lar process

$$e \to e + t^0, \qquad (3)$$

then one obtains a lower limit to the lifetime $\approx 2.0 \times 10^{15}$ years in the case of electrons and $\approx 1.3 \times 10^{13}$ years in the case of free protons. Setting $\mu^2/2m \lesssim 100$ eV, we obtain $\mu \lesssim 4.3 \times 10^5$ and 1.0×10^4 eV for the decays of free protons and electrons, respectively. The limits on the imaginary masses of tachyons derived here are lower than those in Ref. 1 (but lifetimes are smaller, resulting in larger limits to the coupling constant). It must be emphasized, however, that the limits on the neutral tachyon masses set here as well as in Ref. 1 are meaningful only if the coupling constant mediating reactions (1) and (2) is sufficiently large *and*, in the case of reaction (2), if the fluxes of neutral tachyons are adequate.

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Leptonic Decays of Vector Mesons and a Unitary-Singlet Contribution to the Hadronic Electromagnetic Current*

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Following some original ideas of Salam, Mathur, and Okubo, further theoretical consideration is given to a possible singlet contribution to the electromagnetic current of hadrons using spectral sum rules connecting the leptonic decays of vector mesons. Unfortunately, experimental uncertainty prevents meaningful analysis and conclusion.

I. INTRODUCTION

Much attention has centered on the sum rule relating the leptonic decays of the vector mesons ρ, ϕ, and ω:

$$\tfrac{1}{3} m_\rho \Gamma(\rho \to l\bar{l}) = m_\omega \Gamma(\omega \to l\bar{l}) + m_\phi \Gamma(\phi \to l\bar{l}), \qquad (1.1)$$

which is based on vector dominance,[1] Weinberg's first sum rule,[2] and a hadronic electromagnetic current of the usual form,

$$j_\mu^{\text{em}}(x) = V_\mu^3(x) + \frac{1}{\sqrt{3}} V_\mu^8(x), \qquad (1.2)$$

where $V_\mu^a(x)$, $a = 0, 1, \ldots, 8$, are a nonet of vector currents satisfying the chiral algebra.[3] Implications and modifications induced by inclusion of a singlet component in the electromagnetic current can be studied through the sum rules of Weinberg,[2] Oakes and Sakurai,[4] and Das, Mathur, and Okubo,[5] in both the SU(3) and U(3) symmetry limits, by writing the electromagnetic current,

$$j_\mu^{\text{em}}(x) = V_\mu^3(x) + \frac{1}{\sqrt{3}} V_\mu^8(x) + \gamma V_\mu^0(x), \qquad (1.3)$$

with γ some constant to be determined.

II. SUM RULES AND MATRIX ELEMENTS

The vacuum to single-particle current matrix elements are written[6]

$$\begin{aligned}
\langle 0 | V_\mu^{1,2,3}(0) | \rho \rangle &= g_\rho \epsilon_\mu(k), \\
\langle 0 | V_\mu^{4,5,6,7}(0) | K^* \rangle &= g_{K^*} \epsilon_\mu(k), \\
\langle 0 | V_\mu^8(0) | \omega \rangle &= g_\omega \epsilon_\mu(k), \\
\langle 0 | V_\mu^8(0) | \phi \rangle &= g_\phi \epsilon_\mu(k), \qquad (2.1)\\
\langle 0 | V_\mu^0(0) | \omega \rangle &= h_\omega \epsilon_\mu(k), \\
\langle 0 | V_\mu^0(0) | \phi \rangle &= h_\phi \epsilon_\mu(k), \\
\langle 0 | V_\mu^{4,5,6,7}(0) | \kappa \rangle &= f_\kappa k_\mu,
\end{aligned}$$

where the vector mesons $(\rho, K^*, \phi, \omega)$ carry polarization $\epsilon_\mu(k)$ and momentum k. The κ scalar excitation is of second order in the breaking of symmetry, but is retained in what follows. Weinberg's first sum rules for chiral U(3) and SU(3) take the forms, respectively,

$$\int \frac{\rho_{ab}^{(1)}(m^2)}{m^2} + \rho_{ab}^{(0)}(m^2)\,dm^2 = s\delta_{ab}\,,$$
$$\int \frac{\rho_{ab}^{(1)}(m^2)}{m^2} + \rho_{ab}^{(0)}(m^2)\,dm^2 = s\delta_{ab} + s'\delta_{a0}\delta_{b0}\,, \tag{2.2}$$
$$a, b = 0, 1, 2, \ldots, 8$$

where $\rho^{(1)}$ and $\rho^{(0)}$ are the spin-1 and spin-0 spectral functions of the vector currents, diagonal in all indices except 0 and 8. In the meson dominance scheme,

$$\begin{aligned}
&\rho_{1,2,3}^{(1)}(m^2) = g_\rho^{\;2}\delta(m^2 - m_\rho^{\;2})\,,\\
&\rho_{4,5,6,7}^{(1)}(m^2) = g_{K^*}^{\;2}\delta(m^2 - m_{K^*}^{\;2})\,,\\
&\rho_8^{(1)}(m^2) = g_\omega^{\;2}\delta(m^2 - m_\omega^{\;2}) + g_\phi^{\;2}\delta(m^2 - m_\phi^{\;2})\,,\\
&\rho_0^{(1)}(m^2) = h_\omega^{\;2}\delta(m^2 - m_\omega^{\;2}) + h_\phi^{\;2}\delta(m^2 - m_\phi^{\;2})\,,\\
&\rho_{08}^{(1)}(m^2) = \rho_{80}^{(1)}(m^2)\\
&\qquad = g_\omega h_\omega\delta(m^2 - m_\omega^{\;2}) + g_\phi h_\phi\delta(m^2 - m_\phi^{\;2})\,,\\
&\rho_{4,5,6,7}^{(0)}(m^2) = f_\kappa^{\;2}\delta(m^2 - m_\kappa^{\;2})\,.
\end{aligned} \tag{2.3}$$

Applied to U(3) or SU(3), the equations are modified, respectively,

$$\begin{aligned}
&\int \frac{\rho_{ab}^{(1)}(m^2)}{m^2}\,dm^2 = s_1\delta_{ab}\,,\\
&\int \rho_{ab}^{(0)}(m^2)\,dm^2 = s_0\delta_{ab}\,,\\
&\int \frac{\rho_{ab}^{(1)}(m^2)}{m^2}\,dm^2 = s_1\delta_{ab} + s_1'\delta_{a0}\delta_{b0}\,,\\
&\int \rho_{ab}^{(0)}(m^2)\,dm^2 = s_0\delta_{ab} + s_0'\delta_{a0}\delta_{b0}\,,
\end{aligned} \tag{2.4}$$

where, obviously, $s_1 + s_0 = s$, $s_1' + s_0' = s'$. Equations (2.4), thus, are more restrictive than their chiral counterparts, Eqs. (2.2), and are equivalent when $f_\kappa = 0$.[7] From Eqs. (2.2), we find for the case of chiral U(3) symmetry

$$\begin{aligned}
\frac{g_\rho^{\;2}}{m_\rho^{\;2}} &= \frac{g_{K^*}^{\;2}}{m_{K^*}^{\;2}} + f_\kappa^{\;2}\\
&= \frac{g_\phi^{\;2}}{m_\phi^{\;2}} + \frac{g_\omega^{\;2}}{m_\omega^{\;2}}\\
&= \frac{h_\phi^{\;2}}{m_\phi^{\;2}} + \frac{h_\omega^{\;2}}{m_\omega^{\;2}}\,,\\
\frac{h_\phi g_\phi}{m_\phi^{\;2}} &+ \frac{h_\omega g_\omega}{m_\omega^{\;2}} = 0\,,
\end{aligned} \tag{2.5}$$

while identical results hold for chiral SU(3) with the provision that the term involving $h_\phi^{\;2}$ and $h_\omega^{\;2}$ in the first of Eqs. (2.5) is suppressed.

With an electromagnetic current of the form given in Eq. (1.3), the leptonic decay rate of ρ, ω, and ϕ is given by

$$\Gamma(V \to l\bar{l}) = \frac{e^4}{12\pi}\,\frac{c_V^{\;2}}{m_V^{\;3}}\left[1 + O\left(\left(\frac{m_e}{m_V}\right)^4\right)\right]\,, \tag{2.6}$$

with $c_V = g_\rho$, $(1/\sqrt{3})g_\omega + \gamma h_\omega$, $(1/\sqrt{3})g_\phi + \gamma h_\phi$ for $V = \rho$, ω, and ϕ. Using Eqs. (2.5) and (2.6), it follows that Eq. (1.1) is modified by the singlet piece,

$$\tfrac{1}{3}(1+\epsilon)m_\rho\Gamma(\rho \to l\bar{l}) = m_\omega\Gamma(\omega \to l\bar{l}) + m_\phi\Gamma(\phi \to l\bar{l})\,, \tag{2.7}$$

with, for chiral U(3) symmetry,

$$\epsilon = 3\gamma^2 \tag{2.8}$$

and, for chiral SU(3) symmetry,

$$\epsilon = 3\gamma^2\left(\frac{h_\omega^{\;2}}{m_\omega^{\;2}} + \frac{h_\phi^{\;2}}{m_\phi^{\;2}}\right)\frac{m_\rho^{\;2}}{g_\rho^{\;2}}\,. \tag{2.9}$$

The sum rules of Oakes and Sakurai (OS) take the general form for the vector mesons,

$$\int \frac{\rho_{ab}^{(1)}(m^2)}{m^4}\,dm^2 = \Delta_{ab}\,, \tag{2.10}$$

and those of Das, Mathur, and Okubo[5] (DMO),

$$\int \rho_{ab}^{(1)}(m^2)\,dm^2 = \Delta_{ab}\,, \tag{2.11}$$

for Δ_{ab} an isospin-conserving, unitary-symmetry-breaking expression written in generality,[8]

$$\begin{aligned}
\Delta_{ab} = A\delta_{ab} &+ Bd_{8ab} + C\delta_{a0}\delta_{b0}\\
&+ D\delta_{a8}\delta_{b8} + E(\delta_{a0}\delta_{b8} + \delta_{a8}\delta_{b0})\,,
\end{aligned} \tag{2.12}$$

where d_{8ab} are the symmetrical structure constants. Attention will focus on octet breaking, $C = D = E = 0$, but application of other breakings is straightforward. Coupling of either the OS or DMO sum rule to Weinberg's first sum rule gives a reduction in the number of arbitrary parameters and definite predictions for the ratio of leptonic decay rates as a function of γ.

III. RESULTS

Experimentally, one tests Eq. (2.7) for a possible singlet contribution via a deviation from the usual sum rule, Eq. (1.1),

$$\begin{aligned}
\delta &= \tfrac{1}{3}\epsilon m_\rho\Gamma(\rho \to l\bar{l})\\
&= m_\omega\Gamma(\omega \to l\bar{l}) + m_\phi\Gamma(\phi \to l\bar{l}) - \tfrac{1}{3}m_\rho\Gamma(\rho \to l\bar{l})\,,
\end{aligned} \tag{3.1}$$

and examines the ratios

$$\begin{aligned} x &= \frac{\Gamma(\phi \to l\bar{l})}{\Gamma(\rho \to l\bar{l})} \\ &= \left(\frac{m_\rho}{m_\phi}\right)^3 \frac{[(1/\sqrt{3})g_\phi + \gamma h_\phi]^2}{g_\rho^2}, \\ y &= \frac{\Gamma(\omega \to l\bar{l})}{\Gamma(\rho \to l\bar{l})} \\ &= \left(\frac{m_\rho}{m_\omega}\right)^3 \frac{[(1/\sqrt{3})g_\omega + \gamma h_\omega]^2}{g_\rho^2}. \end{aligned} \tag{3.2}$$

The Orsay[9] results give

$$\begin{aligned} \delta_{ex} &= 0.60 \pm 0.78\ \text{MeV}^2, \\ x_{ex} &= 0.21 \pm 0.06, \\ y_{ex} &= 0.15 \pm 0.06, \end{aligned} \tag{3.3}$$

while the Bologna-Cern[10] experiment yields

$$\begin{aligned} \delta_{ex} &= 0.60 \pm 1.40\ \text{MeV}^2, \\ x_{ex} &= 0.27 \pm 0.14, \\ y_{ex} &= 0.06 \pm 0.05, \end{aligned} \tag{3.4}$$

and the DESY[11] results are

$$\begin{aligned} \delta_{ex} &= 0.11 \pm 0.94\ \text{MeV}^2, \\ x_{ex} &= 0.17 \pm 0.08, \\ y_{ex} &= 0.14 \pm 0.06. \end{aligned} \tag{3.5}$$

Within the limits of the error bars, the experiments are compatible. However, the magnitude of the errors in δ_{ex} mitigate against meaningful extrapolation of ϵ and hence γ from the sum rule. Similarly, if we simultaneously solve the Weinberg equation, Eq. (2.5) and the DMO sum rule with octet breaking, Eq. (2.11), we find after some algebra

$$\begin{aligned} & m_\omega^2 = m_\rho^2, \\ & 2m_{K^*}^2 - m_\phi^2 - m_\omega^2 = 2f_\kappa^2 m_{K^*}^2 m_\rho^2 g_\rho^{-2}, \end{aligned} \tag{3.6}$$

and correspondingly for the Weinberg sum rule and the OS sum rule, Eq. (2.10),

$$\begin{aligned} & m_\omega^{-2} = m_\rho^{-2}, \\ & 2m_{K^*}^{-2} - m_\phi^{-2} - m_\omega^{-2} = 2f_\kappa^2 m_{K^*}^{-2} m_\rho^2 g_\rho^{-2}. \end{aligned} \tag{3.7}$$

In the limit $f_\kappa \to 0$, we recover the well known U(3) mass sum rules.[8,12] In both cases, interestingly,

$$\begin{aligned} g_\phi^2 &= g_\rho^2 \left(\frac{2m_\phi^2}{3m_\rho^2}\right), \\ g_\omega^2 &= g_\rho^2 \left(\frac{m_\omega^2}{3m_\rho^2}\right), \\ h_\phi^2 &= g_\rho^2 \left(\frac{m_\phi^2}{3m_\rho^2}\right), \\ h_\omega^2 &= g_\rho^2 \left(\frac{2m_\omega^2}{3m_\rho^2}\right), \\ h_\phi &= -(1/\sqrt{2})g_\phi, \\ h_\omega &= \sqrt{2}\, g_\omega. \end{aligned} \tag{3.8}$$

Substituting in Eqs. (3.2), it follows that,

$$\begin{aligned} x &= \frac{2}{9}\frac{m_\rho}{m_\phi}[1 - \gamma(\tfrac{3}{2})^{1/2}]^2 \\ &= 0.167[1 - \gamma(\tfrac{3}{2})^{1/2}]^2, \\ y &= \frac{1}{9}\frac{m_\rho}{m_\omega}(1 + \gamma\sqrt{6})^2 \\ &= 0.111(1 + \gamma\sqrt{6})^2. \end{aligned} \tag{3.9}$$

Again, uncertainty in measurement prevents meaningful determination of γ.

A discussion of quark-model predictions of γ is given by Mathur and Okubo[13] and is not repeated here, except to point out that the 4-quark model of Maki and Hara,[14] with $\gamma = (\frac{2}{3})^{1/2}$, is ruled out by the present experimental data listed herein. Though the situation is inconclusive now, experimental refinements will hopefully give greater insight into a possible singlet component as seen through Eqs. (3.1) and (3.2).

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[7]Obviously, splitting Weinberg's first sum rule as in Eqs. (2.2) and (2.4) has greater implications for the pseudoscalar and axial-vector mesons than for the

scalar and vector mesons, since little is known about the existence of scalar mesons. Furthermore, to the extent that the axial-vector currents are not conserved, pseudoscalar excitations are of lower order than their corresponding scalar excitations, which are coupled, with the exception of the κ meson, to conserved vector currents.

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Recent High-Energy Multiplicity Distributions in the Context of the Feynman Fluid Analogy

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Recent accelerator data on multiplicity distributions are reexamined within the context of the Feynman fluid analogy. An interpretation of the data put forward is that the diffractive component decreases logarithmically with energy.

The recent data[1] on prong distributions at high energies (50–300 GeV) suggest a reexamination of results based on the Feynman fluid analogy. The previous[2] approach to this problem relied on cosmic-ray data.[3] The available accelerator results differ with the cosmic-ray ones and, presumably, are more reliable. In this note we shall present the results of such a reanalysis together with a possible hint about the energy dependence of the diffractive component of multiparticle production.

We review briefly the method used which is similar to the one of Ref. 2. The reaction studied was $p+p\rightarrow n$ negative particles ($n=0$ includes elastic scattering) at a center-of-mass energy $\sqrt{s}$. Let

$$Y=a\ln(s/s_0)\,. \tag{1}$$

We shall return to the choice of s_0 shortly. Instead of dealing with the cross sections σ_n, we study the partition function

$$Q(z,Y)=\sum z^n\sigma_n(Y)/\sigma_{\text{tot}} \tag{2}$$

and assume that at large Y it has the behavior

$$\ln Q(z,Y)=p(z)Y+s(z)\,. \tag{3}$$

For very large energies the value of s_0 in (1) is irrelevant; however, for present energies it may be important. (The value assigned to a is a scale factor and for our purpose is arbitrary.) A hint as to the value of s_0 may be obtained from the fluid analogy itself. The inelastic average multiplicity, $\langle n\rangle$, is proportional to the length of the plateau in the one-particle-inclusive distribution, which in turn is the analog of the length of the fluid container, Y. Thus it is plausible that the proper extrapolation of Y to present energies is to let

$$Y=\langle n\rangle\approx-2.9+\ln s\,. \tag{4}$$

The analysis presented below makes this identification. Had we chosen $s_0=1\text{ GeV}^2$, as was done in Ref. 2, none of our conclusions would change. With such a choice (3) is not as well satisfied as with choosing (4) and subsequently the errors on $p(z)$ are larger.

The values of $Q(z,Y)$ together with the best fit to (3) are shown in Fig. 1, and the pressure, $p(z)$, is presented in Fig. 2.

One may now speculate on production mechanisms which would yield such a pressure curve. Following the discussion of Ref. 2, we would conclude that the rising part ($z\gtrsim0.8$) of the pressure curve was due to a multiperipheral mechanism, while the relatively straight section ($0<z\lesssim0.8$), one could naively say, was due to a mechanism yielding

$$\sigma_n(Y)=e^{-\eta Y}d_n\,, \tag{5}$$

with $\eta\sim0.2$ and d_n independent of Y.

An energy behavior such as $s^{-0.2}$, which would be implied by a literal interpretation of Fig. 2 and