PHYSICAL REVIEW D VOLUME 7, NUMBER 8 15 APRIL 1973

Relativistic Many-Body Theory for Strongly Interacting Matter

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(Received 19 June 1972)

The relativistic many-body theory for the equation of state describing a strongly interacting system of baryons in flat space-time is presented. After giving the coupled equation of motion for relativistic Green's functions, which include finite density and temperature as boundary conditions, we develop relativistic expressions for the pressure and the energy density of the interacting system in terms of the fermion two-point function. The latter is expressed by an integral equation, which includes the pseudoscalar coupling to lowest order in the coupling constant. The result is regularized, and renormalization to both physical and effective mass and charge is discussed. The expressions obtained serve as the basis for a numerical calculation of the equation of state which is presented in the following paper. A significant feature of our results is a fully relativistic equation of state which includes interactions, and which leads to an asymptotic limit for the speed of sound $v_s \leq c/\sqrt{3}$.

I. INTRODUCTION

One of the most challenging questions facing relativistic astrophysics and general relativity is the problem of stability against gravitational collapse for stars whose mass exceeds the Chandrasekhar mass limit.[1] The fate of such a system poses a problem which involves at one and the same time elementary particle theory, many-body theory, and general relativity. In addition to this issue of principle is the problem of determining the properties of possible final stages of stellar evolution. The neutron star, and its relation to pulsars, is of primary concern in this connection. The problems mentioned above share this one common feature: They depend upon the behavior of relativistic matter at supernuclear densities.

In this and the following paper[2] we shall investigate various properties of relativistic systems of interacting elementary particles. The approach is based on the flat-space-time limit of a relativistic quantum many-body theory developed[3] in an earlier paper. In this paper, we shall develop the limit in terms of a relativistically interacting system of baryons which includes the exchange of pseudoscalar mesons. Particular attention will be given to renormalization and regularization of the theory. Expressions for the pressure, number density, and energy density will be developed in terms of fully relativistic many-body Green's functions. In the second paper, the numerical solution for the equation of state will be given for densities $\rho > 10^{13}$ g/cm^3 and an analytic approximation to these results discussed.

For densities $\rho < 10^{49}$ g/cm^3, the treatment of stellar evolution and stability is usually divided into two distinct parts[4]: the macroscopic influence of gravitation as determined either by Newtonian gravity ($\rho < 10^{15}$ g/cm^3) or Einstein's general theory of relativity (10^{16} g/cm$^3 \leq \rho < 10^{93}$ g/cm^3), and the behavior of matter at nuclear or supernuclear densities ($\rho \geq 10^{15}$ g/cm^3). As long as $\rho < 10^{49}$ g/cm^3, it is generally argued that gravitational contributions to interactions between particles are negligible. We shall restrict attention at this time[5] to such situations. Furthermore it will be assumed that the effects of gravitation, as manifested by the curvature of space-time, are negligible across samples of matter which are microscopically large. As a result, we effect a complete separation between the curvature of space-time and the interaction between particles.

A substantial body of theory exists in the literature concerning the properties of matter up to and slightly above nuclear densities[6] ($\rho_{\text{nuc}} \simeq 2.4 \times 10^{14}$ g/cm^3). In this region, matter consists predominantly of atoms, ions, electrons, protons, neutrons, and nuclei, whose motions are nonrelativistic ($q_F/m < 1$), and between which interactions may be described by phenomenological potentials. The latter, based upon extensive empirical data ranging from the density of iron $\rho_{Fe} = 7.86$ g/cm^3 up to ρ_{nuc}, in conjunction with nonrelativistic many-body theory, have led to a detailed understanding of matter in the nonrelativistic region. In particular, it is suggested that nuclear matter, just as ordinary matter, may exist in collective states which exhibit superconducting, superfluid, or possibly even ferromagnetic behavior.[6] The detailed equations of state derivable from the results mentioned above are crucial to the exact analysis of stellar models, particularly insofar as they determine the mass, radius, and luminosity of stable configurations, whose cores have densities in the nonrelativistic regime. They are of fundamental importance for the analysis of white dwarfs, and possibly for pulsar mechanisms. However, as soon as the density exceeds $\rho_c \simeq 10^{15}$ g/cm^3, or when questions of stability arise, it is necessary to treat matter relativistically[1] – results based upon potentials and nonrelativistic many-body theory become inapplicable. Therefore, in order to obtain a complete understanding of the formation and properties of neutron stars (whose central densities may reach 10^{17} g/cm^3 or more), or to determine whether a star of given mass and composition will evolve through the neutron star stage and enter a state of inevitable gravitational collapse to a curvature singularity, it is necessary to apply relativistic many-body theory to a system of relativistically interacting particles.

As a first step in this direction we have formulated a relativistic many-body theory, and applied it to a system of baryons which includes the exchange of pseudoscalar mesons. The equation of state which we derive is expected to reveal characteristic features of relativistic strongly interacting matter. In particular, it is found that the system described above leads to the restriction[7]

$$v_s \leq \frac{c}{\sqrt{3}}, \tag{1.1}$$

where v_s is the speed of sound in the medium. We hasten to emphasize that this result includes the effects of interactions, and is therefore of a more general nature than similar limits based on noninteracting fields. The results of this investigation will be extended in future work to include the more nearly realistic situation of a system of baryons from the first octet interacting via the exchange of pseudoscalar mesons in an SU(3)-invariant manner.

Before proceeding with the baryon-pseudoscalar meson theory, we review[8] existing methods employed in the analysis of superdense matter. We shall limit this discussion to relativistic systems (since the nonrelativistic regime offers no issues of principle), and focus attention on single-component systems of fermions at zero temperature. A characteristic feature of all stable configurations of fermions is the requirement that the particles obey the Pauli exclusion principle. In the absence of all interactions, such a system will exhibit the well-known pressure due to degeneracy. This led Landau[9] in 1932 to suggest that quantum statistics plays a dominant role in stellar interiors. Similarly, in 1939, Oppenheimer and Snyder[10] demonstrated that a system of neutrons sustained only by pressure due to degeneracy will undergo gravitational collapse to a singularity, unless the mass (at zero temperature) is less than about $0.7 M_\odot$. In 1960, the first detailed relativistic equation of state for a system of noninteracting baryons and nuclear resonances was given by Ambartsumyan and Saakyan.[11] They included in their discussion the baryons from the first octet, the electron, π^-, μ^-, and the nuclear resonances represented by the two isotopic spin states $N^*(1238)$. The equation of state covered a density range 10^{12} g/cm$^3 \lesssim \rho \lesssim 10^{17}$ g/cm^3, with thresholds for the appearance of each particle determined by the requirements that the system have minimum energy, and conserve both electric and baryonic charge. Their results have been extended with the discovery of additional particles and resonances.[12]

A refinement of the noninteracting Fermi gas was given by Gratton and Szamosi,[13] who developed a relativistic treatment of an excluded-volume Fermi gas (the nearest relativistic analog of a hard-core interaction). They find, in particular, an asymptotic speed of sound identical to (1.1).

The problems associated with early cosmologies and high-energy astrophysics discussed previously have recently attracted the interest of elementary particle theorists. As a result, some very intriguing high-density and high-temperature equations of state have been suggested, which are based upon current theories[14] of elementary particles. For example, Regge-pole theory, Veneziano amplitudes, and the dual resonance model,[15] as well as Hagedorn's description of strong interactions at high temperatures,[16] have been used to describe relativistic superdense matter. These theories differ in several fundamental ways from the Green's function approach which we have used. For this reason we shall discuss their characteristic features in order to place our approach in perspective. Each of the first three descriptions of elementary particles mentioned above is based upon a density of states describing the elementary particles (resonances) that occur in the theory. The density of states used to describe the hadron mass spectrum is generally of the form

$$\rho(E) \sim c\, m^a e^{bm}, \tag{1.2}$$

where the constants a, b, and c depend upon the particular theory employed. The resulting mass spectrum is substituted into the expression for the relativistic thermodynamic potential. The pressure and energy density are then evaluated by standard thermodynamic methods. Roughly speaking, this approach replaces elementary particles and their interactions by a system of baryon resonances whose "density" is determined by (1.2). Furthermore it assumes that all such resonances will occur in ultradense matter [a suitable choice of parameters in (1.2) guarantees that the thermodynamic potential converges]. It is argued that this type of analysis, which is free from the detailed dynamics of strong interactions, may lead to a realistic description of superdense matter. It is further argued, though not always convincingly, that any theory which takes into consideration only a finite number of states at asymptotic energies is unrealistic.

Recent work by Sawyer,[17] based in part on a Green's function technique analogous to that used in this paper, suggests that the equations of state calculated from the baryon mass spectrum as described above may not be applicable to superdense matter in its ground state. In fact, he concludes that the population of states above the basic baryon octet may not occur, even at asymptotic densities. To understand this, Sawyer considers the effect of a degenerate Fermi sea of baryons on a single baryonic resonance placed in it. In vacuum the resonance has mass m_0, and is unstable; the presence of filled states of decay products stabilizes the resonance, and the shift in lifetime leads to an increase in effective mass which increases with baryon density. Since the production threshold depends on the mass, the only way to produce the resonance is to increase the density (hence baryon energy). But this leads in turn to a further increase in the resonance's mass, and a regenerative suppression of the excited state follows. Sawyer's calculation, while suggesting that an adequate description of superdense matter in its ground state may depend only on the first baryon octet, serves as a warning: The potential importance of and need to include many-body effects within the framework of a fully relativistic many-body theory is of qualitative significance for the theory of superdense matter.

We turn now to the problem of obtaining a relativistic equation of state for strongly interacting matter. As mentioned above, we shall use the flat-space-time limit of the formalism developed in a previous paper.[3] In Sec. II the interaction is specified and the coupled equations of motion for the relativistic many-body Green's functions given. In Sec. III we develop the appropriate relativistic expressions for the pressure and energy density in terms of the interacting Green's functions of Sec. II. Finally we specify the finite-density boundary conditions, and give expressions for the fermion two-point function to lowest order in the coupling constant. Renormalization and regularization to physical as well as effective masses and charge is discussed, and our approximations justified.

II. RELATIVISTIC MANY-BODY THEORY

The discussion of many-body effects is greatly simplified if it is formulated in terms of Green's functions. As stressed in Sec. I, the properties of superdense matter will be relativistic; their discussion requires a fully relativistic formalism. Therefore, the flat-space-time limit of the approach developed in an earlier paper will be followed. In this approach, interactions are described by a Lagrangian, which determines the equations of motion for the Green's functions. The latter are expressed in a temperature- and density-dependent form. Many-body effects may then be introduced through the homogeneous term in the integral equations for the two-point functions. Although the field-theoretic approach is not necessary to the formalism, it is a convenient intermediate step in determining an appropriate set of coupled Green's function equations with which to work.

The Lagrangian density for a system of spin one-half fermions interacting through a Yukawa coupling in flat space-time is given by[18]

$$\mathcal{L}(x)=\bar{\psi}(x)(i\gamma^\mu\partial_\mu-m)\psi(x)+\tfrac{1}{2}[\partial_\mu\phi(x)\partial^\mu\phi(x)+\mu^2\phi^2(x)] -ig\bar{\psi}(x)\gamma^5\psi(x)\phi(x)+\delta m\,\bar{\psi}(x)\psi(x)-\delta\mu^2\phi^2(x). \tag{2.1}$$

The equations as written are, for example, applicable to a system of neutrons interacting by the exchange of π^0. Mass renormalization counterterms[19] for the two fields have been included; m and μ represent the physical masses of the particles. The matrix γ^5 is defined through the Dirac γ matrices γ^μ:

$$\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}, \tag{2.2}$$

$$\gamma^5\equiv\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{2.3}$$

The following anticommutation relations hold for γ^5:

$$\{\gamma^5,\gamma^\mu\}=0. \tag{2.4}$$

The strong-interaction coupling constant is denoted by g, and all spinor indices have been omitted. The adjoint $\bar{\psi}(x)\equiv\psi^\dagger(x)\gamma^0$, where the dagger denotes Hermitian conjugation. The fermion fields satisfy the usual anticommutation relations, while the boson field satisfies commutation relations.

The equations of motion for $\psi(x)$ and $\phi(x)$ may be derived in the usual manner from Eq. (2.1). They will not be reproduced here. From these it is straightforward to derive a set of coupled integral equations which relate the two-point functions to the three-point function. In this paper we shall be concerned primarily with a system of fermions, and shall assume that the density of antifermions and bosons is zero. The equations will nevertheless be written initially in complete generality; the density of all matter (particles and antiparticles of both spins) will be denoted by ρ, and the temperature by $T=(\beta)^{-1}$, where Boltzmann's constant K has been set equal to unity. The equations of motion for the Green's function are[20]

$$G_F(x-x';\rho,\beta)=S_F(x-x';\rho,\beta)+\int d^4z\int d^4y\,S_F(x-y;\rho,\beta)\Sigma_c(y,z;\rho,\beta)G_F(z-x';\rho,\beta), \tag{2.5}$$

and

$$D_F(x-x';\rho,\beta)=\Delta_F(x-x';\rho,\beta)+\int d^4y\int d^4z\,\Delta_F(x-y,\rho,\beta)\Pi_c(y,z;\rho,\beta)D_F(z-x';\rho,\beta). \tag{2.6}$$

The density and temperature dependence of all quantities has been shown explicitly. The self-energy and polarization operators are denoted by

$$\Sigma_c(x,x';\rho,\beta)=i\gamma^5g^2\int d^4y\int d^4z\,G_F(x-y;\rho,\beta)\Gamma_5(x',y\,|\,z)D_F(x-z;\rho,\beta), \tag{2.7}$$

and

$$\Pi_c(x,x';\rho,\beta)=-ig^2\,\mathrm{tr}\int d^4y\int d^4z\,\gamma^5G_F(x-y;\rho,\beta)\Gamma_5(y,z\,|\,x')G_F(z-x';\rho,\beta), \tag{2.8}$$

respectively. The vertex function, denoted by $\Gamma_5(x,y\,|\,z)$, is related to the three-point function by a well-known identity which will not concern us at this time. It should be noted, however, that it is in general a function of the temperature and density of the system, although to lowest order these effects may be ignored.

The homogeneous terms in the integral equation represent the noninteracting Green's functions. These were discussed in detail for finite-density and -temperature systems in a previous paper.[3] There they were shown to have the following representations in momentum space:

$$S_F(p;\mu\bar{\mu}\beta)=\frac{\not{p}+m}{2E_{\vec{p}}}\left[\frac{1-n_F(p,\beta)}{p_0-E_{\vec{p}}+i\epsilon}+\frac{n_F(p,\beta)}{p_0-E_{\vec{p}}-i\epsilon}-\frac{1-\bar{n}_F(p,\beta)}{p_0+E_{\vec{p}}-i\epsilon}-\frac{\bar{n}_F(p,\beta)}{p_0+E_{\vec{p}}+i\epsilon}\right], \tag{2.9}$$

and

$$\Delta_F(k,\zeta,\bar{\zeta};\beta)=\frac{1}{2\omega_{\vec{k}}}\left[\frac{1+n_B(k,\beta)}{k_0-\omega_{\vec{k}}+i\epsilon}-\frac{n_B(k,\beta)}{k_0-\omega_{\vec{k}}-i\epsilon}-\frac{1+\bar{n}_B(k,\beta)}{k_0+\omega_{\vec{k}}-i\epsilon}+\frac{\bar{n}_B(k,\beta)}{k_0+\omega_{\vec{k}}+i\epsilon}\right]. \tag{2.10}$$

The Fourier transform of any function $f(p)$ of the four-momentum $p^\mu=(p^0,\vec{p})$ is by convention

$$f(x)=\int\frac{d^4p}{(2\pi)^4}f(p)e^{-ip\cdot x}, \tag{2.11}$$

where $p\cdot x=p^\mu x_\mu$. Note that $p^0=p_0$ and $p^i=-p_i$ ($i=1,2,3$). The single-particle energies, in terms of the physical masses m and μ, are

$$E_{\vec{p}}=(|\vec{p}|^2+m^2)^{1/2},\quad \omega_{\vec{k}}=(|\vec{k}|^2+\mu^2)^{1/2}. \tag{2.12}$$

The particle distribution functions are, for each spin degree of freedom,

$$n_F(p,\beta)=\frac{1}{\exp[\beta(E_{\vec{p}}-\mu)]+1} \tag{2.13}$$

for fermions, and

$$n_B(k,\beta)=\frac{1}{\exp[\beta(\omega_{\vec{k}}-\zeta)]-1} \tag{2.14}$$

for bosons. The relationship between the chemical potentials μ and ζ, and the number of particles in the system, is well known.[21] Expressions similar to Eqs. (2.13)–(2.14) are also defined for antiparticles, with μ and ζ replaced by $\bar{\mu}$ and $\bar{\zeta}$. For bosons which may be created in arbitrary numbers, the chemical potentials vanish, $\zeta=\bar{\zeta}=0$. Examination of the interaction term in (2.1) shows that this will be the case. The poles of $S_F(p,\mu,\bar{\mu},\beta)$ in the p_0 plane are shown in Fig. 1, and have been discussed extensively elsewhere.[3]

The anomalous behavior of (2.14) when $\omega_{\vec{k}}=\zeta$ leads to the well-known phenomenon of Bose-Einstein condensation. It is then desirable to separate

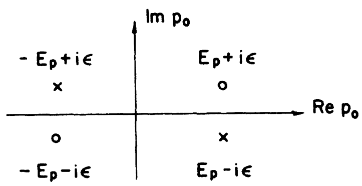

FIG. 1. The poles of $S_F(p, \mu, \bar{\mu}, \beta)$ in the complex p_0 plane. The strength of the singularity is a function of the momentum, the density, and the temperature. States which at $T=(k\beta)^{-1}=0$ correspond to excitations with momenta $|\vec{p}|>p_F$ or $|\vec{p}|>\bar{p}_F$ are denoted by **X**; those corresponding to momenta $|\vec{p}|<p_F$ or $|\vec{p}|<\bar{p}_F$ are denoted by O. At zero density the poles $p_0=\pm(E_{\vec{p}}+i\epsilon)$ do not contribute.

from the Green's function (2.10) that part which describes the condensate.[22] For the applications of this paper it will be assumed that the density of real bosons is strictly zero. In this case, since ζ refers to real particles only, the distribution functions $n_B(k, \zeta)$ and $\bar{n}_B(k, \bar{\zeta})$ vanish identically. Consequently, (2.10) takes its usual form as in relativistic quantum field theory. We shall return in a later paper to the problem of Bose-Einstein condensation in the relativistic region.

III. THERMODYNAMIC VARIABLES

Having defined the fermion and boson two-point functions, it is possible to consider thermodynamic variables for a relativistic system. From the two-point functions one may obtain such information about a many-body system as the pressure, ground-state energy, density of particles, chemical potential, and others. Furthermore, the energies, lifetimes, and effective masses of the elementary excitations of the system are given by the singularities of $G_F(p, \mu, \bar{\mu}; \beta)$ and $D_F(k, \zeta, \bar{\zeta}; \beta)$ in the complex-energy plane. In most instances these follow in a straightforward manner the arguments familiar from nonrelativistic many-body theory, except for the appearance of antiparticle states. The latter result in slight modifications, as will be seen below. The divergences associated with radiative corrections may be easily eliminated as in relativistic quantum field theory,[3] so that finite values for physical observables result. (See Table 1.)

Two examples will be considered below which illustrate the slight changes attendant upon the incorporation of relativistic dynamics for a many-body system. These, the number density and pressure, will be of use later. Finite temperature will be maintained throughout, since the limit $T\to 0$ ($\beta\to\infty$) may be taken later.

Restrict attention to a system of fermions and their antiparticles (although this is by no means a necessary step). From the two-point function $S_F(x-x'; \rho, \beta)$, where ρ denotes the density of particles and antiparticles, it is possible to construct an expression for the number density of fermions in the system. Proceeding as in the nonrelativistic case we define $n_0 \equiv n_0(x)$,

$$n_0(x)=\mathrm{tr}\langle \psi^\dagger(x)\psi(x)\rangle_\beta ,$$

TABLE I. Finite-density Green's function in the nonrelativistic and relativistic regimes compared and contrasted. For simplicity we set $T=0$, and consider spin-$\frac{1}{2}$ fermions.

	Nonrelativistic	Relativistic
Green's function, spin-dependent	2×2 matrix representation in terms of Pauli spin matrices	4×4 matrix representation in terms of Dirac γ matrices
Thermodynamic boundary conditions	Specifies behavior of homogenous part (noninteracting) near singularity in the complex-energy plane	Same
Elementary excitation energies	Measured with respect to nonrelativistic Fermi energy	Measured with respect to relativistic Fermi energy for particles or antiparticles
Number of excitations per momentum state:	Two: one per spin degree of freedom	Four: two for particle and two for antiparticle excitation states
Chemical potential	Measured with respect to zero energy	Includes rest energy of particles in the system. Need not be the same for particles and antiparticles

where

$$\langle A\rangle_\beta \equiv \frac{\mathrm{tr}\hat{\rho}A}{\mathrm{tr}\hat{\rho}},$$

and the statistical density operator $\hat{\rho}$ is defined in this case by

$$\hat{\rho} = \exp[-\beta(H-\mu N-\bar{\mu}\bar{N})].$$

The particle and antiparticle chemical potentials and number operators are denoted by (μ, N) and $(\bar{\mu}, \bar{N})$, respectively, and the Hamiltonian by H. The number density operator appearing above is in fact infinite. To overcome this difficulty we consider the normal-ordered form

$$N_0/V = \mathrm{tr} : \psi^\dagger(x)\psi(x): . \tag{3.1}$$

The double colon refers to normal ordering. It is straightforward to show the equivalence[19]

$$\mathrm{tr} : \psi^\dagger(x)\psi(x): = \tfrac{1}{2}\,\mathrm{tr}[\bar{\psi}(x), \gamma^0\psi(x)] . \tag{3.2}$$

Consequently the (finite) number density is given by

$$n_0(x) = \tfrac{1}{2}\,\mathrm{tr}\langle[\bar{\psi}(x), \gamma^0\psi(x)]\rangle_\beta . \tag{3.3}$$

From the definition of the two-point function

$$G_F(x-x', \rho, \bar{\rho}, \beta) = -i\langle T\psi(x)\bar{\psi}(x')\rangle_\beta , \tag{3.4}$$

where T denotes time ordering of the fields, it is trivial to show[23] that the number density of particles, as given by (3.3) and (3.4) with $\bar{\rho}=0$, is

$$n_0(x,\rho) = -i\left(\lim_{x_0'\to x_0+0} + \lim_{x_0'\to x_0-0}\right)\mathrm{tr}[\gamma^0 G_F(x-x';\rho,0,\beta)] .$$

Using (2.11) we find the corresponding expression in momentum space:

$$n_0(\rho,\beta) = -i\lim_{x_0\to 0^+}\int\frac{d^4p}{(2\pi)^4}(e^{ip_0x^0}+e^{-ip_0x^0}) \times \mathrm{tr}[\gamma^0 G_F(p;\mu,0;\beta)] . \tag{3.5}$$

The factors $\exp(\pm ip^0x_0)$ define the contours of integration in the usual way, and μ parametrizes the density of fermions in the system (for noninteracting systems the number of particles and antiparticles may be separately specified). A similar expression is obtained analogously for antiparticles parametrized by the chemical potential $\bar{\mu}$. From (3.3) and (3.4) with $\rho=0$ we find

$$\bar{n}_0(\bar{\rho},\beta) = -i\lim_{x_0'\to 0}\int\frac{d^4p}{(2\pi)^4}(e^{ip_0x^0}+e^{-ip_0x^0}) \times \mathrm{tr}[\gamma^0 G_F(p;0,\bar{\mu};\beta)] . \tag{3.5'}$$

Two features of the equations above for $n_0(\rho,\beta)$ will be noted. First, a factor of γ^0 occurs in the trace along with the causal propagator. This is a consequence of our having used the adjoint $\bar{\psi}(x)$ rather than the Hermitian conjugate $\psi^\dagger(x)$ in defining the two-point function. The second remark concerns the occurrence of two integration contours in the complex p_0 plane. The latter results from the use of a normal-ordered number operator, and represents the subtraction of infinite effects due to the definition of the vacuum. Apart from these two points (3.5) is formally identical to the nonrelativistic expression for the number density. It is trivial to show that in noninteracting systems (3.5) reduces to the usual expression for the particle number density at temperature β^{-1}:

$$n_0(\rho,\beta) = 2\int\frac{d^3p}{(2\pi)^3}\,n_F(p,\beta) , \tag{3.6}$$

with a similar expression $\bar{n}_0(\bar{\rho},\beta)$ for antiparticles which follows from (3.5′). The factors of two arise in evaluating the trace and represent the particles' spin degeneracy. At zero temperature one readily finds the expressions

$$n_0(p_F) = \frac{p_F^{\,3}}{3\pi^2}, \quad \bar{n}_0 = -\frac{\bar{p}_F^{\,3}}{3\pi^2} . \tag{3.7}$$

The minus sign associated with $\bar{n}_0$ is conventional, and results from our having used the fermion number operator above. We recall that antiparticles have quantum numbers which are the negative of their physically observed values (e.g., negative energy, etc.). Keeping this in mind, we shall omit the minus sign in referring to (3.7), with the observation that the physical number density is to be understood.

A second expression of interest, which also follows from the two-point function, is the pressure. Writing the chemical potential for fermions μ_T (particle and antiparticle) in terms of the total number of fermions $N_T = N-\bar{N}$ and the previously defined chemical potentials μ and $\bar{\mu}$, the pressure is easily shown to be

$$P(\mu_T) = \int_0^{\mu_T} n(\beta,\mu_T')\,d\mu_T' . \tag{3.8}$$

At zero density $P(0)=0$, and $n(\beta,\mu_T')$ is the total fermion number density given by Eq. (3.5). It follows immediately that the pressure is given in terms of the two-point function by the integral

$$P(\mu_T) = -i\int_0^{\mu_T}d\mu_T'\lim_{x^0\to 0^+}\int\frac{d^4p}{(2\pi)^4}(e^{ip_0x^0}+e^{-ip_0x^0})\,\mathrm{tr}[\gamma^0 G_F(p,\mu_T;\beta)] . \tag{3.9}$$

It is instructive to apply this equation to the problem of a noninteracting gas of relativistic fermions, whose particle and antiparticle densities are given by Eqs. (3.7) at zero temperature. The result is the pressure for a relativistic two-component noninteracting gas – a result which is both well known and which follows from simpler arguments than those above. However, in the following paper we shall use the same general method in deriving an approximate result which includes interactions; it is best appreciated when divested of unnecessary algebraic details. Starting with (2.9), and evaluating the trace, we find for the pressure due to particles

$$P(\mu)=\int_0^{\mu}d\mu'\lim_{x_0\to 0^+}\int\frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp_0}{2\pi i}(e^{ip_0x^0}+e^{-ip_0x^0})\frac{p_0}{E_{\vec{p}}^{\mp}}\left[\frac{1-n_F(p)}{p_0-E_{\vec{p}}^{\mp}+i\epsilon}+\frac{n_F(p)}{p_0-E_{\vec{p}}^{\mp}-i\epsilon}-\frac{1}{p_0+E_{\vec{p}}^{\mp}-i\epsilon}\right]. \quad (3.10)$$

The poles of the integrand in the complex p_0 plane are shown in Fig. 1 [note that the singularity $p_0=-E_{\vec{p}}^{\mp}-i\epsilon$ does not occur in (3.10), since $\bar{n}_F(p)=0$ for the particle contribution to the pressure]. It will be noted that for a given momentum only two occur at a time, the position of the positive energy pole with respect to the real axis being determined by the sign of $(p-q_F)$. As a result of the factors $\exp(\pm ip_0x^0)$, the infinite terms $\sim(p_0-E_{\vec{p}}^{\mp}+i\epsilon)^{-1}$ and $(p_0+E_{\vec{p}}^{\mp}-i\epsilon)^{-1}$ cancel, and the expression for the pressure reduces to the form

$$P(\mu,\beta)=2\int_0^{\mu}d\mu'\int\frac{d^3p}{(2\pi)^3}\,n_F(p,\beta). \quad (3.11)$$

At finite temperature the integrals may be evaluated in terms of tabulated functions. For our purposes we consider the zero-temperature limit ($\beta\to\infty$), in which the chemical potential reduces to the Fermi energy. The latter is given by the value of p_0 at which the imaginary part of the Green's function changes sign. From (3.10) we find the particle chemical potential to be [$q_F\le p_F$ where p_F is given by (3.7)]

$$\mu(q_F)=E_F$$
$$=(q_F^{\,2}+m^2)^{1/2}. \quad (3.12)$$

Repeating the analysis for the two-point function $S_F(p,0,\bar{\mu},\beta)$ yields the antiparticle contribution to the pressure of the noninteracting system. The expression analogous to (3.10) is

$$P(\bar{\mu})=\int_0^{\bar{\mu}}d\bar{\mu}'\lim_{x_0\to 0^+}\int\frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dp_0}{2\pi i}(e^{ip_0x^0}+e^{-ip_0x^0})\frac{p_0}{E_{\vec{p}}^{\mp}}\left[\frac{1}{p_0-E_{\vec{p}}^{\mp}+i\epsilon}-\frac{1-\bar{n}_F(p)}{p_0+E_{\vec{p}}^{\mp}-i\epsilon}-\frac{\bar{n}_F(p)}{p_0+E_{\vec{p}}^{\mp}+i\epsilon}\right]. \quad (3.10')$$

The poles at zero temperature are shown in Fig. 1; only two occur for a given value of the momentum, and the singularity $p_0=E_{\vec{p}}^{\mp}+i\epsilon$ is eliminated by the requirement that $n_F(p)=0$. Proceeding as above we find the antiparticle contribution to the pressure of the noninteracting system:

$$P(\bar{\mu},\beta)=-2\int_0^{\bar{\mu}}d\bar{\mu}'\int\frac{d^3p}{(2\pi)^3}\,\bar{n}_F(p,\beta). \quad (3.11')$$

The corresponding antiparticle chemical potential

$$\bar{\mu}(\bar{q}_F)=-\bar{E}_F$$
$$\equiv-(\bar{q}_F^{\,2}+m^2)^{1/2} \quad (3.13)$$

follows from the Green's function appearing in (3.10′). The minus sign in (3.13) corresponds to the appearance of negative energies associated with antiparticles, while that in (3.11′) follows from the contour integration in the p_0 plane. The physical value of $\bar{\mu}(q_F)$ which is positive appears in the upper limit of the integration over $d\mu'$.

The total pressure is given by adding (3.11) and (3.11′). Recalling that the distribution functions at zero temperature are unity below the Fermi momentum, and vanish above it, we have

$$P(p_F,\bar{p}_F)=2\int_0^{\mu(p_F)}d\mu(q_F)\int_0^{q_F}\frac{4\pi p^2dp}{(2\pi)^3}$$
$$+2\int_0^{\bar{\mu}(\bar{p}_F)}d\bar{\mu}(\bar{q}_F)\int_0^{\bar{q}_F}\frac{4\pi p^2dp}{(2\pi)^3}, \quad (3.14)$$

the factors of two arising from spin degeneracy. Integration yields

$$P(p_F,\bar{p}_F)=P(p_F)+P(\bar{p}_F), \quad (3.15)$$

$$P(p_F)=\frac{1}{3\pi^2}\int_0^{p_F}\frac{x^4dx}{(x^2+m^2)^{1/2}}$$
$$=\frac{p_F^{\,3}E_F}{12\pi^2}-\frac{m^2p_FE_F}{8\pi^2}+\frac{m^4}{8\pi^2}\ln\left|\frac{p_F+E_F}{m}\right|. \quad (3.16)$$

The total pressure of the relativistic ideal gas of $n_0=(p_F^{\,3}/3\pi^2)$ particles and $\bar{n}_0=(\bar{p}_F^{\,3}/3\pi^2)$ antiparticles is given by (3.15). It is seen, as expected, that noninteracting particles and antiparticles behave as two distinct perfect gases, each contributing to the total pressure. As is well known, the introduction of interactions results in a shift in the position of the pole of (3.10), though the chemical potential is still given by the value of p_0 at which the imaginary part changes sign.

Equation (3.15) has the familiar asymptotic limit for high densities (omitting the antiparticle contribution)

$$P \sim \frac{p_F{}^4}{12\pi^2}, \tag{3.17}$$

which is characteristic of a noninteracting extreme relativistic gas.

The discussion above demonstrates two features of the relativistic many-body theory: first, the obvious presence of antiparticle contributions as long as their density in the system is nonzero; and the near identity in form of such equations as (3.5) and (3.9) with their nonrelativistic counterpart, the major formal difference being the factor γ^0. The latter is crucial, since it leads to positive values for physical observables calculated from the contribution of poles whose real part is negative.

In concluding this section we note[24] the thermodynamic expression for the total energy density ρ of a system in terms of its pressure P and rest mass-energy density $\rho_0 = n_0 m$, where n_0 is given by the first of (3.7):

$$\rho = m n_0 \left[1 + \frac{9\pi^2}{m} \int_0^{p_F} \frac{P(y) dy}{y^4} \right]. \tag{3.18}$$

The pressure is expressed as a function of the Fermi momentum. This expression for ρ includes, in addition to the rest-energy density (the first term on the right), the contributions to the energy density due to the particles' kinetic energy, interactions (through the pressure), and work of compression. Equation (3.18) may be evaluated after the pressure has been found. Using (3.9) one may express the energy density in terms of the fermion two-point function.

Equations (3.9) for the pressure, (3.18) for the energy density and (3.15) for the number density represent the equation of state in parametric form, entirely in terms of the two-point function. In a similar manner, it is possible to derive expressions for other macroscopic observables describing a relativistic many-body system.

IV. INTERACTING GREEN'S FUNCTION

The results of the previous three sections constitute the basis of a relativistic many-body theory of baryons which includes a pseudoscalar Yukawa coupling. We turn now to a method of solving these equations. In this section we shall specify the boundary conditions which go into our determination of the equation of state for strongly interacting superdense matter. The fermion two-point function will then be set up in terms of these boundary conditions, and includes the effect of interactions to lowest order in the coupling constant. To this degree of approximation the self-energy reduces to a sum of two terms; one is density-dependent, while the other contains the divergence associated with radiative corrections. We discuss regularization of the latter and arrive at a finite expression for the corrected two-point function in terms of the physical parameters (mass and charge) in vacuum. Renormalization to effective parameters is also discussed. Finally we comment on the approximations involved.

In order to formulate the problem of an equation of state for strongly interacting matter, it is necessary to specify the finite density and temperature boundary conditions, and to limit the interactions which will be considered. As a preliminary step in our study of relativistic many-body systems we make the following assumptions:

(i) The system is in its ground state (i.e., zero temperature);

(ii) it consists of N electrically neutral spin-$\frac{1}{2}$ baryons of physical rest mass m;

(iii) interactions between baryons are described by a one-particle exchange (Yukawa coupling) as described by (2.1);

(iv) the density of antiparticles is assumed to be zero, and their subsequent production ignored;

(v) the baryons are assumed to be stable;

(vi) gravitational effects, even in the asymptotic limit, are ignored on the local level.

In a subsequent paper the restrictions (ii) and (iii) will be relaxed, and a system of baryons from the first octet considered, with interactions described by the exchange of members of the first pseudoscalar meson octet.

The rest-mass density of the system $\rho_0 = m n_0$ is parametrized by the Fermi momentum as in Eq. (3.7) [$\bar{n}_0 \equiv 0$ according to (iv) above]:

$$\rho_0 = m n_0 = \frac{m p_F{}^3}{3\pi^2}. \tag{4.1}$$

The noninteracting Green's functions at zero temperature are given by[25]

$$S_F(p, p_F) = \frac{\not{p} + m_0}{2E_{\vec{p}}^+} \left(\frac{1 - n_F(p)}{p_0 - E_{\vec{p}}^+ + i\epsilon} + \frac{n_F(p)}{p_0 - E_{\vec{p}}^+ + i\epsilon} - \frac{1}{p_0 + E_{\vec{p}}^+ - i\epsilon} \right) \tag{4.2}$$

and

$$\Delta_F(k) = \frac{1}{k^2 - \mu_0{}^2 + i\epsilon}, \tag{4.3}$$

where $E_{\vec{p}}$ is given by Eq. (2.12), the 4-momentum $k^\mu = (k^0, \vec{k})$ with $k^2 = k^\mu k_\mu$, and the zero-temperature limit of Eq. (2.13) is

$$n_F(p) = \begin{cases} 1, & |\vec{p}| < p_F \\ 0, & |\vec{p}| > p_F . \end{cases} \tag{4.4}$$

The pressure will be given by (3.9) which, under the assumptions above, may be written in the form

$$P(p_F) = \frac{2}{i}\int_0^{\mu(p_F)} d\mu(q_F) \times \int \frac{d^4p}{(2\pi)^4}\cos(p_0 t)\,\mathrm{tr}[\gamma^0 G_F(p, q_F)] . \tag{4.5}$$

The variable Fermi momentum $0 \leq q_F \leq p_F$ will be discussed later. In the integral over p_0, it is understood that t approaches zero from above.

The strong interactions between baryons will be included to second order in the coupling constant. This implies that the baryon two-point function is determined by the set of equations in momentum space:

$$G_F(p, q_F) = S_F(p, q_F) + S_F(p, q_F)\Sigma_c(p, q_F)G_F(p, q_F), \tag{4.6}$$

where the self-energy is

$$\Sigma_c(p, q_F) = ig_0^{\,2}\int \frac{d^4k}{(2\pi)^4}\gamma^5 S_F(p-k; q_F)\gamma_5 \Delta_F(k) . \tag{4.7}$$

The last equation results from the Fourier transform of (2.7), the approximations in the integrand consistent to the second order

$$G_F(p, q_F) = S_F(p, q_F), \tag{4.8}$$

$$D_F(k) = \Delta_F(k), \tag{4.9}$$

and the ansatz

$$\Gamma_5(p, k) = \gamma_5 . \tag{4.10}$$

In addition to the "exchange term" given by (4.7), and shown diagrammatically in Fig. 2, there results a "direct term" corresponding to the Feynman diagram of Fig. 3 which is also of second order in g_0. Because of the pseudoscalar nature of the interaction, however, the last term vanishes identically.[26]

The finite density expression for the self-energy is obtained by using either Wick's theorem[27] or Schwinger's functional derivative[28] approach. The generalization of either technique to the relativistic many-body formalism encounters no difficulties, and has been discussed elsewhere.[3]

It will be noted that the boson propagator is independent of the density of baryons. From the definition of the polarization operator (2.7) and (2.6) it will be observed that $D_F(k, \rho)$ will in general depend on the baryon density. However, these corrections are proportional to $g_0^{\,4}$ and, insofar as we retain terms of order $g_0^{\,2}$ only, they are dropped. Similar reasoning shows that the density dependence of $\Gamma_5(p, k, q_F)$ is of higher order than the second. Inverting (4.6) yields $G_F(p, q_F)$ in terms of $S_F(p, q_F)$ and the self-energy. The latter is shown diagrammatically in Fig. 2. Using (4.5) it is then possible to construct the pressure. Equation (3.18) may then be integrated to obtain the total energy density of the system. The parametric equations $P = P(\rho_0)$ and $\rho = \rho(\rho_0)$, where $\rho_0 = \rho_0(p_F)$, constitute an equation of state for the system.

At zero density (4.7) reduces to the usual self-energy correction of relativistic quantum field theory. It is to be expected that divergences present in the zero-density theory will arise at finite density as well. Examination of the integral in (4.7) shows this to be the case. It is therefore necessary to regularize the integral, a procedure which may be performed in exactly the same way as in relativistic field theory. As is well known this is accomplished, along with renormalization to observed parameters (mass and charge), by requiring that the two-point function be singular on the mass shell.

In relativistic quantum field theory one assumes that the pole occurs at the physical mass and physical charge.[19] In this section it will be assumed that the pole occurs at the effective mass.[3] Consider for the moment a system at finite temperature containing particles and antiparticles. Defining renormalized quantities in terms of the renormalization constants Z_1, Z_2, and Z_3,

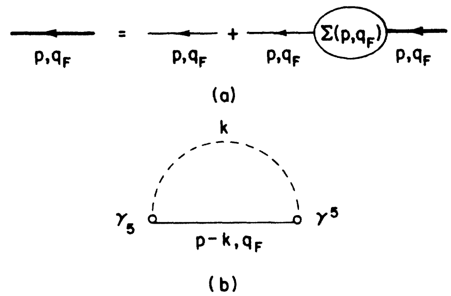

FIG. 2. (a) Baryon Green's function in momentum space (4.6). The propagators and self-energy are density-dependent. (b) The baryon self-energy to second order in g. The baryon propagator is density-dependent, while the boson operator and vertex function are independent of density to second order. Solid lines represent baryons, dashed lines bosons, and circles the elementary vertex. Each line bears in addition to its 4-momentum a density label when apprcpriate.

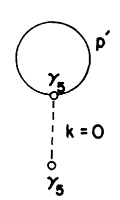

FIG. 3. Second-order contribution to the self-energy shown in Fig. 2(a). This expression vanishes due to the pseudoscalar nature of the interaction.

$$G_F(p;\alpha)=Z_2G_F'(p;\alpha), \qquad (4.11)$$

$$D_F(k;\alpha)=Z_3D_F'(p;\alpha), \qquad (4.12)$$

$$\Gamma_5(p,k;\alpha)=Z_1^{-1}\Gamma_5'(p,k;\alpha), \qquad (4.13)$$

$$g_r=Z_1^{-1}Z_2Z_3^{1/2}g_0, \qquad (4.14)$$

where the renormalized value carries a prime, and the total density and temperature dependence is denoted by α; the requirement that the two-point functions be singular at the effective masses ($p^2 = m_{\text{eff}}^2$ and $k^2=\mu_{\text{eff}}^2$) leads to the equations

$$[\not p - m_{\text{eff}} - \Sigma_c^R(p,\alpha)]\,G'(p,\alpha)=1, \qquad (4.15)$$

$$[k^2-\mu_{\text{eff}}^2-\Pi_c^R(k,\alpha)]D'(k,\alpha)=1, \qquad (4.16)$$

where the completely renormalized self-energy and polarization operators are

$$\Sigma_c^R(p,\alpha)=\Sigma_c(p,\alpha)-\Sigma_c(p,\alpha)| - (\not p - m_{\text{eff}})\frac{d\Sigma_c(p,\alpha)}{d\not p}\Big|_{\not p = m_{\text{eff}}}, \qquad (4.17)$$

$$\Pi_c^R(k,\alpha)=\Pi_c(k,\alpha)-\Pi_c(k,\alpha)|_{k^2=\mu_{\text{eff}}^2} - (k^2-\mu_{\text{eff}}^2)\frac{d\Pi_c(k,\alpha)}{dk^2}\Big|_{k^2=\mu_{\text{eff}}^2}. \qquad (4.18)$$

The effective masses are obtained from the expressions

$$m_{\text{eff}}(\alpha)=m_0+\frac{\Sigma_c'(p,\alpha)}{z_2}\Big|_{\not p = m_{\text{eff}}}, \qquad (4.19)$$

$$\mu_{\text{eff}}(\alpha)=\mu_0+\frac{\Pi_c'(k,\alpha)}{z_3}\Big|_{k^2=\mu_{\text{eff}}^2}. \qquad (4.20)$$

Finally the renormalization constants Z_2 and Z_3 are

$$Z_2(\alpha)=1+\frac{\partial\Sigma_c'(p,\alpha)}{\partial\not p}\Big|_{\not p = m_{\text{eff}}}, \qquad (4.21)$$

$$Z_1(\alpha)=1+\frac{\partial\Pi_c'(p,\alpha)}{\partial k^2}\Big|_{k^2=\mu_{\text{eff}}^2}. \qquad (4.22)$$

The functions Σ_c' and Π_c' are given by Z_1 times the Fourier transform of (2.7) and (2.8), respectively, written in terms of the renormalized quantities appearing in (4.11)–(4.14).

Examination of (4.17)–(4.22) shows that the effective masses, as expected, are functions of the density of all particles in the system to which they may be coupled by the interactions contained in the self-energy and polarization operators. Furthermore, it will be observed that the renormalization constants are functions of the density and temperature as well. By (4.19)–(4.22), and the dependence of $\Sigma(p,\alpha)$ on g^2, it is evident that the leading-order density-dependent corrections to these terms are proportional to the second or higher power of the coupling constant. In particular

$$m_{\text{eff}}(\alpha)=m_0+O(g^2), \qquad (4.23)$$

$$Z_1(\alpha)=1+O(g^2) \qquad (4.24)$$

The last two results are in part justification for the approximations (4.8)–(4.10) above.

The approach to renormalization just outlined is of interest formally, and constitutes the logical conclusion of renormalization of a relativistic many-body theory in flat space-time, that is, in the absence of such effects as gravitation. In actual calculations, however, it may be advisable to renormalize to physical masses which are independent of density, since for many theories [in particular, the one represented by (2.1)] regularization may be accomplished at the same time. This is the approach which will be followed below. The procedure is identical to that discussed above, with the exception that the pole of the two-point function occurs on physical mass shell, and the renormalized quantities are independent of the density. The results, which will be used later, are

$$[\not p - m - \Sigma_c^R(p,q_F)]\,G'(p,q_F)=1, \qquad (4.25)$$

$$\Sigma_c^R(p,q_F)=\Sigma_c(p,q_F)-\Sigma_c(p,0)|_{\not p = m} - (\not p - m)\frac{d\Sigma_c(p,0)}{d\not p}\Big|_{\not p = m}, \qquad (4.26)$$

with similar expressions for the boson two-point function and polarization operator. The baryon physical mass is given by

$$m=m_0+\frac{\Sigma_c'(p,0)}{z_2}\Big|_{\not p = m}, \qquad (4.27)$$

and is independent of density as are Z_1, Z_2, and Z_3. It will be noted that the physical mass is determined by the self-energy at zero density. The results (4.25)–(4.27) apply to the system described by the assumptions at the beginning of this section, and have been written in terms of the density (4.1).

V. CONCLUSION

We have presented and discussed, within the context of a fully relativistic many-body theory, the Green's function for a system of baryons interacting via the relativistic Yukawa coupling to second order in the coupling constant. In the next paper we give the results of a numerical solution of the relativistic pressure (4.5) and energy density (3.18) in terms of the self-energy (4.7), which has been regularized according to (4.26). As a result of this calculation, we will show that the pressure and energy density approach the asymptotic values

$$\lim_{p_F/m\to\infty} P \to \tfrac{1}{4}ap_Fcn\equiv P_\infty, \qquad (5.1)$$

$$\lim_{p_F/m\to\infty} \rho \to 3P_\infty, \quad (5.2)$$

which leads to an asymptotic limit on the velocity of sound[29]

$$v_s \leq \frac{c}{\sqrt{3}}. \quad (5.3)$$

The results of this numerical calculation will be approximated by an analytic expression for P and ρ, and the results discussed. The numerical results are being used as the basis for a calculation of the critical mass and structure of a star at the end point of thermonuclear evolution. This is the first calculation to take into consideration in a detailed manner the relativistic equation of state which includes strong interactions in obtaining the maximum mass above which a cold catalyzed system of baryons is unstable against gravitational collapse to a curvature singularity.

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*This work has been partially supported by National Science Foundation Grant No. GP-34639X.

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[18]We use the metric $g_{\mu\nu}=\mathrm{diag}(1, -1, -1, -1)$. Partial derivatives are denoted by $\partial_\mu=\partial/\partial x^\mu$. Natural units $\hbar=c=1$ are employed. The four-vector inner product is denoted by $p\cdot x=p^\mu x_\mu$, and $\not{p}=\gamma^\mu p_\mu$. Greek indices run from 0 to 3, latin indices from 1 to 3, and all repeated indices are to be summed over the appropriate range. Quantities referring to antiparticles will be denoted by the same generic symbol as used for particles with the addition of a bar; e.g., $\bar{p}_F$ is the antiparticle Fermi momentum.

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[20]In general the system need not be translationally invariant, in which case the Green's functions are given in terms of the two-point functions $G_F(x, x'; \rho, \beta)$, etc. Throughout our discussion translational invariance will be assumed.

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[22]For a nonrelativistic treatment see A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinsky, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, New Jersey, 1963).

[23]The argument parallels the nonrelativistic case (see Ref. 22) except that one deals with four-component spinor fields in place of Pauli spin functions.

[24]This is derived and discussed within the context of relativistic thermodynamics in Ref. 1. In (3.18) we have expressed the pressure as a function of the variable $y=q_F$ for a single-component system (e.g., particles only).

[25]Equations (4.2)-(4.7) are expressed in terms of the bare masses m_0 and μ_0 and the bare charge g_0.

[26]The self-energy corresponding to Fig. 3 contains a factor $\mathrm{tr}\,\gamma^5 S_F(p, q_F)$, which may be decomposed into a sum of two terms, one proportional to γ^5 and the second to $\gamma^5\gamma^\mu$. Equations (2.3)-(2.4) and the cyclical invariance of the trace may be used to show that these terms vanish. Consequently the self-energy vanishes identically.

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[29]The coefficient a, which is a function of the coupling constant, represents the net effect of interactions in the asymptotic regime. This feature will be discussed in greater detail in the following paper.