High-Energy π-π Collisions*

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We have examined the final states $\pi^- + \pi^+ + n$, $\pi^- + \pi^- + \Delta^{++}$, and $\pi^- + \pi^0 + \pi^- + \pi^+ + p$ resulting from the interactions of 25-GeV/c π^- mesons incident on the BNL 80-in. Hydrogen Bubble Chamber to study elastic and inelastic π-π scattering processes. Using the Dürr-Pilkuhn parametrization for off-mass-shell corrections we determine the π^--π^+ and π^--π^- cross sections. The elastic cross sections decrease with increasing dipion energy to a value of 1.5–2.0 mb. From the optical theorem it is found that the total cross sections become about 15 mb in the 2–4-GeV/c^2 region. We have also looked at the properties of the reaction $\pi^- + \pi^0 \to \pi^- + \pi^0 + \pi^- + \pi^+$. The g meson seems to be prominently produced in this reaction.

I. INTRODUCTION

Production processes involving one-pion exchange (OPE) constitute a large fraction of the π^--p interaction cross section at low incident pion energies (7 mb out of a total of 30 mb). At higher energies this fraction is reduced, but one-pion exchange still accounts for a considerable amount of the total π-p cross section. For example, in the 5–10-GeV region processes such as $\pi^- + \pi^+ \to \rho^0$ and $\pi^- + \pi^+ \to f^0$ contribute about 2.5 mb of the total π^--p cross section of about 27 mb, and there is also evidence for $\pi^- + \pi^{0,+} \to (4\pi)^{-,0}$ of about 2 mb. These latter processes are mixed up kinematically with other processes such as baryonic resonance production. The further possibility of having two or more of the final-state pions resonating makes analysis of these processes very complicated.

Studies of π-π scattering in the resonant region below 2 GeV/c^2, primarily by phase-shift analyses, indicate that above the f^0 the scattering becomes quite inelastic and consequently diffractive. We have studied π^--p interactions at 25 GeV/c in order to obtain information about high-energy π-π scattering processes.

Specifically, we have looked at the reactions

$$\pi^- + p \to \pi^- + \pi^+ + n\,, \tag{a}$$

$$\pi^- + p \to \pi^- + \pi^- + \Delta^{++}\,, \tag{b}$$

$$\pi^- + p \to \pi^- + \pi^0 + \pi^- + \pi^+ + p \tag{c}$$

from an analysis of a 100 000-picture exposure of 25-GeV/c π^- mesons in the BNL 80-in. Hydrogen Bubble Chamber.[1,2]

II. EXPERIMENTAL PROCEDURE

The photographic data were obtained in two separate exposures of approximately 50 000 pictures each. The magnitude and direction cosines of the beam momentum for each exposure were determined from a preliminary sample of analyzed four-prong events fitting the four-constraint-fit hypothesis $\pi^- + \pi^- + \pi^+ + p$. These events were then refitted with no beam constraints applied. The resulting beam momenta were averaged for each exposure, with the result

$$p_{\text{in}} = 24.518 \pm 0.275 \text{ GeV}/c, \quad \text{exposure 1}$$

$$p_{\text{in}} = 24.710 \pm 0.300 \text{ GeV}/c, \quad \text{exposure 2}\,.$$

A chamber fiducial volume was established and the film was scanned for all two-, three-, and four-prong events with no associated strange particles which had production vertices in the fiducial volume. Events were not sketched for measurement if fast outgoing tracks had less than $\frac{1}{2}$ cm visible sagitta in space.

Two-prong events without identifiable protons were measured if the sum of the momenta of the outgoing charged tracks, as estimated with a momentum template on the scan table, was at least 15 GeV/c. All three-prong events were measured and fitted to the hypothesis

$$\pi^- + p \to \pi^- + \pi^- + \pi^+ + (p)_{\text{unseen}}\,.$$

All four-prong events with a visible proton were measured. Four-prong events fitting the four-constraint final state $\pi^- + \pi^- + \pi^+ + p$ were found to be unambiguously fitted, with virtually no overlap with fits to the final state of reaction (c), on the basis of fit χ^2's.

Events fitting reaction (a) can be contaminated

by the final states $\pi^- + p + m(\pi^0)$ and $\pi^- + \pi^+ + n + m(\pi^0)$, $m \geq 1$. In fitting events to (a), a vidicon device was used to determine the relative ionization of the positive track. This device can discriminate between π^+ and p up to track momenta of 2 GeV/c.[3] One would expect very few protons with momenta greater than 2 GeV/c to be produced, since the reaction $\pi^- + p \to \pi^- + p + \pi^0$ is generally very peripheral.[2]

To further minimize contamination in fits to (a), events were not accepted if γ's converting in the liquid hydrogen or in a $\frac{1}{4}$-in. Ta plate at the downstream end of the chamber pointed to the vertex of an eligible two-prong event. This procedure is effective in discriminating against events with fast π^0, but the detection efficiency of the chamber is poor for slow π^0. Finally, cuts were imposed on the missing-mass distributions of fits to reactions (a) and (c). Contamination of the event samples for these two reactions is estimated to be less than 20%.

The event samples for each reaction contained the following number of events:

$\pi^- + p \to \pi^- + \pi^+ + n$ 1095 events,

$\pi^- + p \to \pi^- + \pi^- + \pi^+ + p$ 2119 events,

$\pi^- + p \to \pi^- + \pi^- + \pi^+ + p + \pi^0$ 1117 events.

By requiring $M(\pi^+ p) < 1.35$ GeV/c^2 a total of 468 events fitting

$\pi^- + p \to \pi^- + \pi^- + \Delta^{++}$

was obtained for this experiment.

The reaction cross sections were obtained by normalizing the sum of events of all topologies found in the fiducial volume to the total $\pi^- p$ cross-section data of Foley *et al.*[4] Each event topology was corrected for scanning efficiency.

To correct for the unseen elastic cross section a sample of 3150 events fitting the kinematic elastic scattering hypothesis was fitted to an exponential

$$\frac{d\sigma}{dt} \propto e^{at + bt^2}$$

to enable an extrapolation to $t = 0$ to estimate the unseen elastic contribution to the total cross section, with the result

$$a = 8.8 \pm 0.6 \ (\text{GeV}/c)^{-2} ,$$

$$b = 3.5 \pm 1.2 \ (\text{GeV}/c)^{-4} .$$

This compares favorably with the results of Foley *et al.*[5] on π-p elastic scattering at high energies.

The resulting production cross sections for reactions (a) and (b) were

$$\sigma(\pi^- + p \to \pi^- + \pi^+ + n) = 0.33 \pm 0.08 \text{ mb},$$

$$\sigma(\pi^- + p \to \pi^- + \pi^- + \Delta^{++}) = 0.14 \pm 0.03 \text{ mb}$$

at 25-GeV/c incident pion momentum.

III. π^--π^+ AND π^--π^- ELASTIC AND TOTAL CROSS SECTIONS

We deduce the π-π cross sections by comparing the data of reactions (a) and (b) to the predictions of the OPE (one-pion exchange) model. The diagrams and notation for pion emission from the target proton resulting in a final-state nucleon and two pions are illustrated in Fig. 1, and the analysis procedure parallels that of Wolf.[6]

With exchange of a spacelike pion with invariant mass squared $\mu_e^2 = -\Delta^2$ the differential cross section for the processes of Fig. 1 can be written as

$$\frac{d^3\sigma}{d\Delta^2 dm\, d\cos\theta} = \frac{1}{4\pi p^2 s} |A_{NN\pi}|^2 \frac{1}{(\Delta^2 + \mu^2)^2} m^2 k' \frac{d\sigma(m, \mu^2, \mu_e^2)}{d\cos\theta} , \tag{1}$$

where p = incident momentum in the over-all c.m. system, s = c.m. energy squared, m = invariant mass of the dipion system, k' = momentum of incident and exchanged pions in the dipion c.m. system, θ = pion scattering angle in the dipion c.m. system, and μ_e = mass of the exchanged pion. The term $A_{NN\pi}$ includes all of the pion-nucleon coupling dependence, where we assume γ_5 coupling in a parity-conserving P-wave vertex.

For the two-nucleon vertices in Fig. 1, we have

$$A_{pn\pi} = \Delta^2 \pi^2 g^2 ,$$

where $g = 29.2$ is the π-N coupling constant, and

$$A_{p\Delta\pi} = \frac{M^2 (M + M_p)^2 + \Delta^2}{(M + M_p)^2 - \mu^2} K \sigma_{p\pi}(M) ,$$

where $M = \pi^+ p$ invariant mass, and K = momentum of π^+ and p in the Δ^{++} c.m. system. In the case of reaction (b), Eq. (1) is integrated over $M(\pi^+ p)$.

When the π-π scattering takes place in a state of definite angular momentum we can replace $\sigma(m, \mu^2, \mu_e^2)$ by $\sigma^l(m, \mu^2, \mu_e^2)$. The differential cross sections $d\sigma/d\cos\theta$ are then known functions of θ. We use the Born approximation to relate the off-shell cross section to the on-shell cross section ($\mu_e \to \mu$) in a state of angular momentum l

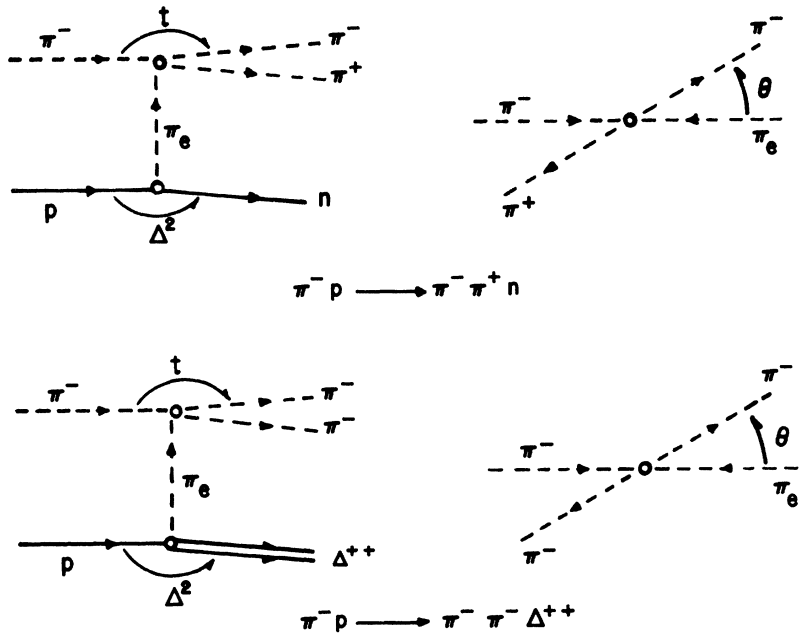

FIG. 1. Feynman diagrams and notation for one-pion exchange in the reactions $\pi^- + p \to \pi^- + \pi^+ + n$ and $\pi^- + p \to \pi^- + \pi^- + \Delta^{++}$, and definition of the π-π scattering angle θ.

(Ref. 7):

$$k'\sigma^I(m, \mu^2, \mu_e{}^2) = \left(\frac{k'}{k}\right)^{2I} k\sigma^I(m, \mu^2, \mu^2) \ , \qquad (2)$$

where

k' = pion momentum in the dipion c.m. system

$= \frac{1}{2}(m^2 - 4\mu^2)^{1/2}$.

The term $\sigma^I(m, \mu^2, \mu^2)$ is the cross section for elastic π-π scattering at a dipion energy m.

We could substitute Eq. (2) into Eq. (1) and compare with the data to extract $\sigma(m, \mu^2, \mu^2)$; however, the results would not be believable because the model fails for several reasons. Except at dipion masses where the scattering is dominated by a single resonant state, and of course at threshold, we do not know the partial-wave composition of the π-π system. This is particularly true above the resonant region. The model fails to reproduce the Δ^2 distribution, particularly at low dipion masses and large Δ^2. This arises from the way in which we eliminate the Δ^2 dependence from the π-π scattering cross section.

We do not understand the nature of the pion emission process at the nucleon, or the structure of the pion propagator and its interaction with the incident pion, in quantitative detail. We absorb all of the unknown in the form-factor approach to correcting the model. A propagator form factor can be used to take into account possible higher-order corrections to the OPE diagram, and vertex form factors can be constructed to make the model fit the data.

We use the method of Dürr and Pilkuhn[8] to modify the Born approximation for the off-shell scattering cross section. It was found to be unnecessary to include a propagator form factor in our expression for the differential cross section. By restricting our analysis to events with small Δ^2, we can effectively ignore the form-factor corrections for large dipion masses, where the scattering is expected to be dominated by states of high angular momentum. In this case all off-shell corrections become negligible and we can use Eq. (1) with the real π-π cross section in the expression for the differential cross section.

In Fig. 2 we plot the dipion mass spectra and absolute value of the nucleon four-momentum-transfer-squared (Δ^2) distributions for reactions (a) and (b). The $\pi^-\pi^+$ invariant-mass distribution exhibits strong ρ^0 and f^0 production and little or no g^0-meson formation. We might expect to see the g with our number of events; however, our lack of mass resolution probably smears the signal from the g.

Using an estimated 20% background, we calculate the ρ^0 and f^0 production cross sections at 25 GeV/c to be

$$\sigma(\pi^- p \to \rho^0 n) = 23.4 \pm 8.0 \ \mu\text{b} \ ,$$

$$\sigma(\pi^- p \to f^0 n) = 19.5 \pm 7.5 \ \mu\text{b} \ .$$

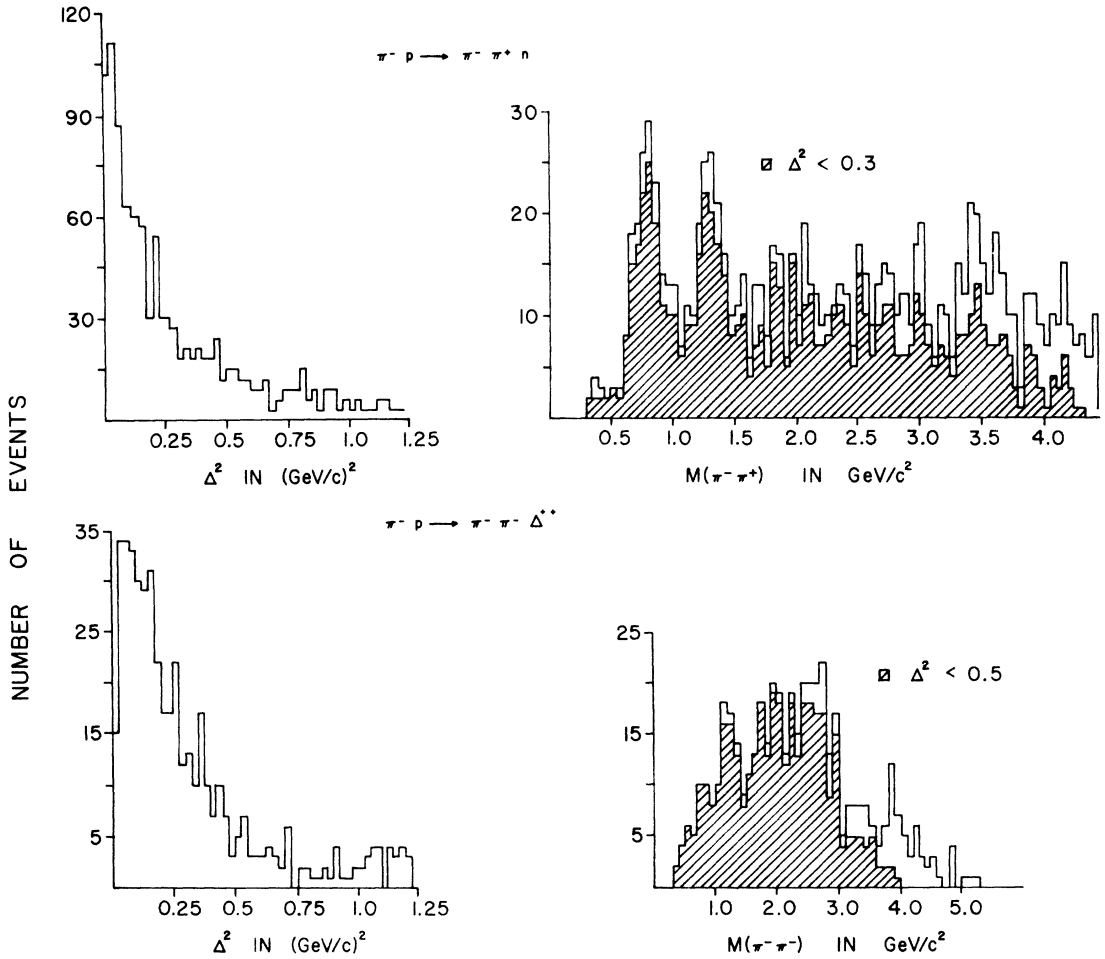

FIG. 2. Plots of $M(\pi\pi)$ and Δ^2 (target, outgoing nucleon) for the reactions $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ and $\pi^- + p \rightarrow \pi^- + \pi^- + \Delta^{++}$. Cross-hatched histograms are $M(\pi\pi)$ for events with $\Delta^2(p,n) < 0.3$ (GeV/c)^2 and $\Delta^2(p, \Delta^{++}) < 0.5$ (GeV/c)^2, respectively, for the above reactions.

We estimate the g^0 production cross section for decay into $\pi^- + \pi^+$ to be no more than 5 μb at this energy.

We plot the $\rho^0 + n$ and $f^0 + n$ production cross sections as a function of the incident π^- momentum from this experiment and previous experiments at lower energies[9] in Fig. 3, together with the absolute predictions of the OPE model. The excellent agreement between the data and the model reflects favorably on our event selection procedure for 1c fits.

To enhance the OPE contribution, we compare the model predictions with the data of reaction (a) for events with $\Delta^2 < 0.3$ (GeV/c)^2, and with the data of reaction (b) for events with $\Delta^2 < 0.5$ (GeV/c)^2, which are illustrated by the cross-hatched histograms in the invariant dipion mass plots of Fig. 2. These two Δ^2 cuts allow the $\pi^-\pi^+$ and $\pi^-\pi^-$ invariant-mass distributions to reach approximately the same maximum mass.

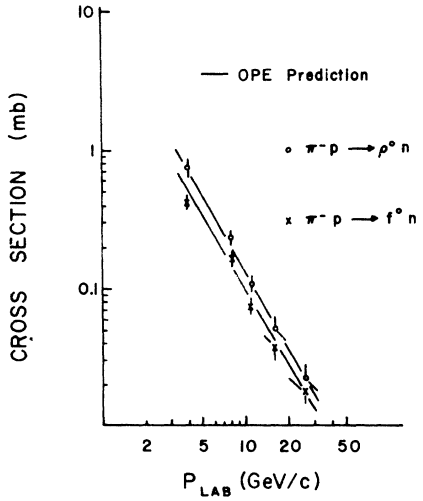

FIG. 3. Cross sections for $\pi^- + p \rightarrow \rho^0 + n$ and $\pi^- + p \rightarrow f^0 + n$ as a function of the incident pion momentum. Curves are the absolute predictions of the OPE model.

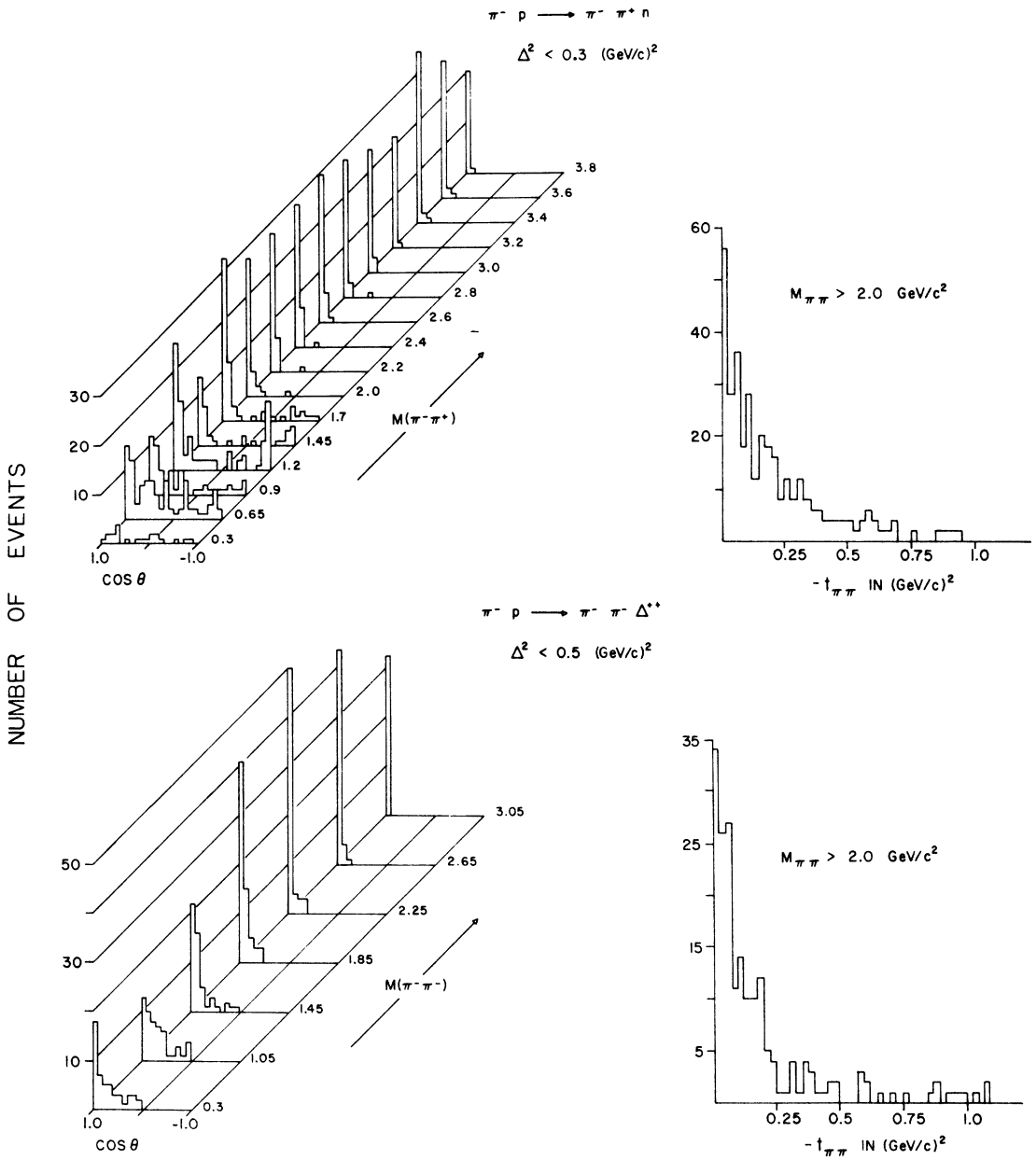

FIG. 4. π-π scattering angle distributions in the dipion rest system (θ), and invariant four-momentum transfer squared in π-π scattering for the reactions $\pi^-+p \rightarrow \pi^-+\pi^++n$ and $\pi^-+p \rightarrow \pi^-+\pi^-+\Delta^{++}$. Note that the scales for the sections of $M(\pi\pi)$ are nonlinear.

In Fig. 4(a) we plot the π-π scattering angle in the dipion rest system ($\cos\theta$) as a function of the dipion mass. The distributions for reaction (b) are folded about $\cos\theta = 0$. Both π^--π^+ and π^--π^- scattering become diffractive above dipion masses of 1.5 GeV/c^2, and it becomes more useful to describe the scattering in terms of the invariant four-momentum transfer in the π-π system

$$|t| = 2k^2(1-\cos\theta) .$$

In Fig. 4(b) we plot the t distributions [for reaction (b) we plot the smaller-magnitude momentum transfer] for events which satisfy the Δ^2 cuts and have $M(\pi\pi) > 2.0$ GeV/c^2. A fit of these distributions to the exponential form

$$\frac{d\sigma^{el}}{dt} \propto e^{\beta t}$$

gave the following exponential slopes for π^--π^+ and

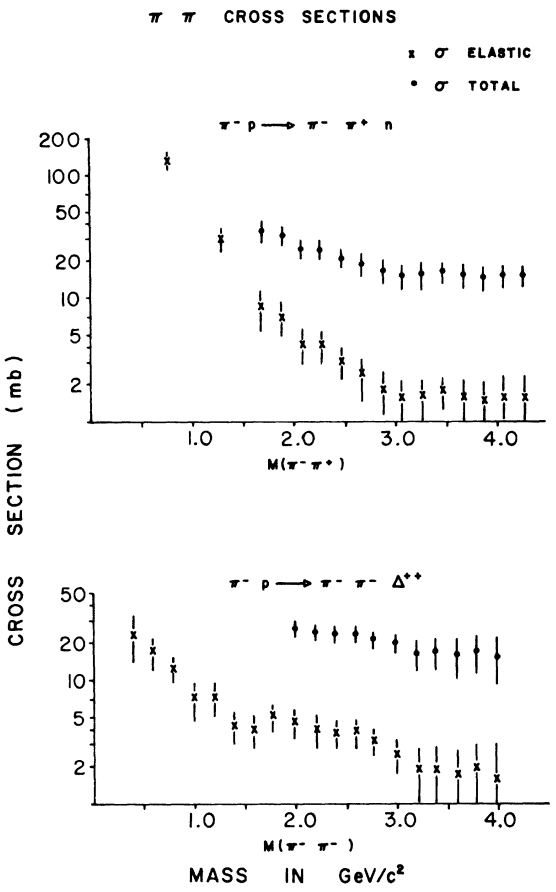

FIG. 5. Elastic and total cross-section determinations for π^--π^+ and π^--π^- scattering as a function of the dipion invariant mass.

π^--π^- scattering:

$$\beta = 5.9 \pm 0.54\ (\mathrm{GeV}/c)^{-2} \quad \text{for } \pi^-\text{-}\pi^+ ,$$
$$\beta = 6.1 \pm 0.51\ (\mathrm{GeV}/c)^{-2} \quad \text{for } \pi^-\text{-}\pi^- . \qquad (3)$$

We relate the real π-π cross sections to the off-shell π-π scattering in a single partial wave by means of the Dürr-Pilkuhn parametrization in the π^--π^+ resonant region. Below $M(\pi^-\pi^-) = 2.0$ GeV/c^2 we use the $I=2$ π-π phase shifts of Morse[10] to do the same for π^--π^- scattering at low dipion energies. In the high-mass, diffractive-scattering region, the off-shell correction factors become negligible for small Δ^2.

Assuming a purely imaginary π-π scattering amplitude for the diffractive-scattering region above dipion masses of 2 GeV/c^2, we can relate the total π-π cross section to the forward elastic scattering cross section:

$$\frac{d\sigma^{\mathrm{el}}}{dt}(\pi\pi) = \frac{d\sigma^{\mathrm{el}}}{dt}(\pi\pi)\Big|_{t=0} e^{\beta t} ,$$

$$\frac{d\sigma^{\mathrm{el}}(\pi\pi)}{dt}\Big|_{t=0} = \frac{1}{16\pi}\sigma^{\mathrm{tot}}(\pi\pi)^2 ,$$

or

$$\sigma^{\mathrm{tot}}(\pi\pi)^2 = 16\pi\beta\sigma^{\mathrm{el}}(\pi\pi) ,$$

where β is given by Eq. (3) for π^--π^+ and π^--π^-, respectively.

In Fig. 5 we plot the π^--π^+ and π^--π^- elastic and total cross-section determinations as a function of the dipion invariant mass, averaged over 200-MeV/c^2 mass intervals. The behaviors of the π^--π^+ and π^--π^- elastic and total cross sections become very similar in the 2–4-GeV/c^2 region of total dipion energy. The elastic cross sections fall to about 1.5 mb, while the total cross sections become constant, or nearly constant, at a value of about 15 mb.

Our result on the total π^--π^+ cross section above 2 GeV/c^2 agrees quite well with that of Caso *et al.*[11] Their data come from a study of reaction (a) at 11 GeV/c, and their dipion mass spectrum extends to 3 GeV/c^2. From 2 to 3 GeV/c^2 of dipion mass they obtain a relatively constant π^--π^+ total cross section of about 15–20 mb.

Biswas *et al.*[12] analyze the same reaction at 8 GeV/c and obtain a total π^--π^+ cross section of about 20 mb in the 1.8–2.4-GeV/c^2 region of dipion mass. In their experiment, Biswas *et al.* also study reaction (c) as well as the reaction

$$\pi^- + p \to \pi^- + \pi^0 + p$$

at 8 GeV/c. Their results on σ^{tot} ($\pi^-\pi^-$) are inconclusive, but they find that for 1.5 GeV/c^2 $< M(\pi^-\pi^0) <$ 2 GeV/c^2 the total $\pi^-\pi^0$ cross section is about 17 ± 5 mb (our estimate).

The cross sections for the three isospin states σ_0, σ_1, and σ_2 can be expressed as

$$\sigma_0 = \sigma(\pi^-\pi^-) + 3[\sigma(\pi^+\pi^-) - \sigma(\pi^-\pi^0)] ,$$
$$\sigma_1 = 2\sigma(\pi^-\pi^0) - \sigma(\pi^-\pi^-) ,$$

and

$$\sigma_2 = \sigma(\pi^-\pi^-) .$$

On the basis of the available data we conclude that $\sigma_0 \approx \sigma_1 \approx \sigma_2 = 15$–$20$ mb.

The high-energy behavior of π-π total cross sections is very similar to the observed behavior of π-N and K-N scattering[13] in that it becomes relatively independent of bombarding energy and also becomes independent of isotopic spin. The π-π scattering seems to obey the Pomeranchuk theorem[14] for high-energy scattering:

$$\sigma^{\mathrm{tot}}(a+b) = \sigma^{\mathrm{tot}}(a+\bar{b}) \text{ at high energies}$$

and

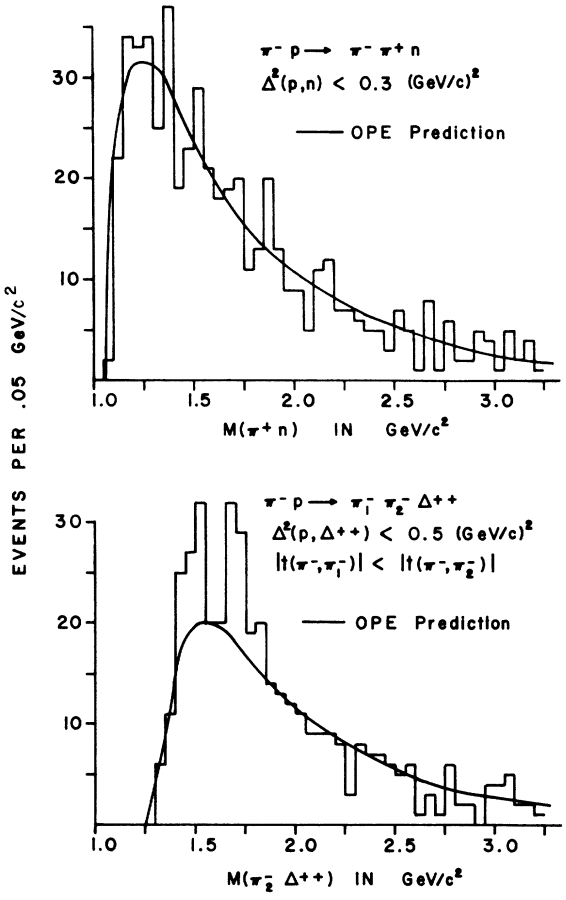

FIG. 6. Plots of $M(\pi^+ n)$ for $\Delta^2(p,n) < 0.3$ (GeV/c)^2 and of $M(\pi^- \Delta^{++})$ for $\Delta^2(p,\Delta^{++}) < 0.5$ (GeV/c)^2 for the reactions $\pi^- + p \to \pi^- + \pi^+ + n$ and $\pi^- + p \to \pi^- + \pi^- + \Delta^{++}$. The curves are OPE predictions.

$$\sigma^{\text{tot}}(a+b) \to \text{constant at high energies}.$$

The value of 15 mb for the high-energy π-π total cross sections which we obtain for π^--π^+ and π^--π^- scattering is in good agreement with the predictions of both the quark model[15] and the Regge model[16] of high-energy π-π collisions, which give 14 mb and 16 mb, respectively. These models also predict a slope for the differential elastic cross section of 6 (GeV/c)$^{-2}$ and 7 (GeV/c)$^{-2}$, respectively, in good agreement with our fitted values of 5.9 and 6.1 (GeV/c)$^{-2}$ for π^--π^+ and π^--π^- scattering.

The diffractive nature of the π-π scattering at large dipion energies results in a forward π^- and a backward π^+ (π^-) going slow relative to the n (Δ^{++}). This results in a peaking at low values of the invariant mass distribution of the $\pi^+ + n$ ($\pi^- + \Delta^{++}$). This is shown in Fig. 6 together with the OPE predictions obtained from our fits to the high-energy π-π cross sections for small Δ^2.

While there is structure in the nucleon system recoiling off of the incident pion, Fig. 6 shows that the shape of the background is described quite well as resulting from a diffractive scattering in an OPE process. This is analogous to the threshold peaking observed in the $\pi\rho$ system from the reaction $\pi N \to \pi\rho N$, and as such may be interpreted as another example of a "Deck-type" kinematic enhancement.[17]

IV. INELASTIC π-π SCATTERING

On the basis of our determination of the elastic and total π^--π^+ cross sections we can determine the inelastic π-π cross section, and from that the cross section for $\pi^- + p \to (\pi + \pi + \cdots)^0 + n$ which goes by one-pion exchange. We estimate a total cross section for such a process of about 1.3 mb. Assuming that the π^--π^0 cross section is the same as the π^--π^+ cross section we then get a total cross section of about 0.65 mb for $\pi^- + p \to (\pi + \pi + \cdots)^- + p$ which goes by exchange of a π^0. This assumption is expected to be valid only when the energy of the pionic system is $\gtrsim$2 GeV. Cross sections for inelastic decay of π^--π^+ and π^--π^0 resonance formation are clearly not expected to be equal; however, at 25 GeV dipion resonance formation is only a small part of the OPE contribution to $\pi^- p$ production reactions.

Another important OPE process is the result of the vertex $p \to \Delta^{++} + \pi^-$. From our results on π^--π^- scattering we deduce a cross section of about 0.67 mb for the reaction $\pi^- + p \to (\pi + \pi + \cdots)^{--} + \Delta^{++}$ which goes by one-pion exchange. There are also OPE processes resulting from $p \to \Delta^0 + \pi^+$ and $p \to \Delta^+ + \pi^0$. These two vertices give a contribution equal to the $p \to \Delta^{++} + \pi^-$ vertex. Thus, such processes giving rise to a $\Delta(1236)$ in the final state from π^--p interactions give a total contribution of about 1.35 mb to OPE reactions.

The sum of these OPE inelastic processes should contribute about 3.2 mb out of the total $\pi^- + p$ inelastic cross section of 21 mb. Because of the ρ, the π-π contribution to the total cross section for $\pi^- + p$ scattering is probably maximum in the 2-GeV/c region of beam momentum. The fraction of the total π-nucleon cross section associated with OPE processes seems to be in the range of $\frac{1}{6}$ to $\frac{1}{4}$ of the total inelastic cross section.

V. CHARACTERISTICS OF $\pi^- + \pi^0 \to \pi^- + \pi^0 + \pi^- + \pi^+$

From the sample of events fitting reaction (c) we have removed those with a Δ^{++} by requiring that $M(\pi^+ p) > 1.35$ GeV/c^2. The cross section for

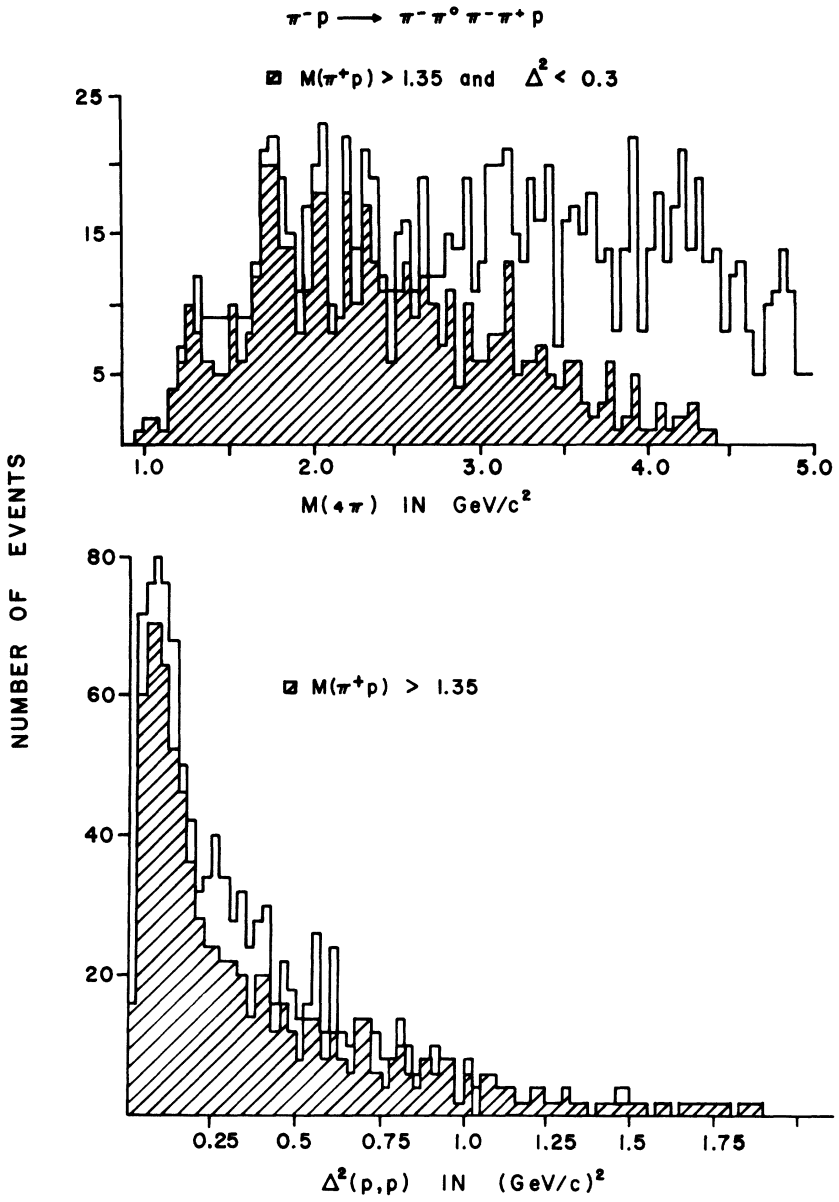

FIG. 7. Plots of $M(4\pi)$ and $\Delta^2(p,p)$ for the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + \pi^- + \pi^+ + p$. The cross-hatched histogram of $\Delta^2(p,p)$ is for events with $M(\pi^+ p) > 1.35$ GeV/c^2. The cross-hatched histogram of $M(4\pi)$ is for events with $M(\pi^+ p) > 1.35$ GeV/c^2 and $\Delta^2(p,p) < 0.3$ (GeV/c)^2.

(c) is 0.34 ± 0.08 mb. After removing Δ^{++} we are left with 862 events. In Fig. 7 we plot the 4π invariant mass and the four-momentum transfer (Δ^2) to the proton for these events. The Δ^2 plot shows the peripheral nature of the production of the 4π system.

The 4π mass plot shows a signal in the g-meson region which is comparable to the f^0 signal in the π^--π^+ spectrum of reaction (a). There is also evidence of possible structure in the 2.0–2.5-GeV/c^2 region. We estimate the production cross section for $g^- \rightarrow \pi^- \pi^- \pi^+ \pi^0$ to be

$$\sigma(\pi^- p \rightarrow g^- p) \approx 20\ \mu\text{b} \searrow \pi^- \pi^- \pi^+ \pi^0 .$$

We find for the ratio

$$\frac{\sigma(\pi^- p \rightarrow g^- p \rightarrow \pi^- \pi^- \pi^+ \pi^0 p)}{\sigma(\pi^- p \rightarrow f^0 n \rightarrow \pi^- \pi^+ n)} \sim 1 .$$

The data of Ballam *et al.* at 16 GeV[18] and of Conte *et al.* at 11 GeV[19] indicate about the same

ratio at these energies, while Oh at 7 GeV[20] finds this ratio to be about $\frac{1}{4}$. This evidence that the 4π signal in the g-meson region grows rapidly in the 5–10-GeV region in relation to the f^0 signal could result from the $M_{4\pi}(1700)$ effect being different from the $g_{2\pi}(1700)$, or could indicate strong absorptive effects in the g region that diminish the 4π signal relative to the elastic π-π system for bombarding energies less than 10 GeV.

We have no information regarding the effects of absorption on the 4π relative to the 2π system. Our studies of reaction (a) show that the absorptive effects in the resonant region of the 2π system are small, insofar as they affect the helicity of the dipion system.[1] In the case of the g meson, we deduce that it is a very inelastic resonance, judging from the fact that it is in a state of $L=3$, yet has a relatively small $\pi^-\pi^+$ signal. At 16 GeV, Ballam *et al.* find a small signal for the g meson in $\pi^-\pi^+$,[9] but no evidence for the g meson in $\pi^-\pi^0$.[18]

To enhance the OPE contribution to reaction (c), we remove events with a $\Delta^{++}(1236)$ by requiring $M(\pi^+p)>1.35$ GeV/c^2 and look only at the very peripheral production of the 4-pion system by requiring $\Delta^2(p,p)<0.3$ (GeV/c)^2. These cuts result in a sample of 463 events. For these events we plot the dipion mass combinations in Fig. 8. We observe ρ^- and ρ^0 but no ρ^+ production. Ballam *et al.* find much the same result in 16-GeV/c $\pi^\pm p$ interactions.[18] This would indicate that $\pi\pi \to A\pi$ is not very strong.

Further quantitative investigation of the four-pion system is hampered by a lack of events and the complexity of four-body kinematics. However, we can compare some aspects of pion production in π-π collisions to similar processes in π-nucleon collisions.

In Fig. 9 we plot the Feynman variable[21] $X=P_L/P_{\max}$ for the three-pion charge states, where P_L = pion longitudinal momentum in the 4π rest system and $P_{\max}$ is the maximum magnitude which P_L can attain; $P_{\max}\approx\frac{1}{2}M(4\pi)$. We assume that we are looking at the reaction $\pi^-+\pi^0\to\pi^-+\pi^0+\pi^-+\pi^+$. The π^+ is the produced pion, and the distribution of the π^+ is essentially symmetric. The presence of the leading π^- can be seen in the forward direction for the X distribution. The falloff with X in the π-π system is less rapid than in the case of the average π-nucleon collision.[22] We do not know what the correct explanation of this effect is. It may arise from the excited states of the π (virtual states) being much above the energy ground state.

In Fig. 10 we plot the distributions of the square of the transverse momentum for each pion charge state in the 4π rest system. The average trans-

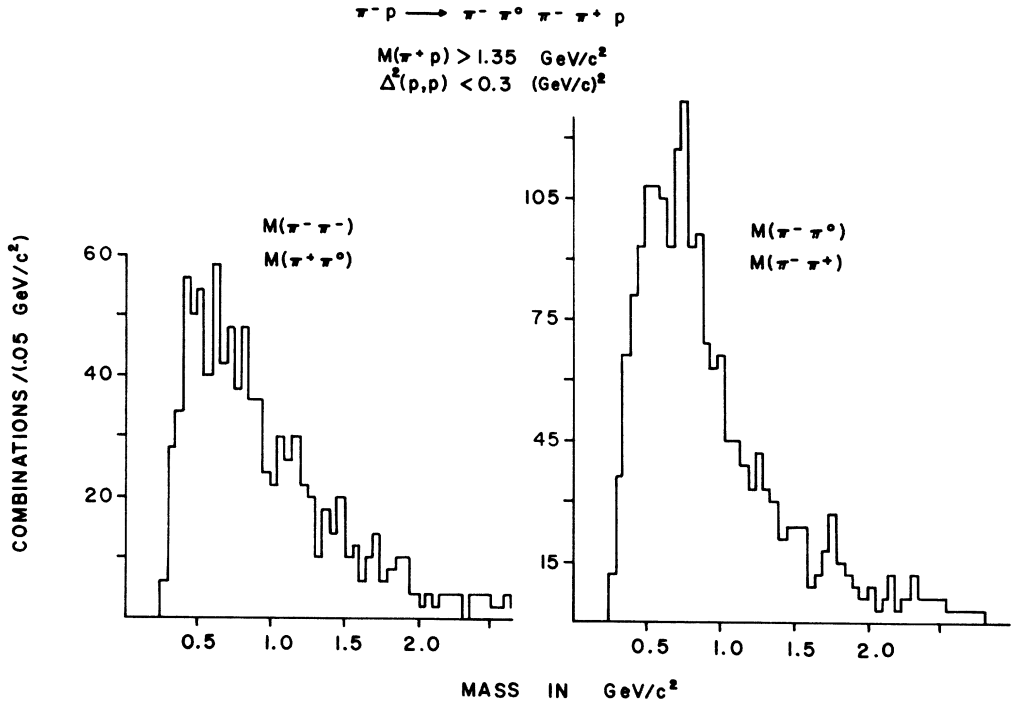

FIG. 8. Plots of $M(\pi^-\pi^-)+M(\pi^+\pi^0)$ and of $M(\pi^-\pi^0)+M(\pi^-\pi^+)$ for events with $M(\pi^+p)>1.35$ GeV/c^2 and $\Delta^2(p,p)<0.3$ (GeV/c)^2 from the reaction $\pi^-+p\to\pi^-+\pi^0+\pi^-+\pi^++p$.

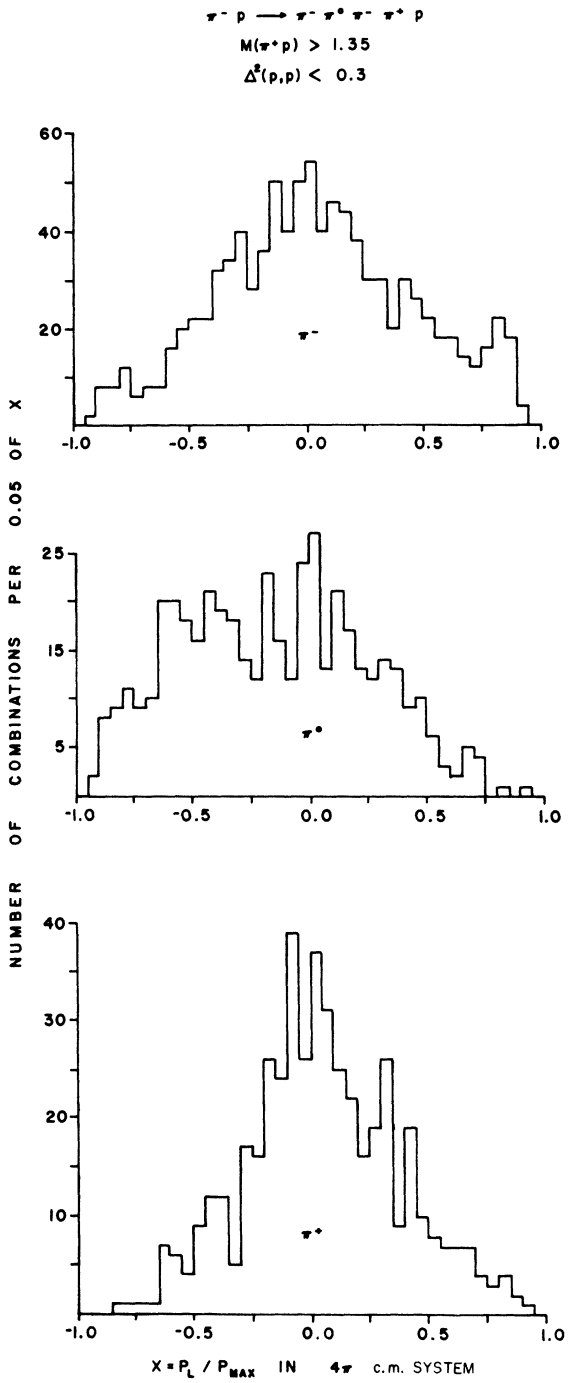

FIG. 9. Plots of $X = P_L/P_{max}$ in the 4π rest system for π^-, π^0, and π^+ for events with $M(\pi^+p) > 1.35$ GeV/c^2 and $\Delta^2(p,p) < 0.3$ (GeV/c)^2 from the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + \pi^- + \pi^+ + p$.

verse momentum of the π's is slightly less than 300 MeV/c, which is consistent with the value for π-nucleon collisions. The distributions are consistent with an exponential dependence of the form

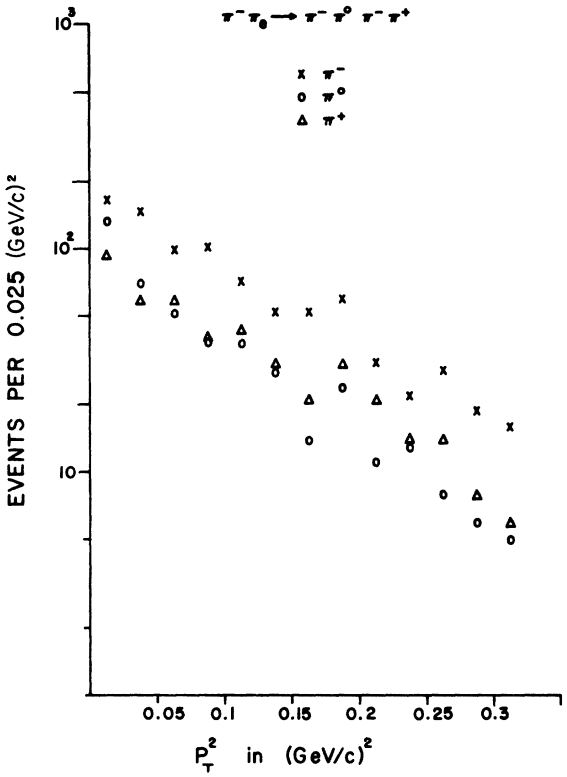

FIG. 10. Plot of the transverse momenta squared of π^-, π^0, and π^+ in the 4π rest system for events with $M(\pi^+p) > 1.35$ GeV/c^2 and $\Delta^2(p,p) < 0.3$ (GeV/c)^2 from the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + \pi^- + \pi^+ + p$.

$$\frac{dN}{dP_T^2} = C\exp(-bP_T^2)\,,$$

with $b = 8$–10 (GeV/c)$^{-2}$.

VI. CONCLUSIONS

In our study of reactions (a) and (b) we have found that the high-energy behavior of the π-π system is very similar to that of other hadron-hadron interactions. The high-energy cross sections become independent of the total dipion energy, or very nearly so, in the 2–4-GeV region. The value obtained for the total π-π cross section, as well as the exponential slope of the elastic differential cross section, is in very good agreement with the predictions of both the quark and Regge models of high-energy π-π scattering. The π-π scattering in the high-energy region also seems to become independent of the isotopic-spin state of the dipion system.

Comparison of the results from reaction (c) to lower-energy data seems to indicate that the 4π state produced from π-p interactions at 25 GeV/c is less affected by absorption than the same state

produced in π-p interactions at 7 GeV/c. Looking at the 4π system as arising from inelastic π-π scattering, we conclude that the falloff in P_L in the π-π collisions is quite different from the effect seen in π-p collisions, while the transverse momentum spectra of the products of the π-π interaction are the same as those seen in π-nucleon collisions.

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[1]W. J. Robertson, Ph.D. thesis, University of Wisconsin, 1971 (unpublished).

[2]J. L. Davis, Ph.D. thesis, University of Wisconsin, 1973 (unpublished).

[3]R. N. Diamond *et al.*, Nucl. Instrum. Methods 89, 45 (1970).

[4]K. J. Foley *et al.*, Phys. Rev. Letters 19, 330 (1967).

[5]K. J. Foley *et al.*, Phys. Rev. Letters 11, 425 (1963).

[6]G. Wolf, Phys. Rev. 182, 1538 (1969).

[7]See for example, H. Pilkuhn, *The Interactions of Hadrons* (North-Holland, Amsterdam, 1967), p. 279.

[8]H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).

[9]4 GeV/c: Aachen-Birmingham-Bonn-Hamburg-London-Munich Collaboration, Nuovo Cimento 31, 279 (1967); 8 GeV/c: J. A. Poirier *et al.*, Phys. Rev. 163, 1462 (1967). 11 GeV/c: C. Caso *et al.*, Nuovo Cimento 62A, 755 (1969); 16 GeV/c: J. Ballam *et al.*, Phys. Letters 31B, 489 (1970).

[10]R. M. Morse *et al.*, University of Wisconsin report 1969 (unpublished).

[11]C. Caso *et al.*, paper submitted to the Fourteenth International Conference on High Energy Physics, Vienna, 1968 (unpublished).

[12]N. N. Biswas *et al.*, Phys. Rev. Letters 18, 273 (1967).

[13]See for example the data in G. Källén, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass., 1964), pp. 72, 73, 247—249.

[14]I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. 34, 758 (1958) [Sov. Phys. JETP 7, 499 (1958)].

[15]E. Shrauner, Phys. Rev. Letters 20, 1258 (1968).

[16]W. J. Abbe, Phys. Rev. 160, 1519 (1967).

[17]R. T. Deck, Phys. Rev. Letters 13, 169 (1964).

[18]J. Ballam *et al.*, Phys. Rev. D 3, 2606 (1971).

[19]F. Conte *et al.*, Phys. Letters 22, 702 (1966).

[20]B. Y. Oh, Ph.D. thesis, University of Wisconsin, 1968 (unpublished).

[21]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[22]See, for example, the review article by Gerald Smith in Argonne National Laboratory Report No. ANL/HEP 7107, 1970 (unpublished).