Charged Version of Tomimatsu-Sato Spinning-Mass Field*

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A solution of the coupled Einstein-Maxwell field equations is constructed from the recently published Tomimatsu-Sato solution of Einstein's vacuum field equations.

The exciting new algebraically general solution of Einstein's vacuum field equations published in this journal by Tomimatsu and Sato[1] may be generalized easily to yield a solution of the coupled Einstein-Maxwell field equations. This generalization bears to the Tomimatsu-Sato solution the same relation as the Kerr-Newman metric bears to the original Kerr metric.

Our charged version of the Tomimatsu-Sato solution corresponds to a line element of the form

$$ds^2 = f^{-1}\left[P^{-2}\left(\frac{dx^2}{x^2-1}+\frac{dy^2}{1-y^2}\right) + (x^2-1)(1-y^2)d\phi^2\right] - f(dT-\omega d\phi)^2 , \tag{1}$$

where

$$f = A/B , \tag{2a}$$

$$(1-q^2)^{1/2}\omega = -4[C(1-y^2)/A]\tan\lambda , \tag{2b}$$

$$P^2 = [(x^2-y^2)^3/A]\cos^4\lambda . \tag{2c}$$

Here λ and q are free real parameters associated, respectively, with the specific angular momentum and the charge-to-mass ratio, while the functions A, B, and C are given by the following expressions:

$$A = [(x^2-1)^2\cos^2\lambda + (1-y^2)^2\sin^2\lambda]^2 - 4(x^2-1)(1-y^2)(x^2-y^2)^2\cos^2\lambda\sin^2\lambda , \tag{3a}$$

$$B = [x^4\cos^2\lambda + y^4\sin^2\lambda - 1 + 2(1-q^2)^{-1/2}x(x^2-1)\cos\lambda]^2 + 4\sin^2\lambda\, y^2[x(x^2-y^2)\cos\lambda + (1-q^2)^{-1/2}(1-y^2)]^2 , \tag{3b}$$

$$C = \tfrac{1}{2}[(1-q^2)^{1/2} + (1-q^2)^{-1/2}]\,\text{even}\,(C_0) + \text{odd}\,(C_0) . \tag{3c}$$

"Even" and "odd" refer to those terms which are respectively even and odd in x in the $q=0$ solution of Tomimatsu and Sato; namely,

$$C_0 = \cos^2\lambda(x^2-1)[(x^2-1)(1-y^2) - 4x^2(x^2-y^2)] - \cos^3\lambda\, x(x^2-1)[2(x^4-1) + (x^2+3)(1-y^2)] + \sin^2\lambda(1+x\cos\lambda)(1-y^2)^3 . \tag{4}$$

Finally, the complex scalar electromagnetic potential is given by

$$\Phi = qe^{i\alpha}D/[N(1-q^2)^{1/2}+D] , \tag{5}$$

where α is an arbitrary phase associated with a duality rotation, and N and D refer, respectively, to the numerator and the denominator of the Tomimatsu-Sato ξ field; namely,

$$N = x^4\cos^2\lambda + y^4\sin^2\lambda - 1 - 2ixy(x^2-y^2)\cos\lambda\sin\lambda , \tag{6a}$$

$$D = 2x(x^2-1)\cos\lambda - 2iy(1-y^2)\sin\lambda . \tag{6b}$$

At this time we feel it is worthwhile to emphasize two points. First, we believe it is likely that other exact solutions exist corresponding to more complicated *rational functions* $\xi(x,y)$ satisfying the basic field equation

$$(\xi\xi^*-1)\left[\frac{\partial}{\partial x}(x^2-1)\frac{\partial\xi}{\partial x} + \frac{\partial}{\partial y}(1-y^2)\frac{\partial\xi}{\partial y}\right] = 2\xi^*\left[(x^2-1)\left(\frac{\partial\xi}{\partial x}\right)^2 + (1-y^2)\left(\frac{\partial\xi}{\partial y}\right)^2\right] . \tag{7}$$

Aided by computer symbolic manipulation capabilities, it should not be too difficult to explore this possibility. Prime candidates would be solutions corresponding to integral values of the δ parameter of Tomimatsu and Sato.

In the second place it would be highly desirable to discover the physical or geometrical significance of the attribute

$$\xi(y, x; \lambda) = -i\xi(x, y; \tfrac{1}{2}\pi - \lambda)^* , \tag{8}$$

which is shared by the Tomimatsu-Sato and Kerr solutions.

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[1]A. Tomimatsu and H. Sato, Phys. Rev. Lett. 29, 1344 (1972).

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Current-Mixing Gauge Theories of Weak and Electromagnetic Interactions of Hadrons*

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Assuming mesons and baryons are composed only of multitriplets of fundamental fermions, we construct renormalizable $SU(2)\otimes U(1)_Y$ gauge theories of weak and electromagnetic interactions of hadrons.

Owing to its renormalizability,[1] the spontaneously broken gauge theory via the Higgs phenomenon[2] offers a possibility to embed the weak and electromagnetic interactions in the framework of quantum field theory. Weinberg has utilized the Higgs phenomenon successfully in constructing a model of leptons[3] from a Yang-Mills theory with the spontaneous breaking of the $SU(2)\otimes U(1)_Y$ gauge invariance. Recently, he extended the model to the case of semileptonic weak interactions.[4] By postulating[4] the existence of four quarks $\mathcal{P}$, $\mathcal{N}$, λ, and $\mathcal{P}'$ of integral charges $+1$, 0, 0, and $+1$, and by an artifice like that of Glashow, Iliopoulos, and Maiani,[4,5]

$$B_L = \begin{pmatrix} \mathcal{P}\cos\theta_C - \mathcal{P}'\sin\theta_C \\ \mathcal{N} \end{pmatrix}_L ,$$

$$B_L' = \begin{pmatrix} \mathcal{P}\sin\theta_C + \mathcal{P}'\cos\theta_C \\ \lambda \end{pmatrix}_L , \tag{1a}$$

$$\mathcal{P}_R,\ \mathcal{P}_R',\ \mathcal{N}_R,\ \lambda_R$$

(where θ_C is the Cabibbo angle, B_L, B_L' are two doublets with $Y_\omega = 1$ [here we define the charge operator $Q \equiv I_3 + \frac{1}{2}Y_\omega$, where $\vec{I}$, Y_ω are the generators of the $SU(2)\otimes U(1)_Y$ gauge group], and R states are singlets), the strangeness-changing neutral current is eliminated from the theory. The $\Delta Y = 0$, $\Delta I = 1$ rule and the $\Delta I = \frac{1}{2}$, $\Delta Y = \Delta Q$ rule in the semileptonic weak interactions are then explained[4] in view of the quark model with integral charges.

But because the four quark states in Eq. (1a) span a rather unnatural symmetry group SU(4) of the strong interactions, in this note we would like to consider alternative schemes to restore the three-dimensional unitary symmetry of the strong interaction.

Let us assume that the lepton world is described either by Weinberg's model[3]

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ,\ e_R^- , \tag{1b}$$

or by the generalized Prentki-Zumino second model[6]

$$E' = \begin{pmatrix} E^+ \\ -\nu_e\sin\alpha + \nu_e'\cos\alpha \end{pmatrix}_L ,$$

$$E = \begin{pmatrix} \nu_e\cos\alpha + \nu_e'\sin\alpha \\ e^- \end{pmatrix}_L , \tag{1c}$$

$$E_R^+,\ e_R^-,\ \nu_{eR}' ,$$

where $\alpha = 45°$ is to be identified with Prentki and Zumino's original model. The muon multiplets are identical to the electron's. Equation (1c) is free of anomalies, while Eq. (1b) is not.

Now we turn to the hadron world and assume that all observed hadrons are composed (as if they were) only of the "uncharmed" multitriplet fundamental fermions $Q_i \equiv (\mathcal{P}_i, \mathcal{N}_i, \lambda_i)$, $i = 1, \ldots, r$ (integers). We shall identify $r = 1$ with the Sakata model[7] or the Gell-Mann–Zweig model,[8] $r = 2$ with the Bacry-Nuyts-Van Hove-Nambu model,[9] and $r = 3$ with the Han-Nambu model,[10] or the SUB model,[11] or the Bardeen–Fritzsch–Gell-Mann model.[12] We