Charge Correlations in High-Energy Multiplicity Processes

P. Rotelli
International Atomic Energy Agency,
and United Nations Educational Scientific and Cultural Organization,
International Centre for Theoretical Physics, Miramare-Trieste, Italy
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It is suggested that a fruitful way of analyzing high-energy multiparticle processes is to search for charge correlations. Certain models predict characteristic patterns independent of detailed dynamical considerations.

I. INTRODUCTION

A great deal of thought has been given in recent years to how best to analyze and study high-multiplicity processes. The reason, of course, is the new range of higher machine energies available or soon to be available to the experimental physicist. To a large extent this problem is determined, or at least strongly constrained, by experimental technology. Nevertheless, some theoretical guidance may be given on which variables (such as rapidity, etc.) are best used. Of particular value in the comparison of theoretical models with experiment are momentum correlations in two- or more-particle inclusive reactions and the probability moments of various multiplicities (charged, total, etc.).

We wish in this paper to advocate the value of another kind of correlation. Consider a scattering process in the center-of-mass frame with total energy $\sqrt{s}$. For each outgoing particle define its Feynman variable[1] x by

$$x=2q_{\parallel}/\sqrt{s}\ , \tag{1}$$

where $q_{\parallel}$ is the longitudinal momentum of the particle. Then label and order these parameters such that $x_1 \geqslant x_2 \geqslant \cdots \geqslant x_n$ (n being the number of outgoing particles). Ignoring those events in which (e.g., because of the experimental accuracy of the momentum resolution) some ambiguity in the ordering of the x's may exist; i.e., assuming that

$$x_1 > x_2 > \cdots > x_n\ , \tag{2}$$

we can associate with each outgoing particle an integer r which labels its x_r and hence its momentum q_r. It may be shown that in models such as the multi-Regge[2] (MRM) and multiperipheral[3] (MPM) the asymptotic ($s \to \infty$) probability of a particular charge (and in general with particular internal quantum numbers) appearing in the rth position is a function of r and the charge of its nearest neighbors. For example, in the simple MRM which we shall describe in the following sections, certain charge patterns are asymptotically "forbidden," where by charge pattern we mean an ordering of particle charges by their index r. Alternatively, in models such as the statistical models[4] (SM) and the uncorrelated jet model[5] (UJM) where by ansatz no charge correlations of the above kind exist, the probability of seeing a forbidden event can be calculated explicitly.

Most of the results we shall quote are applicable also to the charged particles alone and thus this type of analysis is suitable for high-energy bubble-chamber experiments.

In Sec. II we shall present some kinematics of the MRM. In Sec. III the charge correlations of some particularly simple MRM are presented and the question of Regge cuts discussed. We conclude in Sec. IV with a study of the consequences of the uncorrelated ansatz.

II. MRM KINEMATICS

Consider the general multi-Regge diagram shown in Fig. 1. The incoming momenta are labeled by p_1 and p_2 and we shall always be working in the center-of-mass frame, $\vec{p}_1=-\vec{p}_2$. The usual kinematic quantities are defined by

$$\begin{aligned} &s=(p_1+p_2)^2, \qquad p_1^{\,2}=M_1^{\,2} \\ &s_i=(q_i+q_{i+1})^2, \quad p_2^{\,2}=M_2^{\,2} \\ &t_i=(p_1-q_1\cdots-q_i)^2, \quad q_i^{\,2}=m_i^{\,2}, \\ &\qquad\qquad i=1,\ldots,n\ . \end{aligned} \tag{3}$$

Following the procedure of Halliday and Saunders[6] we now introduce the Sudakov variables[7] $\{\alpha, \beta, \kappa\}$ by

$$q_i=\alpha_i p_1'+\beta_i p_2'+\kappa_i\ , \tag{4}$$

where

$$p_1'=p_1-p_2(M_1^{\,2}/s)\ , \tag{5}$$

$$p_2'=p_2-p_1(M_2^{\,2}/s) \tag{6}$$

$[p_1'^2 \sim p_2'^2 \sim M_1^{\,2}(M_2^{\,2}/s)$ as $s\to\infty]$, and κ_i are two-component vectors orthogonal to $\vec{p}_1$, i.e., the trans-

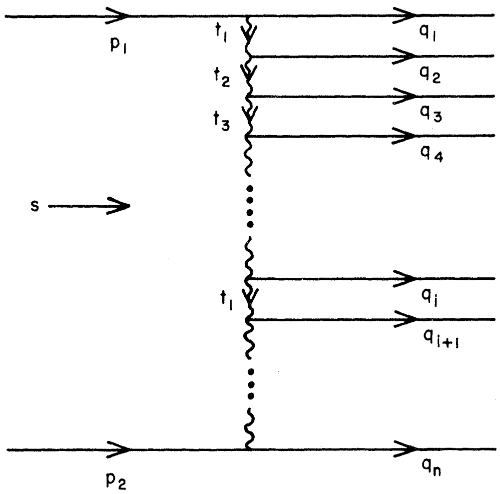

FIG. 1. Standard MRM graph.

verse components of $\vec{q}_i$.

It may now be shown[6] that, because of the multi-Regge restrictions on the kinematics (i.e., t_i finite, $s_i \to \infty$ as $s \to \infty$) and energy-momentum conservation, the parameters $\{\alpha\}$ and $\{-\beta\}$ decrease monotonically as a function of their index i. In addition, the $\vec{\kappa}_i$ are constrained to be finite. Now since

$$\vec{q}_i \sim (\alpha_i - \beta_i)\vec{p}_1 + \vec{\kappa}_i\,, \tag{7}$$

it immediately follows that the longitudinal momenta $q_{i\parallel}$ are *ordered* along the Regge chain (with the ratio of any two with the same sign tending to zero or infinity), as indicated by Fig. 2 in which the length of the particle's line and direction represent the value of the particle's longitudinal momentum. Thus the labeling of the particle momenta is in agreement with that defined in the Introduction. This ordering effect is well known and occurs also in MPM.[8] To this feature we add two ingredients. The first, a simplification, is that apart from the leading particles (first and nth) only pions are produced. This in practice at machine energies appears a reasonable approximation. The consequence of this assumption is that only meson Regge links are involved in the process. The second is that all mesons and hence all meson Regge trajectories are describable by SU(3) octets and singlets, so that all *Regge-pole* links have charge ±1 or 0.

From this it immediately follows that certain charge groupings are forbidden. For example, one cannot produce three or more consecutive π^{+}'s or π^{-}'s without involving one or more Reggeon links with charge z such that $|z| > 1$. Even certain double-charge groupings are forbidden. In the MRM only Regge-Regge cuts (RR), as op-

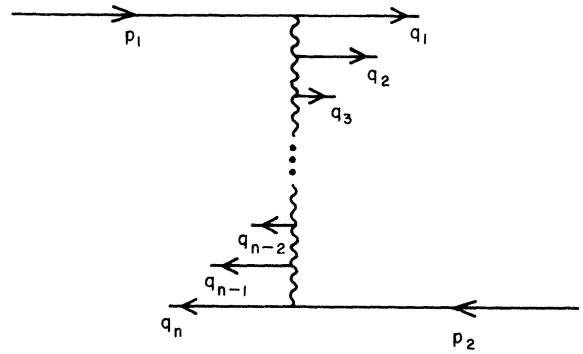

FIG. 2. MRM graph indicating the ordering of the x_r variables.

posed to diffractive ($R\mathcal{P}$) cuts,[9] can produce such groupings as those we have named "forbidden," and in the models we shall discuss in the next section these cuts become negligible asymptotically. However, at finite energies and for those events within the phase space of the MRM, the percentage of forbidden events will yield a simple measure of the cut to pole ratio[10] without recourse to the subtle $\ln(s_i)$ dynamical differences.

III. SOME SIMPLE MRM's

With the above simplifications in mind we now proceed to more explicit versions of the MRM. Consider the process indicated in Fig. 3. This could, for example, refer to

$$pp \to pp + m\pi^{+} + m\pi^{-} + 2l\pi^{0}\,. \tag{A}$$

The Regge links are assumed to be alternate Pomeranchukons ($\mathcal{P}$) and A_1. In particular, the first and last link are assumed to be Pomeranchukons so that the total number of outgoing pions $(n-2)$ is even by G parity. The number of Regge links is therefore odd, with one more Pomeranchukon link than the number of A_1 links. In each link, without loss of generality, we may include an admixture of Regge pole and diffractive cut, e.g., $A_1 \Rightarrow A_1(\text{pole}) + A_1 \times \mathcal{P}(\text{cut})$. Indeed, since the cuts have a flatter slope than the simple pole, and if the Pomeranchukon trajectory satisfies the condition $\alpha_P(0) = 1$ (i.e., total cross sections tend to a constant value), then for all t_i except $t_i = 0$ we expect ultimately that the diffractive cuts will dominate over the Regge pole. In contrast the Regge-Regge cuts such as ρ-A_1 will be of lower order and should asymptotically be negligible. Consequently the produced pions may be grouped in pairs, each pair attached to a single A_1 link. To $A_1(+-0)$ correspond the pairs $\pi^-\pi^+$, $\pi^+\pi^-$, and $\pi^0\pi^0$, respectively. Let us label the charge of the rth produced pion (i.e., that with Feynman parameter x_r) as e_r. Then

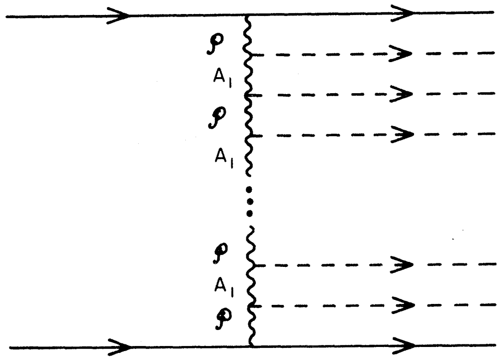

FIG. 3. MRM model with alternative exchange of $\mathcal{P}$ and A_1.

for r odd $[r=2w+1; w=1, \ldots, (n-2)/2]$

$$e_r \equiv e_{2w+1} = \bar{e}_{2w}, \tag{8}$$

where $\bar{e}_r$ is the conjugate charge to e_r. This condition, which is an example of a Markov chain in probability theory, automatically eliminates "forbidden" events. It also holds, even if all neutral π's in process (A) are undetected or, equivalently, left unlabeled.

The probability of a particular charge e_r occurring for *r even* is by isospin invariance equal for all possibilities $e_r = +1, 0, -1$. As an exercise consider now the cases when n is fixed (so that the same number of A_1 links are involved for dynamical equivalence) but the number of charged particles $2m$ varies. By estimating the cross section for a given m, we may independently determine the probability of seeing among this class of events either a π^+ ($\equiv\pi^-$ by charge conservation) or π^0. It can readily be argued that each MRM diagram with different orderings or configurations can be treated as incoherent. Consequently the multiple counting in the integral definition of cross sections exactly cancels the Gibbs factors $(m!\,m!\,2l!)^{-1}$ involved. Thus for a particular m the cross sections $\sigma_{m,l}$ will be proportional to the number of inequivalent orderings of the charged and neutral pion pairs. It also depends upon the number of different configurations obtained by "flipping" a charged pion pair, namely 2^m. Therefore

$$\sigma_{m,l} \propto \binom{m+l}{m} 2^m \equiv \binom{\frac{1}{2}(n-2)}{m} 2^m \tag{9}$$

and

$$\sigma_{\text{tot}} = \sum_{m=0}^{(n-2)/2} \sigma_{m,l} \propto 3^{(n-2)/2} \equiv B\,. \tag{10}$$

Thus it immediately follows that the probabilities of seeing a π^+ (π^-) or π^0 are equal, i.e.,

$$\sum_{m=0}^{(n-2)/2} \frac{m\binom{\frac{1}{2}(n-2)}{m} 2^m}{(n-2)B} = \frac{1}{3} = \sum_{m=0}^{(n-2)/2} \frac{2l\binom{\frac{1}{2}(n-2)}{m} 2^m}{(n-2)B}, \tag{11}$$

in agreement with the isospin argument.

Figure 4 shows another MRM for the same process (A). This has the possible advantage of involving only Pomeranchukon links. The produced pions are created via the isosinglet component (f_1) of the f-f' resonances. Thus they are created in pairs which we shall call "twins" and therefore by isospin again satisfy the above equality [Eq. (11)] of probabilities for all values of n to which the model is applicable. More important, it satisfies the same charge constraint Eq. (8), since asymptotically the x values of the pion twins tend to the same limit.

IV. THE UNCORRELATED ANSATZ

We shall now determine the probability of seeing forbidden events if it is assumed that there is no charge correlation effect. This assumption is implicit, as we have said, in the SM and UJM. Again the distribution of the π^0's in this problem is immaterial; we can concentrate solely upon the charged-pion pattern. Since there are $m\,\pi^+$'s and $m\,\pi^-$'s, the total number of configurations is

$$N = (2m)!/(m!)^2\,. \tag{12}$$

The number of permissible configurations (in the MRM sense) can be counted by noting that each consists of pion pairs and can be converted into any other permissible configuration by a series of charge flips $\pi^+\pi^- \to \pi^-\pi^+$. Let this number be C; then

$$C = 2^m\,. \tag{13}$$

Consequently the number of forbidden configurations $\bar{C} = N - C$ is

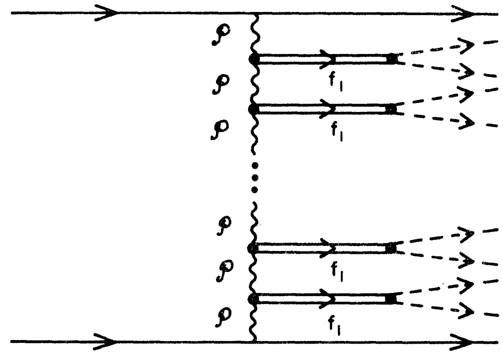

FIG. 4. MRM model with only $\mathcal{P}$ exchanges.

$$\overline{C}=\frac{2^m}{m!}[(2m-1)!!-m!] \quad \text{for } m\geq 1$$
$$=0 \quad \text{for } m=0\ , \tag{14}$$

and the probability of seeing a forbidden event is

$$\frac{\overline{C}}{N}=\frac{2^m m!}{(2m)!}[(2m-1)!!-m!] \quad \text{for } m\geq 1$$
$$=0 \quad \text{for } m=0\ . \tag{15}$$

This rapidly tends to unity as m gets large.

The fact that $\overline{C}$ is zero for $m=0, 1$ means that for processes of type (A) at least six outgoing charged particles are needed before any forbidden configurations can be constructed. However, where charge exchange occurs for both leading particles, i.e.,

$$p+\overline{p}\rightarrow n+\overline{n}+m\pi^+ +m\pi^- +2l\pi^0\ , \tag{A$'$}$$

then forbidden configurations can be constructed for $m=1$. Indeed, as a general rule the *leading charged particle* must have the same charge as the incoming particle (assumed charged) for the configuration to be permissible, after which the same analysis as presented above can be applied to the remaining charged particles.

Experimentally we may test the uncorrelated ansatz for various m by counting the percentage of forbidden events. It is to be supposed that this ansatz is more justified for high m. At the same time charge correlations of the MRM type described earlier may be sought in the low-m events where it is hoped the model gains validity. It is also conceivable that correlations of an unprojected kind may exist, for example, between particle charges and their transverse momentum.

In conclusion, we have noted some characteristic charge patterns associated with various simple MRM and the predicted absence of events which for large m dominate if the uncorrelated ansatz holds. We suggest that not only would a search for such correlations usefully distinguish between various theories but may also yield information about the (RR) cut/pole Regge ratio.

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