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Duality, the Generalized Potential, and Failure of the ρ Bootstrap

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A model, explicitly consistent with duality, is obtained for the generalized potential term $B(s)$ in the partial-wave dispersion relation. We then observe that for the elastic $I = 1$ $\pi\pi$ amplitudes, $B(s) = 0$ (except for the breaking of ρ-f^0 exchange degeneracy). Thus the classic N/D dynamical calculations of the ρ meson using elastic unitarity must fail in this model.

There have been many dynamical calculations of resonances performed by solving partial-wave dispersion relations. The left-hand-cut term or generalized potential $B(s)$ is obtained from crossed-channel processes, and the unitarized amplitude

$$f(s) = B(s) + \int_{\text{physical cut}} \frac{\operatorname{Im} f(s')}{s' - s - i\epsilon} ds' \qquad (1)$$

is calculated using the N/D formalism. Despite the lack of quantitative success of such calculations, they continue to be made with increased sophistication in the hope of obtaining the known properties of resonances.

The duality concept relating Regge exchange in the crossed channels to direct-channel resonances raises the question of whether one has double-counted in such calculations. We present a simple prescription for the potential B starting from the Veneziano representation,[1] which is consistent with duality. We then observe that the classic calculations of the ρ meson using elastic unitarity must necessarily fail in this model.

Consider the explicitly crossing-symmetric, dual Veneziano representation which consists of sums of terms $A(s,t)$, $A(s,u)$, $A(t,u)$, each of the form

$$A_{ab}(s,t) = \text{polynomial}(s,t) \times \frac{\Gamma(1-\alpha_a(s))\Gamma(1-\alpha_b(t))}{\Gamma(1-\alpha_a(s)-\alpha_b(t))}. \qquad (2)$$

More generally, we can take for our A terms the model of Cohen-Tannoudji *et al.*,[2] which, in addition to being dual and crossing-symmetric, has Mandelstam analyticity. Our main observation follows the reasoning behind the successful dual interference model.[3] We observe that both the $A(s,u)$- and $A(s,t)$-type terms are dual to direct-channel resonances and to the Regge-exchange terms for both the real and imaginary parts. Thus, if we start with a partial-wave generalized potential B and solve (1) to include the direct-channel unitarity cut, we should not include any (real) pieces of $A(s,t)$ or $A(s,u)$ in our potential B, in order not to double-count the resonance or direct-channel contribution. Hence our prescription for $B(s)$ is simply to take the jth partial-wave projection of the $A(t,u)$ terms:

$$B(s) = \{A(t,u)\}_{j\text{th partial wave}}\,. \qquad (3)$$

The model (3) gives us a trivial answer to the old problem of generating the ρ meson in an N/D calculation.[4] One determines a potential term B obtained from meson exchanges and unitarizes B by solving (1) using elastic unitarity. For $I=1$ $\pi\pi$ scattering there is no $A(t,u)$ term, since the amplitude must be antisymmetric in $t \leftrightarrow u$ and the $A(t,u)$ term (2) is symmetric due to ρ-f^0 exchange degeneracy. Thus $B=0$ for the elastic channel.

To see this result on a more physical basis, consider ρ and f^0 Regge exchange for the $I=1$ and $I=2$ $\pi\pi$ amplitudes. From crossing, we have for the direct s channel

$$I=2:$$
$$\tfrac{1}{3}[\gamma_f(t)(e^{i\pi\alpha_f(t)}+1)s^{\alpha_f(t)}] - \tfrac{1}{2}[\gamma_\rho(t)(e^{i\pi\alpha_\rho(t)}-1)s^{\alpha_\rho(t)}],$$
$$I=1: \qquad (4)$$
$$\tfrac{1}{3}[\gamma_f(t)(e^{i\pi\alpha_f(t)}+1)s^{\alpha_f(t)}] + \tfrac{1}{2}[\gamma_\rho(t)(e^{i\pi\alpha_\rho(t)}-1)s^{\alpha_\rho(t)}].$$

Since there are no $I=2$ resonances, duality forces the restriction of exact ρ-f^0 exchange degeneracy:

$$\alpha_\rho(t) = \alpha_f(t),$$
$$\gamma_\rho(t) = \tfrac{2}{3}\gamma_f(t),$$

so that for the $I=2$ amplitude the imaginary parts of the Regge exchange from the $e^{i\pi\alpha(t)}$ terms cancel. Now we see that for the $I=1$ amplitude this forces the nonsignatured pieces of the Regge exchanges to cancel. There is, of course, a real part in the $I=1$ amplitude coming from the real part of the "signatured" $e^{i\pi\alpha(t)}$ terms. However, these are dual to the real parts of the direct-channel resonances and must not be included in $B(s)$.

As discussed in Ref. 3, the $A(s,t)$ terms (2) give the signatured $e^{i\pi\alpha(t)}$ part of the Regge exchange and are dual to the direct-channel resonances, whereas the $A(t,u)$ terms give the nonsignatured parts of the Regge exchanges.

In the physical region of the t channel, $t > 4m_\pi^2$, $A(s,t)$ can be expanded as a series of t-channel poles $\sum_n c_n(s)/(t-n)$. If we make a partial-wave projection of each of these pole terms in the s channel, each would contribute a potential term to $B(s)$. However, for the physical s region $s > 4m_\pi^2$, $A(s,t)$ can be expanded as a series of s-channel poles and no $B(s)$ terms. Thus we understand that the old N/D calculations of the ρ, which kept only a few pole terms (or resonances) in the t channel, generated $B(s)$ terms at the expense of losing duality.

We see from (4) that, in our duality model, only the *breaking* of the ρ-f^0 exchange degeneracy would yield a nonsignatured Regge exchange term and hence an elastic $I=1$ $\pi\pi$ potential term.

Thus we conclude that it seems extremely likely that a dynamical calculation of the ρ meson based on elastic unitarity must fail if it is consistent with duality. This, of course, is not to be confused with the consistency-type calculations based on finite-energy sum rules[5] which demonstrate the duality relation for the $I=1$ $\pi\pi$ amplitude between the direct-channel ρ resonance and the Regge exchange amplitudes.

It is possible that the explicit consideration of inelastic higher-mass channels (such as $\pi\pi \to \pi\omega$, $\pi\pi \to N\bar{N}$, etc.) using the multichannel ND^{-1} equation might generate the ρ within the content of our model for the potential B terms.

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