[4] M. J. Duff, ICTP Report No. ICTP/71/19 (unpublished).

[5] R. Arnowitt, S. Deser, and C. W. Misner, in *Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962), Chap. 7.

[6] The function $H(r)$ must vanish at $r = 0$ in order that the derivative of the metric be finite at the origin.

[7] So that the exterior solution is Schwarzschild. More rigorously this quantity must equal the total energy of the system $\int (g_{ij,j} - g_{jj,i}) dS_i$, where dS_i is a two-dimensional surface element at spatial infinity (see Ref. 5).

[8] R. Arnowitt, S. Deser, and C. W. Misner, Phys. Rev. 120, 313 (1960).

[9] The analogous equation for the shell distribution is exactly the Newtonian expression, namely, $m = m_0 - Gm_0^2/2\epsilon_c$. It is curious that this mass formula was first derived by ADM in isotropic coordinates for which $\epsilon_c = H(\epsilon) = (1 + Gm/2\epsilon)^2 \epsilon$, in which case $m = m_0 - Gm^2/2\epsilon$, i.e., in terms of ϵ, rather than $H(\epsilon)$, it is the total mass, m, which appears in the self-energy contribution and not the bare mass, m_0. The appearance of the total mass prompted ADM to interpret this effect as being due to the equivalence principle in general relativity (Ref. 8). However, this phenomenon is merely a fluke of the isotropic frame and does not appear to have any physical significance (Ref. 4).

[10] Provided $\epsilon_c^2 < 8R^2/9$, i.e., provided p remains finite.

[11] This is also true in isotropic but not Schwarzschild, coordinates. Nathan Rosen, Ann. Phys. (N.Y.) 63, 127 (1970).

[12] J. N. Goldberg, Phys. Rev. 111, 315 (1958).

[13] $\mathcal{R}$ is the usual Riemann curvature scalar. The symbol $^3\mathcal{R}$ is reserved for the curvature of the 3-space.

[14] By using the term "noncovariant" we mean that $\mathcal{L}_\Phi$ breaks general covariance but not Lorentz covariance.

[15] E. S. Fradkin and I. V. Tyutin, Phys. Rev. D 2, 2841 (1970).

[16] Bryce S. DeWitt, Phys. Rev. 162, 1239 (1967).

[17] Compare the five terms of Eq. (4.17) with the eleven terms of Eq. (2.6) of Ref. 15.

[18] See B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, *Gravitation Theory and Gravitational Collapse* (University of Chicago Press, Chicago, Ill., 1965).

PHYSICAL REVIEW D VOLUME 7, NUMBER 8 15 APRIL 1973

Experimental Test of Weyl's Gauge-Invariant Geometry[*]

E. G. Harris, Paul G. Huray, F. E. Obenshain, and J. O. Thomson
The University of Tennessee, Knoxville, Tennessee 37916
and Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

R. A. Villecco[†]
University of Tennessee, Knoxville, Tennessee 37916

(Received 30 June 1972; revised manuscript received 1 December 1972)

Weyl's gauge-invariant geometry predicts that the periods of clocks are affected by their previous history in electromagnetic fields. We have used the Mössbauer effect to search for such an effect with negative results.

I. INTRODUCTION

The first attempt to formulate a unified field theory in which both gravitation and electromagnetism were incorporated into a single geometrical structure of a space-time manifold was made by Weyl[1,2] in 1918. In spite of some attractive features this theory was abandoned by its author and others within a few years. As will be discussed presently, at least one of the reasons for this rejection might not seem as compelling in the light of some recent astronomical observations.[3] These observations led us to reconsider Weyl's theory and to subject it to a more stringent experimental test as reported here.

Einstein[4] pointed out in 1918 that a consequence of Weyl's theory would be that the frequencies of spectral lines emitted by atoms would depend on the electromagnetic history of the atoms. Since all atoms of an element in the universe seem to emit the same frequencies the theory seemed to be contradicted by the facts.

However, observations indicate that atoms in distant galaxies have their lines shifted toward the red end of the spectrum[3] as compared with terrestrial spectra. This shift is usually attributed to the Doppler effect and accepted as evidence for expansion of the universe. Some recent astronomical observations of red shifts associated with quasistellar objects as large as $\Delta\nu/\nu \sim 1$ have made this interpretation difficult to maintain.[3] Arp[5] and Burbidge[6] have suggested that it may be necessary to revise our physical theories to account for these observations, and steps in this direction have been taken by Hoyle and Narlikar.[7] These observations might be taken as evidence in favor of Weyl's theory. For example, one possible way in which Weyl's theory could explain the red shifts is the following: In the neighborhood of a luminous star or galaxy, radiation pressure exerts a greater outward force on the electrons than on the ions, while the gravitational force is greater on the ions than on the electrons. As a result there could be a separation of charges such that an electric field is built up which causes both electrons and ion distributions to reach a steady state. Schwartzmann[8] has shown that an electric potential as large as $\phi_e = 10^8$ V might develop. This circumstance would arise if the luminosity of the object is just less than a critical value above which both electrons and ions are blown away by radiation pressure. Within the framework of the Weyl formulation such a potential could lead to an additional frequency shift over that arising from the Doppler mechanism.

In Sec. II we outline the theoretical considerations of Weyl and suggest its relationship to frequency shifts. In Sec. III we describe an experimental procedure to investigate the magnitude of such effects. Finally in Sec. IV we use our experimental results to show that any frequency shifts arising from this mechanism must be many orders of magnitude smaller than those required to explain the quasistellar observations.

II. THEORY

It will be recalled that in Einstein's general theory of relativity,[2] when a vector B^μ is displaced parallel to itself from a point with coordinates x^λ to a neighboring point with coordinates $x^\lambda + dx^\lambda$, the change in the vector is given by

$$\delta B^\mu = -\Gamma^\mu_{\alpha\beta} B^\alpha dx^\beta \ , \tag{1}$$

where $\Gamma^\mu_{\alpha\beta}$ are the components of the affine connection. Einstein assumed that the square of the length of a vector given by $B^2 = g_{\lambda\mu} B^\lambda B^\mu$ is unchanged by a parallel displacement. This assumption establishes the relationship between $\Gamma^\mu_{\alpha\beta}$ and the metric tensor $g_{\lambda\mu}$; namely, $\Gamma^\mu_{\alpha\beta} = \left\{ {\mu \atop \alpha\beta} \right\}$, the Christoffel bracket. In general relativity the affine connection is "integrable" only if there is no gravitational field present. Thus, let us consider two vectors which coincide at one space-time point. If we let them be displaced parallel to themselves along different space-time paths and then be brought together again at a second intersection of their space-time paths, they will be parallel when rejoined only if there is no gravitational field present.

In Weyl's theory[1,2,9,10] it is assumed that when a vector is displaced parallel to itself its change in length is given by

$$\delta B^2 = -B^2 \psi_\mu dx^\mu \ , \tag{2}$$

where ψ_μ is a four-vector. By considering the change in length of a vector as it is displaced around an infinitesimal parallelogram of sides dx_1^μ and dx_2^λ it is found that

$$\Delta B^2 = -B^2 f_{\mu\lambda} dx_1^\mu dx_2^\lambda \ , \tag{3}$$

where $f_{\mu\lambda} = (\partial\psi_\mu/\partial x_\lambda - \partial\psi_\lambda/\partial x_\mu)$. Weyl assumed that ψ_μ was proportional to the electromagnetic potential A_μ, and so $f_{\mu\lambda}$ is proportional to the electromagnetic field tensor. In this theory let us consider two vectors which initially coincide and which are then displaced parallel to themselves along different space-time paths from P to P'. Integrating Eq. (2) over the space-time paths one obtains

$$B^{(1,2)} = B_0 \exp\left(\frac{C}{e} \int_P^{P'} A_\mu^{(1,2)} dx^\mu_{(1,2)} \right) \ , \tag{4}$$

where $-2C/e$ is a coefficient of proportionality between the ψ_μ of Eq. (2) and the electromagnetic potential A_μ. We have introduced the charge of the electron $e = 4.8 \times 10^{-10}$ statvolt cm (or gauss cm^2) into Eq. (4) so as to make C a dimensionless constant. The value of C is not given by Weyl's theory.

Suppose the vector being discussed is cp_μ where p_μ is the energy-momentum vector of a system in its nth quantum state. Here c is the velocity of light. The square of the length of this vector is $E_{n0}{}^2 = E_n{}^2 - c^2p^2$. $E_{n0}{}^2$ is the square of the energy of the system in its rest frame. If two such systems are transferred from P along different space-time paths 1 and 2, and then reunited at P' their energy levels will differ by an amount

$$\frac{\Delta E_{n0}}{E_{n0}} \cong \frac{C}{e} \oint A_\mu dx^\mu \equiv C(\Phi/e) = \frac{C}{e} \left(\int_P^{P'} A_\mu^1 dx_1^\mu + \int_{P'}^{P} A_\mu^2 dx_2^\mu \right) . \tag{5}$$

Consequently, photons emitted or absorbed in transitions between two levels will have frequencies which differ by

$$\frac{\Delta\nu}{\nu} = C\left(\frac{\Phi}{e}\right) = \left(\frac{C}{e}\right) \left(\oint A \cdot dl + c \int_t^{t'} \Delta\phi_e dt \right) , \tag{6}$$

where we have explicitly separated the four-vector integral into its spacelike and timelike components in a suitable coordinate system.

III. EXPERIMENTAL

Our experiment was designed to measure the change in the transition energy between the 14.4125-keV excited state and the ground state in ^{57}Fe for two iron foils which were known to have followed different space-time paths; Eq. (6). This excited state is populated following electron capture in ^{57}Co and it is a suitable choice for the Mössbauer measurement because of the large recoilless fraction for resonant emission or absorption of γ rays at room temperature and because of the narrowness of the resonant line width. The resonance phenomenon allows a very precise determination of small changes in the γ-ray energy (and, hence, in the energy level separation of the nuclear states) which may arise from chemical bonding, thermal and gravitational effects, or in this case by the electromagnetic history of the iron atom as predicted by Weyl's theory.

The equipment used for this experiment was a standard transmission nuclear gamma resonance (NGR) apparatus which has been previously described.[11] Some minor modifications to the electro-mechanical transducer have been made for this work. In order to compare accurately the characteristics of two absorbers the NGR spectra were collected simultaneously using the same moving radioactive source. This source consisted of 20 mc of ^{57}Co diffused into an iron foil 6.35μ thick with the activity over an area of ~1 cm^2 (~0.1 at.% ^{57}Co). For our analysis of the NGR spectra we assumed a uniform distribution of activity throughout the source volume. However, our method of comparing two absorbers with the same source eliminated any intrinsic energy shift of the source with respect to pure Fe as well as instrumental effects, since the absorbers to be compared were periodically interchanged.

To minimize temperature gradients, the source-transducer-absorber-detector system was enclosed in a sealed wooden box, and care was taken after each opening of the box to insure that any thermal perturbations had a chance to die away before beginning a data run. The absorbers to be compared were held securely against a block of aluminum and the temperature differential between the two absorbers was found to be less than 0.01 °C during all data runs. A small magnetic field (~200 G) was applied parallel to the plane of the ferromagnetic source and absorbers so as to enhance the percent absorption near zero Doppler velocity.

The 14.4-keV γ rays were detected by two proportional counters followed by conventional nuclear electronics to select the 14.4-keV γ-ray pulses. The source velocity was synchronized with the address of a dual, zero dead-time multichannel analyzer, used in the multiscalar mode. By this technique velocity spectra of both absorbers could be obtained simultaneously. The range of velocity for most of the runs was ~±0.2 mm/sec, and each spectrum usually consisted of 1–2 millions of counts in each of 512 channels. Additional runs were made at larger velocities to check the quality of the whole 15-line iron spectrum as measured by our system. A typical spectrum with velocity range ~±20 mm/sec is shown in Fig. 1. The small shift of the center line from zero velocity is due to the isomer shift between the ^{57}Co source and the natural iron absorber as well as instrumental nonlinearity for this large velocity range. An absorption spectrum with velocity range ~±0.2 mm/sec is shown in Fig. 2.

The spectra were computer-fitted using a code written by Burton[12] (the solid curves in Figs. 1 and 2). Although the central component line which was studied is really a composite of several overlapping lines (assuming equal hyperfine fields in source and absorber), the center of gravity of the single-line fit gives the centroid position of the whole spectrum to high precision. As may be seen in Fig. 2 this single line provides an adequate description of the data.

In order to examine the effects brought about by differing electrostatic potential-time paths,

$$\Phi = c \int_{t}^{t'} \Delta \phi_e \, dt \,,$$

an iron screw was obtained from within the high voltage terminal of the tandem Van de Graaff accelerator at the Oak Ridge National Laboratory. This screw had logged 6×10^{10} V-hours on the terminal with respect to a similar screw which was taken from the stock room. Foils were fabricated from iron which was chemically removed from each screw. Spectrochemical analyses of the foils showed impurity content below 200 ppm (parts per million), with similar impurity content in both foils.

As an additional test of the effect of the space-like components of the integral, Eq. (6), (in the lab system) another set of experiments was performed wherein the foils to be compared followed paths enclosing differing amounts of magnetic flux. The absorbers used in this set of experiments were cut from the same piece of Armco iron. Several foils were left in the laboratory. Three foils were sent eastward and three foils were sent westward around the world so as to enclose substantial magnetic flux from the earth's magnetic field. Four were carried via mail service and two were on flights by the Pan American Corporation. The magnetic flux enclosed by each of these paths was ~$\pm5\times10^{17}$ G cm^2.

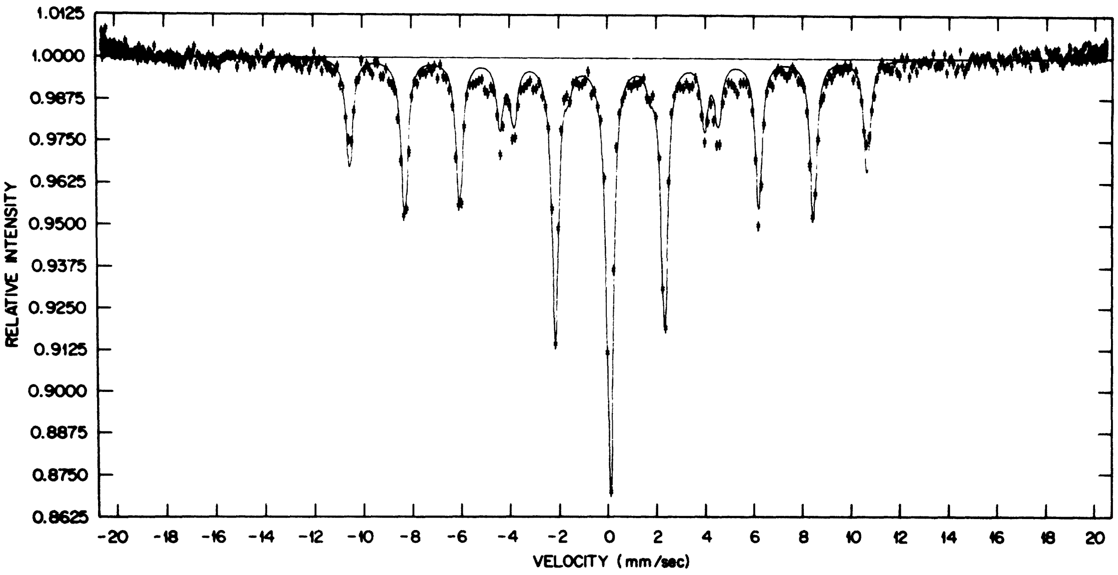

FIG. 1. Typical NGR absorption spectrum for a ^{57}Co in Fe γ source and a pure Fe absorber with a maximum doppler velocity of ± 20 mm/sec. The normal 15 line spectrum has been used in this set of experiments for velocity calibration of the apparatus.

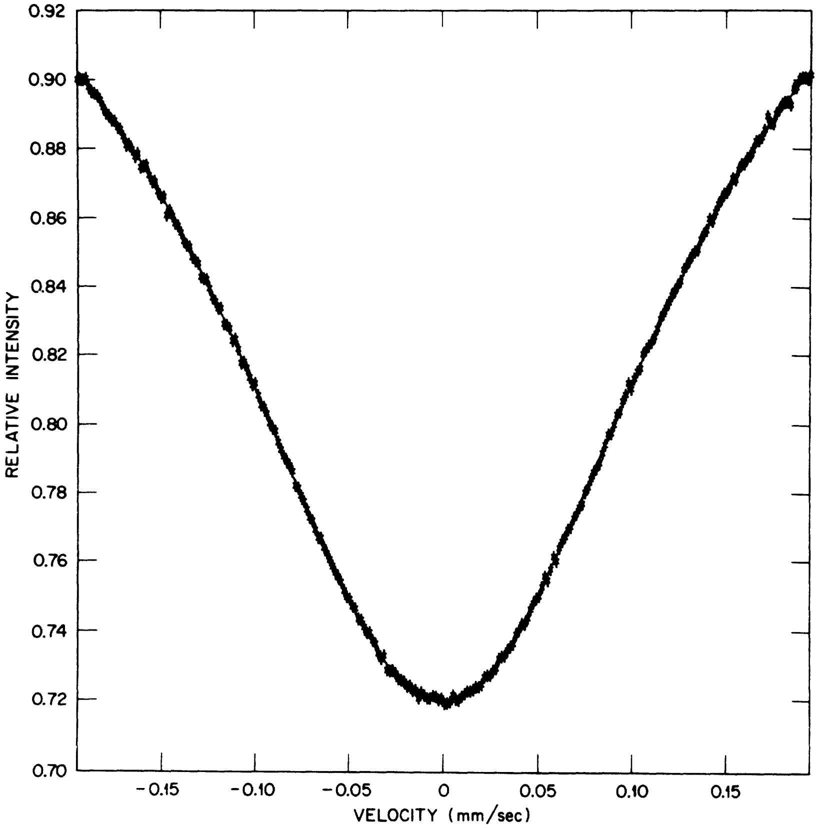

FIG. 2. NGR absorption spectrum for a ^{57}Co in Fe γ source and a pure Fe absorber with a maximum doppler velocity of ±0.2 mm/sec. The solid line drawn through the data points is the computer fit of a theoretical line shape to the data and gives a sensitive measure of any γ-ray frequency shift.

IV. RESULTS

All measured energy shifts of the absorber foils fashioned from the tandem screw and its similar standard, measured with respect to the source were consistent with one another and were different by less than 0.1 ± 0.1 μm/sec. This corresponds to a value of $\Delta E_\gamma / E_\gamma \leq 3 \times 10^{-16}$, and places an upper limit on C due to the Van de Graaff electrostatic effects of 10^{-47}. All measured energy shifts of the absorber foils which enclosed the earth's magnetic flux were also consistent with one another and were different by less than 0.1 ± 0.1 μm/sec. This enclosed flux leads to a value $C \leq 3 \times 10^{-44}$ and is a less stringent limit on the Weyl effect than the electrostatic test.

Our search for the effect predicted by Weyl has yielded negative results. We conclude that the upper limit on the dimensionless constant in Eq. (6) is $|C| < 10^{-47}$. This small limit seems to exclude the mechanism we have discussed as an explanation of the anomalous red shifts of quasistellar objects. Even with the Schwartzmann value of $\phi_e \sim 10^8$ V and a time equal to the lifetime of the universe ($ct \sim 10^{28}$ cm), a value of $C \sim 10^{-43}$ would be required to obtain values of $\Delta\nu/\nu \sim 1$. Our experiments do not, of course, exclude the possibility that C is purely imaginary as suggested in some recent theories.[13]

ACKNOWLEDGMENTS

We would like to thank Professor Shobu Kaneko, Dr. William Gardner, Dr. Abhijit Sen, Professor Bruno Brunelli, Dr. Minal Jhaveri Sen, and Professor K. C. Hines for their help in sending iron foils around the earth, and Dr. C. D. Moak for obtaining the screw from the tandem Van de Graaff. We are also grateful to Pan American Airlines for carrying some of our foils around the earth. We have been helped and encouraged by conversations with our colleagues. Especially valuable suggestions were made by Professor R. D. Present and Dr. O. C. Eldridge.

---

*Research jointly sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation and by the University of Tennessee.

†Work performed as partial fulfillment for the degree of Master of Science at the University of Tennessee.

[1] H. Weyl, Sitzungsber. Preuss. Akad. Wiss. Berl. 465 (1918).

[2] H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity* (Dover, New York, 1952).

[3] The controversy over the interpretation of red shifts is reported Phys. Today 25, 17 (1972).

[4] A. Einstein. This remark appeared in the original appendix of Ref. 1, but not in the reprinted version.

[5] H. C. Arp, Science 151, 1214 (1966).

[6] G. Burbidge, Nature 233, 36 (1971).

[7] F. Hoyle and J. V. Narlikar, Nature 233, 41 (1971).

[8] V. F. Schwartzmann, Astrofizika 6, 309 (1970).

[9] H. Weyl, *Space, Time, Matter* (Dover, New York, 1950).

[10] W. Pauli, *Theory of Relativity* (Pergamon, New York, 1958).

[11] J. E. Tansil, F. E. Obenshain, and G. Czjzek, Phys. Rev. A 6, 2796 (1972).

[12] J. Burton, Oak Ridge National Laboratory Report No. ORNL-4743, 1971 (unpublished), p. 105.

[13] N. H. Cherry, Nuovo Cimento 3B, 183 (1971); 4B, 144 (1971); Lett. Nuovo Cimento 2, 619 (1971); 2, 794 (1971).