multiple production with the index of power $\alpha \ln 10$, with the aid of already established experimental results. The values of α, obtained from fitting procedures, are approximately equal to -1 for nuclear interactions with a sufficient number of charged secondaries in a wide range of the primary energy E_p between 20 and ~10^3 GeV. It is also concluded, from the systematic study of the 1335 30-GeV/c proton jets, that the nature of the target nucleus in the hadron-nucleus collisions does not affect the shape of the angular distribution of the forward-cone secondaries.

I. INTRODUCTION

Previously, attempts have been made to fit $\log_{10}F$ plots[1] of the angular distributions of the charged secondaries, produced in high-energy nuclear interactions of ~10^{12} eV, to several analytical functions of $\eta(\theta)$ [$\cong 0.46 - \ln\tan\theta$].[2,3] The aim of these studies was mainly to organize and to parametrize the *observed* shape of the angular distributions, and the same aim is also carried over to the present study. The $\log_{10}F$ plots for the forward-cone secondaries have been found to fit very well to a linear function of $\eta(\theta)$, i.e.,

$$\log_{10}F = \alpha\eta(\theta)+\gamma\,. \tag{1}$$

Our main interest in the present study is concerned with the trend and its physical significance of the value of α in Eq. (1), obtained from fitting procedures for the angular distributions of forward-cone charged secondaries of high-energy nuclear interactions, produced in nuclear emulsion, as a function of the primary energy E_p of the incoming particle in the laboratory system (LS), the number of charged secondaries n_s, and the number of grey or heavy tracks N_h.

It must be stressed here that our study is based solely on data from the nuclear-emulsion technique and is somewhat different from the theoretical and experimental interest of the "inclusive reactions"[4,5] – reactions in which a single final particle is observed irrespective of whatever else is happening. The nuclear-emulsion technique, as a means of observation for studying the mechanism of multiple production of secondaries in high-energy nuclear interactions, is a superior one particularly for the following reasons:

(a) The relative emission angles of charged secondaries are very accurately measured because of the high spatial resolution of the technique.

(b) All the charged secondaries produced in a jet can be observed *simultaneously* in the analysis.

The procedure and the results of fitting angular distributions of jets to Eq. (1) are described in Sec. II. In Sec. III, it is shown on the basis of the experimentally established facts about multiple production that a good fit of the observed angular distribution to Eq. (1) in the forward cone is closely related to the power-law differential energy spectrum of the secondaries, and α to its power. Physical implication of our findings of the present analysis is explored briefly and compared with predictions of other recent models.

II. EXPERIMENTAL DATA AND ANALYSIS

The observed angular distributions of the produced secondaries are in the form of $\log_{10}F$ plots as a function of $\eta(\theta)$. The accumulative number of secondaries n is counted progressively as a function of $\eta(\theta)$, starting from "one" for the secondary with the smallest emission angle, approximately up to $n \cong \frac{1}{2}n_s$. (The reason why only about half of the total charged secondaries in the forward cone are used in the present analysis is explained in detail in Sec. III.) It follows from its definition[3] that the secondary with the smallest emission angle has the largest value of $\eta(\theta)$. Then $F=n/n_s$ is fitted to Eq. (1) by adjusting α and γ by the least-squares method. The error used in the procedure is statistical, i.e., only the numbers n and n_s in a given bin are involved.

The jets which are used in the present analysis are listed in Table I. These include 20-GeV pion jets,[6] 27-GeV proton jets,[7] 30.5-GeV and 30.9-GeV

TABLE I. Jets used in the analysis.

Reference	The kind of primary	Energy of primary (GeV)	Number of jets	Restrictions imposed for N_h	for n_s
6	pion	20	318	<9	none
7	proton	27	202	0, 1	>3
8	proton	30.5	64	0, 1	≥4
8	proton	30.9	1271	none	none
9	nucleon, fragment	570	14 (individually)	none	>8
			17 (composite)	none	≥5
2, 3, 11	proton, neutron,	$(0.1-8)\times 10^3$	19 (Texas stack)	≤3	>8
	α particle	$(0.6-63)\times 10^3$	20 (Brawley stack)	≤5	>8

proton jets (from a stack of the same exposure),[8] jets produced by nuclear fragments of a primary nucleus of the Brawley No. 1115,[9] and ultrahigh-energy cosmic-ray jets.[2] Also included in the table are the restrictions on N_h and n_s imposed in the selection of the jets used in the analysis.

The procedure and the results of fitting the experimental $\log_{10}F$ distributions with Eq. (1) is illustrated in Fig. 1 for the jets produced by 30-GeV/c protons. The jets are grouped according to N_h and n_s, and the composite distribution for each grouping is analyzed. Tables II–IV show the trends of α for the jets from accelerator-produced hadron primaries. The fits with Eq. (1) are generally good except for those jets with small multiplicity. The values of $(-\alpha)$ for 30-GeV/c proton jets, thus obtained, are shown in Fig. 2 as an example. For jets with small multiplicity, the absolute value of α is generally small compared with unity, but it approaches a constant value of about 1.2 as n_s increases. The dashed lines in Fig. 2 indicate $\alpha = -1.2$.

It is also clear that the trend of the value of α for different N_h is independent of N_h. It is recalled here that the number of evaporation prongs N_h is closely related to the target nucleus of a proton-nucleus collision in nuclear emulsion, whose constituents are H, C, N, O, Br, and Ag atoms. If $N_h > 8$, we can be certain that the target nucleus is one of the heaviest nuclei (Ag, Br) among the constituent nuclei. Thus it should be stressed that, for the jets produced by 20–30 GeV hadrons, the shape of the angular distributions of secondaries in the forward cone in a hadron-nucleus collision seems to be independent of the mass number of the target nucleus. This fact may be related closely to the conclusion in Ref. 6, in which the energy estimate E_{ch} has been found to be the most consistent value for the primary energy E_p, regardless of N_h. Several small-angle tracks in the most forward cone are the main contributors to the estimated value of E_{ch}. It is also found from the present study that these few tracks in the most forward cone seem to be emitted independently of N_h. Thus the consistency of the E_{ch} values found in Ref. 6 must be duly explained by the trend of α as a function of N_h.

The $\log_{10}F$ plots of the composite distribution of the 17 nucleon jets (with $n_s \geq 5$) of the Brawley No. 1115 are shown in Fig. 3. The reason for not im-

TABLE II. The value of $(-\alpha)$ for 20-GeV pion-nucleus interactions (see Ref. 6).

n_s \ N_h	0	1	2–3	4–5	6–8
1	···	0.80± 2.43	···	···	···
2	0.76± 0.31	0.58± 1.16	0.71± 0.46	0.82± 1.54	···
3	0.89± 0.31	1.01± 0.99	0.72± 0.56	0.55± 1.19	1.11± 1.72
4	1.04± 0.46	1.18± 1.47	0.74± 0.44	0.97± 0.50	0.60± 0.93
5	1.37± 0.83	···	1.21± 0.94	1.78± 1.56	1.15± 1.65
6	1.21± 0.70	···	1.39± 0.86	1.51± 0.65	2.00± 1.62
7	···	1.17± 1.24	1.50± 0.90	0.98± 1.10	1.77± 1.39
8	0.91± 2.23	1.21± 0.89	1.01± 0.99	1.15± 1.07	2.19± 1.54
9	0.72± 1.06	···	···	0.76± 1.34	1.01± 1.45
10	···	···	···	1.16± 1.00	···

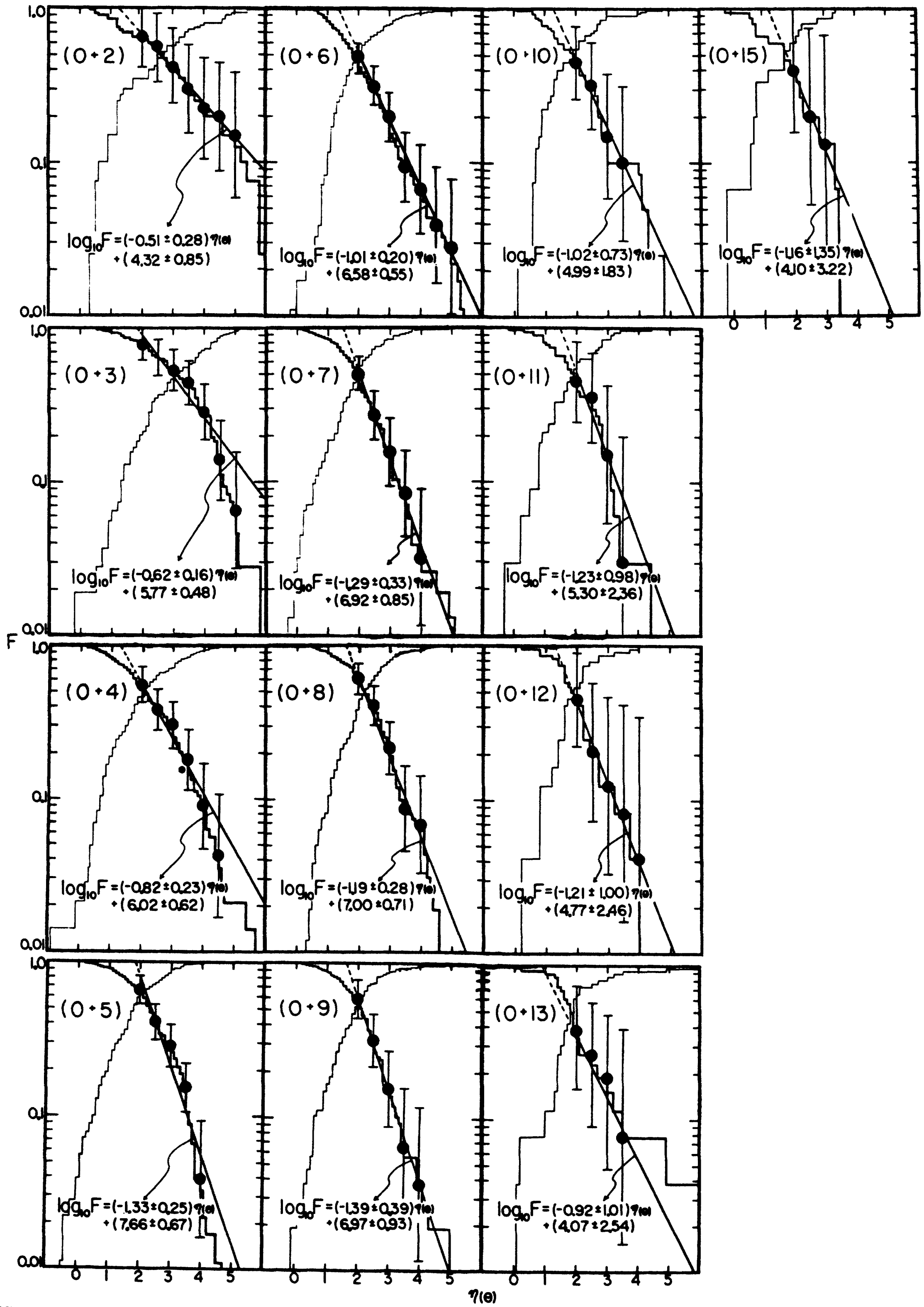

FIG. 1. $\log_{10}F$ plots of $\eta(\theta)$ distributions of 30-GeV/c proton jets. The wide-line histograms show the plots of the forward-cone secondaries and the narrow-line ones those for the backward-cone secondaries. This grouping corresponds to the jets with $N_h=0$.

TABLE III. The value of $(-\alpha)$ for the 27-GeV proton-nucleon interactions ($N_h = 0, 1$ only)

n_s	$(-\alpha)$
3	1.06± 0.30
4	1.29± 0.34
5	1.46± 0.34
6	1.27± 0.24
7	1.37± 0.38
8	1.29± 0.43
9	1.56± 0.43
10	1.24± 0.62
11	1.21± 0.54
12	1.33± 0.24
13	0.83± 1.46

posing the restriction on N_h stems from the consideration of the above finding that the forward-cone particles are emitted almost independently of N_h. The restriction on n_s is imposed also from the consideration of the above that the values of α with small multiplicity seem to be quite different from the trend shown by angular distributions of jets with sufficient multiplicity, say $n_s \geq 5$. The nucleon fragments essentially have the same velocity as the primary nucleus of No. 1115 ($Z = 15$),[9] since the fragments have small velocity with respect to the primary nucleus. The primary energy of the Brawley No. 1115 has been determined from the scattering measurements by the method used by Lohrmann *et al.*[10]: $E_p = (0.57^{+0.07}_{-0.06}) \times 10^3$ GeV/nucleon.[3] In the full range, 3 orders of magnitude, the experimental distribution fits excellently to Eq. (1) as shown in Fig. 3. The value of the parameter of the composite distribution in Fig. 3 is

$$\alpha = -0.91 \pm 0.18\,. \qquad (2)$$

These 17 nucleon fragments are from the same parent nuclei and they are indeed a "pencil beam"

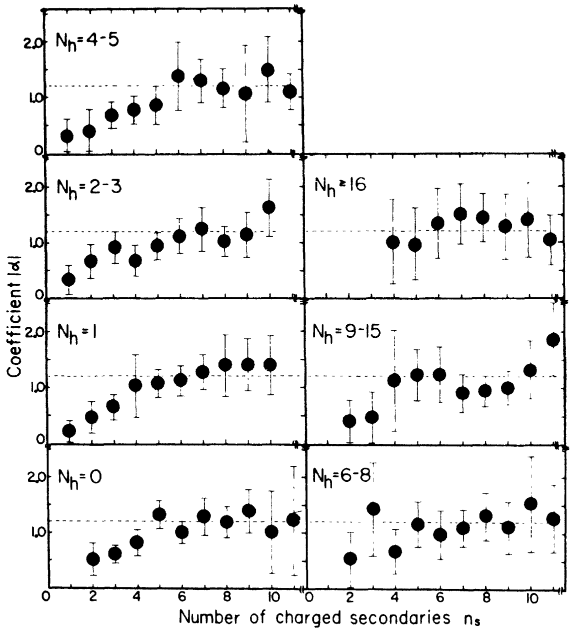

FIG. 2. The trends of the absolute values of α according to groupings of N_h and n_s.

of the primary energy >500 GeV/nucleon. It is significant that a good fit of the angular distribution to Eq. (1) is found for Brawley No. 1115 with almost the same value of α as that found for the accelerator-produced jets.

Values of α from the fitting procedure, individually applied to the 14 jets produced by the fragments of the Brawley No. 1115 with the imposed restriction of $n_s > 8$, are shown in Table V. The weighted average of the values of α for the entire set of 14 jets is

$$\langle \alpha \rangle = -0.63 \pm 0.13\,. \qquad (3)$$

Figures 4(a) and 4(b) show the $\log_{10}F$ plots of

TABLE IV. The value of $(-\alpha)$ for 30-GeV/c proton-nucleon interactions.

n_s \ N_h	0	1	2–3	4–5	6–8	9–15	>16
1	···	0.22± 0.20	0.33± 0.27	0.31± 0.41	···	···	···
2	0.51± 0.28	0.47± 0.29	0.67± 0.30	0.40± 0.59	0.53± 0.73	0.40± 0.58	···
3	0.62± 0.16	0.65± 0.23	0.92± 0.29	0.68± 0.24	1.43± 0.93	0.47± 0.66	···
4	0.82± 0.23	1.03± 0.56	0.67± 0.28	0.78± 0.25	0.67± 0.40	1.13± 0.90	1.01± 0.75
5	1.33± 0.25	1.08± 0.25	0.94± 0.24	0.86± 0.34	1.16± 0.40	1.23± 0.45	0.97± 0.63
6	1.01± 0.20	1.12± 0.27	1.11± 0.31	1.38± 0.61	0.98± 0.43	1.24± 0.49	1.34± 0.61
7	1.29± 0.33	1.27± 0.31	1.25± 0.39	1.30± 0.39	1.09± 0.33	0.90± 0.33	1.50± 0.53
8	1.19± 0.28	1.39± 0.55	1.02± 0.26	1.16± 0.36	1.31± 0.42	0.93± 0.27	1.45± 0.43
9	1.39± 0.39	1.40± 0.47	1.15± 0.40	1.07± 0.87	1.10± 0.46	1.02± 0.29	1.28± 0.58
10	1.02± 0.73	1.39± 0.52	1.63± 0.51	1.50± 0.58	1.53± 0.87	1.32± 0.51	1.40± 0.67
11	1.23± 0.98	···	···	1.09± 0.33	1.27± 0.60	1.86± 0.64	1.04± 0.45
12	1.21± 1.00	0.89± 1.44	···	1.90± 0.69	···	···	···
13	0.92± 1.01	···	···	···	···	···	···
14	···	···	···	···	···	···	···
15	1.16± 1.35	···	···	···	···	···	···

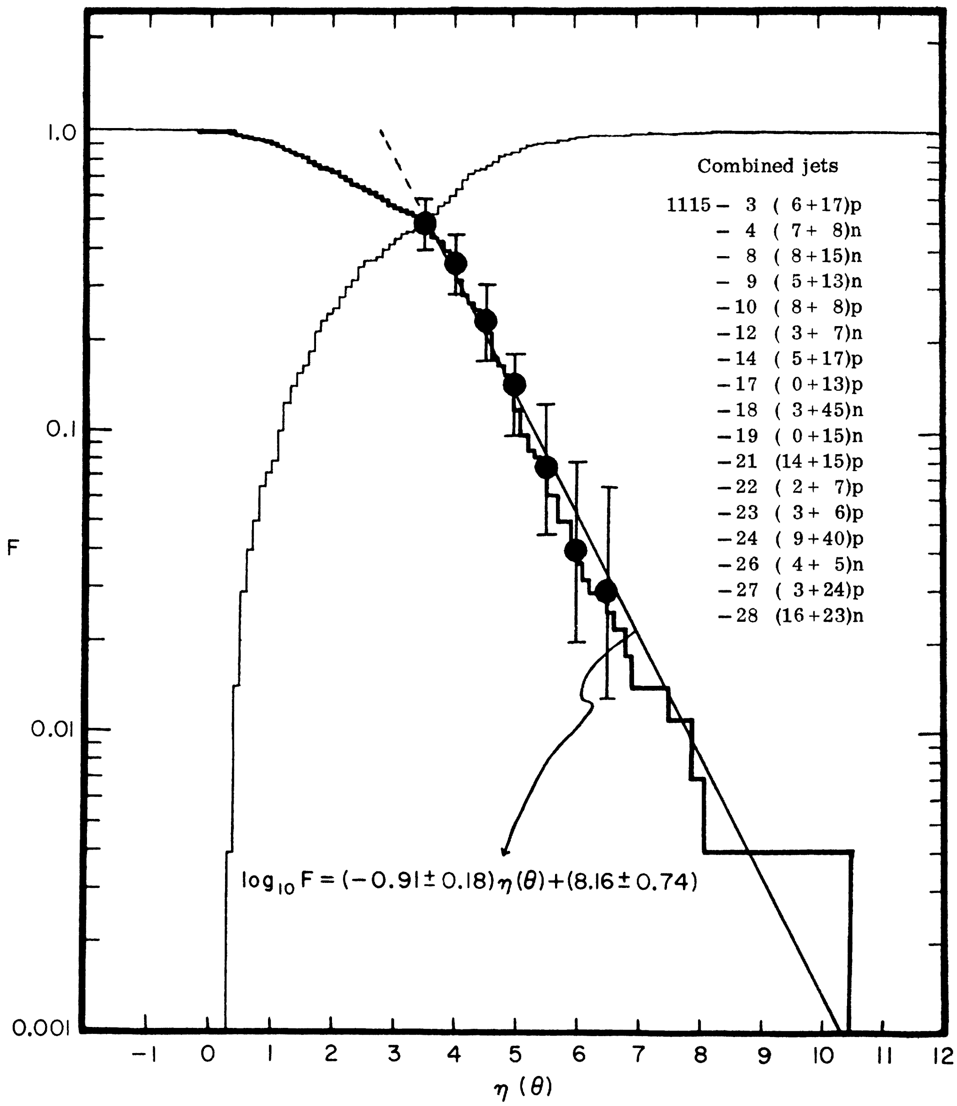

FIG. 3. $\log_{10}F$ plots of the composite distribution of secondaries of jets by nucleon fragments of the primary nucleus No. 1115 of the Brawley stack.

ultrahigh-energy jets, which were found in the Texas and Brawley stacks, respectively. The restrictions of $N_h \leq 3$ and $N_h \leq 5$ which were imposed on the selection of jets analyzed in the previous experiments are carried over to the present analysis.[2] The restriction $n_s > 8$ is further imposed in the present analysis, since enough of the charged secondaries are needed for each fit. As shown in the figures all the $\log_{10}F$ plots show excellent fits to Eq. (1). Table VI lists the primary energy $E(\theta)$,[3] and the values of α, obtained by the fitting procedures. The weighted average of the value of α analyzed for the 39 jets from the Texas and Brawley stacks is

$$\langle \alpha \rangle = -0.56 \pm 0.07 . \qquad (4)$$

The value of $\langle \alpha \rangle$ of Eq. (4) is not obtained from composite distributions, as it is for the accelerator-produced jets and those from Brawley No. 1115, because the primary energies of the 39 jets vary quite widely and they are difficult to measure accurately. Therefore, the average value of α in Eq. (4) should not be compared directly with the values, $\alpha \cong -1.2$, obtained for the accelerator-produced jets, or with Eq. (2). However, it may not be surprising if the value of $|\alpha|$ for the nuclear interactions at $E_p \approx 10^3$ GeV turns out to be quite close to unity, just as for accelerator-produced jets and the nucleon jets produced by the fragments of the Brawley No. 1115. This situation may be indirectly inferred from the case of 570-GeV jets, i.e., from the comparison between Eqs. (2) and (3).

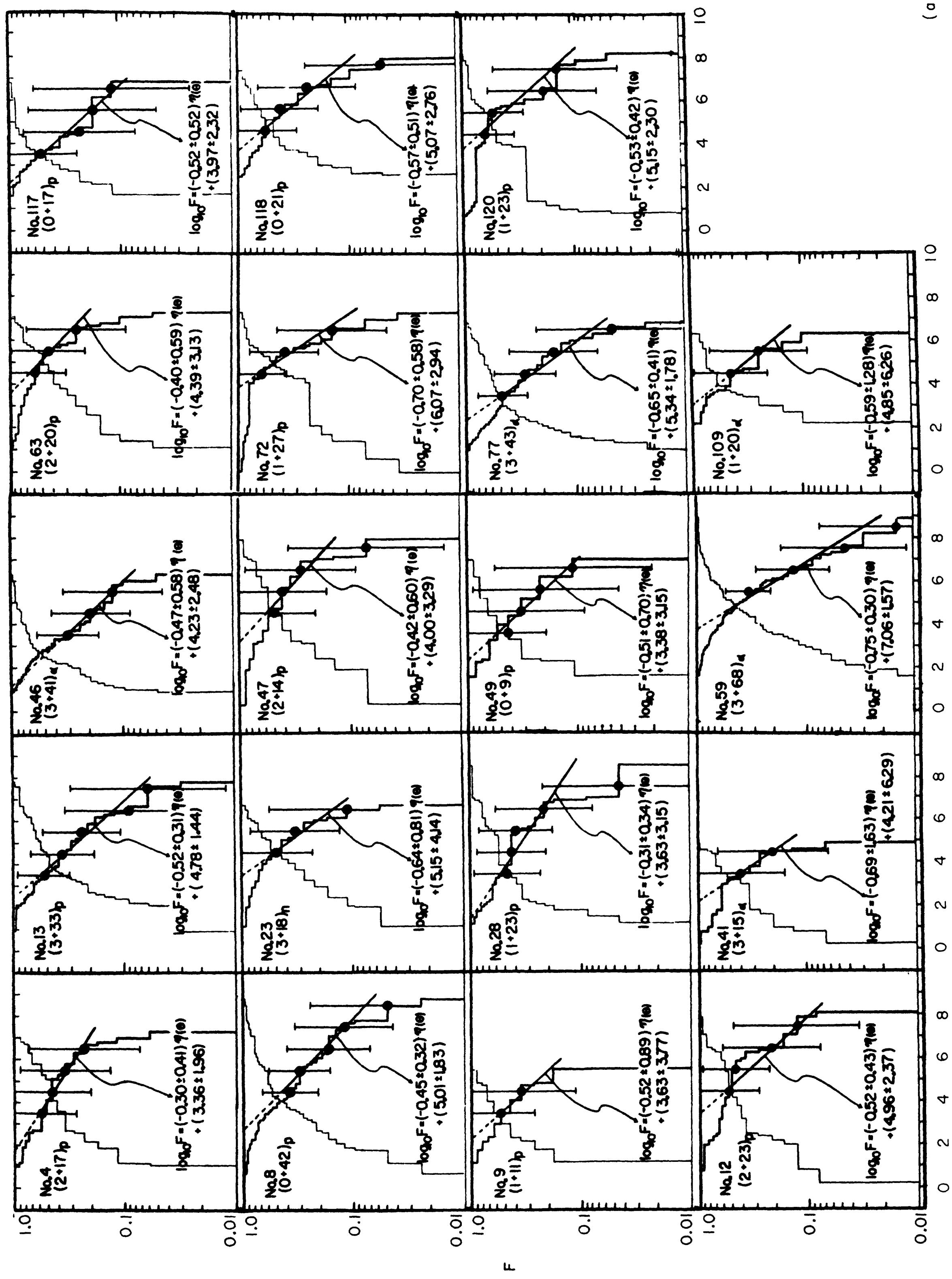

FIG. 4. (Continued on following page)

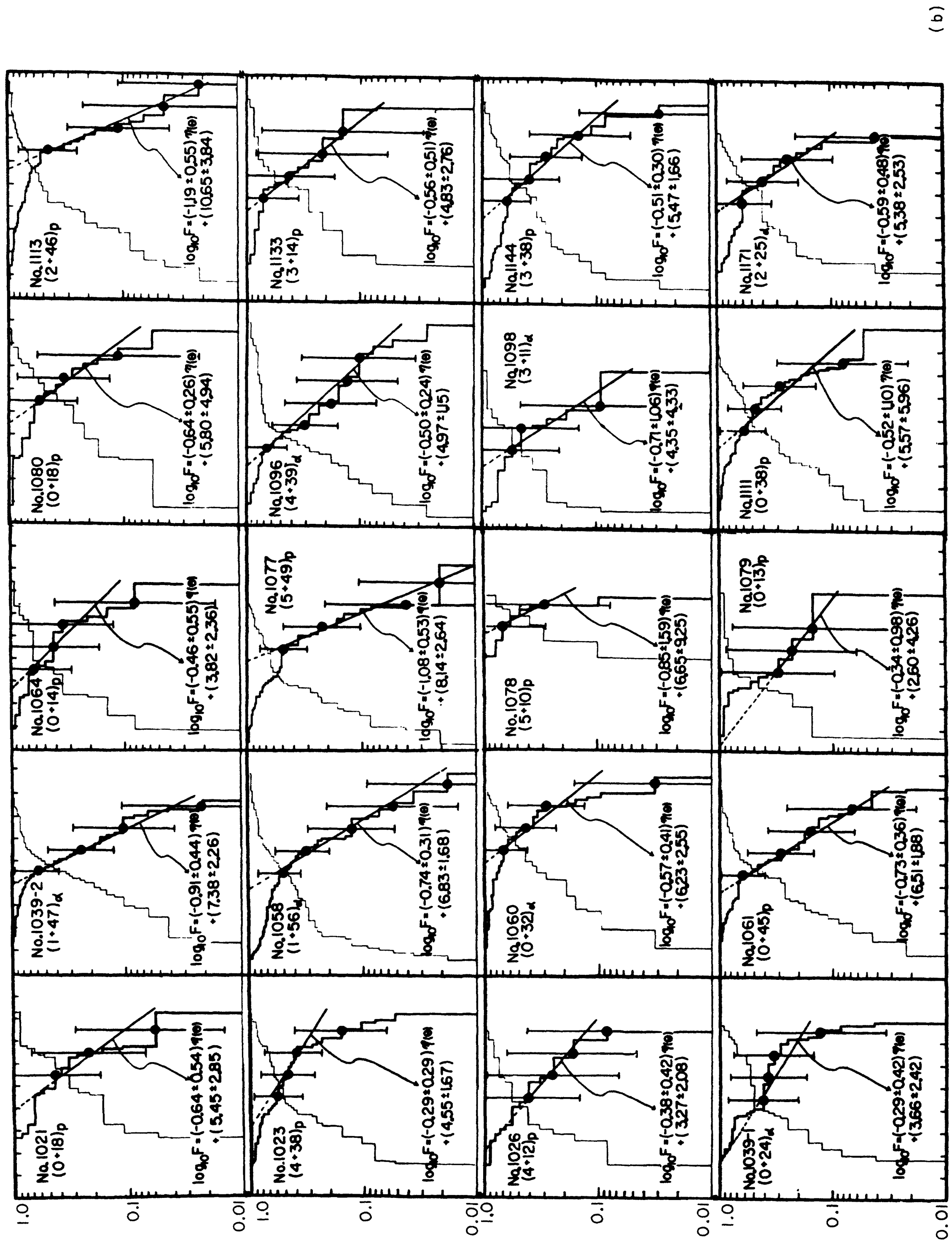

FIG. 4. (a) $\log_{10}F$ plots of individual distributions of p, n, and α jets in the Texas stack. (b) $\log_{10}F$ plots of individual distributions of p, n, and α jets in the Brawley stack.

TABLE V. The value of $(-\alpha)$ for jets produced by fragments of the primary nucleus of Brawley No. 1115 group ($E_p = 570$ GeV).

Event No.	Type	$(-\alpha)$
2	$(11+44)_H$	0.65 ± 0.38
3	$(6+17)_p$	0.79 ± 0.94
6, 7	$(13+65)_H$	0.52 ± 0.26
8	$(8+15)_n$	1.38 ± 1.28
9	$(5+13)_n$	1.02 ± 1.09
14	$(5+18)_p$	0.44 ± 0.86
17	$(0+13)_p$	0.59 ± 0.71
18	$(3+45)_n$	0.97 ± 0.55
19	$(0+15)_n$	0.37 ± 0.54
20	$(0+44)_{Li}$	0.70 ± 0.38
21	$(14+15)_p$	0.41 ± 0.41
24	$(9+40)_p$	1.65 ± 0.77
27	$(3+24)_p$	0.59 ± 0.54
28	$(16+23)_n$	0.57 ± 0.55

But especially for the ultrahigh-energy jets, it should be additionally kept in mind that the effect of the experimental practice to determine the primary's direction from the centroid of the target diagram may distort the shape of the angular distribution in the forward cone and may also affect the value of α considerably in the forward cone.

III. DISCUSSION

The essential features of multiple production are best manifested in the great number of secondaries produced in a single interaction. Moreover, since the p_t distribution seems to be insensitive both to the primary energy, E_p, and energies of secondaries, E, theoretical models are compared primarily with angular distributions or longitudinal-momentum distributions. However, because of the presence of so many particles in an interaction, various kinematic constraints such as the conservation of energy and momentum become less restrictive and the effect of the "leading" particles, which may be the originally incoming particles slowed down, becomes negligible.[11] At present, experimentally "exclusive n-body experiments" are almost impossible, since neutral particles are difficult to detect and full information on produced particles is not attainable. Thus, among all the properties of hadron collisions, the single-particle distributions

$$d^3N = f(\vec{p}, E_p)d^3p/E \tag{5}$$

are inclined to be the main target of study by experimentalists as well as by theorists.[12] In other words, the single-particle distributions for high-energy hadronic collisions are set out to be searched for to represent the law of multiple production in high-energy nuclear interactions, just like the Planck radiation formula for photons in the black body. The good fits of Eq. (1) to the data on angular distributions in high-energy nuclear interactions, as shown in the previous section, suggest a form of Eq. (5). This aspect is first dealt with in Sec. III A. The physical significance of our findings in the present study is briefly discussed in conjunction with other experiments and predictions of various models of multiple production in Sec. III B.

TABLE VI. The value of $(-\alpha)$ for proton, neutron, and α jets.

Event No.	Type	$E(\theta)$ (10^3 GeV)	$(-\alpha)$
	A. Texas stack		
4	$(2+17)_p$	2.38	0.30 ± 0.41
8	$(0+42)_p$	2.78	0.45 ± 0.32
9	$(1+11)_p$	0.56	0.52 ± 0.89
12	$(2+23)_p$	4.59	0.52 ± 0.43
13	$(3+33)_p$	1.45	0.52 ± 0.31
23	$(3+18)_n$	3.24	0.64 ± 0.81
28	$(1+23)_p$	2.09	0.31 ± 0.34
41	$(3+15)_\alpha$	0.117	0.69 ± 1.63
46	$(3+41)_\alpha$	0.223	0.47 ± 0.58
47	$(2+14)_p$	4.60	0.42 ± 0.60
49	$(0+9)_p$	1.21	0.51 ± 0.70
59	$(3+68)_\alpha$	4.80	0.75 ± 0.30
63	$(2+20)_p$	4.40	0.40 ± 0.59
72	$(1+27)_p$	4.90	0.70 ± 0.58
77	$(3+43)_\alpha$	0.441	0.65 ± 0.41
109	$(1+20)_\alpha$	1.74	0.59 ± 1.28
117	$(0+17)_p$	1.02	0.52 ± 0.52
118	$(0+21)_p$	8.20	0.57 ± 0.51
120	$(1+23)_p$	5.90	0.53 ± 0.42
	B. Brawley stack		
1021	$(0+18)_p$	9.5	0.64 ± 0.54
1023	$(4+38)_p$	18.9	0.29 ± 0.29
1026	$(4+12)_p$	3.5	0.38 ± 0.42
1039-1	$(0+24+\alpha)_\alpha$	6.8	0.29 ± 0.42
1039-2	$(1+47)_\alpha$	6.1	0.91 ± 0.44
1058	$(1+56)_\alpha$	2.2	0.74 ± 0.31
1060	$(0+32)_\alpha$	63	0.57 ± 0.41
1061	$(0+45)_p$	5.7	0.73 ± 0.36
1064	$(0+14)_p$	1.8	0.46 ± 0.55
1077	$(5+49)_p$	1.5	1.08 ± 0.53
1078	$(5+10)_p$	45.2	0.85 ± 1.59
1079	$(0+13)_p$	1.3	0.34 ± 0.98
1080	$(0+18)_p$	17	0.64 ± 0.26
1096	$(4+39)_\alpha$	1.7	0.50 ± 0.24
1098	$(3+11)_\alpha$	0.55	0.71 ± 1.06
1111	$(0+38)_p$	14.1	0.52 ± 1.10
1113	$(2+46)_p$	35	1.19 ± 0.55
1133	$(3+14)_p$	16	0.56 ± 0.51
1144	$(3+38)_p$	6.4	0.51 ± 0.30
1171	$(2+25)_\alpha$	6.3	0.59 ± 0.48

A. The Angular Distribution in the Forward Cone and the Power-Law Energy Spectrum

The parameter $\eta(\theta)$ [$\cong 0.46 - \ln\tan\theta$] in the present analysis was originally introduced to approximate the "rapidity" parameter $\eta = \operatorname{arctanh}(\beta\cos\theta)$, where β denotes the velocity and θ the emission angle of a secondary in the LS.[3,4] With respect to Lorentz transformations, the parameter possesses a simple property,[3]

$$\eta = \eta_c + \eta^* , \tag{6}$$

where $\eta_c = \operatorname{arctanh}\beta_c$ and $\eta^* = \operatorname{arctanh}(\beta^*\cos\theta^*)$. The velocity β_c is that of a frame of reference moving in the direction of the incident primary in the LS, where the secondary's velocity and emission angle are represented, respectively, as β^* and θ^*. Thus, it is clear from Eq. (6) that the differences between the values of η of the secondaries produced in a jet are Lorentz-invariant in the frames of reference that move in the direction of the primary.

It is interesting to show further that the Lorentz-invariant phase space in Eq. (5), d^3p/E, may be simplified in terms of the two independent parameters, η and p_t, the transverse momentum, as follows:

$$\begin{aligned} d^3p/E &= \frac{2\pi p^2 dp \sin\theta\, d\theta}{E} \\ &= \frac{2\pi p^2 \sin\theta\, |J|\, d\eta\, dp_t^2}{E} \\ &= \pi d\eta\, dp_t^2 , \end{aligned} \tag{7}$$

where

$$|J| = \begin{vmatrix} \dfrac{\partial p}{\partial \eta} & \dfrac{\partial p}{\partial p_t} \\ \dfrac{\partial \theta}{\partial \eta} & \dfrac{\partial \theta}{\partial p_t} \end{vmatrix} .$$

Because of the cylindrical symmetry of the problem (which eliminates the dependence on the variable φ), and because of the relation $m^2 = E^2 - p^2$, only two components of the energy-momentum four-vector are independent. Therefore, the single-particle distribution of Eq. (5) may be expressed in the form

$$d^3N = f_1(p, \theta, E_p)\, d^3p/E , \tag{8}$$

and, with the aid of Eq. (7), it may be transformed into the convenient form

$$d^3N = F(\eta, p_t^2, E_p)\, d\eta\, dp_t^2 . \tag{9}$$

It has been experimentally established that the distribution of p_t in high-energy multiple production is almost independent of E_p as well as of the longitudinal momentum p_l.[12,13] Thus, as has been done frequently before, the function $F(\eta, p_t^2, E_p)$ may be factored into separate functions of η and p_t to the extent that the rapidity parameter η is independent of p_t. [This statement is approximately equivalent to saying that p_t is independent of p_l, as can be seen from Eq. (13) in the following.[14]] We speculate that good fits of angular distributions to the form of Eq. (1) with almost the same index α throughout a range of E_p from 20 to ~10^3 GeV may well imply the following form of the function:

$$F(\eta, p_t^2, E_p)\, d\eta\, dp_t^2 = C \exp(a\eta + bp_t^2)\, d\eta\, dp_t^2 , \tag{10}$$

where C is a constant which may have some dependence on E_p, and where

$$a = \alpha \ln 10 \tag{11}$$

and b are independent of E_p.[16]

Equation (11) needs to be explained. The present analysis deals only with about half of the produced secondaries in the extreme forward cone, mainly because of our consideration of the approximation valid in the LS,

$$p_l \cong E , \tag{12}$$

i.e., the longitudinal momentum and total energy of a secondary in the LS are almost equal for the secondaries produced in the forward cone. Equation (12) is equivalent to the relation

$$(m^2 + p_t^2)^{1/2}/p_l \cong 0 , \tag{12$'$}$$

since $E = (m^2 + p_l^2 + p_t^2)^{1/2}$. In high-energy nuclear interactions, the secondaries are mostly composed of pions,[11,17] and their average transverse momentum, $\langle p_t \rangle$, is small and constant over a wide range of E_p, which justifies Eq. (12′). In such cases, we have a new approximation formula for η,[18]

$$\eta \cong \ln E - \ln[\tfrac{1}{2}(m^2 + p_t^2)^{1/2}] \tag{13}$$

or

$$\eta \cong \ln p_l - \ln[\tfrac{1}{2}(m^2 + p_t^2)^{1/2}] . \tag{13$'$}$$

The second terms in the above equations are small and also insensitive to the variation of E and p_l for the reasons already mentioned above.

The simple single-particle distribution of the form $e^{a\eta} d\eta$ indicates essentially a power-law energy spectrum in the LS of the form $E^{a-1}dE$ from Eq. (13). If Eq. (10) is assumed as the differential spectrum, our method of constructing $\log_{10} F$ plots is essentially that we integrate the single-particle distribution of Eq. (10) as

$$\begin{aligned} F &\propto \int_{\eta_0}^{\eta} e^{a\eta} d\eta \\ &= (1/a)[e^{a\eta} - e^{a\eta_0}] \\ &\cong (1/a) e^{a\eta} , \end{aligned} \tag{14}$$

where η_0 is the largest value of η permitted by the kinematical limit in an interaction, and it is assumed to be sufficiently large in high-energy nuclear interactions with a negative value of a. Thus we expect a linear dependence

$$\log_{10}F \cong \alpha\eta + \gamma, \tag{15}$$

which is equivalent to Eq. (1), and Eq. (11) follows. Thus with the aid of experimentally established facts on high-energy nuclear interactions, our finding about the angular distribution can be best characterized by the differential power law of the energy spectrum of secondaries with the power index $a \approx -2$.

B. Discussion and Comparison to Various Models of Multiple Production

The multiperipheral model of Amati, Bertocchi, Fubini, Stanghellini, and Tonin[19] (ABFST) and its revised versions[20] as well as the work of Cheng and Wu[21] and the model by Takagi *et al.*[22] essentially predict the single-particle distribution to be[16,23]

$$\begin{aligned} d^3N &= C\exp(-A p_t^2)d^3p/E \\ &= C'\exp(-A p_t^2)d\eta dp_t^2 \end{aligned} \tag{16}$$

in the region of small value of η in the center-of-mass system (CMS) of the two colliding hadrons. The constants C and C' may be dependent on E_p. On the other hand, the prediction of the parton model of Feynman[4] in the limit of the "scaling behavior of the single-particle distribution" may be represented in the form[24]

$$d^3N = f(x)\exp(-A p_t^2)d^3p/E, \tag{17}$$

where $x = 2p_l/s^{1/2}$. Here $f(x)$ is independent of s, which is as usual the square of the total CMS energy of the collision, and unity for $|x| \lesssim 0.06$, and decreases rapidly to zero as x increases to the kinematical limit at $x = \pm 1$. Therefore, in the region of small values of η in the CMS, both of the above distributions are very similar. The limiting-fragmentation hypothesis considered by Yang *et al.*[5] predicts almost the same distribution.[25] They further speculate that the distribution for large p_t is[26]

$$d^3N = Cd^3p/(Ep_t^{\beta}), \tag{18}$$

with $\beta = 1$ or 2.

The gross features of the above theories have been reported to be confirmed by various experiments[27] and the physical picture of the models is explained especially by Bøggild *et al.*[16] and by Stodolsky.[28] The result of our analysis, shown grossly in Eq. (10), also confirms features of Eqs. (16) and (18) [especially with $\beta \cong 2$ in Eq. (18)]. However, the scaling behavior, shown in Eq. (17), does not seem to be borne out in our analysis because of the constancy of the value of α in the range of E_p from 20 to ~10^3 GeV. A similar deduction has been reported by von Lindern *et al.*[29]

It is of great interest that the fact that the values of α and the consistency of the estimate E_{ch} in Ref. 6, as a function of N_h, show their independence of N_h may imply that *the limiting distributions of the projectile fragments in the forward cone are independent of the nature of the target nucleus of the high-energy collision.*

There have been numerous theories and models for multiple production. Their predictions on the production-energy spectrum in high-energy collisions will be only briefly mentioned here. The statistical theory[30] initiated by Fermi predicts the single-particle distribution as

$$d^3N = Cd\eta dp_t^2, \tag{19}$$

which corresponds essentially to the power-law of energy spectrum $E^{-1}dE$. The thermodynamical model[30] may be represented typically for pion secondaries as

$$d^3N = C[\exp(E/kT) - 1]^{-1}d^3p, \tag{20}$$

which is the same form of function as Eq. (19) in the range $E \ll kT$, and has the dominant dependence of the exponential function in the range $E \gg kT$. The Bose-type distribution with

$$d^3N = \frac{Cd^3p}{E[\exp(E/kT) - 1]} \tag{20'}$$

has been compared with experiment by Hoang *et al.*[31]

The theory of Heisenberg[32] predicted the energy spectrum to be approximately as $(1/p^2)dp$, which is almost equivalent to Eq. (18).

The CKP model[33] assumes the longitudinal part of the single-particle distribution to be $\exp(-ap_l) \times dp_l$, which has been compared especially by Ko *et al.*[34]

The hydrodynamical theory of Landau[35] predicts an angular distribution of the form

$$\frac{dN}{d\lambda} \propto \exp(L^2 - \lambda^2)^{1/2}, \tag{21}$$

where $\lambda = -\ln\tan(\frac{1}{2}\theta) \cong \bar{\eta}(\bar{\theta})$ and $L = \frac{1}{2}\ln(E_p/2Mc^2)$, in which M is the nucleon mass. The three-dimensional hydrodynamical theory by Milekhin[36] predicted the single-particle distribution in terms of the rapidity parameter η as

$$\frac{dN}{d\eta} = (2\pi L)^{-1/2}\exp(-\eta^2/2L), \tag{22}$$

where $L = 0.56\ln(E_p/M) + 1.6$ for proton-proton jets.

Our group studied the angular distributions of charged secondaries of jets of $E_p = $~$10^{12}$ eV by fitting them to the following forms of the single-particle distribution in the CMS for the rapidity parameter[2]:

$$\frac{dN}{d\eta}=C\left(\frac{d}{d\eta}\right)[\exp(-a\eta^2)] \tag{23}$$

and also

$$\frac{dN}{d\eta}=C\eta^{1.5}\exp(-a\eta)\,. \tag{24}$$

The distribution represented by Eq. (1) in the present work has been the outcome of the above work in which the simplest form looked the best.

Actually we mention here that the observed double-peaked feature in the $\ln\tan\theta$ distribution of charged secondaries produced in jets (especially those jets with small multiplicity at ultrahigh energy[37]) led to the proposition of the two-fireball model.[38]

IV. CONCLUSION

The angular distributions of the charged secondaries, produced in the forward cone of high-energy jets, have been found to fit very well to a linear function of $\eta(\theta)$, i.e.,

$$\log_{10}F=\alpha\eta(\theta)+\gamma\ ,$$

where the absolute value of α is close to unity in the whole range of primary energies investigated in the present analysis: For $E_p=20-30$ GeV, the value of α approaches a constant, $\alpha\cong-1.2$, as n_s increases; for $E_p=570$ GeV/nucleon, $\alpha=-0.91$; for $E_p\approx10^3$ GeV, $\alpha=-0.6$.

We suggest that the gross features of the above results can be well represented by a power-law energy spectrum with constant power index $a=\alpha\ln10$. An exponential law of the energy spectrum such as found in the CKP model[33] may thus be excluded from our analysis, at least for the region of large p_t in the CMS.

It is significant that the value of α is independent of N_h, while it depends on n_s, in the analysis of 1335 proton jets of 30 GeV/c by grouping them according to N_h and n_s. It is interpreted that the target nucleus in the high-energy hadron-nucleus collisions does not affect the shape of the angular distribution of the forward-cone secondaries. This may suggest that the limiting distributions of the projectile fragments in the forward cone are independent of the nature of the target nucleus of the high-energy collision.

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$$\eta = \operatorname{arccosh}\,[E/(m^2+p_t{}^2)^{1/2}]$$

$$= \ln\,[(E+p_l)/(m^2+p_t{}^2)^{1/2}].$$

So, when $p_l \cong E$, Eq. (13) or (13′) results.

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