New Finite-Energy Sum Rules Based on ρ and ω Exchange*

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New finite-energy sum rules relate the isospin dependence of low-energy s-channel amplitudes to the ratio of ρ- and ω-exchange contributions in the Regge region. Applications to NN, KN, $K\Delta$, and $N\Delta$ scattering are discussed. Results include the isospin dependence of $\bar{N}N$ annihilation, SU(6) symmetry breaking in the couplings of strange baryon resonances to the KN system, and predictions for exotic production in Δ - exchange reactions.

I. INTRODUCTION

Superconvergence relations were originally written for amplitudes that had no contribution from the leading Regge trajectories because they carried exotic quantum numbers.[1] They have also been written for linear combinations of amplitudes, each of which has nonvanishing contributions from the leading Regge trajectories, in situations in which the coefficients of the linear combination are chosen to make these contributions cancel one another. Such sum rules require some input to give the relation between the different Regge residues so that a cancellation can be constructed. Universality relations between the couplings between different hadrons and the same trajectory have been used as such an input and can be justified either on theoretical or experimental grounds.[2] This paper proposes new sum rules for odd-signature amplitudes dominated at high energies by exchanges of the ρ and ω trajectories. The sum rules are obtained by choosing linear combinations of amplitudes for which the ρ- and ω-exchange contributions cancel one another. If the ρ and ω trajectories are degenerate, one can construct linear combinations of amplitudes in which the ρ- and ω-exchange contributions cancel at all energies in the Regge region. Such linear combinations satisfy a superconvergence sum rule. If the ρ and ω are not degenerate, an energy-independent cancellation in the Regge region cannot be obtained, but a finite-energy sum rule can be written for a linear combination of amplitudes chosen to make the ρ and ω contributions to the right-hand side of the sum rule cancel one another. The particular linear combination will then depend on the upper limit of the integral in the finite-energy sum rule. Since degeneracy of the ρ and ω trajectories seems to be a very good approximation, the approach will be to consider primarily the degenerate case but to indicate which conclusions are unaffected by a departure from degeneracy.

Like all superconvergence or finite-energy sum rules, these new sum rules relate the low-energy behavior of an amplitude in the resonance region to the parameters of the Regge trajectories that dominate the high-energy behavior.[3] When Regge parameters are used as input, conclusions regarding the properties of low-energy s-channel amplitudes and possible resonances are obtained. This sum rule differs from previous sum rules in that its input is the ratio of the ρ- and ω-exchange contributions – i.e., the ratio of t-channel amplitudes having different isospins. The output is expressed in the low-energy region as a ratio of integrals of s-channel amplitudes having different isospins. The resulting expression relates the isospin dependence of a t-channel amplitude in the Regge region to the isospin dependence of the corresponding s-channel amplitude in the resonance region. These isospin dependences are sometimes predicted by models and symmetry schemes. Such predictions could be tested by the new sum rules, by analogy with the use of finite-energy sum rules in duality treatments in which the absence of exotic states in one channel is used to obtain relations between couplings in the cross channel.[4] However, the input is not absence of exotic states, but rather the experimental value of the ratio of the ρ and ω nonflip couplings to the nucleon, and this value does not agree with the predictions of any known model or symmetry.[5] The use of the experimental value as input therefore includes symmetry-breaking effects which are not simply described by any model. The sum rule thus connects these symmetry-breaking effects in the Regge region with phenomena in the resonance region and can give some insight into the symmetry breaking there.

The smallness of the ratio of the ρ to ω nonflip couplings to the nucleon is one of the unexplained mysteries of hadron dynamics. Both SU(6) symmetry and SU(3)-symmetric quark models predict a value of $\frac{1}{3}$ for this ratio.[6,7] The experimental value[8] is closer to $\frac{1}{5}$ or $\frac{1}{6}$. Although the deviation from the symmetry prediction can be attributed to symmetry-breaking effects, there is no successful prediction for the magnitude of this symmetry breaking. The ratio of the ρ and ω couplings to the nucleon is thus an interesting experimental number which awaits a theoretical explanation.

The smallness of this ratio has recently been used as a new input to a finite-energy sum rule. In the extreme limit of neglecting the ρ-nucleon coupling altogether, Finkelstein[9] has obtained a superconvergence sum rule for the ρ-exchange contribution to ρ-nucleon scattering. One can question the approximation of setting the admittedly small ρ-exchange contribution equal to zero. There is therefore interest in finding better sum rules that incorporate the exact value and not merely the fact that it is small. Such sum rules should include both isoscalar and isovector contributions. The sum rule for ρ-nucleon scattering is not easily extended to include isoscalar exchanges as well as ρ exchange because G parity requires the isoscalar and isovector contributions to have opposite signature. They are therefore not easily incorporated in the sum rule. The sum rules considered in this paper include nontrivial contributions from both ρ and ω exchange and can test the validity of the Finkelstein approximation in which ρ exchange is neglected. However, they are not applicable to the scattering of nonstrange mesons, which are eigenstates of G. We therefore restrict our consideration to scattering amplitudes for kaons and baryons.

Nucleon-nucleon and nucleon-antinucleon scattering are considered in Sec. II, and the ratio of the ρ and ω couplings is shown to place constraints on the ratio of isovector to isoscalar annihilation at low energies. Kaon-nucleon scattering is considered in Sec. III, and the breaking of SU(6) symmetry predictions in the ratio of the ρ and ω couplings is shown to be related to and consistent with the experimentally observed breaking of SU(6) predictions for the couplings of the $\bar{K}N$ system to isovector and isoscalar resonances. The general case of KN scattering, where K denotes any strange nonexotic boson, is considered in Sec. IV, and the results of Sec. III are shown to apply equally to all K^* resonances and trajectories, as well as to pseudoscalar kaons. This is in qualitative agreement with the experimental observation that SU(6) symmetry breaking in the production of strange baryons via strangeness exchange seems to be universal and independent of which K or K^* is exchanged. Section IV considers $K\Delta$ scattering in which the $I=\frac{3}{2}$ s channel is exotic, and the result being that the ρ and ω couplings must satisfy SU(3) symmetry with the canonical mixing angle if the exotic amplitude is required to vanish. Section V considers $N\Delta$ and $\bar{N}\Delta$ scattering and finds that the exotic $I=2$ s-channel amplitude must be of the same order of magnitude as the nonexotic $I=1$ amplitude. This is the familiar "baryon-antibaryon catastrophe," which requires the existence of exotic contributions to satisfy finite-energy sum rules. However, the new sum rule provides a quantitative estimate for the exotic contribution relative to the nonexotic contribution. This can be used in a quantitative and conclusive search for exotic states.

II. NUCLEON-NUCLEON AND NUCLEON-ANTINUCLEON SCATTERING

We first consider nucleon-nucleon scattering. Under the assumption that these amplitudes are dominated in the Regge region by the ρ and ω exchanges, we can write the finite-energy sum rule

$$\int_0^N \nu^n\{[A(\bar{p}p)-A(pp)](1-\kappa^2) - [A(\bar{p}n)-A(pn)](1+\kappa^2)\}\,d\nu=0\,, \quad (1a)$$

where A denotes a scattering amplitude at some value of t and κ is a parameter chosen to make the right-hand side vanish. If the ρ and ω trajectories are assumed to be degenerate, κ is just the ratio of the ρ and ω couplings to the nucleon, i.e.,

$$\kappa=g_{N\bar{N}\rho}/g_{N\bar{N}\omega}\,. \quad (1b)$$

In this case the upper limit of the integral in the sum rule (1a) can be taken to be infinite to give a superconvergence sum rule for a suitable value of n. However, even if the ρ and ω trajectories are not degenerate, κ is a well-defined number determined by the behavior of the scattering amplitudes in the Regge region and can be determined either theoretically or experimentally and substituted into the sum rule.

The sum rule (1a) can be rewritten

$$\int_0^N \nu^n[A(\bar{p}p)-A(pp)]\,d\nu = \left(\frac{1+\kappa^2}{1-\kappa^2}\right)\int_0^N \nu^n[A(\bar{p}n)-A(pn)]\,d\nu\,. \quad (2)$$

Let us now consider the case in which A is the imaginary part of the forward scattering amplitude. The sum rule (2) then applies to the total cross sections and the couplings appearing in Eq. (1b) are the nonflip couplings. For this case in which the experimental value is $\kappa=\frac{1}{5}$, it is a reasonable approximation to neglect κ^2 in Eq. (2). This leads to the surprising result that the difference between the antiproton and proton total cross sections on a proton target is equal to the corresponding difference on a neutron target when suitably averaged over energy, i.e.,

$$\langle \sigma_{\text{tot}}(\bar{p}p) - \sigma_{\text{tot}}(pp)\rangle \approx \langle \sigma_{\text{tot}}(\bar{p}n) - \sigma_{\text{tot}}(pn)\rangle . \quad (3)$$

This result is surprising since the difference between the total cross sections can be assumed to have a large contribution from annihilation at low energies and the $\bar{p}n$ system is a state of isospin 1 while the $\bar{p}p$ system is an equal mixture of isospin 0 and isospin 1. This gives the conclusion that nucleon-antinucleon annihilation in the isospin-0 and isospin-1 states are approximately equal when averaged suitably over energy, or that there are additional nonannihilation contributions that exactly compensate for any isospin dependence in annihilation. In models in which the annihilation is described by a simple s-channel mechanism, the sum rule (1) requires some kind of exchange degeneracy of isoscalar and isovector s-channel contributions. Thus, Rubinstein's model that nucleon-antinucleon annihilation is dominated by the Regge recurrences of the pion[10] could not give the whole story. An additional isoscalar trajectory, possibly exchange-degenerate with the pion, is needed to enable this mechanism to satisfy the sum rule (1). If this isoscalar trajectory had odd G parity (e.g., if it is the isoscalar member of the octet containing the B meson), annihilation into nonstrange mesons would still be dominated by odd-G states and there would be a systematic difference between annihilation into even and odd numbers of pions as predicted by Rubinstein.

If one attempts to saturate the sum rule by resonances (admittedly a rather dubious procedure for baryon-antibaryon scattering), degenerate or nearly degenerate isoscalar and isovector resonances would occur. Some evidence for this has been noted in the case of proton-antiproton annihilation into kaons.[11]

III. KAON-NUCLEON SCATTERING

We now apply the same approach to KN scattering and obtain the sum rule

$$\int_0^N \nu^n\{[A(K^-p) - A(K^+p)](1-\kappa) - [A(K^-n) - A(K^+n)](1+\kappa)\}\,d\nu = 0 . \quad (4)$$

If factorization of Regge residues is assumed and SU(3) is used to give equal couplings of the ρ and ω to kaons, the parameter κ in the sum rule (4) is the same as that appearing in the sum rule (1a) and is given by Eq. (1b). Note that when the symmetry prediction $\kappa=\frac{1}{3}$ is substituted into Eq. (4), the linear combination of amplitudes appearing in the integral is just the one required to vanish by the Johnson-Treiman relation[6] for kaon-nucleon scattering. The disagreement between this particular relation and experiment is well known, and the value of κ obtained from the data[8] is about $\frac{1}{5}$ or $\frac{1}{6}$. Note that the Finkelstein approximation[9] of neglecting ρ exchange completely corresponds to $\kappa=0$. This sum rule can be tested both by comparison with experimental data and by resonance saturation.

The sum rule (4) can be expressed in a more convenient form by introducing amplitudes corresponding to a definite isospin in the kaon-nucleon system. These are

$$A(K^-p) = \tfrac{1}{2}(\bar{A}_0 + \bar{A}_1) , \quad (5a)$$

$$A(K^-n) = \bar{A}_1 , \quad (5b)$$

$$A(K^+p) = A_1 , \quad (5c)$$

$$A(K^+n) = \tfrac{1}{2}(A_0 + A_1) , \quad (5d)$$

where A_i and $\bar{A}_i$ are amplitudes for KN and $\bar{K}N$ scattering in the state of isospin i. Substituting these expressions (5) into the sum rule (4) yields

$$\int_0^N \nu^n\{\bar{A}_0(1-\kappa) - \bar{A}_1(1+3\kappa) + A_0(1+\kappa) - A_1(1-3\kappa)\}\,d\nu = 0 . \quad (6)$$

One would expect resonance saturation to be reasonably good for this sum rule since the nonresonant background should be the same for the KN and $\bar{K}N$ amplitudes and therefore cancel one another. Since the KN system is exotic and has no resonances, resonance saturation of the sum rule (6) gives a relation between integrals over isoscalar and isovector kaon-nucleon resonances, namely,

$$\frac{\int \bar{A}_0^R \nu^n d\nu}{\int \bar{A}_1^R \nu^n d\nu} = \frac{1+3\kappa}{1-\kappa} , \quad (7)$$

where $\bar{A}_i^R$ denotes the resonance contribution to the amplitude. Table I gives values for the right-hand side of the expression (7) for some typical values of κ. We see that the SU(6) limit and the extreme approximation of completely neglecting

TABLE I. SU(6) symmetry-breaking factors.

	κ	$(1+3\kappa)/(1-\kappa)$
Finkelstein limit	0	1
Real world (?)	$\frac{1}{5}$	2
SU(6) limit[a]	$\frac{1}{3}$	3

[a] Johnson and Treiman, Ref. 6.

the isovector exchange give quite different relative strengths for the isovector and isoscalar resonance contributions – and that the real world is somewhere in between. Thus the use of this sum rule may give interesting information about the parameter κ.

As a crude approximation, one might attempt to saturate the sum rule (7) with the strange-baryon resonances in the lowest-lying SU(6) $\underline{56}$. In this case the sum rule could be applied to the K-exchange contribution to the reactions

$$A+p\to B+(\Lambda, \Sigma^0, Y^{*0}), \tag{8}$$

where A and B are any hadrons. For any given pair A and B, application of the sum rule (7) relates the cross sections through the expression

$$\frac{\int \bar{A}_0^R(Kp)\nu^n d\nu}{\int \bar{A}_1^R(Kp)\nu^n d\nu}=\frac{1+3\kappa}{1-\kappa} = \frac{\bar\sigma(\Lambda)}{\bar\sigma(\Sigma^0)+\bar\sigma(Y^{*0})}, \tag{9}$$

where $\bar\sigma$ denotes the cross section for the reaction (8), corrected for the usual kinematic and phase-space factors. Although the result (9) has been derived only for the contribution of K exchange, we shall see below that it is reasonable to expect it to hold as well for other exchanges, since similar sum rules can be written for the scattering of the various K^* resonances that would provide the other exchanges.

In the SU(6) limit, Table I shows that our sum rule predicts the value 3 for the ratio of cross sections in Eq. (9). This is in fact a well-known SU(6) prediction obtained by considering only the reaction (8) without invoking any finite-energy sum rules.[12] This result thus shows the internal consistency of using SU(6) to relate couplings at low and at high energies and to saturate the sum rule relating the two quantities with a set of resonances appearing in a single SU(6) multiplet. The experimental value of the ratio (9) is in strong disagreement with the SU(6) prediction and is considerably lower.[13] This is in qualitative agreement with the SU(6) breaking determined at high energies by the experimental value of κ.

IV. UNIVERSAL SU(6) BREAKING IN $K^*\bar{Y}N$ COUPLINGS

One of the peculiar features of the SU(6) breaking observed experimentally in the reactions (8) is that the deviation from the SU(6) predictions seem to be qualitatively the same regardless of the exchange mechanism.[14] Different exchange mechanisms have been studied extensively for the case in which B is a vector meson and the contributions of different types of exchanges (e.g., flip vs nonflip or natural parity vs unnatural parity) can be separated by observing the density matrices for vector-meson polarization. That SU(6) should be broken is no great surprise, but it is very peculiar that it should be broken in the same way for all possible exchanges. This implies that all relevant couplings have the same D/F ratio for the baryon octet and that the ratio of octet and decuplet couplings is also independent of the exchange.

The use of the finite-energy sum rule (4) provides a possible insight into the universal coupling ratios to baryons of the strange K^* trajectories. If finite-energy sum rules are written for the scattering of these strange Reggeons on nucleons, the high-energy behavior of the odd-signature amplitudes can be expected in all cases to be dominated by the ρ and ω trajectories. The ratio of isovector to isoscalar exchange in all cases depends only on the same parameter, the ratio of the ρ and ω nonflip couplings to nucleons, since the relative value of the ω and ρ couplings to kaons is uniquely predicted by SU(3) and is the same for any meson octet. This universality can be seen explicitly by examining the case in which two strange bosons K_α and K_β belonging to different octets scatter on nucleons. In the Regge region where the scattering is expected to be dominated by the ρ and ω trajectories, we can write down the expressions

$$[A(K_\alpha^- p)-A(K_\alpha^+ p)]-C_{\alpha\beta}^p[A(K_\beta^- p)-A(K_\beta^+ p)]=0, \tag{10a}$$

$$[A(K_\alpha^- n)-A(K_\alpha^+ n)]-C_{\alpha\beta}^n[A(K_\beta^- n)-A(K_\beta^+ n)]=0, \tag{10b}$$

where the coefficients $C_{\alpha\beta}^p$ and $C_{\alpha\beta}^n$ can be considered to be defined by the relations (10).

We now assume the SU(3) relation for the three-boson vertex, namely[12]

$$\frac{g_{K_\alpha\bar{K}_\alpha\rho}}{g_{K_\alpha\bar{K}_\alpha\omega}}=\frac{g_{K_\beta\bar{K}_\beta\rho}}{g_{K_\beta\bar{K}_\beta\omega}}. \tag{11}$$

Note that exact SU(3) gives the value unity for the ratio (11). However, we do not need this exact value for our treatment. Thus our result is insensitive to SU(3) breaking if it is the same for K_α and K_β. One example of such a breaking would be a departure of the ω from the canonical expression with the ideal mixing angle. If we further assume that the expressions (10) are dominated by the ρ and ω trajectories, then it follows immediately from the relation (11) that

$$C_{\alpha\beta}^p=C_{\alpha\beta}^n = g_{K_\alpha\bar{K}_\alpha\omega}/g_{K_\beta\bar{K}_\beta\omega}. \tag{12}$$

Note that this result does not require degeneracy for the ρ and ω trajectories. Even if they have different intercepts, the energy dependence cancels in the definition of the coefficients (12). We can therefore write finite-energy sum rules for the expressions (10), eliminate the coefficients (12) between the two sum rules, introduce s-channel isospin amplitudes analogous to the expressions (5), and assume resonance saturation. We thus obtain the expression

$$\frac{\int \overline{A}_0^R(\overline{K}_\alpha p)\nu^n d\nu}{\int \overline{A}_1^R(\overline{K}_\alpha p)\nu^n d\nu}=\frac{\int \overline{A}_0^R(\overline{K}_\beta p)\nu^n d\nu}{\int \overline{A}_1^R(\overline{K}_\beta p)\nu^n d\nu}\,. \tag{13}$$

We thus find that the ratio of isovector to isoscalar resonance contributions to these sum rules is a universal quantity related to the ratio κ of the ρ to ω nonflip nucleon couplings and is independent of the strange meson. This suggests that if saturation by the Λ, Σ, and $Y^*(1385)$ is a good approximation, then the relation (9) holds for all exchanges:

$$\frac{{g_{K_\alpha \bar{p}\Sigma}}^2+{g_{K_\alpha \bar{p} Y^*}}^2}{{g_{K_\alpha \bar{p}\Lambda}}^2}=\frac{{g_{K_\beta \bar{p}\Sigma}}^2+{g_{K_\beta \bar{p} Y^*}}^2}{{g_{K_\beta \bar{p}\Lambda}}^2}\,, \tag{14}$$

$$\frac{\overline{\sigma}_\alpha(\Sigma^0)+\overline{\sigma}_\alpha(Y^{*0})}{\overline{\sigma}_\alpha(\Lambda)}=\frac{\overline{\sigma}_\beta(\Sigma^0)+\overline{\sigma}_\beta(Y^{*0})}{\overline{\sigma}_\beta(\Lambda)}\,. \tag{15}$$

Note that although the relation (9) depends upon degeneracy of the ρ and ω trajectories, the universality relations (14) and (15) are independent of this degeneracy assumption.

Difficulties are encountered if the universality relation is carried too far. The relation (13) has been derived for the nonflip amplitude for any helicity state of the kaon or nucleon. Consider the forward scattering amplitude for a K^* with helicity $+1$ and a nucleon with helicity $-\frac{1}{2}$. The total angular momentum of the system therefore has a projection of $+\frac{3}{2}$ on the direction of the incident kaon. This amplitude can have a contribution from the $Y^*(1385)$ but cannot have any contribution for spin-$\frac{1}{2}$ resonances or from baryons such as the Λ or Σ. Thus, resonance saturation limited to the baryon $\underline{56}$ is clearly invalid for this amplitude. One might question the use of such resonance saturation for all amplitudes if it definitely fails in a particular case. On the other hand, it is known that there are difficulties in fitting polarization data with simple Regge models. One might argue that the simple description in terms of ρ and ω poles may not be adequate to describe the spin dependence of the amplitudes but may be adequate for a nonflip amplitude that is averaged over the nucleon spin direction. This is the amplitude whose imaginary part is related to the total cross section on an unpolarized target, the case for which the experimental agreement with the simple ρ and ω model has been best. One might consider then that if the sum rules (9) and (13) are to be saturated with a small number of resonances, then they should be applied only to these spin-averaged amplitudes.

V. $K\Delta$ SCATTERING

The same approach can be applied to $K\Delta$ scattering. By analogy with Eq. (4) we can write the sum rule

$$\int_0^N \nu^n\{[A(K^-\Delta^{++})-A(K^+\Delta^{++})](1-\lambda) - [A(K^-\Delta^-)-A(K^+\Delta^-)](1+\lambda)\}d\nu=0\,. \tag{16}$$

We assume as before that the dominant contributions in the Regge region come from degenerate ρ and ω trajectories. The parameter λ is the ratio of the nonflip couplings of these two trajectories to the Δ. However, we encounter a difficulty if we attempt to leave λ a free parameter to be determined by experiment and introduce resonance saturation of the sum rule (16). Only one of the four amplitudes appearing in the sum rule is nonexotic, namely $A(K^-\Delta^{++})$. Thus resonance saturation combined with the assumption that there are no exotic resonances determines the value of λ. The result is

$$\lambda=1\,, \tag{17}$$

which is just the SU(6) prediction.

We thus find that in the KN and NN systems, in which all s-channel isospin values are nonexotic, it is possible to break the SU(6) relation between the ρ and ω couplings to the nucleon without encountering difficulties; and indeed experiments seem to indicate such breaking. However, in the case of $K\Delta$ scattering, in which one of the s-channel isospin values is exotic, the requirement of absence of exotic resonances in this channel cannot be satisfied unless the ρ and ω couplings to the Δ satisfy the SU(6) relation (17).

VI. NUCLEON-Δ SCATTERING

A similar sum rule can be written for nucleon-Δ scattering. Again we assume that the high-energy behavior of the odd-signature amplitude is dominated by ρ and ω exchange. The ratios of the ρ and ω contributions are determined by the parameters κ and λ defined by KN and $K\Delta$ scattering. Thus without any new parameters we obtain the sum rule

$$\int_0^N \nu^n\{[A(\bar{p}\Delta^{++}) - A(p\Delta^{++})](1-\kappa\lambda) - [A(\bar{p}\Delta^-) - A(p\Delta^-)](1+\kappa\lambda)\}d\nu = 0\,. \quad (18)$$

Here again there is only one nonexotic amplitude, namely $A(\bar{p}\Delta^{++})$. Thus if all exotic amplitudes vanish, the product $\kappa\lambda$ must be equal to unity. However, it is evident that this is not true either in the symmetry limit or in the real world as can be seen from Table I.

This is just another manifestation of the well-known baryon-antibaryon catastrophe,[15] which implies either that resonance saturation of finite-energy sum rules is not valid for the baryon-antibaryon system or that there are exotic contributions. Let us assume that there are indeed exotic-resonance contributions with isospin 2 and zero baryon number. In that case, resonance saturation of the relation (18) gives

$$\int_0^N \nu^n A^R(\bar{p}\Delta^-)d\nu = \frac{1-\kappa\lambda}{1+\kappa\lambda}\int_0^N \nu^n A^R(\bar{p}\Delta^{++})d\nu\,. \quad (19)$$

In the same way that the reactions (8) test the sum rule (4), the sum rule (19) can be tested by looking at nucleon-antinucleon annihilation via Δ exchange. Consider for example the two double-charge-exchange reactions

$$\bar{p}+p \to \pi^+ + M^-\,, \quad (20a)$$

$$\bar{p}+n \to \pi^+ + X^{--}\,, \quad (20b)$$

where the meson M^- is any negatively charged meson while X^{--} is a doubly charged exotic meson. These reaction can be assumed to go via Δ^{++} exchange when the π^+ goes in the direction of the incident antiproton. If the states M^- and X^{--} are used to saturate the sum rule (19), one sees that the cross sections (20a) and (20b) are of the same order of magnitude since the product $\kappa\lambda$ should be small. It would therefore be of considerable interest to look for the reactions (20a) and (20b). A similar situation obtains for the pion-nucleon reactions obtained from the reactions (20) by line reversal,

$$\pi^- + p \to p + M^-\,, \quad (21a)$$

$$\pi^- + n \to p + X^{--}\,. \quad (21b)$$

A detailed analysis of the application of the sum rule (19) to the search for exotics in reactions (20) and (21),[16] shows that the sum rule makes possible a conclusive quantitative test of the argument that exotic resonances provide the answer to the baryon-antibaryon catastrophe.

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