Study of Dalitz-Plot Distributions of the Decays $\eta \to \pi^+\pi^-\pi^0$ and $\eta \to \pi^+\pi^-\gamma$*

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Based on 80 884 events, we obtained the matrix element squared for the decay $\eta \to \pi^+\pi^-\pi^0$: $|M(x,y)|^2 = 1 - (1.08 \pm 0.014)y + (0.03 \pm 0.03)y^2 + (0.05 \pm 0.03)x^2$ where x and y are the usual Dalitz-plot coordinates. The γ-ray energy spectrum was measured for 18 150 $\eta \to \pi^+\pi^-\gamma$ events. ρ dominance of the $\pi^+\pi^-$ final state is strongly favored over a simple gauge-invariant matrix element.

We measured the Dalitz-plot distributions of 80 884 events of the decay $\eta \to \pi^+\pi^-\pi^0$ and 18 150 events of the decay $\eta \to \pi^+\pi^-\gamma$. The experimental techniques have been reported elesewhere and are briefly summarized here.[1] Those aspects of the analysis pertinent to the measurements of the matrix element are described in more detail.

η mesons were produced in the reaction $\pi^- p \to \eta n$. The incident pion beam had a momentum of 730 MeV/c with a spread of $\pm\frac{1}{2}\%$. The η's were identified by the time of flight of the associated neutron, and the momenta of the two outgoing charged pions were measured in two six-gap sonic spark chambers located in a magnetic field.

Our sample of 5.5 million reconstructable events consisted primarily of the following reactions:

$$\pi^- p \to \pi^+\pi^- n, \quad (1)$$

$$\to \pi^+\pi^-\pi^0 n, \quad (2)$$

$$\to \pi^+\pi^-\gamma n. \quad (3)$$

To eliminate reaction (1) and to separate reactions (2) and (3), we considered three fictitious reactions:

$$\pi^- p \to \pi^+\pi^- Y^0, \quad (4a)$$

$$\to \pi^\pm n Z^\mp, \quad (4b)$$

$$\to \pi^+\pi^- n X^0. \quad (4c)$$

In reaction (1), one has $M_Y^2 = M_n^2$, $M_Z^2 = M_\pi^2$, and $E_X = 0$. M_Y and M_Z are the masses of the Y and Z particles defined in (4a) and (4b), and E_X is the energy of the X particle defined in (4c). M_n and M_π are the masses of the neutron and pion. We eliminated those events for which $M_Y^2 < 1.15 M_n^2$, $M_Z^2 < 1.8 M_\pi^2$, and $E_X < 50$ MeV. These cuts eliminated none of reaction (2), 17% of reaction (3), and most of reaction (1), leaving 413 000 events. Further geometric cuts reduced the sample to the 257 000 events used in the asymmetry measurement.[1] For the reasons discussed below, we found it necessary to impose further cuts to obtain the sample of 99 000 events used in the matrix-element analysis.

The usual method for correcting the data for the variation in detection efficiency over the Dalitz plot is to simulate the experiment with a Monte Carlo computer program. The program included the resolution of the apparatus, multiple Coulomb scattering, $\pi \to \mu$ decays, etc. The program generated only η events and assumed that the spark chambers were uniformly efficient over their fiducial area. From the data we attempted to get a clean sample of η decays and also to obtain data in such a way that the effects of inefficiency of the chambers were eliminated.

Neutrons associated with η production form a sharp peak in the time-of-flight spectrum, while those from reaction (1) form a broad distribution. The remaining small amount of reaction (1) was removed by comparing the neutron time-of-flight spectrum with that for a known pure sample of η events.[2] Since studies of the data indicated that more background events were to be found at lower pulse heights, we imposed a more stringent pulse-height cut than for the asymmetry analysis. Independence of our results on this cut indicated that the background subtraction was correctly done.

To eliminate effects due to gap inefficiency, we eliminated the outer six cm of each gap from the analysis, and we required that each pion traverse at least five gaps. The number of gaps was found not by counting sparks but by fitting a helix through any track with three or more sparks and extrapolating the track. In this way, we kept a track even if two gaps failed. The efficiency of each gap was everywhere > 90% so that the probability of losing three gaps is less than 0.1%. In Fig. 1, we compare the spark distribution in the 6th gap and the event distribution along the target with the Monte Carlo. The agreement is very good.

The above analysis reduced the number of events to the final number of 99 000. We did not include the 810-MeV/c data[1] in the results.

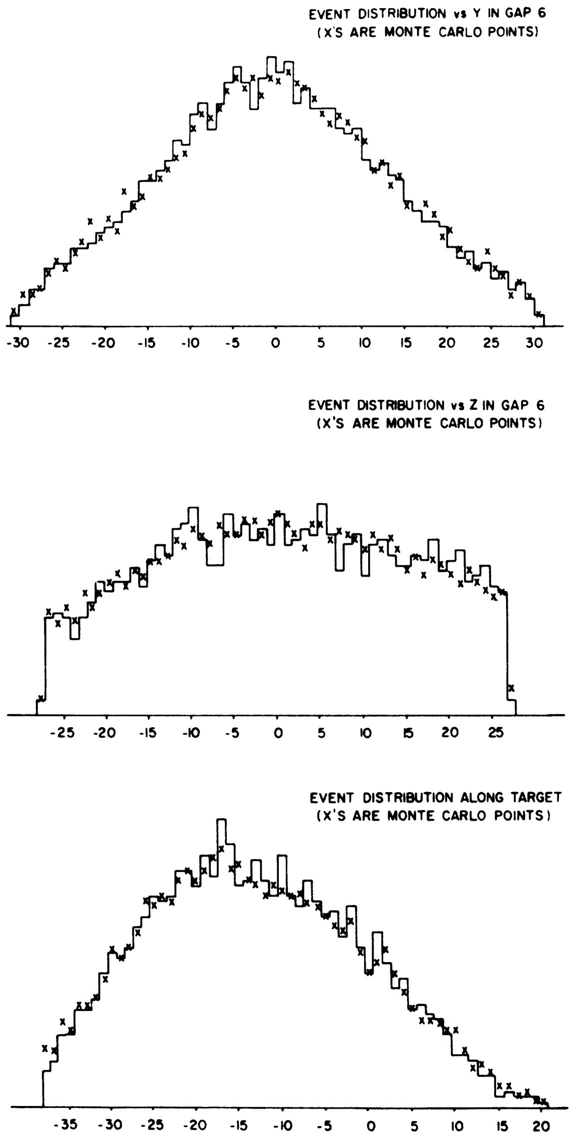

FIG. 1. (a), (b) Spark distribution in gap 6 and (c) vertex distribution along the target compared with Monte Carlo distributions.

On the basis of the null result for the asymmetry,[1] we limited the form of the expression for the $\eta\to\pi^+\pi^-\pi^0$ matrix element squared to

$$|M(|x|,y)|^2 = N_0(1+ay+by^2+cx^2).$$

We considered the number of events in each of 78 Dalitz-plot bins to be

$$N_{ij} = N_0\epsilon_{ij}\int |M(x_i,y_j)|^2 dx_i dy_j ,$$

where the integration is over the area of each bin. Here $x=\sqrt{3}\,(T_+ - T_-)/Q$, $y=(3T_0/Q)-1$, T_+, T_-, and T_0 is the kinetic energy of the π^+, π^-, and π^0 in the η rest system, respectively, and $Q=T_+ + T_- + T_0$. We assumed the efficiency ϵ_{ij} to be constant over the area of the bin. The integrals $\int dx_i dy_j$, $\int y dx_i dy_j$, etc. were numerically evaluated for bins on the boundary. The coefficients a, b, c, and N_0 were then obtained by least-squares fitting. Since only relative efficiencies were used, N_0 has no meaning.

The results are shown in Table I along with fits to other forms of $|M(|x|,y)|^2$. The coefficients b and c are consistent with zero. All the fits are equally good, except for the fit to a linear matrix element. Fits were also made to data with a cut on the neutron pulse height that eliminated about 50% of the events, and a cut that eliminated the outer 12 cm of each gap from the analysis. The results are essentially the same for all cuts. Figure 2 shows our fit to $|M(|x|,y)|^2 = 1+ay$ for the data projected on the y axis.

Previous measurements of the 3π matrix element are shown in Table II.[4]

Inclusion of Coulomb corrections[3] makes a small change in a and b:

$$a:\ -1.080 \to -1.086,$$

$$b:\ 0.030 \to 0.021.$$

There is no change in c. Both corrections are smaller than our error.

TABLE I. Fits to the $\pi^+\pi^-\pi^0$ decay matrix element.

$\lvert M(\lvert x\rvert,y)\rvert^2$	a	b	c	χ^2/degrees of freedom
$1+ay+by^2+cx^2$	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031	51.7/50
$1+ay+by^2+cx^2$ [a]	-1.08 ± 0.03	0.08 ± 0.06	0.03 ± 0.07	42 /47
$1+ay+by^2+cx^2$ [b]	-1.10 ± 0.03	0.05 ± 0.05	0.02 ± 0.06	42 /48
$1+ay$	-1.07 ± 0.013			53.8/52
$1+ay+by^2+cy^3$	-1.118 ± 0.027	0.048 ± 0.029	0.102 ± 0.058	51.2/50
$1+2ay+a^2y^2$	-0.52 ± 0.007			103 /52

[a] With a stricter neutron pulse-height cut which eliminated 50% of the events.
[b] With a stricter neutron pulse-height cut and a 12-cm fiducial cut.

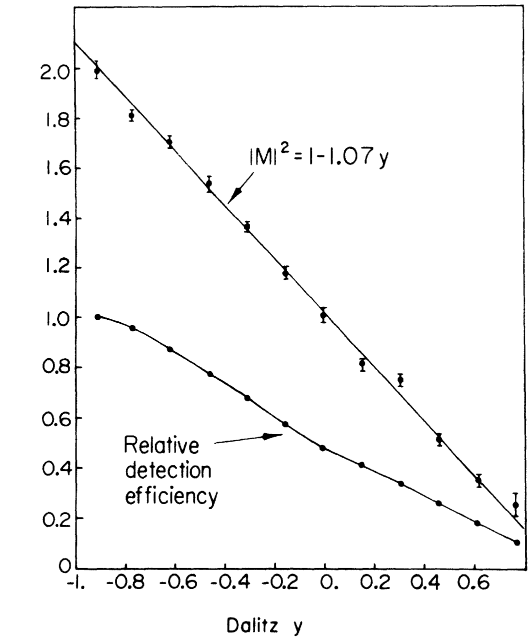

FIG. 2. $\eta \rightarrow \pi^+\pi^-\pi^0$ data projected onto the Dalitz y axis with a linear fit to the matrix element squared. The detection efficiency is also shown as a function of y.

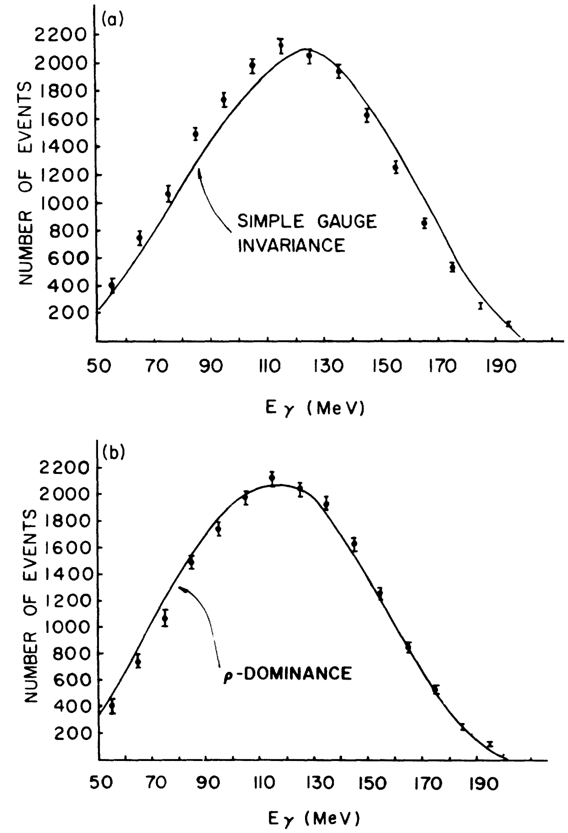

FIG. 3. Fit to the γ energy distribution in $\eta \rightarrow \pi^+\pi^-\gamma$ for (a) the simplest gauge-invariant matrix element, and (b) the ρ-dominant matrix element.

One possible source of error is a systematic error in the π^0 energy measurement which could be caused by an error in the incident pion momentum or by an error in the magnetic field measurement. In reaction (1), all three outgoing particles were detected allowing a 4-constraint fit by imposing conservation of momentum and energy. We calculated p_{in} to be 730.7 ± 0.5 MeV/c and the error on the magnetic-field normalization to be less than 0.2%. The largest changes that could be caused by these errors are:

a: $-1.080 \rightarrow -1.080$,

b: $0.034 \rightarrow 0.040$,

c: $0.046 \rightarrow 0.041$.

These changes are smaller than our errors.

TABLE II. Previous measurements of the $\pi^+\pi^-\pi^0$ decay matrix element.

Experiment [a]	No. of events	Fit	Coefficients
Baglin *et al.*	526	$\lvert M(y)\rvert^2 = 1 + 2ay$	$a = -0.41 \pm 0.04$
Larribe *et al.*	765	$\lvert M(y)\rvert^2 = 1 + 2ay$	$a = -0.46 \pm 0.03$
Carpenter *et al.*	1138	$\lvert M(y)\rvert^2 = 1 + 2ay$	$a = -0.47 \pm 0.04$
Bubble Chamber Collaboration	1300	$M(y) = 1 + ay$ [b]	Re$a = -0.48 \pm 0.04$, Im$a = 0.05 \pm 0.39$
Kim	1215	$M(y) = 1 + ay$ [b]	Re$a = -0.54 \pm 0.04$, Im$a = 0.00 \pm 0.22$
Cnops *et al.*	7170	$M(y) = 1 + ay$ [b]	Re$a = -0.55 \pm 0.02$, Im$a = 0.00 \pm 0.11$
Gormley *et al.*	30 905	$\lvert M(x,y)\rvert^2 = 1 + ay + by^2 + cx$	$a = -1.18 \pm 0.02$
			$b = 0.19 \pm 0.03$
			$c = 0.05 \pm 0.02$

[a] See Ref. 4.

[b] Fit was made with complex a.

TABLE III. Previous fits to the γ-ray energy spectrum in $\eta \to \pi^+\pi^-\gamma$.

Experiment [a]	No. of events	χ^2 for simplest matrix element	χ^2 for ρ dominant M.E.	Degrees of freedom
Crawford *et al.*	33	13.6	5.9	6
Kim	250	13.8	4.4	7
Cnops *et al.*	1088	48.0	7.6	8
Gormley *et al.*	7257	145	11.5	13

[a] See Ref. 7.

The procedure followed for the $\pi\pi\gamma$ events was similar to that used for the 3π, except that to parametrize the Dalitz plot, we used the variables k and $\cos\theta$, where k is the γ-ray energy in the η rest system, and θ is the angle between the π^+ and the γ ray in the dipion rest system. There is no evidence of any angular momentum state of the dipion other than $l=1$.[5] Thus, the mass distribution of the dipion is sensitive to the π^+-π^- p-wave phase shift in a region well removed from the ρ meson:

$$4M_\pi{}^2 \leq M_{2\pi}{}^2 \leq M_\eta{}^2 .$$

The simplest gauge-invariant matrix element for the decay can be written[6]:

$$\mathfrak{M} = \frac{f(s)}{M^3}\, \epsilon_{\mu\nu\sigma\tau}\, e_\gamma^\mu p_\gamma^\nu p_+^\sigma p_-^\tau , \qquad (5)$$

where $s = M_{2\pi}{}^2$, $\epsilon_{\mu\nu\sigma\tau}$ is the totally antisymmetric fourth-rank tensor, e_γ^μ is the photon polarization, and p_γ, p_+, p_- are the photon, π^+, and π^- momenta, respectively. If there is no final-state interaction between the pions, $f(s)=1$. If the phase shift of the two pions is dominated by the ρ meson, then $f(s)$ becomes

$$f(s) = M_\rho{}^2/(s - M_\rho{}^2) .$$

The effect of the ρ is to enhance $\mathfrak{M}$ for large s or small E_γ.

The results showing the comparison between the data and the two forms of $f(s)$ are in Figs. 3(a) and 3(b), where the data and f are presented as a function of $E_\gamma = \frac{1}{2}(M_\eta - s/M_\eta)$. For the simple matrix element, $\chi^2 = 113$ for 14 degrees of freedom, while for ρ dominance, $\chi^2 = 25.3$.

Similar to the discussion for 3π, we know that the systematic error on E_γ due to an error in the incident pion momentum, is less than 0.1 MeV, and causes a change of 1.1 in χ^2.

Previous measurements of the γ energy spectrum are listed in Table III.[7]

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[1] J. J. Thaler *et al.*, Phys. Rev. Letters 29, 313 (1972); J. G. Layter *et al.*, *ibid.* 29, 316 (1972); J. G. Layter, Columbia University Ph.D. thesis, Nevis Report No. 192 (unpublished); J. J. Thaler, Columbia University Ph.D. thesis, Nevis Report No. 194 (unpublished).

[2] A pure sample of reaction (2) was obtained by imposing strict requirements on $M_Y{}^2$, $M_x{}^2$, $M_z{}^2$, and E_x, and making a background subtraction. See Ref. 1.

[3] A. Neveu and J. Scherk, Phys. Letters 27B, 384 (1968).

[4] Measurements of the $\pi^+\pi^-\pi^0$ decay matrix element were made by Columbia-Berkeley-Purdue-Wisconsin-Yale Collaboration, Phys. Rev. 149, 1044 (1966); J. Kim, Columbia University Ph.D thesis, Nevis Report No. 170 (unpublished); A. M. Cnops *et al.*, Phys. Letters 27B, 113 (1968); M. Gormley *et al.*, Phys. Rev. D 2, 501 (1970); C. Baglin *et al.*, Phys. Letters 29B, 445 (1969); A. Larribe *et al.*, *ibid.* 23, 600 (1966); D. W. Carpenter *et al.*, Phys. Rev. D 1, 1303 (1970).

[5] In Ref. 1, the fit to $dN/d|\cos\theta| = A \sin^2\theta(1+\beta\cos^2\theta)$ gave $\beta = 0.12 \pm 0.06$. We do not believe this to be an indication of the existence of a d-wave contribution, because the dependence of β on E_γ is inconsistent with the theoretical prediction. See J. J. Thaler, Nevis Report No. 194 (unpublished).

[6] See for example, B. Barrett and T. Truong, Phys. Rev. 147, 1161 (1966).

[7] Measurements of the $\pi\pi\gamma$ matrix element were made by F. S. Crawford *et al.*, Phys. Rev. Letters 16, 333 (1966); J. Kim, Columbia University Ph.D. thesis, Nevis Report No. 170 (unpublished); A. M. Cnops *et al.*, Phys. Letters 26B, 398 (1968); M. Gormley, Phys. Rev. D 2, 501 (1970).