Relative Decay Rates of $\eta\to\pi^+\pi^-\pi^0$, $\eta\to\pi^+\pi^-\gamma$, $\eta\to\pi^+\pi^-\pi^0\gamma$, and $\eta\to\pi^+\pi^-$ *

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We have measured the branching ratio $(\eta\to\pi^+\pi^-\gamma)/(\eta\to\pi^+\pi^-\pi^0)$ to be 0.209 ± 0.004. The rare decays $\eta\to\pi^+\pi^-\pi^0\gamma$ and $\eta\to\pi^+\pi^-$ were searched for and not found. The upper limit on the branching ratio $(\eta\to\pi^+\pi^-\pi^0\gamma)/(\eta\to\pi^+\pi^-\pi^0)$ is 0.0024 at 90% confidence level, and the upper limit on the width of the decay $\eta\to\pi^+\pi^-$ is 4 eV. We then have a limit on the rate of a *P*- and *T*-violating strong interaction:

$$\frac{\text{Rate (}P\text{- and }T\text{-violating strong interaction)}}{\text{Rate (conventional strong interaction)}} < 4\times10^{-8}.$$

The Dalitz-plot distributions of the decays $\eta\to\pi^+\pi^-\pi^0$ and $\eta\to\pi^+\pi^-\gamma$ are reported in the preceding paper.[1] The experimental techniques and the methods of analysis have been described elsewhere.[2] This paper describes the method by which the relative detection efficiencies for these two decay modes were determined, and how the two rare decay modes $\eta\to\pi^+\pi^-\pi^0\gamma$ and $\eta\to\pi^+\pi^-$ were sought.

A. The Branching Ratio $(\eta\to\pi^+\pi^-\gamma)/(\eta\to\pi^+\pi^-\pi^0)$

We express the branching ratio as the product of two terms:

$$R\left(\frac{\eta\to\pi^+\pi^-\gamma}{\eta\to\pi^+\pi^-\pi^0}\right)=\frac{\epsilon(\pi^+\pi^-\pi^0)}{\epsilon(\pi^+\pi^-\gamma)}\,\frac{N(\pi^+\pi^-\gamma)}{N(\pi^+\pi^-\pi^0)}\ ,$$

where the first term is the ratio of the detection efficiencies and the second term is the ratio of the numbers of observed events. $N(\pi^+\pi^-\gamma)$ and $N(\pi^+\pi^-\pi^0)$ are the numbers of events used in the matrix-element calculation[1]; they are 18 150 and 80 884, respectively. The first term is obtained from Monte Carlo analysis. In addition to problems encountered in the matrix-element calculation we need to account for two sources of systematic error:

1. uncertainty in the decay matrix element,
2. uncertainty in the resolution of our detectors.

For the decay $\eta\to\pi^+\pi^-\gamma$, a part of the Dalitz plot (E_γ<50 MeV) was entirely eliminated from our analysis. The number of events lost because of this cut is estimated by an extrapolation of the matrix element. The fraction of the γ-ray spectrum which lies below 50 MeV is, however, very small and quite insensitive to the $\eta\to\pi^+\pi^-\gamma$ matrix element. For the ρ-dominant matrix element, which we used in our calculation, 3.0% of the events have E_γ less than 50 MeV. For the simple gauge-invariant matrix element, this loss would be 1.8%, and the branching ratio would be reduced by 1.2%. Our data are consistent with a 30% contribution from the simple gauge-invariant matrix element. This may cause a 0.4% change in the branching ratio.

The number of events lost also depends on the γ-ray energy resolution, which is ~15 MeV. Because the γ-ray energy distribution has a positive slope in the vicinity of the cut at 50 MeV, the resolution causes 1.8% of the $\eta\to\pi^+\pi^-\gamma$ events to be lost that would not be if E_γ were exactly known. Since we know the resolution to within 20%,[3] the maximum error of the branching ratio due to this source is 0.4%.

As discussed in the preceding paper,[1] we know the systematic error of the γ-ray energy to be less than 0.1 MeV. This can affect the branching ratio by at most 0.02%.

These considerations did not affect the efficiency for 3π events since we accepted events from the entire Dalitz plot. We obtain

$$\epsilon_{\pi\pi\gamma}=0.0351\pm0.0002,$$

$$\epsilon_{3\pi}=0.0327\pm0.0002.$$

The errors are purely statistical. The relative efficiency is not affected by the various geometric cuts imposed, since the cuts were applied equally to the two decay modes.

The two decay modes were separated by looking at the mass distribution of the undetected neutral particle (γ or π^0) and observing two peaks, one at 0 and the other at the π^0 mass. The amount of tail from one peak that fell under the other was calculated by Monte Carlo technique and was small. For

TABLE I. Previous measurements of the branching ratio $(\eta \to \pi^+\pi^-\gamma)/(\eta \to \pi^+\pi^-\pi^0)$.

Experiment [a]	Branching ratio
Crawford *et al.*	0.30 ± 0.06
Foster *et al.*	0.20 ± 0.04
Baltay *et al.*	0.28 ± 0.04
Litchfield *et al.*	0.25 ± 0.035
Gormley *et al.*	0.201 ± 0.006

[a] See Ref. 4.

the $\pi\pi\gamma$ events, we removed about 200 3π events. An error of 50 events in this subtraction would change the branching ratio by 0.3%. The combination of all the systematic errors gives

$$\Delta R < 0.011\, R.$$

For the branching ratio we get

$$R\left(\frac{\eta \to \pi^+\pi^-\gamma}{\eta \to \pi^+\pi^-\pi^0}\right) = \frac{0.0327 \pm 0.0002}{0.0351 \pm 0.0002} \times \frac{18150 \pm 175}{80884 \pm 360}$$

$$= 0.209 \pm 0.004\ .$$

Previous measurements of the branching ratio are listed in Table I.[4]

B. $\eta \to \pi^+\pi^-\pi^0\gamma$

Search for this decay mode is complicated by the presence of large numbers of $\eta \to \pi^+\pi^-\pi^0$ events. About 0.67% of the 3π events produce a photon (hereafter called γ_b) of energy greater than 9 MeV by inner bremsstrahlung. We are interested in $\eta \to \pi^+\pi^-\pi^0\gamma$ proceeding through other mechanisms.

Separation of $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^0\gamma_b$, and $\pi^+\pi^-\pi^0\gamma$ events is done by looking at the mass distribution

$$X^2 = M^2(\pi^0+\gamma)/M_{\pi^0}{}^2.$$

The 3π events peak at $X^2 = 1.0$. To calculate the bremsstrahlung distribution, we used the matrix element[5]

$$\mathfrak{M}_{3\pi\gamma_b}(p_+, p_-, p_0, k_b)$$
$$= \mathfrak{M}_{3\pi}(p_+, p_-, p_0)$$
$$\times \left[\frac{(2p_+^\mu + k_b^\mu)(ie)}{(p_+^\mu + k_b^\mu)^2 + m_+{}^2} + \frac{(2p_-^\mu + k_b^\mu)(-ie)}{(p_-^\mu + k_b^\mu)^2 + m_-{}^2}\right]\epsilon_\mu\ ,$$

where p_+, p_-, p_0, and k_b are the four-momenta of the three pions and the photon, $\mathfrak{M}_{3\pi}$ is the measured 3π matrix element, ϵ_μ is the photon polarization, m_+ and m_- are the pion masses, and e is the charge of the electron. This matrix element is valid for small k_b.

To calculate the $3\pi\gamma$ distribution, we used a simple gauge-invariant matrix element for the lowest allowed angular momentum state of the 3π system:

$$\mathfrak{M}_{3\pi\gamma} = \epsilon_{\mu\nu\rho\sigma}\epsilon^\mu p_{3\pi}^\nu k^\rho (\epsilon^{\sigma\alpha\beta\delta} p_{+\alpha} p_{-\beta} p_{0\delta})\ ,$$

where $p_{3\pi} = p_+ + p_- + p_0$, and $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric fourth-rank tensor. This gives

$$d\Gamma_{3\pi\gamma} \propto \frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\frac{d^3p_0}{2E_0}\frac{d^3k}{2k}(\vec{k}\cdot\vec{q})^2,$$

where $\vec{q} = \vec{p}_+ \times \vec{p}_-$ in the 3π center-of-mass system.

The effect of the bremsstrahlung is to broaden the 3π peak, giving it a tail at large X^2 as shown in Fig. 1. The $3\pi\gamma$ X^2 distribution is very broad, extending from 1.0 to 3.0, with a maximum at ~1.5.

We looked for the $3\pi\gamma$ events in the region $X^2 > 2.2$. The region $X^2 < 2.2$ contains the tails of the 3π and bremsstrahlung X^2 distributions, which necessitate a large subtraction in order to get the number of $3\pi\gamma$ events. This gives a large error on the number of events.

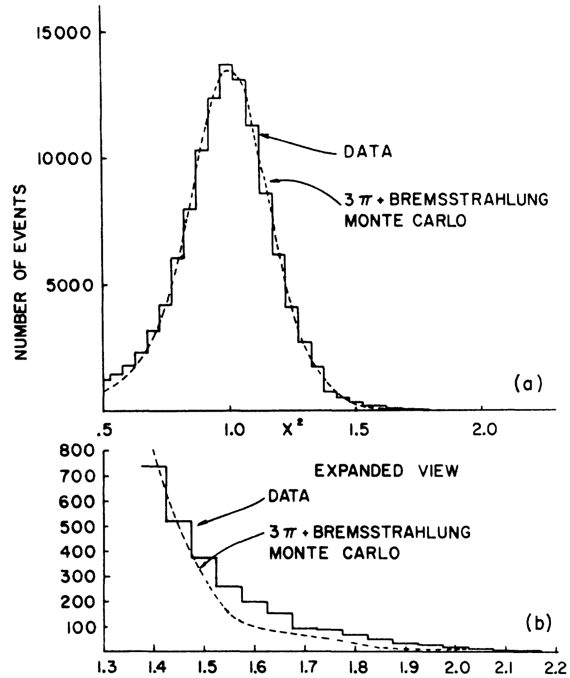

FIG. 1. (a) X^2 distribution in the region where $\eta \to \pi^+\pi^-\pi^0\gamma$ events are expected. The dashed line indicates a Monte Carlo-generated distribution which includes $\eta \to \pi^+\pi^-\pi^0$ and bremsstrahlung events. (b) An expanded view of the large-X^2 region.

In the region $X^2 > 2.2$, there were 96 events. There is no evidence that there are any $\eta \to \pi^+\pi^-\pi^0\gamma$ events in this region from examination of the neutron time-of-flight spectrum. There were 56 events in the 4.8-nsec-wide region of the time of flight where $\pi^- p \to \eta n$ events should be.

Monte Carlo calculation indicated that 14% of the detected $3\pi\gamma$ events would be in the region $X^2 > 2.2$. Since there were 110 000 3π events,[6] with a relative detection efficiency for 3π and $3\pi\gamma$ events of $\epsilon_{3\pi\gamma}/\epsilon_{3\pi} = 0.41$, we get the upper limit

$$R\left(\frac{\eta \to \pi^+\pi^-\pi^0\gamma}{\eta \to \pi^+\pi^-\pi^0}\right) < \frac{15}{0.41 \times 0.14 \times 110000} = 0.0024$$

at 90% confidence level.

C. $\eta \to \pi^+\pi^-$

The observation of the decay $\eta \to \pi^+\pi^-$ would imply the existence of a P- and T-violating strong interaction. A recent model[7] incorporating a P-violating strong interaction suggests a partial width for $\eta \to \pi^+\pi^-$ of the same order of magnitude as that for $\eta \to \pi^+\pi^-\pi^0$.

The decay was sought by looking for a peak in the neutron time-of-flight spectrum for a sample of the reaction $\pi^- p \to \pi^+\pi^- n$. The following cuts were made: $Y^2 < 1.10 M_n^2$, $|X^2| < 0.25$ MeV/c, $|\Delta E| < 35$ MeV, $|\Delta p_z| < 60$ MeV/c, $|\Delta p_x|$ and $|\Delta p_y| < 50$ MeV/c, where Y^2 is the missing mass of the outgoing neutrals, ΔE and Δp represent the missing energy and momentum, and M_n is the neutron mass. The sample was further restricted to 12 spark events. A Monte Carlo study indicated that about 1.6% of the detected $\pi\pi\gamma$ events would survive these cuts, a total of 400 events.[8]

We obtained 1 064 780 events, of which 330 897 lie in a 4.1-nsec region around the η mass. Fits to the time-of-flight distribution, over a somewhat larger region, are consistent with no enhancement at the η mass. If we set an upper limit of 3σ, where σ is the statistical fluctuation of the number of events in the η region, we have 1700 events. From this, we calculate the upper limit for the branching ratio to be 0.15%, taking the relative efficiency for 12-spark 2π events to 3π events to be 1.9 and the number of 12-spark 3π events to be 140 000. From the total decay width of the η of 2.36 keV,[9] we obtain a partial width for $\eta \to \pi^+\pi^-$ of 4 eV.

This result is clearly inconsistent with the model mentioned above. If η were allowed to decay strongly into $\pi^+\pi^-$, we would expect its width to be ~ 100 MeV. We then have a limit on the P and T violating strong interaction:

$$\frac{\text{Rate}(P \text{ and } T \text{ violating strong interaction})}{\text{Rate(conventional strong interaction)}} < 4 \times 10^{-8}.$$

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[1]J. G. Layter *et al.*, preceding paper, Phys. Rev. D 7, 2565 (1973).

[2]J. G. Layter *et al.*, Phys. Rev. Letters 29, 316 (1972); J. J. Thaler *et al.*, Phys. Rev. Letters 29, 313 (1972); J. G. Layter, Columbia University Ph.D. thesis, Nevis Report No. 192 (unpublished); J. J. Thaler, Columbia University Ph.D. thesis, Nevis Report No. 194 (unpublished).

[3]This estimate is based on the comparison of the widths of Monte Carlo generated distributions with the data.

[4]Measurements of the $(\eta \to \pi^+\pi^-\gamma)/(\eta \to \pi^+\pi^-\pi^0)$ branching ratio have been made by: F. S. Crawford and L. R. Price, Phys. Rev. Letters 16, 333 (1966); M. Foster *et al.*, Phys. Rev. 138, B652 (1965); C. Baltay *et al.*, Phys. Rev. Letters 19, 1498 (1967); P. J. Lichtfield *et al.*, Phys. Letters 24B, 486 (1967); M. Gormley *et al.*, Phys. Rev. D 2, 501 (1970).

[5]N. Christ, private communication.

[6]We impose looser cuts on the 3π data than we did in Sec. A since our errors here are limited by statistics.

[7]Y. Yamaguchi, Univ. of Tokyo Report No. UT-105 (unpublished).

[8]There is an error in the discussion of this point by J. G. Layter, Ph.D. thesis (see Ref. 2).

[9]Particle Data Group, Phys. Letters 39B, 1 (1972).