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Focusing of Gravitational Radiation into the Galactic Plane

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We consider rays of gravitational radiation emitted near the event horizon of a maximally rotating black hole at the galactic center, aligned with the galactic rotation. If the emitting matter is concentrated in the equatorial plane of the black hole, the focusing effect of the metric will be concentrated in the galactic plane. An average intensification of about an order of magnitude results at the earth.

The bursts of gravitational radiation reported by Weber[1] seem to originate at the galactic center. If so, the assumption that the radiation is isotropic implies a galactic mass-loss rate ($dM/dt \sim 10^3 M_\odot/\text{yr}$) one or two orders of magnitude too large to be consistent with observed galactic dynamics ($dM/dt \lesssim 70 M_\odot/\text{yr}$).[2] Source models which radiate preferentially into the galactic plane, with which the earth is closely aligned, avoid this problem. A class of such models recently proposed by Misner *et al.*[3] involves gravitational synchrotron radiation (GSR) from galactic matter moving near a large ($M \sim 10^7 - 10^8 M_\odot$) centrally located black hole (BH). If the matter moves at relativistic speeds in the galactic plane near the event horizon, then the GSR effect will both concentrate the radiation in the plane and increase its frequency to match the kilohertz frequency of Weber's detector. Bardeen[4] has pointed out, however, that, because of the accretion of rotating galactic matter, a central BH would have nearly the maximum angular momentum (aligned with that of the galaxy) consistent with collapse. This introduces the difficulty that stable, circular orbits near the event horizon are not sufficiently relativistic to produce GSR. Thus one must rely on matter plunging into the event horizon.

It is generally assumed that matter which could give rise to radiation in the neighborhood of a BH lies in a disk rotating in the equatorial plane.[4,5] If this is so, then focusing would arise from the well-known gravitational lens effect,[6] the equatorial concentration of the emitting matter giving focusing of the radiation into the galactic plane. To discuss this we consider the behavior of null geodesics near the equatorial plane of a maximally rotating Kerr metric[7] in Boyer-Lindquist coordinates[4,8]:

$$ds^2 = \rho^2 \Delta^{-1} dr^2 + \rho^2 d\theta^2 - \rho^{-2} \sin^2\theta [M dt - (r^2 + M^2) d\phi]^2 - \rho^{-2} \Delta (dt - M \sin^2\theta \, d\phi)^2 , \quad (1)$$

where $\rho^2 = r^2 + M^2 \cos^2\theta$, $\Delta = (r - M)^2$ and where $\theta = \frac{1}{2}\pi$ is presumed to correspond to the galactic plane. We consider a ray of gravitational radiation emitted from a point $\theta = \frac{1}{2}\pi$ and $r = M + \delta$, $\delta \ll M$. For the radiation to emerge without a very strong red shift ($\sim M/\delta$), it must do so with z component of angular momentum/unit energy $\sim 2M$.[4,9] We thus consider a ray emitted at a small angle $\epsilon_0 \ll 1$ with respect to the equatorial plane *when observed in a locally nonrotating, inertial frame of reference at the point of emission.*[10] The purpose of this choice of initial condition is to separate focusing effects due to the emitter's velocity relative to the inertial frame from the effect of the gravitational focusing. The ray will then follow a null geodesic of the metric,[11] which will be given by the equations of motion of Carter[12] and Wilkins.[13] These give the equation

$$\frac{d\epsilon}{dr} = \pm 2M\epsilon_0(1 - 3\epsilon^2/4\epsilon_0^2)^{1/2} \times [r(r+2M)(r-M)^2]^{-1/2} \quad (2)$$

to lowest order in ϵ, ϵ_0, where $\theta - \frac{1}{2}\pi \equiv \epsilon \ll 1$. The sign of Eq. (2) depends on initial conditions, and changes each time ϵ^2 reaches its maximum value $\frac{4}{3}\epsilon_0^2$. Equation (2) is easily integrated over the intervals $0 \leq \epsilon \leq \epsilon_1$ and $M + \delta \leq r \leq \infty$ to obtain ϵ_1, the asymptotic value of ϵ:

$$\epsilon_1 = (2\epsilon_0/\sqrt{3}) \sin[\ln(M/\delta)] \; . \quad (3)$$

Radiation emitted between ϵ_0 and $\epsilon_0 + d\epsilon_0$ will emerge at infinity between ϵ_1 and $\epsilon_1 + d\epsilon_1$. Thus from Eq. (3) we find the intensification of radiation at infinity compared to that at $r = M$ observed by the locally nonrotating, inertial observer:

$$I \equiv \left| \frac{d\epsilon_0}{d\epsilon_1} \right| = \tfrac{1}{2}\sqrt{3}\,|\csc[\ln(M/\delta)]| \; . \quad (4)$$

It is interesting to note here that one can repeat this calculation for the case of a ray emitted in a Schwarzschild metric from a point $r = 3M + \delta$, with asymptotic angular momentum $3\sqrt{3}\,M$. One then finds that $I_{\text{Kerr}}/I_{\text{Sch}} = \frac{1}{2}\sqrt{3} < 1$, so that the Kerr case focuses less strongly than the Schwarzschild. This seems connected to the fact that the energy of the ray is red-shifted between the nonrotating inertial frame at emission and infinity by a factor of 2 in the Kerr case and by a factor of $\sqrt{3}$ in the Schwarzschild case. A discussion of the focusing of radiation by a Schwarzschild metric has recently been given by Campbell and Matzner.[14] Although their general approach is rather different from ours, their results are consistent with those of the present work.

Radiation concentrated within an angle of the galactic plane less than the earth's galactic latitude will not be detectable at the earth. Thus the infinities of Eq. (4), which arise from the geometrical optics approximation, will be cut off at an appropriate maximum intensification I_m, which will depend on the galactic latitude of the earth (ϵ_E).

As $\delta \to 0$, the maxima of I will be very close together, so that a finite source in the zone in which it is most likely to radiate should do so over a number of maxima, and averaging is required. Since the cosecant function is unwieldy, we approximate it by a series of δ-function peaks, which are appropriately weighted to take account of the intensification cutoff required by the earth's location. Thus we write

$$|\csc[\ln(M/\delta)]| \sim W \sum_{n=-\infty}^{\infty} \delta(\ln(M/\delta) - n\pi) \, , \quad (5)$$

with

$$W = \int_{\xi}^{\pi-\xi} dx \csc x = 2 \ln|\cot \xi/2| \; , \quad (6)$$

where $\sin\xi = \sqrt{3}/2I_m$. We find the average by integrating δ over several δ-function peaks and dividing by the appropriate interval:

$$\langle I \rangle \sim \tfrac{1}{2}\sqrt{3}\; W \, . \quad (7)$$

Let us now consider the case in which the source, either by GSR or some other mechanism, emits essentially all of its radiation within the angle ϵ_0 of the galactic plane as seen by the locally nonrotating, inertial observer. At infinity the radiation will emerge between the galactic latitudes $\pm 2\epsilon_0/\sqrt{3}$. If the earth is so located that $|\epsilon_E| \geq 2\epsilon_0/\sqrt{3}$, the radiation will not be detectable. Thus $I_m = \epsilon_0/\epsilon_E$. If we write $\alpha \equiv 2\epsilon_0/\sqrt{3}\,\epsilon_E$, Eqs. (6) and (7) give

$$\langle I \rangle = \sqrt{3}\, \ln[\alpha + (\alpha^2 - 1)^{1/2}] \, . \quad (8)$$

If I_T represents the intensity of the radiation observed at the earth divided by that which would be observed if the radiation escaped isotropically, then

$$I_T \sim (1/\alpha\epsilon_E) \ln[\alpha + (\alpha^2 - 1)^{1/2}] \, . \quad (9)$$

I_T is maximized for $\epsilon_0 \sim \epsilon_E$, where $I_T \sim 1/\epsilon_0$.

If, on the other hand, $\epsilon_0 \gg \epsilon_E$, then

$$\langle I \rangle \sim 4 \log_{10}(\epsilon_0/\epsilon_E) \, . \quad (10)$$

Averaging this over ϵ_E from zero to a fixed value of ϵ_E gives $\langle I \rangle_{\text{av}} \sim \langle I \rangle$. Thus we need not worry about large amounts of undetected radiation at smaller galactic latitudes than the earth's. For $\epsilon_0/\epsilon_E \sim 10^3$ we have $\langle I \rangle \sim 10$. In this case, nearly all of the intensification is due to the focusing effect. (Note that $\epsilon_E \lesssim 10^{-3}$ rad.[15]) In such a situation, one might envision, with Bardeen,[4] that large amounts of matter exist in circular orbits about the BH with $\theta=\frac{1}{2}\pi$, $r=M+\delta$, $\delta\ll 1$. This matter might include many sources of gravitational radiation with natural frequencies in the kilohertz range (e.g., collapsing stars). The focusing effect would be sufficiently large that only a small GSR-type effect would be necessary to bring Weber's results into line with estimated galactic mass-loss rates. Such a situation would also be desirable in that the resulting radiation should be less strongly polarized than in the pure GSR case. Analysis of Weber's data seems to indicate a lack of polarization.[16] Finally, the author's attention has been called to recent related work of Bardeen and Cunningham[17] concerning the optical appearance of isotropically radiating sources orbiting in the maximally rotating Kerr metric.

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[1]J. Weber, Phys. Rev. Letters 25, 180 (1970); Sci. Am. 224, 22 (1971).

[2]D. W. Sciama, G. F. Field, and M. J. Rees, Phys. Rev. Letters 23, 1514 (1969).

[3]C. W. Misner, Phys. Rev. Letters 28, 994 (1972); C. W. Misner, R. A. Breuer, D. R. Brill, P. R. Chrzanowski, H. G. Hughes III, and C. M. Pereira, *ibid.* 28, 998 (1972).

[4]J. M. Bardeen, Nature (London) 226, 64 (1970).

[5]D. Lynden-Bell, Nature (London) 223, 690 (1969).

[6]S. Liebes, Phys. Rev. 133, B835 (1964).

[7]R. P. Kerr, Phys. Rev. Letters 11, 522 (1963).

[8]R. H. Boyer and R. W. Lindquist, J. Math. Phys. 8, 265 (1967).

[9]J. M. Bardeen and R. V. Wagoner, Astrophys. J. 167, 359 (1971).

[10]J. M. Bardeen, Astrophys. J. 162, 71 (1970).

[11]R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965), p. 220.

[12]B. Carter, Phys. Rev. 174, 1559 (1968).

[13]D. C. Wilkins, Phys. Rev. D 5, 814 (1972).

[14]G. A. Campbell and R. A. Matzner, J. Math. Phys. 14, 1 (1973).

[15]C. S. Gum, F. J. Kerr, and G. Westerhout, Monthly Notices Roy. Astron. Soc. 121, 132 (1960).

[16]J. A. Tyson and D. H. Douglass, Phys. Rev. Letters 28, 991 (1972).

[17]C. T. Cunningham and J. M. Bardeen, Astrophys. J. 173, L137 (1972).