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Radiation of Massive Gravitation

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The radiation problem in massive and massless linear Einstein gravitation is analyzed for mass discontinuities in the limit of vanishing graviton mass m. It is found that in this limit: (i) the radiation into the massive modes of helicity ± 2 becomes equal to $\frac{3}{4}$ of the total massless Einstein radiation, (ii) the radiation into the massive modes of helicity ± 1 tends to zero, and (iii) the radiation into the massive helicity-0 mode stays nonzero and is model-dependent. Two models illustrate this: (i) An oscillating point mass radiates for $m \to 0$ into the modes of helicity ± 2 and 0, and its total radiation approaches in this limit the massless Einstein radiation; (ii) a pulsar radiates for $m \to 0$, but only into the helicity-0 mode: Birkhoff's theorem is broken in linear massive gravitation. It is also broken in massive Maxwell theory, but holds in massless Maxwell theory.

I. INTRODUCTION

Recently it was noted[1] that in the linear approximation of Einstein gravitation, a nonzero graviton mass, however small, results in a bending angle of light near the sun's edge which is $\frac{3}{4}$ of Einstein's value. Since experimentally Einstein's prediction has been confirmed to within 10%,[2] one is tempted to conclude that Einstein's theory of general relativity allows no massive neighbors, however small their mass.

An analysis of the free linear gravitational field reveals[3] that here mass discontinuities are absent, canceling in the same subtle way as in electromagnetism.[4]

In this article the radiation problem in massive and massless linear Einstein theory is considered. The interest of such a calculation is that radiation represents the intermediate case between free fields and virtual fields exchanged between two sources (as in the case of light bending).

The result is that there are also mass discontinuities in linear gravitational radiation.

If one scales the coupling constants of both massive and massless linear gravitation such that the Newtonian limit holds, then the radiation which is emitted in the massive case into the modes of helicity ± 2 approaches in the limit of vanishing graviton mass $\frac{3}{4}$ of the total radiation in the massless case. This mass discontinuity $\frac{3}{4}$ has the same origin as the factor $\frac{3}{4}$ in light bending. Moreover, in the massive case the radiation into the helicity modes ± 1 tends to zero in proportion to (graviton mass)^2. These results are to be expected since a massive spin-S ($S=1,2$) helicity-λ particle becomes in the limit of vanishing mass a genuine spin-$|\lambda|$ particle. Hence the helicity modes ± 1 can only couple to a vector source and the only available vector source is $\partial_\mu t_{\mu\nu}$ which is zero. Correspondingly, in electromagnetism the longitudinal fields decouple since in the limit of vanishing photon mass they can only couple to $\partial_\mu j_\mu$ which is also zero. Hence, in linear Einstein gravitation and Maxwell electromagnetism, the maximal helicity modes approach in the limit of vanishing mass those of the massless theory (up to the factor $\frac{3}{4}$ in gravitation), and the modes of helicity $\pm(S-1)$ decouple.

A second mass discontinuity in the radiation of gravitation occurs in the helicity-0 mode. That this mode does not decouple is not astonishing: The scalar source $t_{\mu\mu}$ is in general nonzero. But both angular dependence and intensity of this mode are independent of the other modes and model-dependent. In the massless case on the other hand one can always choose a gauge such that only helicities ± 2 remain.

These general results are illustrated in two models. The first model is a harmonically oscillating point mass. Here the helicity-0 mode carries in the massive case for vanishing graviton mass $\frac{1}{4}$ of the total massless Einstein radiation; hence the total massive radiation in this case approaches in the limit as $m \to 0$ precisely the total massless Einstein result, both in angular dependence and in intensity. The second model is a spherically symmetric pulsating source (pulsar). In this case only the helicity-0 mode carries energy. The Birkhoff theorem[5] asserts that in massless (even nonlinear) Einstein theory the radiation from any spherically symmetric source is zero; hence the Birkhoff theorem is broken in massive linear gravitation. Consequently, detection of gravitational radiation from pulsars would contradict the measurements on light-beam bending.

The mass discontinuities in the helicity-0 mode can be displayed in a particularly simple way in the rest frame of the massive graviton. The massive Einstein fields, having spin 2, must satisfy outside sources $\partial_\mu \psi_{\mu\nu} = \psi_{\mu\mu} = 0$, and hence have in the rest frame only space components with space-trace zero. On the other hand, precisely and only in the case of plane waves, one can choose a gauge for massless Einstein fields such that they satisfy $\partial_\mu \varphi_{\mu\nu} = \varphi_{\mu\mu} = 0$. It follows that in the massive case one must make the fields which propagate from the energy tensor [$\psi^{(0)}_{\mu\nu}$, see Eq. (42)] space-traceless, whereas in the massless case only the transverse components remain. Since the Poynting vector is the same in massive and massless linear gravitation, in contrast to electromagnetism, the calculation of radiation in particular models is simple.

An analysis of massless and massive Maxwell theory reveals that no mass discontinuities occur in the limit of vanishing photon mass. From the usual Maxwell equations, a Birkhoff theorem for massless spin-1 photons follows but it will be shown that for massive photons, the Birkhoff theorem breaks down in the quadrupole limit.

The explanation of the aforementioned mass discontinuity factor $\frac{3}{4}$ is the following. Massive linear gravitation gives an effective source-source interaction of the form $V = TDT$ with

$$V = G^{(\mu)} T^{(1)}_{\rho\sigma}(T^{(2)}_{\rho\sigma} - \tfrac{1}{3}\delta_{\rho\sigma} T^{(2)})/(k^2 + \mu^2)\ ,$$

whereas the usual linear Einstein theory gives

$$V = G^{(0)} T^{(1)}_{\rho\sigma}(T^{(2)}_{\rho\sigma} - \tfrac{1}{2}\delta_{\rho\sigma} T^{(2)})/k^2\ .$$

The Newtonian limit requires $\frac{2}{3}G^{(\mu)} = \frac{1}{2}G^{(0)} =$ Newton's constant, since for static sources $(k^2+\mu^2)^{-1}$ becomes in x space a Yukawa potential. On the other hand, light, having a vanishing trace of its energy tensor ($T^{(2)} = 0$), is scattered in the massive theory over only $\frac{3}{4}$ of Einstein's angle. The factors $\frac{1}{3}$ and $\frac{1}{2}$ in the expressions for TDT are obtained by summing over five and two polarizations of the graviton, respectively.

In order to bring out clearly where in the case of gravitation mass discontinuities are generated, the radiation in massless and massive linear Einstein gravitation is compared with the radiation in massless and massive Maxwell electromagnetism. In Sec. II massive Maxwell theory is briefly treated and a Birkhoff theorem is derived for massless spin-1 fields. In Sec. III the radiation of massive photons is calculated in the two models and in an example the Birkhoff theorem is shown to break down for massive Maxwell fields. In Sec. IV the theory of linear Einstein gravitation to which is added the Fierz-Pauli mass term, is treated and a simple proof for the Birkhoff theorem in linear gravitation is given. In Sec. V the radiation of massive linear gravitation is calculated in two models and the Birkhoff theorem is shown to break down for massive linear Einstein gravitation. Finally, in Sec. VI the results for massless and massive Maxwell and linear Einstein radiation are compared and the conclusions are drawn, some of which have already been given in this introduction.

II. MASSIVE ELECTROMAGNETISM

In this section the Lagrangian field theory is discussed for a massive photon which is coupled to a point charge.

The Lagrangian for a free massive vector meson is unique. If the vector meson is uncharged, the Lagrangian density, as given by Proca,[6a] reads

$$\mathcal{L}^{q,\mathrm{ph}} = \frac{1}{4\pi}(-\tfrac{1}{4}F_{\mu\nu}{}^2 - \tfrac{1}{2}\mu^2 A_\mu{}^2)\ . \tag{1}$$

For comparison with results from ordinary electromagnetism cgs units are used; the photon mass m is given by $\mu = mc/\hbar$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Any Lagrangian inequivalent to Eq. (1), such as one with a term added proportional to $(\partial_\mu A_\mu)^2$, implies the existence of a scalar ghost particle. Upper limits on m follow for instance from the absence of color phenomena in distant eclipsing binaries[7a] and from the laws of Planck and Wien[7b]; the lowest limit to date is obtained by studying the effects that a massive photon would have on the magnetic field of the earth and gives $m \leq 10^{-48}$ g.[7c]

For massless photons ($\mu = 0$), gauge invariance[8] requires that the field A_μ be coupled to a conserved current.[9,10] Conventionally one couples to the electric current which is conserved due to phase invariance[8] of the matter part of the total Lagrangian.

For massive photons, coupling to a nonconserved current can be consistent[11] – it merely implies a relation between the Lorentz condition and current conservation,

$$\mu^2(\partial_\mu A_\mu) = \frac{4\pi}{c}(\partial_\mu j_\mu)\,. \tag{2}$$

In this article, the massive photons will be coupled to the same minimal conserved current as the massless photons.[12] This minimal current is defined by requiring that the source of the photon be that current which is conserved by phase invariance.

The equations of motion for the massive photon are

$$\partial_\mu F_{\mu\nu} - \mu^2 A_\nu = -\frac{4\pi}{c} j_\nu\,, \tag{3}$$

where

$$j_\nu = \delta \mathcal{L}^{(m+\gamma)}/\delta A_\nu , \quad (4)$$

and where the total matter Lagrangian density $\mathcal{L}^{(m+\gamma)}$ is obtained from the matter Lagrangian without photons by the minimal substitution $\partial_\mu \to \partial_\mu - (ie/c)A_\mu$. The corresponding massive Maxwell equations are

$$\vec{b} = \text{curl}\,\vec{A}, \quad \text{curl}\,\vec{e} = -\frac{1}{c}\frac{\partial \vec{b}}{\partial t}, \quad \text{div}\,\vec{e} = 4\pi\rho - \mu^2 A_0,$$

$$\vec{e} = -\vec{\nabla}\phi - \frac{1}{c}\frac{d\vec{A}}{dt}, \quad \text{div}\,\vec{b} = 0, \quad (5)$$

$$\text{curl}\,\vec{b} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{e}}{\partial t} - \mu^2 \vec{A}.$$

From these equations, a Birkhoff theorem[5] for massless spin 1 will be derived. A symmetric source gives $\vec{b} = 0$, hence no radiation is possible. Moreover, $\vec{e}$ depending only on r and t, $\vec{e} = \vec{\nabla}\chi$, hence outside sources $\nabla^2 \vec{e} = 0$, so $\vec{e} = \vec{\alpha}(t)/r$. Finally, $\vec{b} = 0$ gives $d\vec{\alpha}/dt = 0$: The field $\vec{e}$ is static. In the next section, it will be shown in an example that the Birkhoff theorem is broken in the case of massive spin 1.

A point particle with charge e and prescribed[13] trajectory $[\vec{R}(t), R_4(t) \equiv ict]$ gives a source current

$$j_\nu(\vec{r}, t) = e\frac{dR_\nu(t)}{dt}\delta^3(\vec{r} - \vec{R}(t)) . \quad (6)$$

The correct Lorentz behavior is manifest from the representation

$$j_\nu(\vec{r}, t) = \int_{-\infty}^{+\infty} e\frac{dR_\nu(\tau)}{d\tau}\delta^4(x_\mu - R_\mu(\tau))d\tau , \quad (7)$$

where τ is the proper time.

It has been shown in the literature that in electromagnetism the limit $\mu \to 0$ exists and coincides with the case $\mu \equiv 0$. Stückelberg has shown this for the Hamiltonian[14] and Deser for the action.[4] They introduce rescaled canonical variables. This means only that one can say "one photon has an energy $\hbar\omega$." In calculating a classical quantity which does not depend explicitly on the number of photons, like the radiated energy, one needs no canonical formalism. This will be of advantage in the case of gravitation. No redundant field variables (like A_0) are eliminated and the helicity properties of the field components follow directly from their Lorentz behavior.

III. RADIATION OF MASSIVE PHOTONS

In this section in two models the amount of energy is calculated which is radiated into the three helicity modes of the massive photon. Subsequently, corresponding results for massless Maxwell theory are given. The two models are: (i) an electric dipole oscillator and (ii) an electric pulsar.

The symmetric energy tensor for the free massive photon is

$$T^{0,\text{ph}}_{\mu\nu} = \frac{1}{4\pi}[F_{\mu\lambda}F_{\nu\lambda} + \mu^2 A_\mu A_\nu - \delta_{\mu\nu}(\tfrac{1}{4}F_{\rho\sigma}{}^2 + \tfrac{1}{2}\mu^2 A_\rho{}^2)]. \quad (8)$$

The mass term breaks gauge invariance and the total matter part contributes the rest of the total energy tensor such that the latter is conserved. For example a free electron gives

$$T^{0,\text{el}}_{\mu\nu} = \tfrac{1}{4}[(D^*_\nu\bar{\psi})\gamma_\mu\psi - \bar{\psi}\gamma_\mu D_\nu\psi + \mu \longleftrightarrow \nu] + \delta_{\mu\nu}(ie\bar{\psi}A\!\!\!/\psi) \quad (9)$$

with covariant derivative $D_\mu = \partial_\mu - (ie/c)A_\mu$, $e < 0$. The interaction term gives

$$T^{\text{int}}_{\mu\nu} = -\delta_{\mu\nu}(ie\bar{\psi}A\!\!\!/\psi), \quad (10)$$

and $T^{0,\text{el}}_{\mu\nu} + T^{\text{int}}_{\mu\nu}$ is gauge-invariant.

In canonical formalism, the decomposition of the Hamiltonian and Poynting vector for a massive photon proceeds as follows.

The total Hamiltonian density in terms of canonical variables $(-\vec{E}, \vec{A})$ is given by[15]

$$\mathcal{H} = \frac{1}{8\pi}(\vec{E}^2 + \vec{B}^2) + \frac{\mu^2}{8\pi}(\vec{A}^2 + A_0{}^2) - \vec{j}\vec{A} + \mathcal{H}^{0,\text{matter}}, \quad (11)$$

where A_0 is eliminated by Eq. (3),

$$A_0 = \frac{-1}{\mu^2}(\text{div}\vec{E} - 4\pi\rho). \quad (12)$$

The Poynting vector is

$$S_k = cT_{k0} = \frac{c}{4\pi}(\vec{E}\times\vec{B} + \mu^2\vec{A}A_0)_k . \quad (13)$$

Decomposing $\vec{E} = \vec{E}^T + \vec{E}^L$ with $\vec{E}^L = \vec{\nabla}E^L$ and $\text{div}\vec{E}^T = 0$ and similarly decomposing $\vec{A}$ shows that the mass term in Eq. (13) describes pure longitudinal radiation [use Eq. (12)] while the term $\vec{E}\times\vec{B}$ describes pure transverse radiation ($\vec{E}^L\times\vec{B}$ is parallel to the surface of a large sphere around the source).

As argued before, the radiation problem will be considered without canonical formalism. In this case one has for the Poynting vector

$$S_k = \frac{c}{4\pi}(F_{k\lambda}F_{0\lambda} + \mu^2 A_k A_0) . \quad (14)$$

Boosting along the radius $\vec{r}$ to the massive photon rest system shows that the longitudinal and scalar components of A_μ have helicity 0 while the transverse components have helicity ±1. Hence it again follows that the first term in Eq. (14) has helicity ±1, while the second has helicity 0.

The Green's function for massive photons

$$(\Box-\mu^2)G(x-x')=-\frac{1}{4\pi}\delta^4(x-x') \qquad (15)$$

with a harmonic source with frequency ω is

$$G(x-x')=\frac{\delta(t-t'-|\vec{r}-\vec{r}'|/c')}{|\vec{r}-\vec{r}'|}, \qquad (16)$$

where the velocity c' is given by the group velocity corresponding to ω,

$$c'=(kc/\omega)c\,,\quad \omega^2=(kc)^2+(\mu c)^2\,. \qquad (17)$$

For conserved currents, one has with Eqs. (2) and (3)

$$(\Box-\mu^2)A_\mu=-\frac{4\pi}{c}j_\mu \qquad (18)$$

with solution

$$A_\mu(x)=\frac{1}{c}\int G(x-x')j_\mu(x')d^4x' \qquad (19)$$

since indeed $\partial_\mu A_\mu=0$. Current conservation gives for large r in the dipole limit

$$\vec{A}(\vec{r},t)=\frac{1}{cr}\left(\frac{1}{c}\frac{d}{dt}\int\vec{r}'j_0(\vec{r}',t'=t-r/c')d\vec{r}\right), \qquad (20)$$

where t' is the retarded time, and A_0 follows from the Lorentz conditions

$$A_0(\vec{r},t)=\left(\frac{kc}{\omega}\right)\frac{\vec{r}\cdot\vec{A}}{r}\,. \qquad (21)$$

The Poynting vector in Eq. (14) is diagonal in helicities

$$S_k=\frac{c}{4\pi}\left[\tfrac{1}{2}F_{k,1+i2}(F_{0,1+i2})^*+\tfrac{1}{2}F_{k,1-i2}(F_{0,1-i2})^*+\mu^2A_kA_0\right] \qquad (22)$$

in an obvious notation.

The *electric dipole oscillator* radiates into the transverse modes (p is the dipole moment)

$$\frac{d^2E^{\mathrm{hel}(+1)}}{d\Omega dt}=\frac{d^2E^{\mathrm{hel}(-1)}}{d\Omega dt}=\frac{\omega^2}{2}\left(\frac{kc}{\omega}\frac{\omega^2p^2}{8\pi c^3}\right)\sin^2\theta \quad \text{(massive case)}, \qquad (23)$$

while the longitudinal mode carries an amount of energy

$$\frac{d^2E^{\mathrm{hel}(0)}}{d\Omega dt}=(\mu c)^2\left(\frac{kc}{\omega}\frac{\omega^2p^2}{8\pi c^3}\right)\cos^2\theta \quad \text{(massive case)}. \qquad (24)$$

The total rate of massive radiation is

$$\frac{dE}{dt}=\frac{\omega^4p^2}{3\pi c^3}\left[1-\frac{3}{8}\left(\frac{\mu c}{\omega}\right)^4+O(\mu^6)\right] \quad \text{(massive case)}. \qquad (25)$$

The absence of terms of order μ^2 is no accident; in the dipole limit the total rate of massive radiation never contains terms proportional to μ^2. On the other hand, it will be shown in the next example that the total rate of massive radiation can contain terms proportional to μ^2 in the quadrupole limit. For massless photons one can choose a gauge such that the radial component of $\vec{A}$ vanishes, which at the same time removes A_0. The remainder is then transverse and gives for the electric dipole oscillator

$$\frac{d^2E}{d\Omega dt}=\omega^2\left(\frac{\omega^2p^2}{8\pi c^3}\right)\sin^2\theta \quad \text{(massless case)} \qquad (26)$$

and

$$\frac{dE}{dt}=\frac{\omega^4p^2}{3\pi c^3} \quad \text{(massless case)}\,. \qquad (27)$$

The *electric pulsar* is defined as a spherically symmetric charge cloud which pulsates harmonically with amplitude R and frequency ω with total charge Q. It never radiates in massless Maxwell theory [see after Eq. (5)], nor does it radiate in the dipole limit in massive Maxwell theory [in Eq. (20) its dipole moment vanishes], but it radiates in the quadrupole limit in massive Maxwell theory. Expanding the Liénard-Wiechert potentials in powers of $1/c$, one finds spherically symmetric emission into the helicity-0 mode with total rate

$$\frac{dE}{dt}=\frac{(\mu R)^2}{6}\frac{\omega^4P^2}{3\pi c^3}\quad (P\equiv QR)\,. \qquad (28)$$

Hence here terms linear in μ^2 do not vanish and at the same time Eq. (28) proves that the Birkhoff theorem for massive spin-1 fields is broken.

IV. MASSIVE GRAVITATION

In this section the Lagrangian field theory is discussed for a massive graviton which is coupled to a point mass.

The action for a free massive uncharged tensor meson is unique and the corresponding Lagrangian density is given by

$$\mathcal{L}^{0,\mathrm{gr}}=-\tfrac{1}{4}\left[h_{\mu\nu,\lambda}{}^2-2h_\mu{}^2+2h_\mu h_{,\mu}-h_{,\mu}{}^2+\mu^2(h_{\mu\nu}{}^2-h^2)\right], \qquad (29)$$

where $h_\mu\equiv h_{\mu\nu,\nu}$, $h\equiv h_{\mu\mu}$, and $h_{\mu\nu}$ must be symmetric *a priori*. In contrast to electromagnetism Eq. (29) does not follow from the requirement that the free field contains only spin 2 but can be obtained by requiring that the propagator is given by the spin-2 projection operator.[16] Nor does the existence of lower-spin parts in the most general Lagrangian for a tensor meson always imply the existence of a ghost, again in contrast to electromagnetism. However, the linear part of the Einstein action gives Eq. (29) with $\mu=0$, and the only mass term which does not introduce ghosts is the Fierz-Pauli[11] mass term in Eq. (29). The correct factor $-\frac{1}{4}$ in Eq. (29) is determined by the canonical form "$\frac{1}{2}p\dot{q}$" of the kinetic energy[17] and is needed for the Rosenfeld prescription for the energy tensor.[18a]

For massless gravitation gauge invariance requires that the field $h_{\mu\nu}$ be coupled to a conserved tensor and one couples conventionally to the energy tensor of matter[10] which is covariantly conserved by phase invariance (Lorentz invariance). Strictly, the linear theory of gravitation cannot consistently be coupled to a dynamical matter tensor (although it is a useful approximation) and only prescribed external functions are permitted.[19]

Massive gravitons can be coupled to a nonconserved tensor – it merely implies

$$\mu^2\partial_\mu(h_{\mu\nu}-\delta_{\mu\nu}h)=\partial_\mu t_{\mu\nu}\,. \tag{30}$$

As in the case of electromagnetism, the massive particles will be coupled to the same source as the massless particles. Then the requirement that the source of the field $h_{\mu\nu}$ be the energy tensor of the nongravitational world leads to universal minimal coupling. Hence, as in electromagnetism, the mass term does not break universal minimal coupling nor source conservation.

It is possible to derive by means of the Rosenfeld prescription the energy tensor of a point particle. There exists however a powerful (but little known) theorem by Tulczyjew[20] which proves that the only covariantly conserved symmetric point tensor which does not contain derivatives of δ functions is given by

$$t_{\mu\nu}(x)=\frac{M}{\sqrt{-g}}\frac{dR_\mu(t)}{dt}\frac{dR_\nu(t)}{dt}\frac{dt}{d\tau}\delta^3(\vec{r}-\vec{R}(t))\,, \tag{31}$$

where τ is the proper time, $\vec{R}(t)$ the trajectory of the particle $[R_4(t)\equiv ict]$, and $-g$ is the determinant of the metric $g_{\mu\nu}$. Its correct Lorentz behavior follows from a representation like Eq. (7) and its correct general relativistic behavior is manifest since $\delta^4(x)$ is a scalar density like $\sqrt{-g}$. In the aforementioned linear approximation one has

$$\mathcal{L}^{\rm int}=\tfrac{1}{2}(2\kappa)^{1/2}h_{\mu\nu}t_{\mu\nu}\,,\quad \kappa=\frac{8\pi G^{(\mu,0)}}{c^4}\,, \tag{32}$$

where $G^{(\mu)}$ and $G^{(0)}$ are the coupling constants of the massive and massless theory.

The equations of motion for the symmetric field $h_{\mu\nu}$ follow from Eqs. (29) and (32),

$$2G^L_{\mu\nu}+\mu^2(h_{\mu\nu}-\delta_{\mu\nu}h)=(2\kappa)^{1/2}t_{\mu\nu}\,, \tag{33}$$

where $G^L_{\mu\nu}$ is the linear part of the Einstein tensor

$$2G^L_{\mu\nu}\equiv-\Box h_{\mu\nu}-(\partial_\mu\partial_\nu-\delta_{\mu\nu}\Box)h+h_{\mu,\nu}+h_{\nu,\mu}-\delta_{\mu\nu}h_{\alpha,\alpha} \tag{34}$$

satisfying identically (not by means of the equations of motion)

$$\partial_\mu G^L_{\mu\nu}=0\,,\quad G^L_{\mu\mu}=\partial_\mu(h_\mu-h_{,\mu})\,. \tag{35}$$

Inserting $\partial_\mu G^L_{\mu\nu}$ into Eq. (33) leads with $\partial_\mu t_{\mu\nu}=0$ to

$$\partial_\mu(h_{\mu\nu}-\delta_{\mu\nu}h)=0\,, \tag{36}$$

and Eq. (36) together with the expression in Eq. (35) for $G^L_{\mu\mu}$ inserted into Eq. (33) leads to the fifth constraint – leaving 5 degrees of freedom,

$$-3\mu^2h=(2\kappa)^{1/2}t\,. \tag{37}$$

Inserting Eqs. (37) and (36) into Eq. (33) finally gives[21]

$$(\Box-\mu^2)h_{\mu\nu}=(2\kappa)^{1/2}\left(-t_{\mu\nu}+\tfrac{1}{3}\delta_{\mu\nu}t-\frac{\partial_\mu\partial_\nu}{3\mu^2}t\right)\,. \tag{38}$$

It is at this point that the origin of the mass discontinuities lies. For the relation $\partial_\mu G^L_{\mu\nu}$ is in the massless case no constraint on the field $h_{\mu\nu}$, but on the source ($\partial_\mu t_{\mu\nu}=0$). Consequently, the second relation in Eq. (35) stays an equation of motion instead of leading to a constraint. It is not possible to choose a gauge such that $G^L_{\mu\mu}$ vanishes in the massless case in order to obtain still a relation which resembles Eq. (37), since $G^L_{\mu\mu}$ is gauge-invariant [it is the linear curvature, and equal to $\frac{1}{2}(2\kappa)^{1/2}t$ in the massless case]. Of course, the correct number of degrees of freedom (2) for massless graviton follows from gauge invariance; as in the case of electromagnetism, twice as many degrees of freedom (8 here) are eliminated by the number of gauge functions (4 here).

Defining the conserved field $\psi_{\mu\nu}$,

$$\psi_{\mu\nu}=h_{\mu\nu}-\delta_{\mu\nu}h\quad\text{(massive case)}, \tag{39}$$

one has with Eq. (38) in the massive case

$$\begin{aligned}&(\Box-\mu^2)\psi_{\mu\nu}=-(2\kappa)^{1/2}\left(t_{\mu\nu}+\frac{\partial_\mu\partial_\nu-\delta_{\mu\nu}\Box}{3\mu^2}t\right)\,,\\ &\psi_{\mu\mu}=[(2\kappa)^{1/2}/\mu^2]t\,,\quad \partial_\mu\psi_{\mu\nu}=0\,.\end{aligned} \tag{40}$$

In terms of the Green's function $G(x-x')$ in Eqs. (15) and (16), one has

$$\psi_{\mu\nu}=\psi^{(0)}_{\mu\nu}+\psi^{(1)}_{\mu\nu}\,, \tag{41}$$

where[22]

$$\psi_{\mu\nu}^{(0)}=\frac{(2\kappa)^{1/2}}{4\pi}\int D(x-x')t_{\mu\nu}(x')d^4x' \tag{42}$$

and outside sources

$$\psi_{\mu\nu}^{(1)}=\frac{(2\kappa)^{1/2}}{4\pi}\left(\frac{\partial_\mu\partial_\nu-\delta_{\mu\nu}\mu^2}{3\mu^2}\right)\int D(x-x')t(x')d^4x' \tag{43}$$

since indeed $\psi_{\mu\nu}$ satisfies the constraints in Eq. (40). Linear massless Einstein theory on the other hand gives for

$$\varphi_{\mu\nu}=h_{\mu\nu}-\tfrac{1}{2}\delta_{\mu\nu}h \tag{44}$$

in the gauge $\partial_\mu\varphi_{\mu\nu}=0$ the following equation[22]:

$$\Box\varphi_{\mu\nu}=-(2\kappa)^{1/2}t_{\mu\nu}\,,\quad \partial_\mu\varphi_{\mu\nu}=0\,. \tag{45}$$

Hence, $\varphi_{\mu\nu}=\psi_{\mu\nu}^{(0)}(\mu\to 0)$. On the other hand $\psi_{\mu\nu}^{(1)}$ blows up for $\mu\to 0$, but in the Poynting vector it will give a finite but nonzero result.

If one would have taken instead of the Fierz-Pauli mass term the following mass term in the Lagrangian,

$$-\tfrac{1}{4}\mu^2({h_{\mu\nu}}^2-\tfrac{1}{2}h^2)\,, \tag{46}$$

then the massive and massless fields would have limited to each other in the limit $\mu\to 0$. Moreover, also in this case the Poynting vectors are equal for harmonic sources (see the next section) and $\psi_\mu=\varphi_\mu=0$; however, since in that case $\psi\neq 0$ in the massive theory, there is a scalar particle present. It is a ghost and cancels precisely the radiation into the spin-2, helicity-0 mode in the limit $\mu\to 0$ (for example it has the same angular dependence of radiation as the helicity-0 mode). The Fierz-Pauli mass term on the other hand leaves the helicity-0 mode, and since the latter tends for vanishing graviton mass to a scalar particle, one ends up with a Brans-Dicke theory of gravitation.

This section will be concluded by a simple proof of the Birkhoff theorem for linear massless gravitation. Spherical symmetry implies (i) that $h_{ij}=\delta_{ij}H_1+\partial_i\partial_jH_2$, $h_{0i}=\partial_iH_3$, and $h_{00}=H_0$, where $H_i=H_i(r,t)$ and (ii) that there are only two symmetry-conserving gauge functions, $\chi_\mu=(x^i\chi,\chi_0)$. Choosing a gauge such that H_2 and H_3 vanish, one finds from the field equations $H_{1,0i}=0$, $\nabla^2(H_0-H_1)+3H_{1,00}=0$, and $(H_0-H_1)_{,ij}=0$. Hence H_1 is stationary, $H_0-H_1=A(t)/r$, and $H_0=H_1$, respectively. So, in this gauge the fields are all stationary and cannot radiate.

V. RADIATION OF MASSIVE GRAVITONS

In this section, the amount of energy will be calculated which is radiated into the five helicity modes of the massive graviton by (i) a harmonically oscillating point mass M (gravitational dipole)[23] and (ii) a spherically pulsating mass distribution (pulsar or gravitational monopole). Finally, corresponding results for linear Einstein theory will be derived.

A symmetric and conserved energy tensor can be obtained by adding to the massless energy tensor[18b] the mass term which follows from the Rosenfeld prescription. One finds for the energy tensor of the free massive graviton field

$$T_{\mu\nu}^{0,\mathrm{gr}}=\tfrac{1}{2}[h_{\alpha\beta,\mu}h_{\alpha\beta,\nu}+2h_{\mu\sigma,\tau}h_{\nu\sigma,\tau}-h_{,\mu}h_{,\nu}-h_{\sigma\mu,\nu}h_\sigma-h_{\sigma\nu,\mu}h_\sigma+2\mu^2(h_{\mu\sigma}h_{\nu\sigma}-h_{\mu\nu}h)]+\delta_{\mu\nu}\mathcal{L}^{0,\mathrm{gr}} \tag{47}$$

with $\mathcal{L}^{0,\mathrm{gr}}$ given in Eq. (29). There are other symmetric conserved energy tensors. For example, the Rosenfeld prescription applied on the full Eq. (29) gives a tensor with second derivatives of the fields. All such tensors differ however by superpotential terms[3] and, when averaged over the volume of a wavelength, give identical results for the rate of emission.

The Poynting vector S follows from Eq. (47); it is $-icT_{k4}$. Outside sources, the massive field $\psi_{\mu\nu}\equiv h_{\mu\nu}-\delta_{\mu\nu}h$ satisfies

$$\partial_\mu\psi_{\mu\nu}=0\,,\quad \psi_{\mu\mu}=0 \text{ (constraints, massive case)}. \tag{48}$$

On the other hand, far away from the scattering region the emitted fields become plane waves, and precisely in this case one can choose a gauge such that[18c] locally the same relations hold for massless fields. Denoting from now on these regauged massless fields by $\varphi'_{\mu\nu}$, one has

$$\partial_\mu\varphi'_{\mu\nu}=0\,,\quad \varphi'_{\mu\mu}=0 \text{ (gauge, massless case)}. \tag{49}$$

Since our sources are harmonic, the Klein-Gordon equation for the fields leaves, together with the constraint and gauge relations in Eqs. (48) and (49), the same Poynting vector for massive fields in terms of $\psi_{\mu\nu}$ and for massless fields in terms of $\varphi'_{\mu\nu}$,

$$S_k=-\tfrac{1}{2}\chi_{\alpha\beta,k}\chi_{\alpha\beta,0}\,,\quad \chi_{\alpha\beta}=\psi_{\alpha\beta}\text{ or }\varphi'_{\alpha\beta}\,. \tag{50}$$

This result is identical to that in Ref. 24a after choosing a gauge there such that $\varphi'_{\mu\mu}=0$ as well. It follows from the contraction over α and β that this Poynting vector is diagonal in helicities.

It is now clear from Eq. (50) that the only difference in emission of radiation between massive and massless gravitational fields comes from the different way in which $\psi_{\mu\mu}=0$ and $\varphi'_{\mu\mu}=0$ is obtained. From $\partial_\mu\psi_{\mu\nu}=0$, it follows that in the rest frame of the massive graviton only the space components of $\psi_{\mu\nu}$ can be different from zero. The absence of lower spin moreover requires that $\sum^3_{i=1}\psi_{ii}=0$; and this is precisely what $\psi^{(1)}_{\mu\nu}$ [see Eq. (43)] does:

$$\psi^{(1)}_{ij}=-\tfrac{1}{3}\delta_{ij}\sum_{k=1}^{3}\psi^{(0)}_{kk}$$

in the graviton rest system and the field $\psi^{(1)}$ eliminates a nonghost spin-0 particle from $\psi^{(0)}_{\mu\nu}$. In the limit of vanishing graviton mass, however, the spin-2 helicity-0 component becomes a nonghost spin-0 particle leading to a tensor-scalar mixing of a Brans-Dicke type. On the other hand, in the massless case the gauge $\partial_\mu\varphi'_{\mu\nu}=\varphi'_{\mu\mu}=0$ leaves only the regauged transverse fields $\varphi'_{11}=-\varphi'_{22}=\frac{1}{2}(\varphi_{11}-\varphi_{22})$; $\varphi'_{12}=\varphi_{12}$ and $\varphi_{\mu\nu}=\psi^{(0)}_{\mu\nu}(\mu\to 0)$.

Hence $\varphi'_{\mu\nu}$ and $\psi_{\mu\nu}$ are calculated directly from $\psi^{(0)}_{\mu\nu}$ of Eq. (42) in the graviton rest frame. For conserved sources $t_{\mu\nu}$ one has[24b] in the quadrupole limit[25]

$$\int t_{ij}(x)d_3x=\frac{1}{2c^2}\frac{\partial^2}{\partial t^2}\int x^ix^jt_{00}(x)d_3x \qquad (51)$$

leading far away from the source to

$$\psi^{(0)}_{ij}=\frac{(2\kappa)^{1/2}}{4\pi r}\left(\frac{1}{2c^2}\frac{\partial^2}{\partial t^2}\int x^ix^jt_{00}(\vec{r},t-r/c)d_3x\right). \qquad (52)$$

The Lorentz condition then gives

$$k_i\psi^{(0)}_{ij}=\frac{\omega}{c}\psi^{(0)}_{0j}\,,\quad k_i\psi^{(0)}_{i0}=\frac{\omega}{c}\psi^{(0)}_{00}\,. \qquad (53)$$

The same relations hold for $\varphi_{\mu\nu}$. The relation between $\vec{k}=\vec{k}\cdot\vec{r}/r$ and ω is given by the Klein-Gordon equation[26]

$$\omega^2=(kc)^2+(\tfrac{1}{2}\mu c)^2\,. \qquad (54)$$

With the Poynting vector of Eq. (50), the fields $\psi^{(0)}_{ij}$ of Eqs. (52) and (53), the energy tensor of Eq. (31), and the above-derived prescription how to obtain in the graviton rest frame $\psi_{\mu\nu}$ and $\varphi'_{\mu\nu}$ from $\psi^{(0)}_{\mu\nu}$, the radiation in the two models will now be calculated.

The *gravitational dipole oscillator* has a trajectory $\vec{R}(t)=\vec{R}_0\sin\omega t$ with $\vec{R}_0$ along the z axis and a mass M. One finds for the massive field $\psi^{(0)}_{\mu\nu}$

$$\psi^{(0)}_{33}=\left(\frac{-(2\kappa)^{1/2}(MR_0{}^2)\omega^2}{4\pi r}\right)\cos[2(kr-\omega t)]\,,$$

$$\psi^{(0)}_{03}=\psi^{(0)}_{30}=\left(\frac{kc}{\omega}\cos\theta\right)\psi^{(0)}_{33}\,, \qquad (55)$$

$$\psi^{(0)}_{00}=\left(\frac{kc}{\omega}\cos\theta\right)^2\psi^{(0)}_{33}\,.$$

In the graviton rest frame with Cartesian axes in the plane $(\vec{R}_0,\vec{r})$, perpendicular to $\vec{r}$ and $\vec{R}_0$, and along $\vec{r}$, one has in matrix notation

$$\psi^{(0)}_{\mu\nu}=\begin{pmatrix}\sin^2\theta & 0 & (\mu/4\omega)\sin 2\theta & 0\\ 0&0&0&0\\ (\mu/4\omega)\sin 2\theta & 0 & (\mu^2/4\omega^2)\cos^2\theta & 0\\ 0&0&0&0\end{pmatrix}\psi^{(0)}_{33}\,, \qquad (56)$$

while

$$\psi^{(1)}_{\mu\nu}=\begin{pmatrix}-\frac{1}{3}&0&0&0\\0&-\frac{1}{3}&0&0\\0&0&-\frac{1}{3}&0\\0&0&0&0\end{pmatrix}\left(\sin^2\theta+\frac{\mu^2}{4\omega^2}\cos^2\theta\right)\psi^{(0)}_{33}\,. \qquad (57)$$

Indeed, $\psi^{(1)}_{\mu\nu}$ eliminates the space trace of $\psi^{(0)}_{\mu\nu}$. The disentangling of the five helicity components is in this frame reduced to ordinary quantum mechanics for a spin-2 particle. For $\varphi'_{\mu\nu}$ one finds

$$\varphi'_{\mu\nu}=\begin{pmatrix}\frac{1}{2}\sin^2\theta&0&0&0\\0&-\frac{1}{2}\sin^2\theta&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\psi^{(0)}_{33}\,. \qquad (58)$$

Inserting $\varphi'_{\mu\nu}$ and $\psi_{\mu\nu}=\psi^{(0)}_{\mu\nu}+\psi^{(1)}_{\mu\nu}$ into the Poynting vector, boosting back to the laboratory frame, and putting $\frac{2}{3}G^{(\mu)}=\frac{1}{2}G^{(0)}=\gamma=$ Newton's constant in Eq. (32) (see Introduction), one has with $I\equiv MR_0{}^2$

$$\frac{d^2E^{\text{hel}\pm 2}}{d\Omega dt}=\frac{3}{4}\left(\frac{kc}{\omega}\right)\left(\frac{\gamma\omega^6I^2}{2\pi c^5}\right)\sin^4\theta\,,$$

$$\frac{d^2E^{\text{hel}\pm 1}}{d\Omega dt}=\frac{3}{4}\left(\frac{\mu^2}{4\omega^2}\frac{kc}{\omega}\right)\frac{\gamma\omega^6I^2}{2\pi c^5}\sin^2 2\theta\,,$$

(massive case) (59)

$$\frac{d^2E^{\text{hel}\,0}}{d\Omega dt}=\frac{3}{4}\left(\frac{1}{3}\frac{kc}{\omega}\right)\left(\frac{\gamma\omega^6I^2}{2\pi c^5}\right)\left(\sin^2\theta-\frac{\mu^2}{2\omega^2}\cos^2\theta\right)^2.$$

The positive and negative helicities radiate equally.

The radiation in the massless case is

$$\frac{d^2E}{d\Omega dt}=\frac{\gamma\omega^6I^2}{2\pi c^5}\sin^4\theta \quad \text{(massless case)} \qquad (60)$$

and the differential rate of massive radiation approaches the differential massless rate while terms linear in μ^2 are absent in the total rate

$$\frac{dE}{dt}=\frac{16\gamma\omega^6I^2}{15c^5}+O(\mu^4) \qquad (61)$$

using

$$\langle \sin^4\theta \rangle = \tfrac{8}{15}\,, \quad \langle \sin^2\theta \cos^2\theta \rangle = \tfrac{2}{15}\,, \quad \langle \cos^4\theta \rangle = \tfrac{1}{5}\;.$$

The *gravitational pulsar* has a spherically symmetric harmonically pulsating mass distribution. Let the inertial tensor be $I_{ij} = \delta_{ij} I$. Then, after boosting to the graviton rest frame

$$\psi^{(0)}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & (\mu c/2\omega)^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \left(\frac{(2\kappa)^{1/2}\omega^2 I}{4\pi r} \right) \cos(\mu t)\;. \tag{62}$$

Hence in this system $\psi_{\mu\nu}$ tends for small graviton mass to

$$\psi_{\mu\nu}(\mu \to 0) = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \left(\frac{(2\kappa)^{1/2}\omega^2 I}{4\pi r} \right) \cos(\mu t)\,, \tag{63}$$

while $\varphi'_{\mu\nu} = 0$. The latter result is an illustration of the Birkhoff theorem which says that in general relativity or its linear approximation no spherically symmetric source can radiate. The radiation in the massive case into the helicity-0 mode is isotropic and has a total rate

$$\frac{dE^{\text{hel}\,0}}{dt} = \frac{32\gamma^2\omega^6 I^2}{3c^5}\left[1 - \left(\frac{\mu c}{2\omega}\right)^2\right]^2\;. \tag{64}$$

Hence, in this case the total rate contains terms linear in μ^2.

This concludes the calculation in the two models. In general, the rate of massive radiation is always and in all directions greater than the corresponding rate in the massless theory. This follows from the fact that any two-dimensional projection of the three-dimensional inertial tensor is necessarily semi-positive-definite. For one-dimensional sources (dipole oscillator) the lower bound (equality of both rates) is reached.

VI. CONCLUSIONS

The radiation of massive linear Einstein gravitational energy differs by a factor $\frac{3}{4}$ in the modes of helicity ± 2 from the usual (massless) Einstein results and has an independent degree of freedom for the helicity-0 mode. In particular:

(i) The radiation into the massive modes of helicity ± 2 approaches, in the limit as $m \to 0$, $\frac{3}{4}$ of the total massless radiation. The massless fields can be taken in a gauge such that they have only helicity ± 2.

(ii) The radiation into the massive modes of helicity ± 1 tends to zero in proportion to the square of the graviton mass, which reflects the property of a spin-2, helicity ± 1 massive particle that it becomes for vanishing graviton mass equal to a spin-1 particle which can only couple to $\partial_\mu t_{\mu\nu} = 0$.

(iii) The radiation into the massive helicity-0 mode does not tend to zero for vanishing graviton mass. The reason is that the spin-2 helicity-0 graviton becomes for vanishing mass a genuine spin-0 particle which couples to $t_{\mu\mu} \neq 0$, thus leading to a Brans-Dicke type of gravitation.

(iv) The constraints on the massive fields $(\partial_\mu \psi_{\mu\nu} = \psi_{\mu\mu} = 0)$ leave in the massive graviton rest frame only the space components ψ_{ij} with vanishing space trace, whereas the gauge for the massless fields, leading locally to $\partial_\mu \varphi'_{\mu\nu} = \varphi'_{\mu\mu}$, leaves only the transverse components.

(v) The Birkhoff theorem is shown in the example of the pulsar to be broken in massive linear gravitation already in the lowest possible approximation (quadrupole limit), even for vanishing graviton mass. Experimental detection of radiation from pulsars would contradict the light-bending experiments.

(vi) The total rate of massive gravitational radiation can (pulsar) have terms linear in μ^2, but need not have such terms (gravitational dipole oscillator).

(vii) The total rate of radiation in massive linear gravitation is always greater than or equal to the corresponding rate of massless radiation. The lower limit is reached in the case of the dipole oscillator.

(viii) The Poynting vectors in massive and massless theory for harmonic sources are equal and diagonal in helicities.

The radiation of massive Maxwell fields on the other hand limits smoothly to the radiation of usual (massless) Maxwell fields. In particular:

(i) The radiation into the massive transverse modes (of helicity ± 1) smoothly approaches the total massless radiation in the limit as $m \to 0$. A gauge can be chosen for the massless field such that only components of helicity ± 1 are present.

(ii) The longitudinal radiation is damped in proportion to the square of the photon mass, which reflects the property of a spin-1 helicity-0 massive particle that it becomes for vanishing mass a genuine spin-0 particle which can only couple to $\partial_\mu j_\mu = 0$.

(iii) A Birkhoff theorem for massless spin-1 particles holds; hence a spherically symmetric source cannot radiate massless photons. On the other hand, an electric pulsar does emit massive radiation in the quadrupole limit.

(iv) In the dipole limit, the total rate of massive radiation never contains terms proportional to μ^2, due to the presence of a mass term in the Poynting vector. This was illustrated in the case of the dipole oscillator.

(v) The Poynting vector is diagonal in helicities.

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