PHYSICAL REVIEW D VOLUME 7, NUMBER 7 1 APRIL 1973

# Experimental Tests for Exotic States in the Baryon-Antibaryon System*

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(Received 28 July 1972)

New experiments are proposed in which negative as well as positive results are significant. A new finite-energy sum rule based on dominance of $\rho$ and $\omega$ exchange for high-energy odd-signature amplitudes gives quantitative estimates for production of doubly charged exotic meson resonances via $\Delta$ exchange in nucleon-antinucleon and pion-nucleon reactions. Absence of exotic resonances with predicted cross sections would indicate serious disagreement with two-component duality.

The search for exotic resonances has made little progress since the first suggestion that resonances with exotic quantum numbers need not be coupled to the common two-body meson-meson and meson-baryon systems.[1] Search experiments can be either (1) *conclusive* or (2) *inconclusive*. The exotic-resonance search experiments suggested on theoretical grounds[1-4] can be conclusive *only if a positive result is obtained*. So far all results have been negative and inconclusive. In this note experiments are proposed which are always conclusive, *even if negative results are obtained*. Any result thus leads to some progress in the understanding of the exotic-resonance problem.

Strong theoretical arguments for the existence of some kind of exotic contribution in baryon-antibaryon scattering have been given by finite-energy sum rules. These require an exotic $s$-channel contribution[2-4] to amplitudes that have no diffractive component at high energy – e.g., charge-exchange amplitudes.[5] Whether this exotic component is resonant or nonresonant is still unknown. Three possible answers to this "baryon-antibaryon catastrophe" have been suggested.

1. *Exotic resonances.*[2] These are required in the "two-component-duality description,"[6] which has nonresonant background appearing in the imaginary parts of only those amplitudes that have a diffractive component at high energies. This description is supported by experimental data in nonexotic cases, in which it has been shown that the imaginary part of a nondiffractive amplitude can be approximated very well with resonances alone.[7]

2. *A nonresonant "third component."* This might be associated with annihilation in baryon-antibaryon scattering.[3] The success of the $\omega$ universality relation,[8] which links nucleon-antinucleon and kaon-nucleon scattering in the Regge region, implies that such a third component cannot exhibit anomalous behavior at high energies, but must combine with the resonances at low energies to satisfy a FESR (finite-energy sum rule) together with the conventional Regge component at high energies.

3. *The null solution.*[9] The finite-energy sum rule is trivially satisfied if all nondiffractive baryon-antibaryon amplitudes involving decuplet baryons vanish.

So far there has been no experimental test of these alternatives. Experimental investigation of baryon-antibaryon exotics has been difficult. The accessible $N\bar{N}$ system has no exotic channels; and while exotics have been proved to exist[5] in the $\Delta\bar{\Delta}$ system, it is not directly accessible to experiment. Between these two extremes is the $\bar{N}\Delta$ system which has an exotic $I=2$ channel accessible to experimental study in reactions involving $\Delta$ exchange on nucleon targets. However, one consistent solution of the finite-energy sum rules for this system has a vanishing exotic amplitude, and this possibility has thus far prevented any conclusive quantitative investigation of exotics in this system.[10]

The purpose of this note is to show how a new type of superconvergence sum rule[11,12] enables one to test the above three alternatives by means of a quantitative experimental search for exotic states. This sum rule is very nearly model-independent and does not assume SU(3); and by use of it in the discussion below, the cross sections for the production of exotic states by $\Delta$ exchange are shown to be comparable to cross sections for the production of nonexotic resonances by $\Delta$ exchange in related reactions accessible to experiment. The sum rule is based on the experimental observation that odd-signature-exchange amplitudes are dominated at high energies by exchange of the $\rho$ and $\omega$ trajectories. A finite-energy sum rule can therefore be written for a linear combination of amplitudes with contributions from both $\rho$ and $\omega$ exchange, with coefficients adjusted to make the $\rho$ and $\omega$ contributions cancel on the right-hand side. Thus for the $N\Delta$ system, the amplitudes $A(N\Delta)$ and $A(\bar{N}\Delta)$ in different charge states are related by the sum rule

$$\int_0^N \nu^n\{[A(\bar{p}\Delta^{++}) - A(p\Delta^{++})](1-\tau) - [A(\bar{p}\Delta^{-}) - A(p\Delta^{-})](1+\tau)\}d\nu = 0\,, \tag{1}$$

where the parameter $\tau$ is chosen to make the right-hand side vanish as a result of the cancellation of the $\rho$ and $\omega$ exchange contributions.

If the $\rho$ and $\omega$ trajectories are degenerate, as they seem to be to a good approximation, then $\tau$ is given by the ratio of the products of Regge residues. That is,

$$\tau = \beta_{\rho N\bar{N}}\,\beta_{\rho\Delta\bar{\Delta}} / \beta_{\omega N\bar{N}}\,\beta_{\omega\Delta\bar{\Delta}}\,. \tag{2}$$

In this case $\tau$ is independent of $N$, and $N$ can be taken to be infinite to give a superconvergence sum rule. However, the treatment below *does not require* exact degeneracy of the $\rho$ and $\omega$ trajectories. In the absence of degeneracy $\tau$ depends upon $N$ and is not given by Eq. (2). However, for a given fixed value of $N$, the value of $\tau$ is well defined by $N$ and the Regge parameters of the $\rho$ and $\omega$ trajectories.

It is conventional to assume that the nondiffractive component of the imaginary part of an amplitude can be approximated by resonances[6] in such sum rules. Since the diffractive contribution cancels in the difference between the $\bar{N}\Delta$ and $N\Delta$ amplitudes, we obtain

$$\int_0^N A^R(\bar{p}\Delta^-)\nu^n\,d\nu = \frac{1-\tau}{1+\tau}\int_0^N A^R(\bar{p}\Delta^{++})\nu^n d\nu\,, \tag{3}$$

where $A^R$ denotes the contribution of resonances to the amplitude and we make the conventional assumption that there are no dibaryon resonances.

Equation (3) relates an exotic contribution on the left-hand side to a nonexotic contribution on the right-hand side with a coefficient of order unity. The amplitudes on the two sides of the relation (3) can be measured experimentally in the pairs of reactions

$$\bar{p} + p \to \pi^+ + M^-\,, \tag{4a}$$

$$\bar{p} + n \to \pi^+ + X^{--}\,, \tag{4b}$$

where $M$ denotes a negatively charged meson resonance, $X^{--}$ denotes a doubly-charged exotic state. Both reactions are assumed to go via $\Delta$ exchange when the outgoing pion is in the direction of the incident antiproton with small momentum transfer. In this kinematic region, the cross sections for these two reactions should satisfy the relation (3). The $\Delta$-exchange contributions shown in Fig. 1 differ only in the lower vertex which is described by the relation (3), since the $(\Delta\bar{p}\pi)$ vertex is the same for the two reactions. Present

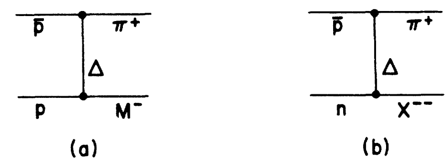


FIG. 1. Contribution of $\Delta$ exchange to reactions (4a) and (4b).

experimental data on reaction (4a) show it to have a non-negligible cross section. Reaction (4b) should therefore be examined to determine the nature of $X^{--}$. If exotic resonances provide the answer to the baryon-antibaryon catastrophe[2] they should appear in this reaction. They are not expected to be coupled to two pions[1,2] and should be observed as $3\pi$ or $4\pi$ states, depending on $G$ parity.[13] The most plausible decay modes are

$$X^{--} \to \pi^- + \pi^- + \pi^0\,, \tag{5a}$$

$$X^{--} \to \pi^- + \pi^- + \pi^+ + \pi^-\,, \tag{5b}$$

$$X^{--} \to K^- + K^0 + \pi^-\,. \tag{5c}$$

If no exotic resonances are found, the nature of the exotic contribution that satisfies the sum rule (3) can be investigated by examining the forward inclusive $\pi^+$ production in the reactions (4). The inclusive cross sections as a function of the missing mass should show peaks associated with quasi-two-body final states on a background due to multiparticle final states. The sum rule (3) for the reactions (4a) and (4b) requires that the cross sections integrated over missing mass should be of the same order of magnitude. Two-component duality together with the sum rule (3) require the quasi-two-body portion of the cross sections integrated over missing mass also to be of the same order of magnitude. Thus if appreciable production of resonances in quasi-two-body reactions is found for the case (4a), while no exotic resonances are found in the exotic case (4b), *there is a clear experimental proof of the violation of two-component duality*. If the nonresonant cross section for the reaction (4b) is comparable to the cross section for (4a), this might be evidence for a third component to satisfy the sum rule. However, if the nonresonant background (final states that are not quasi-two-body) is large for the reaction (4a), the third component in the reaction (4b) would be masked by this nonresonant component that is associated with diffraction at high energy. This component corresponds to the Pomeranchukon-exchange contribution in the diagrams of Fig. 2, which shows the reactions (4) in the triple-Regge region.

The Pomeranchukon-exchange contribution can

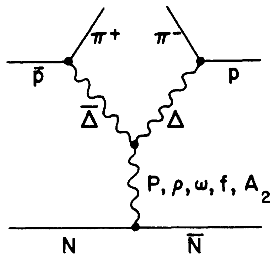


FIG. 2. Reactions (4) in the triple-Regge region.

be eliminated by considering also the reactions

$$p+p\to\pi^-+(BB)^{+++}, \tag{4c}$$

$$p+n\to\pi^-+(BB)^{++}, \tag{4d}$$

where $(BB)$ denotes a dibaryon ($B=2$) state. In the triple-Regge region, the reactions (4c) and (4d) have the same $N\pi\Delta$ vertex as the reactions (4a) and (4b), while the remaining portion of the diagram describes $\Delta N$ scattering, rather than $\bar{\Delta}N$. These reactions thus all have the same Pomeranchukon contribution. The usual exchange-degeneracy arguments require the nondiffractive component to vanish in baryon-baryon amplitudes.[6] Thus the reaction (4d) should be purely diffractive and should give a direct measure of the Pomeranchukon component in the reaction (4b). The difference between the two cross sections then gives a measure of the nondiffractive part of the reaction (4b), and this part should satisfy the sum rule (3).

If exchange-degeneracy arguments break down for the exotic baryon-baryon case, there could be a non-Pomeranchukon contribution to the reaction (4d). In this case the four reactions (4a), (4b), (4c), and (4d) can be used in the original sum rule (1) rather than in the simplified sum rule (3). These reactions provide all the four amplitudes appearing in the sum rule, and their investigation can determine which of the exotic reactions (6b), (6d), and (6e) give contributions to the FESR to balance the production of nonexotic meson resonances $M^-$ in the reaction (4a).

Another set of reactions with similar $\Delta$ exchanges are obtainable by line reversal from the reactions (4).

$$\pi^-+p\to p+M^-, \tag{6a}$$

$$\pi^-+n\to p+X^{--}, \tag{6b}$$

$$\pi^++p\to\bar{p}+(BB)^{+++}, \tag{6c}$$

$$\pi^++n\to\bar{p}+(BB)^{++}, \tag{6d}$$

$$\pi^++n\to n+M^+, \tag{6e}$$

$$\pi^++p\to n+X^{++}, \tag{6f}$$

where the outgoing baryon is in the direction of the incident pion with a small momentum transfer. Reactions (6e) and (6f) are charge-symmetry mirrors of (6a) and (6b) and may be more convenient for experimental comparisons. Reactions (6a), (6f), and (6c) all have proton targets and can be used to test the sum rule (3) under the assumption that the baryon-baryon amplitude is purely diffractive. The reactions (6c), (6d), (6e), and (6f) are in principle all measurable in the same $\pi^+d$ experiment, and can be used in the sum rule (1).

The above argument breaks down if $\tau=1$ and the sum rule (1) is trivially satisfied with no exotic contributions. However, it seems reasonable to overlook this pathological value. By symmetry arguments, the ratio of the nucleon residues appearing in Eq. (2) has been predicted to be considerably smaller than unity, and this is checked by experimental observation. Thus $\tau=1$ implies both that $g_{\rho\Delta\bar{\Delta}}/g_{\omega\Delta\bar{\Delta}}\gg1$ and that this ratio has a peculiar accidental value. Although there are no experimental measurements of these couplings, this accident appears highly unlikely. Furthermore, symmetry schemes and quark models predict[14,15] that $g_{\rho\Delta\bar{\Delta}}=g_{\omega\Delta\bar{\Delta}}$, and this value is also required to avoid exotics in the $K\Delta$ system[15] for which a sum rule analogous to (1) can be written with the ratio $g_{\rho\Delta\bar{\Delta}}/g_{\omega\Delta\bar{\Delta}}$ replacing the parameter $\tau$. The pathological value $\tau=1$ eliminates the difficulty with exotic states in the $\bar{N}\Delta$ sum rule (1), but the analogous $K\Delta$ sum rule then requires the existence of doubly charged $Y^*$ resonances coupled to the $K\Delta$ system, while still keeping the baryon-antibaryon exotic states in the $\Delta\bar{\Delta}$ system. Although there is no conclusive proof against this possibility, it does seem rather farfetched.

In any case, experimental investigation of the reactions (4) and (6) should give new insight into the question of what exotic contributions to these reactions enable the baryon-antibaryon system to satisfy the finite-energy sum rules.

It is a pleasure to thank G. F. Chew for stimulating discussions and for calling Ref. 12 to my attention. Discussions with M. Jacob, S. Meshkov, and J. L. Rosner are also gratefully acknowledged.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†On leave from Weizmann Institute of Science, Rehovoth, Israel.

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[13]Equal contributions from even-$G$ and odd-$G$ exotic states are required by the FESR for the reaction $\bar{N}\Delta \to N\bar{\Delta}$ in the $I=2$ state. The vanishing of the sum rule, required by the exotic dibaryon exchange, is achieved by having equal and opposite contributions from even- and odd-$G$ resonances.

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PHYSICAL REVIEW D VOLUME 7, NUMBER 7 1 APRIL 1973

# Duality, the Generalized Potential, and Failure of the $\rho$ Bootstrap

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(Received 21 August 1972)

A model, explicitly consistent with duality, is obtained for the generalized potential term $B(s)$ in the partial-wave dispersion relation. We then observe that for the elastic $I=1$ $\pi\pi$ amplitudes, $B(s)=0$ (except for the breaking of $\rho$-$f^0$ exchange degeneracy). Thus the classic $N/D$ dynamical calculations of the $\rho$ meson using elastic unitarity must fail in this model.

There have been many dynamical calculations of resonances performed by solving partial-wave dispersion relations. The left-hand-cut term or generalized potential $B(s)$ is obtained from crossed-channel processes, and the unitarized amplitude

$$f(s) = B(s) + \int_{\text{physical cut}} \frac{\operatorname{Im} f(s')}{s' - s - i\epsilon} ds' \tag{1}$$

is calculated using the $N/D$ formalism. Despite the lack of quantitative success of such calculations, they continue to be made with increased sophistication in the hope of obtaining the known properties of resonances.

The duality concept relating Regge exchange in the crossed channels to direct-channel resonances raises the question of whether one has double-counted in such calculations. We present a simple prescription for the potential $B$ starting from the Veneziano representation,[1] which is consistent with duality. We then observe that the classic calculations of the $\rho$ meson using elastic unitarity must necessarily fail in this model.

Consider the explicitly crossing-symmetric, dual Veneziano representation which consists of sums of terms $A(s,t)$, $A(s,u)$, $A(t,u)$, each of the form

$$A_{ab}(s,t) = \text{polynomial}(s,t) \times \frac{\Gamma(1-\alpha_a(s))\Gamma(1-\alpha_b(t))}{\Gamma(1-\alpha_a(s)-\alpha_b(t))}. \tag{2}$$

More generally, we can take for our $A$ terms the model of Cohen-Tannoudji *et al.*,[2] which, in addition to being dual and crossing-symmetric, has Mandelstam analyticity. Our main observation follows the reasoning behind the successful dual interference model.[3] We observe that both the $A(s,u)$- and $A(s,t)$-type terms are dual to direct-channel resonances and to the Regge-exchange terms for both the real and imaginary parts. Thus, if we start with a partial-wave generalized potential $B$ and solve (1) to include the direct-channel unitarity cut, we should not include any (real) pieces of $A(s,t)$ or $A(s,u)$ in our potential $B$, in