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Attempts to Calculate the Electron Mass*

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In conventional quantum electrodynamics both the electron mass and the muon mass are free parameters, so that their ratio is arbitrary. In a unified theory of weak and electromagnetic interactions, this need not be the case. We examine models in which only the muon mass appears as a counterterm. The electron is massless in zeroth order, but develops a finite and calculable mass of order αm_μ. Unfortunately, the precise value of the electron mass depends on details of the models, and in particular on masses of unobserved vector bosons.

I. INTRODUCTION

The lepton mass spectrum is one of the longstanding mysteries of theoretical physics. The size of the electron-muon mass ratio suggests that the electron mass is entirely electromagnetic in origin, but, until recently, it has not been possible to implement this idea within the context of local quantum field theory. Typically, in renormalized perturbation theory, the electromagnetic mass is either zero, because of some unbroken symmetry, or infinite, in which case the infinity must be canceled by a counterterm, leaving the mass a free parameter. However, in the recently developed class of renormalizable models with spontaneously broken gauge symmetry,[1] there is a third possibility: In such theories, the only counterterms necessary for the cancellation of all infinities in the theory are those allowed by the gauge symmetry. If a zeroth-order electron mass and mass counterterm are forbidden by the gauge structure, then the higher-order contributions to the mass must be finite.

This third possibility is an example of a phenomenon which is quite common in theories with spontaneously broken gauge symmetry. The gauge symmetry imposes certain relations among the coupling constants in the theory. When the symmetry is spontaneously broken higher-order effects may change these zeroth-order relations, but because the theory can be renormalized in a gauge-invariant way, the corrections must be finite. For instance, consider the couplings of the gauge fields. If the gauge group is simple, there are many zeroth-order relations because the gauge couplings are characterized by a single parameter, the gauge coupling constant. By the same token, the infinities associated with the renormalization of the gauge coupling constant can be canceled by a single counterterm, with the same structure as the zeroth-order couplings. If the symmetry is spontaneously broken, higher-order effects may contribute to the couplings of the gauge fields in some complicated way, but the divergent parts must have the zeroth-order form, since there is only one counterterm to cancel the infinities. The corrections to the zeroth-order relations must be finite.

Weinberg[2] and the authors[3] have studied the consequences of the above considerations for fermion masses. In this paper, we give a review of the earlier work. We analyze two unsuccessful but instructive attempts to apply these ideas to the electron-muon mass ratio. Finally, we show that it is possible to implement the idea that the electron mass is entirely electromagnetic within the framework of unified theories of weak and electromagnetic interactions. In particular, we will exhibit models in which the electron-muon mass ratio is calculable and of order α.

II. ZEROTH-ORDER MASS RELATIONS

In a theory with spontaneously broken gauge symmetry, the general zeroth-order fermion mass matrix is a sum of a bare-mass term and a tadpole term coming from the Yukawa coupling proportional to the zeroth-order vacuum expectation values of the spinless meson fields. A zeroth-order mass relation is a relation among the masses in the zeroth-order mass matrix which is left unchanged by arbitrary (but sufficiently small) changes in the renormalized parameters. We distinguish four types of zeroth-order mass relations[4]:

(0) mass relations determined by an unbroken subgroup of the symmetry of the Lagrangian,

(1) mass relations determined by the representation content of the spinless meson multiplet,

(2) mass relations involving accidental symmetry, and

(3) mass relations which arise due to the con-

straints imposed on the Lagrangian by the requirement of renormalizability.

Type (0) is the familiar exact mass relation associated with an unbroken symmetry. Such a relation will be maintained in higher orders. The vanishing of the neutrino mass might be a type-(0) mass formula. But this type of relation cannot explain the small, but nonzero, electron mass.

A type-(1) mass relation can occur when the Yukawa couplings are incomplete. If the fermions transform under a representation D of the symmetry group of the Lagrangian, a spinless meson multiplet can couple to fermions if it transforms according to any irreducible component of $D\times D^*$. If there are no spinless mesons in one or more of these representations then the Yukawa coupling terms cannot give the most general zeroth-order mass matrix no matter what vacuum expectation values the spinless meson fields take. For example, if there is no spinless meson which couples to $\bar{e}e$ and no bare-electron-mass term, then the masslessness of the electron in zeroth order is a type-(1) mass relation. This type of mass relation was first analyzed by Weinberg.[2]

A Lagrangian is said to have an accidental symmetry if the most general renormalizable Yukawa couplings and couplings of the spinless meson fields among themselves have a larger invariance group than the full Lagrangian. This type of model has been discussed by Weinberg[5] and by Coleman and Weinberg.[6] Such a Lagrangian gives a natural set of zeroth-order mass relations for the spinless mesons; when the symmetry is spontaneously broken there are "pseudo-Goldstone bosons,"[5] massless in zeroth order, associated with the accidental symmetry. We have not found any application of type-(2) mass relations to the problem of the electron mass, so we will not discuss this type any further.

Type-(3) mass relations can occur because the Lagrangian is required to be renormalizable, so that only quadratic, cubic, and quartic couplings of the spinless meson fields are allowed. It is possible for the zeroth-order vacuum expectation values, obtained by maximizing $\mathcal{L}$, to be qualitatively different from the true vacuum expectation value, obtained by minimizing the classical potential. A simple example of this phenomenon was given by the authors in an earlier paper[3]: Consider a gauge theory based on a U(1) gauge group with four complex spinless meson fields, ϕ_1, ϕ_2, ϕ_3, and ϕ_8 which transform as $\phi_n \to e^{in\theta}\phi_n$. It is easy to write a Lagrangian invariant under this group which is maximized at $\phi_1 = \phi_2 = \phi_8 = 0$, $\phi_2 \neq 0$, for a range of all the parameters. But this is only possible because renormalizability forbids the coupling $\phi_2{}^4\phi_8^*$. Such terms will be induced in the classical potential by higher-order effects and will give rise to a calculable vacuum expectation value for the ϕ_8 field. If there is a fermion in the theory coupled to ϕ_8, then the vanishing of its mass is an example of a type-(3) mass relation.

III. INSTRUCTIVE FAILURES

A contribution to the electron mass of order αm_μ can presumably come from a diagram like that illustrated in Fig. 1, where W is some massive intermediate vector boson with couplings of electromagnetic strength. For such a diagram to exist, the electron and muon must be combined into an irreducible representation of the gauge group. Weinberg[7] has suggested a model of this type based on a gauge group of chiral SU(3)×SU(3), with the observed leptons in a Konopinski-Mahmoud triplet (μ^+, ν, e^-). Before discussing this model, we will examine a simpler model based on the gauge group SU(3).

We assign the right-handed leptons, ψ_R, to a $\underline{3}$, and the left-handed leptons, ψ_L, to a $\underline{\bar{3}}$:

$$\psi_R = \begin{pmatrix} \mu^+ \\ \nu \\ e^- \end{pmatrix}_R , \quad \psi_L = \begin{pmatrix} e^- \\ \nu \\ \mu^+ \end{pmatrix}_L . \tag{1}$$

Under an infinitesimal gauge transformation, $1 + i\omega^i T_i$, where T_i for $i = 1$ to 8 are the gauge group generators, the fermions transform as follows:

$$\begin{aligned} \delta\psi_R &= i\omega^i \lambda_i \psi_R , \\ \delta\psi_L &= -i\omega^i \lambda_i^* \psi_L , \end{aligned} \tag{2}$$

where λ_i are the usual SU(3) matrices. The electric charge is $\frac{1}{2}(T_3 + \sqrt{3}\, T_8)$ and the weak charge is $T_1 + i\, T_2$. There is also an intermediate vector boson coupled to the wrong-helicity weak charge $T_6 + i\, T_7$. The contribution from the exchange of this vector boson is suppressed by "superstrong symmetry breaking."[7] That is, there is a spinless meson multiplet in the theory whose vacuum

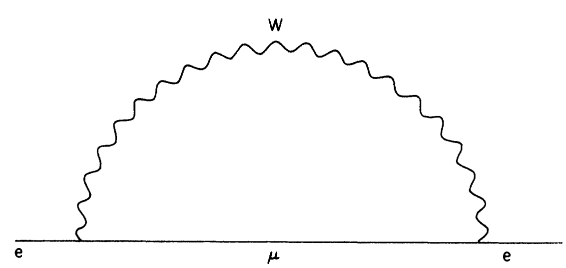

FIG. 1. Feynman diagram which could lead to an electron mass of order αm_μ.

expectation value breaks the symmetry down to the SU(2)×U(1) of the original Weinberg model and gives a very large mass to the other four vector bosons. This superstrong breaking can be done with an octet of mesons.

In this model, there is a doubly charged vector boson coupled to $\overline{e\mu^+}\gamma^\mu\gamma_5 e^-$, so that the diagram in Fig. 1 exists; but it is easy to see that it cannot contribute to a calculable electron mass. The zeroth-order lepton mass matrix comes entirely from the Yukawa couplings of a $\underline{\overline{3}}$ and a $\underline{6}$ of spinless mesons. The vacuum expectation value of the $\underline{6}$ contributes equally to the muon and electron masses, while the $\underline{\overline{3}}$ contributes to the masses in equal magnitude but with opposite signs. If the zeroth-order muon mass is nonzero, the zeroth-order electron mass can only vanish due to an accidental cancellation between the contributions from the $\underline{\overline{3}}$ and the $\underline{6}$, which will not be stable under small changes in the parameters; so there is no zeroth-order mass relation. To put it another way, there are two independent mass counterterms from the $\underline{\overline{3}}$ and $\underline{6}$, so both the muon and electron masses are free parameters.

The trouble with Weinberg's model[7] is considerably more subtle. The gauge group is SU(3) ×SU(3), the right-handed fermions are a (3, 1), and the left-handed fermions are a $(1,\overline{3})$.[8] Under the infinitesimal gauge transformation $1+i\omega_R^i T_i^R + i\omega_L^i T_i^L$ the fermions transform like

$$\begin{aligned}\delta\psi_R &= i\omega_R^i \lambda_i \psi_R\ , \\ \delta\psi_L &= -i\omega_L^i \lambda_i^* \psi_L\ ,\end{aligned} \tag{3}$$

with ψ_R and ψ_L as in (1). The observed weak charge is $T_1^R + T_1^L + iT_2^R + iT_2^L$, and the superstrong breaking can be done with a $(3,\overline{3})$ of spinless mesons.

Because of the chiral nature of the gauge group, there is no bare-lepton-mass term and the only meson representation which can couple to leptons is a (3, 3). We describe this representation by a complex 3-by-3 matrix field ϕ which transforms like

$$\delta\phi = i\omega_R^i \lambda_i \phi + i\phi\omega_L^i \lambda_i^*\ . \tag{4}$$

The gauge-invariant Yukawa coupling is $f\overline{\psi}_R\phi\psi_L$ +H.c. It is possible for the ϕ field to develop a stable zeroth-order vacuum expectation value of the form

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5}$$

Small changes in the parameters will change the value of a but not the form of the vacuum expectation value. The zeroth-order muon mass $m_\mu = fa$ is a free parameter, but the zeroth-order electron mass vanishes. Since there *is* a scalar meson which couples to $\overline{e}e$, this is an example of a type-(3) mass relation.

Unfortunately, no matter what other spinless meson multiplets are included in the model, the diagram in Fig. 1 cannot contribute to a calculable electron mass. To see this, we analyze the contribution in the original gauge-symmetric Lagrangian (rather than, say, the unitary gauge), with the gauge symmetry breaking introduced explicitly in the form of tadpoles. Alternatively, one can think of the diagrams in this language as contributions to the effective action, where the tadpoles represent the classical fields of the spinless mesons. If the diagram in Fig. 1 is to contribute, there must be a contribution from the diagram in Fig. 2. This is obvious because only the piece of the muon propagator proportional to the muon mass contributes to the electron mass. Therefore a W_L^{--} must be emitted at one vertex and a W_R^{--} reabsorbed at the other. Since there are no other doubly charged vector bosons in the theory, there must be a direct mass mixing between the two, which can only come from a four-pronged vertex involving two tadpoles.

Now assume that the diagram in Fig. 2 gives a nonvanishing contribution. Then there is also a nonzero contribution from the diagram in Fig. 3, where we have just closed the fermion line and coupled in the meson coupled to $\overline{e}e$ (chirality forbids the closing of the electron loop without this coupling). But this diagram gives a contribution to the effective action which forces the scalar meson coupled to $\overline{e}e$ to have a nonzero vacuum expectation value. Furthermore, it is a gauge-invariant quartic polynomial in the classical fields, so such a term must be present as a counterterm in the original Lagrangian. But then the meson coupled to $\overline{e}e$ develops a zeroth-order vacuum expectation value and there is an electron-mass counterterm, so the electron mass is a free pa-

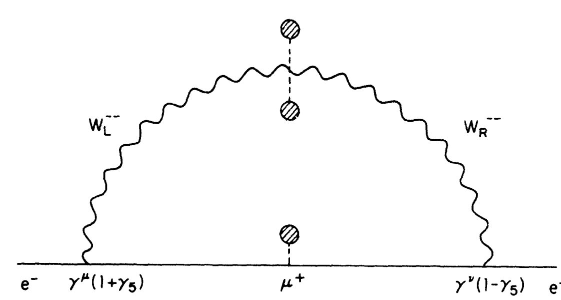

FIG. 2. A contribution to the electron mass in Weinberg's SU(3)×SU(3) model.

rameter. We have shown that if the diagram in Fig. 1 contributes to the electron mass in Weinberg's theory, then the mass is not calculable.

This argument does not rule out two-loop contributions to the electron mass or contributions not involving vector-boson exchange.[3] Such contributions are not obviously of order αm_μ. In this paper, we will be concerned only with one-loop contributions involving vector-boson exchange. We will always choose the parameters in the Lagrangians so that other contributions are small.

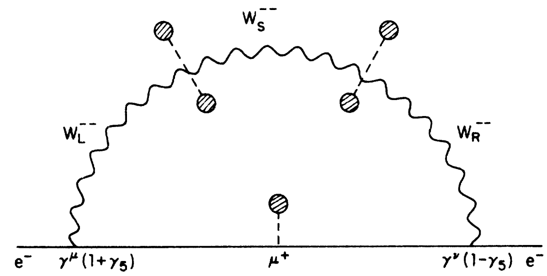

FIG. 4. Feynman diagram leading to a calculable electron mass in our SU(3)×SU(3)×SU(3) model.

IV. SU(3) × SU(3) × SU(3)

It is easy to modify Weinberg's theory so that the argument given above does not apply. The trick is to enlarge the gauge group so that there is another doubly charged vector boson. We will consider a model based on the gauge group SU(3) ×SU(3)×SU(3). In analogy with the Weinberg model, we assign the right-handed leptons ψ_R to a (3, 1, 1), the left-handed leptons ψ_L to a $(1, \overline{3}, 1)$, and the spinless mesons ϕ having Yukawa couplings to the leptons to a (3, 3, 1). Under an infinitesimal gauge transformation $1+i\omega_R^i T_i^R + i\omega_L^i T_i^L + i\omega_S^i T_i^S$, these fields transform as in (3) and (4). The Yukawa coupling is as before, and the zeroth-order vacuum expectation value of ϕ is again given by (5).

To do the superstrong breaking, we need two more complex 3-by-3 matrix fields χ_1 and χ_2, which are a $(3, 1, \overline{3})$ and a $(1, 3, \overline{3})$, respectively. They transform according to

$$\delta\chi_1 = i\omega_R^i \lambda_i \chi_1 - i\chi_1 \omega_S^i \lambda_i \ ,$$

$$\delta\chi_2 = i\omega_L^i \lambda_i \chi_2 - i\chi_2 \omega_S^i \lambda_i \ .$$

It is convenient to impose a peculiar charge conjugation invariance on the unshifted Lagrangian. In a representation in which the γ matrices are imaginary, the transformation is $W_R \to W_L$, $\psi_L \to \psi_R^*$, $\psi_R \to \psi_L^*$, $\phi \to \phi^T$, and $\chi_1 \leftrightarrow \chi_2$. This invariance forces the gauge couplings of the R and L vector bosons to be equal. It is spontaneously broken by the ϕ vacuum expectation value.

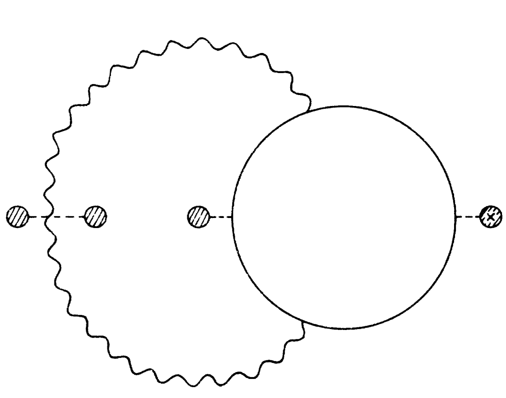

FIG. 3. Feynman diagram leading to the loss of the zeroth-order masslessness of the electron in Weinberg's theory; X marks the scalar meson coupled to $\bar{e}e$.

The χ_1 and χ_2 vacuum expectation values are diagonal and have the form

$$\begin{pmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} + O(a) \ ,$$

where $|b|$, $|b-c|$, and $|b+c|$ are $\gg |a|$. The terms of order a are different for χ_1 and χ_2 because of the spontaneous breakdown of the charge-conjugation invariance. These vacuum expectation values give very large masses to all the vector bosons except the four which are coupled to the SU(2)×U(1) subgroup generated by $\vec{T}_R + \vec{T}_L + \vec{T}_S$, $T_R^8 + T_L^8 + T_S^8$, which in the lepton system is the group of the original Weinberg model.

In this model, the masslessness of the neutrino can be imposed as an exact type-(0) mass relation. We require that the Lagrangian possess an exact global U(1) symmetry wherein $\phi \to e^{2i\beta}\phi$, $\psi_L \to e^{-i\beta}\psi_L$, $\psi_R \to e^{i\beta}\psi_R$, and all other fields are unchanged; this symmetry is obtained by omitting $\det\phi$ from the Lagrangian. After the spontaneous symmetry breaking, there will remain an exact gauge U(1) – just the electromagnetic gauge group – and, in addition, an exact global U(1) corresponding to a chiral transformations on the neutrino field. This symmetry keeps the neutrino massless in all orders of perturbation theory.

Finally we consider the electron mass. Now the diagram in Fig. 2 cannot contribute because there is no direct mass mixing between W_R^{--} and W_L^{--}. Instead, both these vector bosons can mix with W_S^{--} so that there is a contribution from the diagram in Fig. 4. Closing the lepton loop, we again obtain a vacuum expectation value for the meson coupled to $\bar{e}e$, but here this vacuum expectation value is calculable because the invariant which contributes is not allowed in the original renormalizable Lagrangian: It is a sixth-order polynomial.

To calculate the contribution of the diagram in Fig. 1, it is convenient to work with the shifted Lagrangian in the Landau gauge, so that there is no mixing between vector boson and the Goldstone mesons. If the gauge coupling constant of the R and L vector bosons is $\sqrt{2}\,e/\cos\theta$ and the coupling constant for the S vector bosons is $e/\sin\theta$, so that the photon couples to the correct electric current, the leading contribution to the electron mass is

$$\alpha m_\mu \frac{3}{16\pi\cos^2\theta}\left(\ln\frac{M^4}{M_+^{\,2}M_-^{\,2}}+\cos 2\xi \ln\frac{M_-^{\,2}}{M_+^{\,2}}\right),$$

where M and $M_\pm$ are masses of the doubly charged vector bosons, and ξ is the mixing angle between W_S^{--} and $(W_R^{--}+W_L^{--})/\sqrt{2}$. In terms of the other parameters,

$$M^2=\frac{4e^4}{\cos^2\theta}(b^2+c^2)+O(e^2a^2),$$

$$M_\pm^{\,2}=\frac{4e^2}{\cos^2\theta\sin^2\theta}\times\{b^2+c^2\pm[(b^2+c^2)^2\cos^2 2\theta+b^2c^2\sin^2 2\theta]^{1/2}\}+O(e^2a^2),$$

$$\cos 2\xi=\frac{-(b^2+c^2)\cos 2\theta}{[(b^2+c^2)^2\cos^2 2\theta+b^2c^2\sin^2 2\theta]^{1/2}}.$$

Other contributions are smaller by a factor of a^2/b^2 or more.

While this model has the property we have been after, it is not economical. It requires 24 vector bosons, and 31 spinless mesons survive after spontaneous symmetry breaking. This kind of proliferation seems to be a necessary consequence of the type-(3) mass relation because the diagram in Fig. 4 which contributes to the electron mass is so complicated. The last model we describe overcomes this problem.

V. SU(3) × U(1)

We modify the SU(3) model in Sec. II by including another charged lepton, which is an SU(3) singlet, s. In terms of the eventual mass eigenstates, the lepton fields are

$$\psi_R=\begin{pmatrix}\cos\rho\,\mu^+ +\sin\rho\,X^+\\ \nu\\ e^-\end{pmatrix}_R,$$

$$\psi_L=\begin{pmatrix}e^-\\ \nu\\ \cos\lambda\,\mu^+ -\sin\lambda\,X^+\end{pmatrix}_L,$$

$$s_R=\cos\rho\,X_R^+ -\sin\rho\,\mu_R^+,$$

$$s_L=\cos\lambda\,X_L^+ +\sin\lambda\,\mu_L^+.$$

Under an infinitesimal gauge transformation $(1=i\omega^i T_i+i\omega^9 T_9)$, ψ_R and ψ_L transform as in (2), while the singlets transform according to

$$\delta s=i\omega^9 s.$$

The electric charge generator is $(T_3+\sqrt{3}\,T_8)/2+T_9$. The only spinless mesons in the model are two triplets with U(1) quantum numbers ±1. They transform according to

$$\delta\phi_1=i\omega^i\lambda_i\phi_1+i\omega^9\phi_1,$$

$$\delta\phi_2=i\omega^i\lambda_i\phi_2+i\omega^9\phi_2.$$

The Yukawa couplings are

$$f\bar{s}_L\phi_1^\dagger\psi_R+f'\bar{\psi}_L\phi_2^* s_R+\text{H.c.},$$

and there is an invariant bare-mass term $m\bar{s}_L s_R$ + H.c. The vacuum expectation values are

$$\langle\phi_1\rangle=\begin{pmatrix}a\\0\\0\end{pmatrix},\quad\langle\phi_2\rangle=\begin{pmatrix}0\\0\\b\end{pmatrix}.$$

In terms of the observable masses, m_μ and m_X, and the angles ρ and λ, we have

$$af=m_X\sin\rho\cos\lambda+m_\mu\cos\rho\sin\lambda,$$

$$bf'=-m_X\cos\rho\sin\lambda-m_\mu\sin\rho\cos\lambda,$$

$$m=m_X\cos\rho\cos\lambda-m_\mu\sin\rho\sin\lambda,$$

and the relation

$$m_\mu\cos\rho\cos\lambda=m_X\sin\rho\sin\lambda. \tag{6}$$

No additional spinless mesons are needed to implement the superstrong breaking. If $|b|\gg|a|$, the wrong-helicity current and doubly charged current contributions are suppressed. This is a model with a photon and eight massive vector bosons: Weinberg's $W^\pm$ and Z^0, and five superheavy vector bosons. Only four spinless mesons, two neutrals and a doubly charged pair, remain after spontaneous symmetry breaking. As in the SU(3)×SU(3)×SU(3) model, there is a type-(0) mass relation guaranteeing the masslessness of the neutrino. There is a global invariance of the original Lagrangian under $\phi_{1,2}\to e^{i\beta}\phi_{1,2}$, $\psi_R\to e^{i\beta}\psi_R$, $\psi_L\to e^{-i\beta}\psi_L$. After spontaneous symmetry breaking, there is still a global chiral invariance on the neutrino field.

Since there is no spinless meson coupled to $\bar{e}e$, this model has a type-(1) mass relation, keeping the electron massless in zeroth order. Contributions to the electron mass come from the diagrams in Fig. 1 and in Fig. 5. In the symmetric-gauge language, the contribution comes from diagrams with three or more tadpoles on the internal lepton line.

To calculate the electron mass, we again work with the shifted Lagrangian in Landau gauge. Because of (6), the logarithmic divergences from the diagrams in Figs. 1 and 5 cancel. If the gauge coupling constant of the octet of gauge bosons is $e/\cos\theta$ and the coupling constant of the singlet is $e/\sin\theta$, the electron mass is

$$\alpha m_\mu \cos\rho \cos\lambda \frac{3}{16\pi \cos^2\theta} \frac{m_X^2}{m_D^2 - m_X^2} \ln\frac{m_D^2}{m_X^2} + O\left(\frac{m_\mu^2}{m_D^2}\right).$$

Here $m_D^2 = 2e^2(a^2+b^2)/\cos^2\theta$ is the squared mass of the doubly charged vector boson. In this model, the unobserved lepton must be extremely massive. In fact, for (6) to give a reasonable value for the electron mass, m_X must be of the same order of magnitude as m_D, which is at least a few hundred GeV.

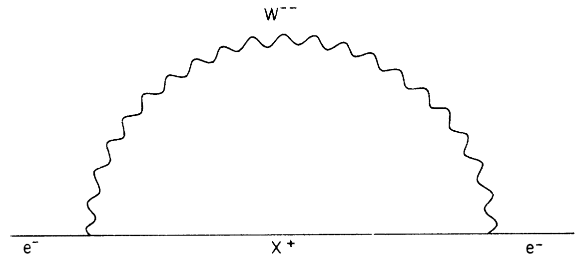

FIG. 5. Together with Fig. 1, this Feynman diagram leads to a calculable electron mass in our SU(3)×U(1) model.

VI. COMMENTS AND CONCLUSIONS

We have only discussed models in which the leptons are combined into representations of SU(3). This restriction is not necessary. We can, for instance, add an extra neutral lepton and use the four-dimensional representation of O(5). Models of this type are interesting because they are automatically anomaly-free and because they can lead to a very small, calculable, but nonzero neutrino mass. Another possibility is to add two charged leptons and use the five-dimensional representation of O(5). Such theories are natural extensions of the Lee-Prentki-Zumino model.[9] But all these models seem more complicated than the SU(3) schemes described in this paper.

The SU(3) models do have anomalies, and are difficult to extend to hadrons. It may be possible, however, to eliminate both these problems by building hadrons out of three [conventional strong-interaction SU(3)] triplets of quarks with the unconventional charge assignments $(\frac{5}{3}, \frac{2}{3}, \frac{2}{3})$, $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, and $(-\frac{1}{3}, -\frac{4}{3}, -\frac{4}{3})$. The quarks can transform under the weak SU(3) gauge group like three triplets: one with charges $(\frac{5}{3}, \frac{2}{3}, -\frac{1}{3})$ and two with charges $(\frac{2}{3}, -\frac{1}{3}, -\frac{4}{3}.)$ It is then possible to arrange the lepton and hadron anomalies to cancel. These models are quite complicated and rather artificial, and we will not discuss them further here.

The most annoying feature of the models we have presented is that the predicted value of the electron mass depends more or less sensitively on many parameters which, while measurable in principle, are well out of reach of contemporary experimental technique. It certainly seems unlikely that the ideas discussed in this paper can lead to a parameter-free prediction of the electron-muon mass ratio, but it may, at least, be possible to do slightly better. In particular, it would be nice to eliminate the need for more than one gauge coupling constant. This can probably be done in the SU(3)×SU(3)×SU(3) model if one adds another lepton multiplet transforming as a triplet under the S group and enough spinless mesons so that there are more charge-conjugation symmetries; however, the cost in extra fields seems prohibitive.

We cannot claim to have given an explication of the lepton mass spectrum, but we have solved an interesting technical problem: We have found rational, if implausible, models in which the electron mass is finite, calculable, and of order αm_μ. Hopefully, the ideas presented in this paper will be useful in the search for better models.

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SU(2)×U(1) Gauge Theories with Han-Nambu Quarks*

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Several SU(2)×U(1) gauge theories of weak and electromagnetic interactions based on Han-Nambu quarks are presented which satisfy all known selection rules of semileptonic and nonleptonic weak interactions, excluding the $\Delta I = \frac{1}{2}$ rule.

Several possibly renormalizable unified theories of weak and electromagnetic interactions have recently been proposed.[1-6] In fact, a large number of such mathematically consistent models exist,[7] but the imposition of a few physical constraints, specifically the absence of $\Delta S = 1$ neutral currents,[8] the absence of $\Delta S = 2$ nonleptonic processes, the cancellation of Adler-type anomalies,[9] the suppression of induced neutral currents in semileptonic processes and in the K_L^0 - K_S^0 mass difference,[10] the criterion that the theory gives the correct sign and magnitude for the decay $\pi^0 \to 2\gamma$, and the apparent suppression of $\bar{\nu}\nu$ couplings,[11] reduces the number of viable theories considerably. We present here several models based on the group SU(2)×U(1) which satisfy the above constraints.[12] Hadrons in these models are described by the Han-Nambu three-triplet quark model,[13] which incorporates both the phenomenology of the naive quark model and a sensible spin-statistics assignment for the quarks. As Lipkin[14] has emphasized, such an approach is less likely to result in a clash between the weak and strong symmetry structures. Georgi and Glashow[6] have recently presented a gauge theory based on Han-Nambu quarks for the group SO(3). Despite the added economy of such a model, the lack of a weak hypercharge may lead to difficulties in eventually incorporating strong interactions into a renormalizable theory,[15] so SU(2)×U(1) models may provide useful alternatives.

We begin with a description of the hadrons. The three Han-Nambu triplets have the charge assignments

$$Q_1 = \begin{pmatrix} \mathcal{P}_1^0 \\ \mathcal{N}_1^- \\ \lambda_1^- \end{pmatrix}, \quad Q_2 = \begin{pmatrix} \mathcal{P}_2^+ \\ \mathcal{N}_2^0 \\ \lambda_2^0 \end{pmatrix}, \quad Q_3 = \begin{pmatrix} \mathcal{P}_3^+ \\ \mathcal{N}_3^0 \\ \lambda_3^0 \end{pmatrix}.$$

It is assumed that these nine quarks span a $(3, \bar{3})$ representation of the strong-interaction group SU(3)×SU(3)′ and that all observed hadrons are SU(3)′ singlets. Because there are four charged quarks, one can choose the SU(2) gauge multiplets to be either four doublets and one singlet or two triplets and three singlets.[16] In the former case, the weak currents will contain many terms and the constraints on the mixing parameters that result from imposition of the observed selection rules are mathematically intractable. The two-triplet case is easier to handle and in fact has been analyzed in detail for the group SO(3) by Georgi and Glashow.[6] The SU(2)×U(1) analog of their parametrization is

$$T_{L1} = \begin{pmatrix} \mathcal{P}_2 \\ \sin\beta\,\mathcal{N}_2(\theta) + b\lambda_3(\chi) + \sin\beta\cos\phi\,\mathcal{P}_1 \\ \mathcal{N}_1(\theta-\phi) \end{pmatrix}_L,$$

$$T_{L2} = \begin{pmatrix} \mathcal{P}_3 \\ \sin\beta\,\mathcal{N}_3(\theta) + b'\lambda_2(\chi') + \sin\beta\sin\phi\,\mathcal{P}_1 \\ \lambda_1(\theta-\phi) \end{pmatrix}_L,$$

$$S_{L1}, \quad S_{L2}, \quad S_{L3},$$

$$\mathcal{N}_i(\xi) \equiv \mathcal{N}_i \cos\xi + \lambda_i \sin\xi,$$

$$\lambda_i(\xi) \equiv \lambda_i \cos\xi - \mathcal{N}_i \sin\xi,$$