Inclusive Vector-Meson Production at Small t in the Dual Resonance Model*

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Starting from the B_8 of Chan and Tsou and using the results of DeTar, Kang, Tan, and Weis, we obtain expressions for the cross section and density-matrix elements of vector mesons produced in inclusive reactions. The behavior of these quantities as functions of energy, missing mass, and momentum transfer is explored.

I. INTRODUCTION

With experiments of the type $a+b \to c+d+X$ (anything) at ultrahigh energy imminent, it is of some theoretical interest to investigate predictions for density-matrix elements of resonances produced in inclusive processes. The cross section for this process can be written as a discontinuity in the forward four-to-four amplitude. The dual resonance model (DRM) provides a natural framework for our investigation, because it specifies the eight-point amplitude as well as a simple prescription for focusing on the case where two pairs of particles form resonances. The resulting amplitude (for $\sigma\sigma$"ρ"$\to \sigma\sigma$"ρ") is a linear combination of B_6's, and we can use the results of DeTar, Kang, Tan, and Weiss (DKTW)[1] to take the desired discontinuity.

The paper is divided into five sections. Following the Introduction we define our notation and set forth the approximations and assumptions made, along with some motivation for these assumptions and approximations. Section III contains a description of the calculation; results obtained are in Sec. IV. Finally, we comment upon the results.

II. BACKGROUND

A. Notation and Kinematic Region

For the most part, we follow the notation used in Ref. 1. In considering $a+b\to x+X$ (anything) we are interested in the amplitude for $a+b+\bar{x} \to a+b+\bar{x}$, where the bar denotes the antiparticle. We use

$$\alpha_{ijk} \equiv \alpha((p_i+p_j+p_k)^2) \tag{2.1}$$

and let

$$\alpha_{ab\bar{x}} = \alpha(M^2) \equiv \alpha. \tag{2.2}$$

Here M^2 is the missing mass squared. We also use s for the center-of-mass energy squared and t for the momentum transfer squared between particles a and x.

We deviate from this notation when obtaining from the eight-point function the combination of six-point functions which constitute the amplitude for the case of two external vector mesons. Then, since we shall be dealing solely with momenta of groups of particles adjacent in our diagrams, we adopt the notation of Chan and Tsou,[2]

$$\alpha_{ij}(i<j) = \alpha((p_i+p_{i+1}+\cdots+p_j)^2)\,, \tag{2.3}$$

$$x_{ij} = -\alpha_{ij} - 1. \tag{2.4}$$

Also, throughout the paper, in all DRM diagrams, all momenta are ingoing.

The kinematic region in which we shall be working is the fragmentation region of particle a. In this region

$$s, M^2 \to \infty \tag{2.5}$$

while t is held fixed. We do not assume that s/M^2 is large. (However, see comments at the end of Sec. III.)

B. Assumptions and Approximations

We start from the results of DKTW. They find that of the 60 different diagrams contributing to the six-point amplitude, only four contribute to the discontinuity in the fragmentation region. These four are shown in Fig. 1(a). Therefore, in obtaining the amplitude for $V\sigma\sigma \to V\sigma\sigma$ from eight-point functions, we need only consider the pairs of poles in two-particle subenergies which yield combinations of B_6's with external particles ordered as in one of these four diagrams. Hence, the B_8's which we consider are those shown in Fig. 1(b). The amplitudes of these graphs we denote by A', B', C', and D', from left to right.

Linear trajectories are assumed. In the numerical evaluation, all internal trajectories are taken to be the "π" trajectory, $\alpha(s)=s$, with $\alpha(m_\sigma^2)=0$, $\alpha(m_V^2)=1$, except where it is obvious that what should be used is the actual mass of the π or the ρ – as in the calculation of the invariants, for example ($s=2m_\pi^2+2p_a\cdot p_b$ not $s=2m_\sigma^2+2p_a\cdot p_b$). This assumption is made for simplicity. If one desired, one could insert a more "realistic" vacu-

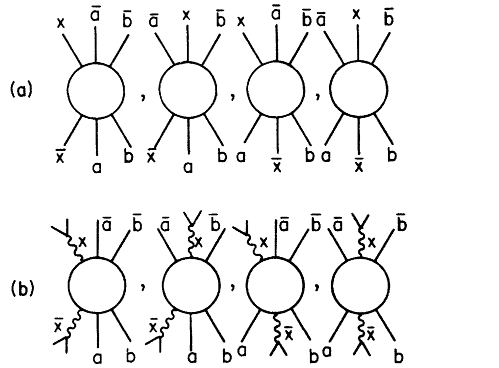

FIG. 1. (a) The four DRM diagrams which contribute to $a+b\rightarrow x+X$ in the fragmentation region of a. (b) The four DRM diagrams which contribute to $a+b\rightarrow V+X$ in the fragmentation region of a.

um trajectory in the appropriate channels. Since our main interest lies in t dependence, however, we can tolerate a vacuum trajectory with intercept zero. We comment further on this in the final section.

III. CALCULATION

It is easy to show that a simple application of the Mueller[3] technique to reactions involving the production and subsequent decay of vector mesons yields the following expression for elements of the density matrix of the vector meson x in the reaction $a+b\rightarrow x+X$:

$$\rho_{\lambda\lambda'} = (1/N)\,\mathrm{Disc}_{M^2}\langle ab\bar{x}(\lambda')|T|ab\bar{x}(\lambda)\rangle\ , \tag{3.1}$$

where

$$N = \sum_{\lambda}\mathrm{Disc}_{M^2}\langle ab\bar{x}(\lambda)|T|ab\bar{x}(\lambda)\rangle\ . \tag{3.2}$$

The one-particle spectrum is

$$\frac{1}{\sigma_{ab}}E_x\frac{d\sigma}{dp_x^3} = \frac{\Gamma(\alpha_{\mathrm{vac}}+1)}{\pi(\alpha_{ab})^{\alpha_{\mathrm{vac}}}}N\ , \tag{3.3}$$

where we include the total cross section σ_{ab} to maintain scaling even with a vacuum trajectory with intercept zero.

Consequently, in the fragmentation region of particle a we wish to compute the DRM diagrams shown in Fig. 2. We denote these four diagrams by A, B, C, and D from left to right. To obtain the expressions for A, B, C, and D, we first need the expressions for A', B', C', and D', which are obtained by taking the residue of B_8 at the appropriate poles.

Consider first A'. We are interested in the $\alpha_{12}=\alpha_{16}=1$ $(x_{12}=x_{16}=-2)$ pole. We find it convenient to use the multiperipheral configuration of B_8, represented by the duality diagram of Fig. 3(a). Using the expression of Chan and Tsou,[2]

$$\begin{aligned}B_8 = \int_0^1 & du_{12}du_{13}du_{14}du_{15}du_{16}\,u_{12}^{x_{12}}u_{13}^{x_{13}}u_{14}^{x_{14}}u_{15}^{x_{15}}u_{16}^{x_{16}}(1-u_{12})^{x_{23}}(1-u_{13})^{x_{34}}(1-u_{14})^{x_{45}}\\ &\times(1-u_{15})^{x_{56}}(1-u_{16})^{x_{67}}(1-u_{12}u_{13})^{x_{24}-x_{23}-x_{34}-1}(1-u_{13}u_{14})^{x_{35}-x_{34}-x_{45}-1}(1-u_{14}u_{15})^{x_{46}-x_{45}-x_{56}-1}\\ &\times(1-u_{15}u_{16})^{x_{57}-x_{56}-x_{67}-1}(1-u_{12}u_{13}u_{14})^{x_{25}+x_{34}-x_{24}-x_{35}}(1-u_{13}u_{14}u_{15})^{x_{36}+x_{45}-x_{35}-x_{46}}\\ &\times(1-u_{14}u_{15}u_{16})^{x_{47}+x_{56}-x_{46}-x_{57}}(1-u_{12}u_{13}u_{14}u_{15})^{x_{26}+x_{35}-x_{25}-x_{36}}\\ &\times(1-u_{13}u_{14}u_{15}u_{16})^{x_{37}+x_{46}-x_{36}-x_{47}}(1-u_{12}u_{13}u_{14}u_{15}u_{16})^{x_{27}+x_{36}-x_{26}-x_{37}}\ ,\end{aligned} \tag{3.4}$$

we first expand terms containing u_{12} and/or u_{16} in Taylor series in those variables. We then do the u_{12} and u_{16} integrations and take the residue at the double pole $x_{12}=-2$, $x_{16}=-2$. [Details of this procedure and of steps leading from Eq. (3.5) to Eq. (3.6) are given in Appendix A.] We thus obtain

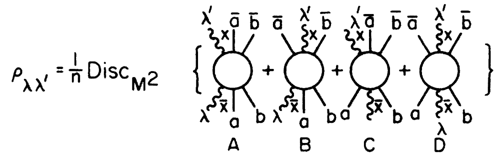

FIG. 2. Diagrammatic representation of density-matrix elements.

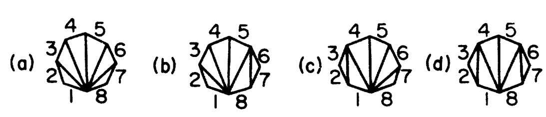

FIG. 3. (a)– (d) The duality diagrams for the B_8's used in obtaining A, B, C, and D, respectively.

TABLE I. Expressions for A, B, C, and D.

$$(1/\alpha'^2)A=-\epsilon\cdot p_a\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x},\alpha,\alpha_{\bar a x},\alpha_{ab},\alpha_{b\bar b},\alpha_{\bar b\bar a},\alpha_{\bar a x\bar x},\alpha_{ax\bar x},\alpha_{x\bar x})+p_{\bar b}B_6(\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\ldots,\alpha-1,\alpha_{\bar a x}-1,\ldots)+p_aB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_b\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots)+p_{\bar b}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)+p_aB_6(\alpha_{a\bar x}-2,\alpha-1,\alpha_{\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_{\bar b}\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\alpha-1,\ldots)+p_{\bar b}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\alpha-2,\alpha_{\bar a x}-1,\ldots)+p_aB_6(\alpha_{a\bar x}-2,\alpha-2,\alpha_{\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_{\bar a}\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)+p_{\bar b}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-2,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\alpha-2,\alpha_{\bar a x}-2,\ldots)+p_aB_6(\alpha_{a\bar x}-2,\alpha-2,\alpha_{\bar a x}-2,\ldots)]$$
$$-(\epsilon\cdot\epsilon'^*/2\alpha')B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)\ .$$

$$(1/\alpha'^2)B=-\epsilon'^*\cdot p_{\bar a}\epsilon\cdot[p_aB_6(\alpha_{a\bar x},\alpha,\alpha_{\bar a x},\alpha_{ab},\alpha_{b\bar b},\alpha_{\bar b x},\alpha_{\bar a x\bar x},\alpha_{a\bar a\bar x},\alpha_{\bar a\bar x})+p_bB_6(\alpha_{a\bar x}-1,\ldots)$$
$$+p_{\bar b}B_6(\alpha_{a\bar x}-1,\alpha-1,\ldots)+p_xB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)]$$
$$-\epsilon'^*\cdot p_{\bar x}\epsilon\cdot[p_aB_6(\ldots,\alpha_{\bar a x}-1,\ldots)+p_bB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_{\bar b}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)+p_xB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-2,\ldots)]$$
$$-\epsilon'^*\cdot p_b\epsilon\cdot[p_aB_6(\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)$$
$$+p_{\bar b}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)$$
$$+p_xB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-2,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)]$$
$$-\epsilon'^*\cdot p_a\epsilon\cdot[p_aB_6(\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)+p_bB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)$$
$$+p_{\bar b}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)+p_xB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-2,\ldots,\alpha_{\bar a x\bar x}-1,\ldots)]$$
$$+(\epsilon\cdot\epsilon'^*/2\alpha')[B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)-B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-2,\ldots)]\ .$$

$$\begin{aligned}A'=(\alpha_{23}+1)&[(\alpha_{67}+1)B_6(\alpha_{13},\alpha_{14},\alpha_{15},\alpha_{34},\alpha_{45},\alpha_{56},\alpha_{35},\alpha_{46},\alpha_{36})+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\ldots,\alpha_{15}-1,\ldots)\\
&+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\ldots,\alpha_{14}-1,\alpha_{15}-1,\ldots)+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)]\\
+(\alpha_{24}-\alpha_{23}-\alpha_{34})&[(\alpha_{67}+1)B_6(\alpha_{13}-1,\ldots)+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots)\\
&+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)\\
&+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots)]\\
+(\alpha_{34}+\alpha_{25}-\alpha_{24}-\alpha_{35})&[(\alpha_{67}+1)B_6(\alpha_{13}-1,\alpha_{14}-1,\ldots)+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)\\
&+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\alpha_{13}-1,\alpha_{14}-2,\alpha_{15}-1,\ldots)\\
&+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-2,\alpha_{14}-2,\alpha_{15}-1,\ldots)]\\
+(\alpha_{26}+\alpha_{35}-\alpha_{25}-\alpha_{36})&[(\alpha_{67}+1)B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-2,\ldots)\\
&+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\alpha_{13}-1,\alpha_{14}-2,\alpha_{15}-2,\ldots)\\
&+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-2,\alpha_{14}-2,\alpha_{15}-2,\ldots)]\\
+(\alpha_{27}+\alpha_{36}-\alpha_{26}-\alpha_{37})&B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots),\end{aligned}\tag{3.5}$$

TABLE I *(Continued)*

$$(1/\alpha'^2)C=-\epsilon\cdot p_a\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x},\alpha,\alpha_{\bar a x},\alpha_{b\bar x},\alpha_{b\bar b},\alpha_{\bar a\bar b},\alpha_{a\bar a x},\alpha_{ax\bar x},\alpha_{ax})+p_{\bar b}B_6(\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\ldots,\alpha-1,\alpha_{\bar a x}-1,\ldots)+p_{\bar x}B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_x\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots)+p_{\bar b}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)+p_{\bar x}B_6(\alpha_{a\bar x}-2,\alpha-1,\alpha_{\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_{\bar b}\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)$$
$$+p_{\bar b}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)$$
$$+p_{\bar x}B_6(\alpha_{a\bar x}-2,\alpha-1,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)]$$
$$-\epsilon\cdot p_{\bar a}\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)+p_{\bar b}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)+p_{\bar x}B_6(\alpha_{a\bar x}-2,\alpha-1,\alpha_{\bar a x}-1,\ldots,\alpha_{ax\bar x}-1,\ldots)]$$
$$+(\epsilon\cdot\epsilon'^*/2\alpha')[B_6(\alpha_{a\bar x}-1,\alpha-1,\alpha_{\bar a x}-1,\ldots)-B_6(\alpha_{a\bar x}-2,\alpha-1,\alpha_{\bar a x}-1,\ldots)]\,.$$

$$(1/\alpha'^2)D=-\epsilon\cdot p_a\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x},\alpha,\alpha_{\bar a x},\alpha_{b\bar x},\alpha_{b\bar b},\alpha_{\bar b x},\alpha_{a\bar a x},\alpha_{a\bar a\bar x},\alpha_{a\bar a})+p_aB_6(\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a x}-1,\ldots)+p_{\bar x}B_6(\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{a\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_{\bar a}\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots)+p_aB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a x}-1,\ldots)$$
$$+p_{\bar x}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{a\bar a x}-1,\ldots)]$$
$$-\epsilon\cdot p_{\bar b}\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a\bar x}-1,\ldots)$$
$$+p_aB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a\bar x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-2,\ldots,\alpha_{a\bar a x}-1,\alpha_{a\bar a\bar x}-1,\ldots)$$
$$+p_{\bar x}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a x}-1,\alpha_{a\bar a\bar x}-1,\ldots)]$$
$$-\epsilon\cdot p_x\,\epsilon'^*\cdot[p_{\bar a}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{a\bar a\bar x}-1,\ldots)+p_aB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{a\bar a\bar x}-1,\ldots)$$
$$+p_bB_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a x}-1,\alpha_{a\bar a\bar x}-1,\ldots)$$
$$+p_{\bar x}B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{a\bar a x}-1,\alpha_{a\bar a\bar x}-1,\ldots)]$$
$$+(\epsilon\cdot\epsilon'^*/2\alpha')[B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{b\bar b}-1,\ldots,\alpha_{a\bar a x}-1,\alpha_{a\bar a\bar x}-1,\ldots)$$
$$-B_6(\alpha_{a\bar x}-1,\ldots,\alpha_{\bar a x}-1,\ldots,\alpha_{a\bar a x}-1,\alpha_{a\bar a\bar x}-1,\ldots)]\,.$$

where $B_6(\alpha_{13},\alpha_{14},\alpha_{15},\alpha_{34},\alpha_{45},\alpha_{56},\alpha_{35},\alpha_{46},\alpha_{36})$ is the standard six-point function[2] and the ellipses indicate the omitted α_{ij}'s are unchanged. This amplitude receives contributions from all the daughter trajectories. In order to restrict ourselves to the leading trajectory (i.e., the vector meson) we employ the method of Ref. 4. Having done so, it is easy to pick off the $V\sigma\sigma$ vertices to obtain the expression for A given in Table I (see Appendix A). In the same manner, the expressions for B, C, and D given in Table I are obtained. (Appendix B contains an outline of the main steps.) From this point we shall neglect all $\epsilon\cdot p_x$ and $\epsilon'^*\cdot p_{\bar x}$ terms since in the end we set $p_{\bar x}=-p_x$.

Next we take the discontinuity in $(A+B+C+D)$. To do so we use the results of Ref. 1.

$$\mathrm{Disc}_{M^2}B_6(\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7,\alpha_8,\alpha_9)$$
$$=\frac{\pi(\alpha_2)^{\alpha_5}}{\Gamma(1+\alpha_5)}\left(-\frac{\alpha_4}{\alpha_2}\right)^{\alpha_1}\left(-\frac{\alpha_6}{\alpha_2}\right)^{\alpha_3}I(\alpha_2/\alpha_4,\,\alpha_2/\alpha_6;\alpha_1,\,\alpha_3,\,\alpha_1+\alpha_9-\alpha_8,\,\alpha_2+\alpha_9-\alpha_7,\,-\alpha_5-\alpha_9+\alpha_7+\alpha_8,\,\alpha_5),\tag{3.6}$$

where

$$I(\alpha_1,\beta_1;\alpha_2,\beta_2,\alpha_3,\beta_3,\gamma,\delta)$$
$$=\int_0^\infty dy_1dy_2\,y_1^{-\alpha_2-1}y_2^{-\beta_2-1}(1-\alpha_1y_1)^{\alpha_3}(1-\beta_1y_2)^{\beta_3}(1-\alpha_1y_1-\beta_1y_2)^{\gamma}(1-y_1-y_2)^{\delta}\theta(1-y_1-y_2)\,.\tag{3.7}$$

TABLE II. Expressions for the discontinuities in A, D, and $B+C$.

$$\frac{\pi}{\alpha'^2}\,\mathrm{Disc}_{M^2}A=\epsilon\cdot p_a\epsilon'^*\cdot p_a\Bigg[\left(\frac{\alpha_{ab}}{\alpha}\right)^{2\alpha_{a\bar x}}I\left(\frac{\alpha}{\alpha_{ab}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x},\alpha_{a\bar x},\alpha_{a\bar x},\alpha_{a\bar x},0,0\right)$$

$$-2\left(\frac{\alpha_{ab}}{\alpha-1}\right)^{2\alpha_{a\bar x}-2}I\left(\frac{\alpha-1}{\alpha_{ab}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,0,0\right)$$

$$+\left(\frac{\alpha_{ab}}{\alpha-2}\right)^{2\alpha_{a\bar x}-4}I\left(\frac{\alpha-2}{\alpha_{ab}},\frac{\alpha-2}{\alpha_{ab}};\alpha_{a\bar x}-2,\alpha_{a\bar x}-2,\alpha_{a\bar x}-2,\alpha_{a\bar x}-2,0,0\right)\Bigg]$$

$$+\epsilon\cdot p_b\epsilon'^*\cdot p_b\Bigg[\left(\frac{\alpha_{ab}}{\alpha}\right)^{2\alpha_{a\bar x}-2}DI\left(\frac{\alpha}{\alpha_{ab}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,0,0\right)$$

$$-\left(\frac{\alpha_{ab}}{\alpha-1}\right)^{2\alpha_{a\bar x}-2}DI\left(\frac{\alpha-1}{\alpha_{ab}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,0,0\right)\Bigg]$$

$$+(\epsilon\cdot p_a\epsilon'^*\cdot p_b+\epsilon\cdot p_b\epsilon'^*\cdot p_a)\Bigg[-\left(\frac{\alpha_{ab}}{\alpha}\right)^{2\alpha_{a\bar x}-1}DI\left(\frac{\alpha}{\alpha_{ab}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x},\alpha_{a\bar x}-1,\alpha_{a\bar x},\alpha_{a\bar x}-1,0,0\right)$$

$$+\left(\frac{\alpha_{ab}}{\alpha-1}\right)^{2\alpha_{a\bar x}-3}DI\left(\frac{\alpha-1}{\alpha_{ab}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-2,\alpha_{a\bar x}-1,\alpha_{a\bar x}-2,0,0\right)\Bigg]$$

$$-\frac{\epsilon\cdot\epsilon'^*}{2\alpha'}\left(\frac{\alpha_{ab}}{\alpha-1}\right)^{2\alpha_{a\bar x}-2}I\left(\frac{\alpha-1}{\alpha_{ab}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,0,0\right).$$

$$\frac{\pi}{\alpha'^2}\,\mathrm{Disc}_{M^2}D=\epsilon\cdot p_a\epsilon'^*\cdot p_a\Bigg[\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}}I\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x},\alpha_{a\bar x},\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,2,0\right)$$

$$-2\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}-1}I\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x},\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-2,2,0\right)$$

$$+\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}-2}I\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-2,\alpha_{a\bar x}-2,2,0\right)\Bigg]$$

$$+\epsilon\cdot p_b\epsilon'^*\cdot p_b\Bigg[\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}-2}DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,2,0\right)$$

$$-\left(\frac{-\alpha_{b\bar x}}{\alpha-1}\right)^{2\alpha_{a\bar x}-2}DI\left(\frac{\alpha-1}{\alpha_{b\bar x}},\frac{\alpha-1}{\alpha_{b\bar x}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,2,0\right)\Bigg]$$

$$-(\epsilon\cdot p_a\epsilon'^*\cdot p_b+\epsilon\cdot p_b\epsilon'^*\cdot p_a)\Bigg[\left(\frac{\alpha}{-\alpha_{b\bar x}}\right)\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}}DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x},\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,2,0\right)$$

$$-\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}-2}DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-2,\alpha_{a\bar x}-1,2,0\right)\Bigg]$$

$$-\frac{\epsilon\cdot\epsilon'^*}{2\alpha'}\left(\frac{-\alpha_{b\bar x}}{\alpha}\right)^{2\alpha_{a\bar x}-2}\Bigg[I\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,0,0\right)$$

$$-DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{b\bar x}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,1,0\right)\Bigg],$$

$$DI\left(\frac{\alpha}{\alpha_{ab}},\frac{\alpha}{\alpha_{ab}};\alpha_2,\beta_2,\alpha_3,\beta_3,\gamma,\delta\right)\equiv I\left(\frac{\alpha}{\alpha_{ab}},\frac{\alpha}{\alpha_{ab}};\alpha_2,\beta_2,\alpha_3,\beta_3,\gamma,\delta\right)$$

$$-\left(\frac{\alpha}{\alpha-1}\right)^{\alpha_2+\beta_2}I\left(\frac{\alpha-1}{\alpha_{ab}},\frac{\alpha-1}{\alpha_{ab}};\alpha_2,\beta_2,\alpha_3,\beta_3,\gamma,\delta\right).$$

TABLE II *(Continued)*

$$\frac{\pi}{\alpha'^2}\,\mathrm{Disc}_{M^2}(B+C)=2\cos(\pi\alpha_{a\bar x})\epsilon\cdot p_a\epsilon'^*\cdot p_a$$

$$\times\Bigg[\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{\alpha^2}\right)^{\alpha_{a\bar x}} I\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x},\alpha_{a\bar x},\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-1,1-\alpha_{ax},0\right)$$

$$-\left(\frac{\alpha}{-\alpha_{b\bar x}}\right)\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{\alpha^2}\right)^{\alpha_{a\bar x}} I\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x},\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-1,-\alpha_{ax},0\right)\Bigg]$$

$$+2\cos(\pi\alpha_{a\bar x})\epsilon\cdot p_b\epsilon'^*\cdot p_b$$

$$\times\Bigg[\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{\alpha^2}\right)^{\alpha_{a\bar x}-1} DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-2,1-\alpha_{ax},0\right)$$

$$-\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{(\alpha-1)^2}\right)^{\alpha_{a\bar x}-1} DI\left(\frac{\alpha-1}{\alpha_{b\bar x}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-2,1-\alpha_{ax},0\right)\Bigg]$$

$$-(\epsilon\cdot p_a\epsilon'^*\cdot p_b e^{i\pi\alpha_{a\bar x}}+\epsilon\cdot p_b\epsilon'^*\cdot p_a e^{-i\pi\alpha_{a\bar x}})$$

$$\times\Bigg[\frac{\alpha}{\alpha_{ab}}\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{\alpha^2}\right)^{\alpha_{a\bar x}} DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x},\alpha_{a\bar x}-1,\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-2,1-\alpha_{ax},0\right)$$

$$-\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{\alpha^2}\right)^{\alpha_{a\bar x}-1} DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-2,-\alpha_{ax},0\right)\Bigg]$$

$$-(\epsilon\cdot p_b\epsilon'^*\cdot p_a e^{i\pi\alpha_{a\bar x}}+\epsilon\cdot p_a\epsilon'^*\cdot p_b e^{-i\pi\alpha_{a\bar x}})$$

$$\times\left(\frac{\alpha}{-\alpha_{b\bar x}}\right)\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{\alpha^2}\right)^{\alpha_{a\bar x}} DI\left(\frac{\alpha}{\alpha_{b\bar x}},\frac{\alpha}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x},\alpha_{a\bar x}+\alpha_{ax},\alpha_{a\bar x}+\alpha_{ax}-1,1-\alpha_{ax},0\right)$$

$$-\frac{\epsilon\cdot\epsilon'^*\cos(\pi\alpha_{a\bar x})}{\alpha'}$$

$$\times\Bigg[\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{(\alpha-1)^2}\right)^{\alpha_{a\bar x}-1} I\left(\frac{\alpha-1}{\alpha_{b\bar x}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-1,\alpha_{a\bar x}-1,\alpha_{a\bar x}+\alpha_{ax}-1,\alpha_{a\bar x}+\alpha_{ax}-2,1-\alpha_{ax},0\right)$$

$$-\left(\frac{\alpha-1}{-\alpha_{b\bar x}}\right)\left(\frac{-\alpha_{b\bar x}\alpha_{ab}}{(\alpha-1)^2}\right)^{\alpha_{a\bar x}-1} I\left(\frac{\alpha-1}{\alpha_{b\bar x}},\frac{\alpha-1}{\alpha_{ab}};\alpha_{a\bar x}-2,\alpha_{a\bar x}-1,\alpha_{a\bar x}+\alpha_{ax}-2,\alpha_{a\bar x}+\alpha_{ax}-2,1-\alpha_{ax},0\right)\Bigg].$$

Note that I is symmetric under simultaneous interchange of α_1 and β_1, α_2 and β_2, and α_3 and β_3.

Before we take the discontinuity we rewrite the B_6's which contain $\alpha_{b\bar b}-1$ or $\alpha_{b\bar b}-2$ by use of recursion relations of the form

$$B_6(\alpha_{a\bar x},\alpha,\alpha_{\bar a x},\alpha_{b\bar x},\alpha_{b\bar b}-1,\alpha_{\bar b x},\alpha_{a\bar a x},\alpha_{a\bar a\bar x},\alpha_{a\bar a})=B_6(\alpha_{a\bar x},\alpha,\alpha_{\bar a x},\alpha_{b\bar x},\alpha_{b\bar b},\alpha_{\bar b x},\alpha_{a\bar a x}+1,\alpha_{a\bar a\bar x}+1,\alpha_{a\bar a}+1)$$
$$-B_6(\alpha_{a\bar x},\alpha-1,\alpha_{\bar a x},\alpha_{b\bar x},\alpha_{b\bar b},\alpha_{\bar b x},\alpha_{a\bar a x}+1,\alpha_{a\bar a\bar x}+1,\alpha_{a\bar a}+1). \quad (3.8)$$

This gives us an expression with all $\alpha_{b\bar b}$'s unchanged, thus enabling us to factor out that Γ function which arises in taking the discontinuity and also to avoid difficulties which would otherwise arise from our choice of $\alpha(0)=0$.

We now take the discontinuity (being careful to evaluate α_{ab} at $s+i\epsilon$ and $\alpha_{\bar a\bar b}$ at $s-i\epsilon$), use the symmetry of I, and insert our trajectory $[\alpha(s)=s,\ \alpha(m_a^2)=\alpha(m_b^2)=0,\ \alpha(m_x^2)=1]$ to obtain the expressions listed in Table II.

We can now evaluate these expressions numerically. In doing so it is convenient to use the fact that the last argument of each I is zero. This enables us to write

$$
\begin{aligned}
I(\alpha_1,\beta_1;\alpha_2,\beta_2,\alpha_3,\beta_3,\gamma,0) &= \int_0^\infty dy_1dy_2\theta(1-y_1-y_2)y_1^{-\alpha_2-1}y_2^{-\beta_2-1}(1-\alpha_1y_1)^{\alpha_3}(1-\beta_1y_2)^{\beta_3}(1-\alpha_1y_1-\beta_1y_2)^{\gamma} \\
&= \sum_{j,k=0}^{\infty}\frac{\Gamma(j-\beta_3)\Gamma(k-\gamma)}{\Gamma(-\beta_3)\Gamma(j+1)\Gamma(-\gamma)\Gamma(k+1)(j+k-\beta_2)}\beta_1^{j+k} \\
&\quad\times\int_0^1 dy_1y_1^{-\alpha_2-1}(1-\alpha_1y_1)^{\alpha_3+\gamma-k}(1-y_1)^{-\beta_2+j+k} \\
&= \sum_{j,k}\frac{\Gamma(j-\beta_3)\Gamma(k-\gamma)\beta_1^{j+k}}{\Gamma(-\beta_3)\Gamma(j+1)\Gamma(-\gamma)\Gamma(k+1)(j+k-\beta_2)} \\
&\quad\times\frac{\Gamma(k+j-\beta_2+1)\Gamma(-\alpha_2)}{\Gamma(k+j-\alpha_2-\beta_2+2)}\,{}_2F_1(k-\gamma-\alpha_3,-\alpha_2,j+k-\alpha_2-\beta_2+1;\alpha_1), \qquad (3.9)
\end{aligned}
$$

where ${}_2F_1$ is a generalized hypergeometric series.

We should at this point comment upon the statement that s/M^2 need not be large. The expression (3.10) for the discontinuity in the B_6's is correct for large M^2 and large s, regardless of their ratio. However, it neglects terms of order $1/M^2$. In calculating some of our amplitudes, the factors multiplying $\mathrm{Disc}B_6$ can be large. In particular, p_b is of the order of $(s-M^2)/m_x$, DI is typically of the order $(1/M^2)I$, and terms containing two p_b's also contain at least two factors of M^2/s. Consequently, for s/M^2 small enough, problems can arise if the larger terms should cancel for some reason. Such cancellation does in fact occur for $\cos[\pi\alpha(t)]=-1$.

In practice, our expressions seem adequate, even in the region of cancellation, for s/M^2 down to four in the density-matrix elements, and down to two for the cross section. Hence our evaluation of the density matrix does not extend to as large a value of M^2 as does our evaluation of the cross section.

IV. RESULTS

A. Cross Section

In Fig. 4 we plot the differential cross section

$$(1/\sigma_{ab})d^2\sigma/dtdM^2$$

as a function of $t'=t-t_{\max}$. The cross section is plotted at different values of $\xi\equiv M^2/s$, since for large s the quantity

$$(s/\sigma_{ab})d^2\sigma/dtdM^2$$

has scaled and depends only on ξ and t. It should be noted that our differential cross section satisfies

$$\int\frac{1}{\sigma_{ab}}\frac{d^2\sigma}{dtdM^2}\,dtdM^2=\langle N_x(s)\rangle\ , \qquad (4.1)$$

where $\langle N_x(s)\rangle$ is the average number of vector mesons x produced. Using variables which are more accessible experimentally, we display in Fig. 5 plots of the one-particle spectrum $(E_x/\sigma_{ab})d\sigma/dp_x^3$ as a function of $p_{x\perp}^2$ for fixed $p_{x\parallel}/p_{\mathrm{inc}}$(lab). The one-particle spectrum also depends only on this ratio and $p_\perp^2$ for large s, and so we do not specify s.

General features of note, besides the expected rapid decrease as t ($p_\perp$) increases, are the development of structure around $t'=-0.5$ or -0.6 GeV$^2/c^2$, the broadening of the distribution as M^2 increases, and the gradual change of slope as t

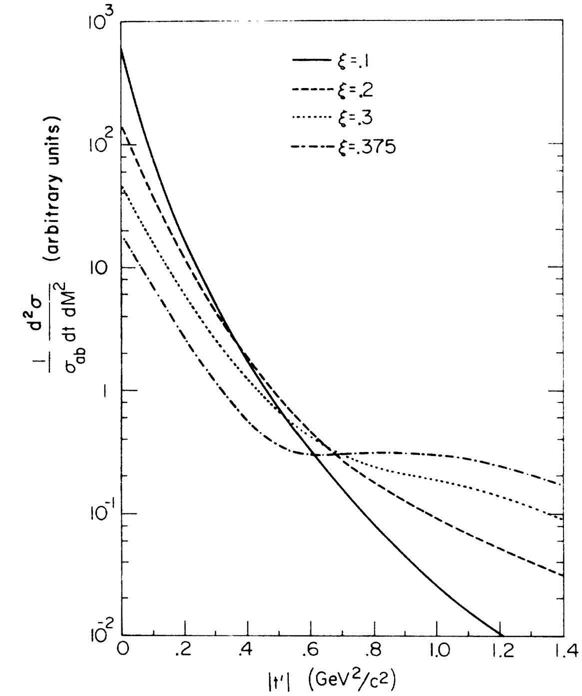

FIG. 4. $(1/\sigma_{ab})d^2\sigma/dt\,dM^2$ as a function of t' for $\xi=M^2/s=0.1,\ 0.2,\ 0.3,\ 0.375$.

increases. The dip which develops as M^2 increases is a signature effect[5] due to the $\cos[\pi\alpha(t)]$ factor contained in $\text{Disc}_{M^2}(B+C)$. The dip occurs near the nonsense wrong-signature point $t=-1$ GeV$^2/c^2$ or $t'\simeq -0.6$ GeV$^2/c^2$. For smaller M^2, the steeper slope of the other factors obscures the cosine modulation. The broader distribution for larger M^2 is expected from the factors of $(\alpha_{ab}/\alpha)^{\alpha(t)}$ and/ or $(-\alpha_{b\bar{x}}/\alpha)^{\alpha(t)}$. The slope change is due in part to the $\cos[\pi\alpha(t)]$ and in part to the fact that A, B, and C, which have steeper slopes, are fading away compared to D.

B. Density Matrix

The independent elements of the decay density matrix of the vector meson in the Gottfried-Jackson frame are plotted as functions of t' at fixed M^2 and s in Fig. 6. As expected, ρ_{00} starts near one at $t'=0$. It decreases to a minimum between $t'=-0.5$ GeV$^2/c^2$ and $t'=-1$ GeV$^2/c^2$, and then increases again. It continues its gradual rise (with local minima near $t=3, 5, 7, \ldots$ GeV$^2/c^2$) and eventually ($t'\simeq -8$ GeV$^2/c^2$) seems to level off at a value of 0.82 for $M^2/s=0.2$. This is in qualitative agreement with the results of Fenster and Uretsky and of Kang and Shen.[6] Since we are graphing the density-matrix elements as functions of t', whereas the minimum is related to the factor $\cos[\pi\alpha(t)]$, the minimum in ρ_{00} migrates to the left as M^2 (and consequently $|t_{\max}|$) increases.

C. $\sigma\sigma\to\sigma x$

This has already been considered in depth by Bebel *et al.*, by Thomas, and at intermediate energies by Kang and Shen.[5] We reproduce our results for

$$(E_x/\sigma_{ab})d\sigma/dp_x^3$$

at fixed $p_{x\parallel}(\text{lab})/p_{\text{inc}}(\text{lab})$ for completeness and purposes of comparison to the same calculation with different internal trajectories (see Fig. 7).[7]

V. COMMENTS

The question arises whether these model calculations can be taken seriously. Without even considering possible consequences of the DRM's unitarity problems, the phenomenological shortcomings of the model are well established.[8] Besides these problems common to all DRM calculations, our results are additionally suspect owing to our insistence on using the pion trajectory everywhere. In light of these questions we comment upon the expected significance of our three sets of results.

We feel that the most significant results for the cross section are the qualitative features as functions of t or $p_\perp^2$. The rapid decrease as $p_\perp^2$ increases (hardly startling), the development of a dip as the slope decreases (M^2 increases), and the change in slope as a function of $p_\perp^2$ are regarded as legitimate predictions; whereas the s and M^2 dependence [since $\alpha_{\text{vac}}(0)=\alpha_{b\bar{b}}(0)$ was chosen equal to zero] and more quantitative features such as the actual slopes and when the dip begins to appear would require a more detailed

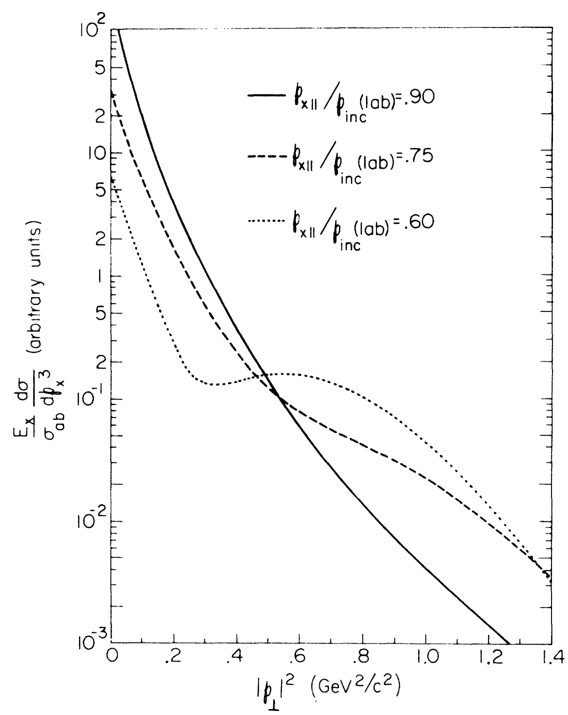

FIG. 5. $(E_x/\sigma_{ab})d\sigma/dp_x^3$ as a function of $p_\perp^2$ for $p_{x\parallel}/p_{\text{inc}}=0.6$, 0.75, 0.9.

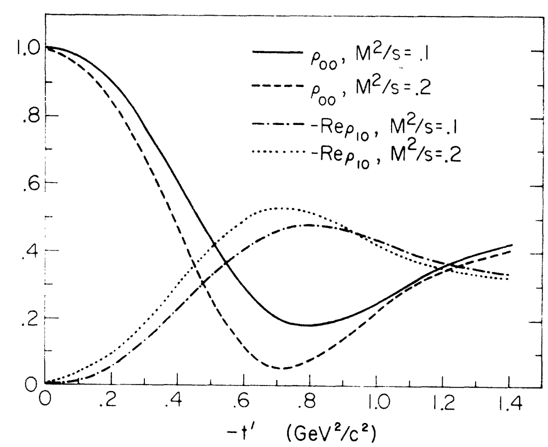

FIG. 6. ρ_{00} and ρ_{10} in the Gottfried-Jackson frame at $\xi=0.1$, 0.2.

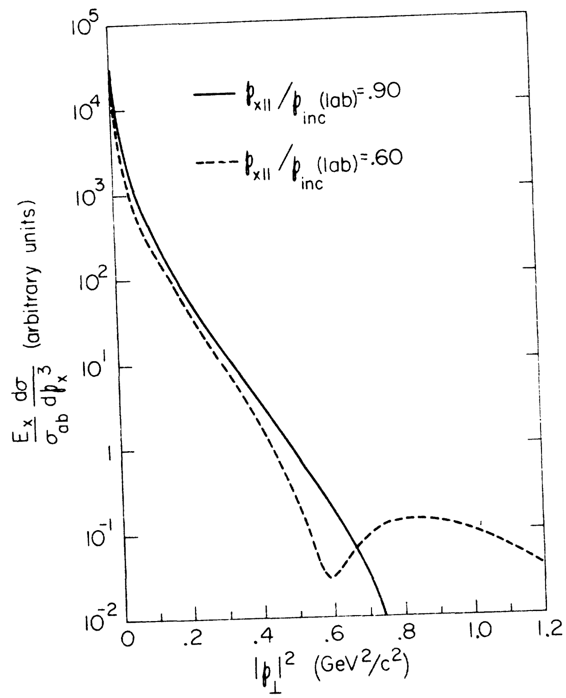

FIG. 7. $(E_x/\sigma_{ab})d\sigma/dp_x^3$ for $\sigma\sigma\to\sigma X$; $p_{x\parallel}/p_{\rm inc}=0.6,\ 0.9$.

analysis – if, indeed they *can* be predicted with confidence. We should mention here that the dip occurs in the amplitude from a particular t-channel trajectory and contributions from other trajectories could obscure it (although in $\pi^-p\to\rho^0X$ one would expect to see predominantly π exchange, based upon exclusive processes). Also, integration over M^2 would wash out the dip since its position in $p_\perp^2$ varies with M^2. The slope change would be expected to persist, even after such an integration.

Qualifications on our results for the density matrix are less necessary than for the cross section. In particular, a major part of the M^2 and s dependence $[(M^2/s)^{\alpha_{\rm vac}(0)}]$ cancels and consequently we are inclined to take the M^2 and s dependence more seriously than in the cross section. General features predicted for ρ_{00} are a rather rapid decrease to a minimum around $t'\simeq -0.7$ to -0.9 GeV$^2/c^2$ and then a gradual rise to a "large" value as the central region is approached. This contrasts with the exclusive calculation of Jones and Wyld[4] in which ρ_{00} decreases monotonically in the region considered. The dip we observe is not present in the exclusive calculation because as one goes to the Regge limit the signature factor factors out of all the amplitudes. (Consequently, Jones and Wyld's neglect of the crossed graph does not affect the density matrix.) In the $s/M^2\to\infty$ limit of the inclusive reaction, the dip in ρ_{00} also disappears.

Comments made for the $\sigma\sigma\to VX$ cross section are also applicable to the $\sigma\sigma\to\sigma X$ cross section. The situation for quantitative predictions can be expected to be worse here since the t channel will not even be a pionlike trajectory ordinarily. The qualitative features, however, are again considered valid predictions.

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APPENDIX A

Starting from

$$B_8=\int_0^1 du_{12}du_{13}du_{14}du_{15}du_{16}u_{12}^{x_{12}}u_{13}^{x_{13}}u_{14}^{x_{14}}u_{15}^{x_{15}}u_{16}^{x_{16}}(1-u_{12})^{x_{23}}(1-u_{13})^{x_{34}}(1-u_{14})^{x_{45}}(1-u_{15})^{x_{56}}(1-u_{16})^{x_{67}}$$
$$\times(1-u_{12}u_{13})^{x_{24}-x_{23}-x_{34}-1}(1-u_{13}u_{14})^{x_{35}-x_{34}-x_{45}-1}(1-u_{14}u_{15})^{x_{46}-x_{45}-x_{56}-1}(1-u_{15}u_{16})^{x_{57}-x_{56}-x_{67}-1}$$
$$\times(1-u_{12}u_{13}u_{14})^{x_{25}+x_{34}-x_{24}-x_{35}}(1-u_{13}u_{14}u_{15})^{x_{36}+x_{45}-x_{35}-x_{46}}(1-u_{14}u_{15}u_{16})^{x_{47}+x_{56}-x_{46}-x_{57}}$$
$$\times(1-u_{12}u_{13}u_{14}u_{15})^{x_{26}+x_{35}-x_{25}-x_{36}}(1-u_{13}u_{14}u_{15}u_{16})^{x_{37}+x_{46}-x_{36}-x_{37}}(1-u_{12}u_{13}u_{14}u_{15}u_{16})^{x_{27}+x_{36}-x_{26}-x_{37}},\tag{A1}$$

we make a Taylor expansion in u_{12} and u_{16} and integrate over these variables to obtain

$$B_8=\sum_{i,j,k,l,m}\ \sum_{i',j',k',l'}\gamma(i,x_{23})\gamma(i',x_{67})\gamma(j,x_{24}-x_{23}-x_{34}-1)\gamma(j',x_{57}-x_{56}-x_{67}-1)\gamma(k,x_{25}+x_{34}-x_{24}-x_{35})$$
$$\times\gamma(k',x_{47}+x_{56}-x_{46}-x_{57})\gamma(l,x_{26}+x_{35}-x_{25}-x_{36})$$
$$\times\gamma(l',x_{37}+x_{46}-x_{36}-x_{47})\gamma(m,x_{27}+x_{36}-x_{26}-x_{37})$$

$$\times \int_0^1 du_{13}du_{14}du_{15}u_{13}{}^{x_{13}+j+k+l+l'+m}u_{14}{}^{x_{14}+k+k'+l+l'+m}u_{15}{}^{x_{15}+j'+k'+l'+l+m}(1-u_{13})^{x_{34}}(1-u_{14})^{x_{45}}$$
$$\times(1-u_{15})^{x_{56}}(1-u_{13}u_{14})^{x_{35}-x_{34}-x_{45}-1}(1-u_{14}u_{15})^{x_{46}-x_{45}-x_{56}-1}(1-u_{13}u_{14}u_{15})^{x_{36}+x_{45}-x_{35}-x_{46}}$$
$$\times\int_0^1 du_{12}du_{16}u_{12}{}^{x_{12}+i+j+k+l+m}u_{16}{}^{x_{16}+i'+j'+k'+l'+m}\,, \tag{A2}$$

where

$$\gamma(i,x)=\frac{\Gamma(i-x)}{\Gamma(-x)\Gamma(i+1)}\;. \tag{A3}$$

Performing the u_{12} and u_{16} integrations and noticing that the remaining integral is a B_6, we can write

$$B_8=\sum_{i,j,k,l,m}\;\sum_{i',j',k',l'}\gamma(i,x_{23})\gamma(i',x_{67})\gamma(j,x_{24}-x_{23}-x_{34}-1)\gamma(j',x_{57}-x_{56}-x_{67}-1)\gamma(k,x_{25}+x_{34}-x_{24}-x_{35})$$
$$\times\gamma(k',x_{47}+x_{56}-x_{46}-x_{57})\gamma(l,x_{26}+x_{35}-x_{25}-x_{36})\gamma(l',x_{37}+x_{46}-x_{36}-x_{47})$$
$$\times\gamma(m,x_{27}+x_{36}-x_{26}-x_{37})(x_{12}+1+i+j+k+l+m)^{-1}(x_{16}+1+i'+j'+k'+l'+m)^{-1}$$
$$\times B_6(x_{13}+j+k+l+l'+m,\,x_{14}+k+k'+l+l'+m,\,x_{15}+j'+k'+l'+l+m,\,x_{34},x_{45},x_{56},x_{35},x_{46},x_{36}). \tag{A4}$$

If we take the residue at $x_{12}=x_{16}=-2$ we obtain Eq. (3.4):

$$A'=(\alpha_{23}+1)[(\alpha_{67}+1)B_6(\alpha_{13},\alpha_{14},\alpha_{15},\alpha_{34},\alpha_{45},\alpha_{56},\alpha_{35},\alpha_{46},\alpha_{36})+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\ldots,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\ldots,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)]$$
$$+(\alpha_{24}-\alpha_{23}-\alpha_{34})[(\alpha_{67}+1)B_6(\alpha_{13}-1,\ldots)+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots)]$$
$$+(\alpha_{34}+\alpha_{25}-\alpha_{24}-\alpha_{35})[(\alpha_{67}+1)B_6(\alpha_{13}-1,\alpha_{14}-1,\ldots)+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\alpha_{13}-1,\alpha_{14}-2,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-2,\alpha_{14}-2,\alpha_{15}-1,\ldots)]$$
$$+(\alpha_{26}+\alpha_{35}-\alpha_{25}-\alpha_{36})[(\alpha_{67}+1)B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+(\alpha_{57}-\alpha_{56}-\alpha_{67})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-2,\ldots)$$
$$+(\alpha_{47}+\alpha_{56}-\alpha_{46}-\alpha_{57})B_6(\alpha_{13}-1,\alpha_{14}-2,\alpha_{15}-2,\ldots)$$
$$+(\alpha_{37}+\alpha_{46}-\alpha_{36}-\alpha_{47})B_6(\alpha_{13}-2,\alpha_{14}-2,\alpha_{15}-2,\ldots)]$$
$$+(\alpha_{27}+\alpha_{36}-\alpha_{26}-\alpha_{37})B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)\,. \tag{A5}$$

[Note: We use $B_6(x_{13},\ldots)$ and $B_6(\alpha_{13},\ldots)$ interchangeably.] In order to eliminate contributions from daughters, we write the coefficients of the B_6's in terms of particle momenta and retain only those terms linear in both (p_2-p_1) and (p_8-p_7). These are the only terms which result when both resonances (in s_{12} and s_{78}) are vector mesons. This yields A' without any daughter effects:

$$A''=-\alpha'^2 q\cdot p_3[q'\cdot p_6B_6(\alpha_{13},\ldots)+q'\cdot p_5B_6(\ldots,\alpha_{15}-1,\ldots)+q'\cdot p_4B_6(\ldots,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+q'\cdot p_3B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)]$$
$$-\alpha'^2 q\cdot p_4[q'\cdot p_6B_6(\alpha_{13}-1,\ldots)+q'\cdot p_5B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots)+q'\cdot p_4B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+q'\cdot p_3B_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots)]$$
$$-\alpha'^2 q\cdot p_5[q'\cdot p_6B_6(\alpha_{13}-1,\alpha_{14}-1,\ldots)+q'\cdot p_5B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)$$
$$+q'\cdot p_4B_6(\alpha_{13}-1,\alpha_{14}-2,\alpha_{15}-1,\ldots)+q'\cdot p_3B_6(\alpha_{13}-2,\alpha_{14}-2,\alpha_{15}-1,\ldots)]$$

$$
\begin{aligned}
&-\alpha'^2 q\cdot p_6[q'\cdot p_6 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots)+q'\cdot p_5 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-2,\ \ldots)\\
&\qquad +q'\cdot p_4 B_6(\alpha_{13}-1,\ \alpha_{14}-2,\ \alpha_{15}-2,\ \ldots)+q'\cdot p_3 B_6(\alpha_{13}-2,\ \alpha_{14}-2,\ \alpha_{15}-2,\ \ldots)]\\
&-\tfrac{1}{2}\alpha' q\cdot q' B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots),
\end{aligned}
\tag{A6}
$$

where $q=p_1-p_2$, $q'=p_7-p_8$, and α' is the slope of the trajectory. Replacing q by $\epsilon(\lambda)$ and q' by $\epsilon(\lambda')^*$ factors off the $V\sigma\sigma$ vertex and yields the equation for A found in Table I, after identifying numerical labels with the appropriate literal labels.

APPENDIX B

1. Diagram B

The duality diagram corresponding to the most convenient configuration for us is found in Fig. 3(b). The B_8 is obtained from that used for A by a simple transformation of variables:

$$
\begin{aligned}
B_8=-\int_0^1 & du_{12}du_{13}du_{14}du_{15}du_{67}u_{12}{}^{x_{12}}u_{13}{}^{x_{13}}u_{14}{}^{x_{14}}u_{15}{}^{x_{15}}u_{67}{}^{x_{67}}(1-u_{12})^{x_{23}}(1-u_{13})^{x_{34}}(1-u_{14})^{x_{45}}\\
&\times(1-u_{15})^{x_{57}}(1-u_{67})^{x_{16}}(1-u_{12}u_{13})^{x_{24}-x_{23}-x_{34}-1}(1-u_{13}u_{14})^{x_{35}-x_{34}-x_{45}-1}\\
&\times(1-u_{14}u_{15})^{x_{46}-x_{45}-x_{56}-1}(1-u_{15}u_{67})^{x_{26}-x_{27}-x_{16}-1}(1-u_{12}u_{13}u_{14})^{x_{25}-x_{24}+x_{34}-x_{35}}\\
&\times(1-u_{13}u_{14}u_{15})^{x_{36}-x_{35}+x_{45}-x_{46}}(1-u_{12}u_{13}u_{14}u_{15})^{x_{26}-x_{25}-x_{36}+x_{35}}\{1-u_{15}[u_{67}(1-u_{14})+u_{14}]\}^{x_{56}+x_{47}-x_{57}-x_{46}}\\
&\times\{1-u_{15}[u_{67}(1-u_{13}u_{14})+u_{13}u_{14}]\}^{x_{46}+x_{37}-x_{47}-x_{36}}\{1-u_{15}[u_{67}(1-u_{12}u_{13}u_{14})+u_{12}u_{13}u_{14}]\}^{x_{27}-x_{26}+x_{36}-x_{37}}.
\end{aligned}
\tag{B1}
$$

Making a Taylor expansion in u_{12} and u_{67} and performing those two integrations yields

$$
\begin{aligned}
B_8=-\sum_{i,j,k,l,m}\ \sum_{i',j',k',l'} & \gamma(i,x_{23})\gamma(j,x_{24}-x_{23}-x_{34}-1)\gamma(k,x_{25}-x_{24}+x_{34}-x_{35}+m)\gamma(l,x_{27}-x_{37}+x_{35}-x_{25}-m)\\
&\times\gamma(i',x_{16})\gamma(j',x_{26}-x_{27}-x_{16}-1)\gamma(k',x_{56}-x_{57}+x_{47}-x_{46})\gamma(l',x_{46}-x_{47}+x_{37}-x_{36})\\
&\times\gamma(m,x_{27}-x_{26}+x_{36}-x_{27})(x_{12}+1+i+j+k+l)^{-1}(x_{67}+1+i'+j'+k'+l'+m)^{-1}\\
&\times B_6(x_{13}+j+k+l,\ x_{14}+k+l,\ x_{15}+j'+k'+l'+l+m,\ x_{34},\ x_{45}+k',\ x_{57},\ x_{35}+k'+l',\ x_{47},\ x_{37}).
\end{aligned}
\tag{B2}
$$

Taking the residue at $x_{12}=x_{67}=-2$, disregarding terms not linear in both (p_1-p_2) and (p_6-p_7), and factoring off the $V\sigma\sigma$ vertex gives us

$$
\begin{aligned}
(1/\alpha'^2)B=&-\epsilon\cdot p_3[\epsilon'^*\cdot p_8 B_6(\alpha_{13},\ \alpha_{14},\ \alpha_{15},\ \alpha_{34},\ \alpha_{45},\ \alpha_{57},\ \alpha_{35},\ \alpha_{47},\ \alpha_{37})+\epsilon'^*\cdot pB_6(\ldots,\ \alpha_{15}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_4 B_6(\ldots,\ \alpha_{15}-1,\ \ldots,\ \alpha_{45}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_3 B_6(\ldots,\ \alpha_{15}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)]\\
&-\epsilon\cdot p_4[\epsilon'^*\cdot p_8 B_6(\alpha_{13}-1,\ \ldots)+\epsilon'^*\cdot pB_6(\alpha_{13}-1,\ \ldots,\ \alpha_{15}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_4 B_6(\alpha_{13}-1,\ \ldots,\ \alpha_{15}-1,\ \ldots,\ \alpha_{45}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_3 B_6(\alpha_{13}-1,\ \ldots,\ \alpha_{15}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)]\\
&-\epsilon\cdot p_5[\epsilon'^*\cdot p_8 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \ldots)+\epsilon'^*\cdot pB_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_4 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots,\ \alpha_{45}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_3 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)]\\
&-\epsilon\cdot p'[\epsilon'^*\cdot p_8 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots)+\epsilon'^*\cdot pB_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-2,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_4 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-2,\ \ldots,\ \alpha_{45}-1,\ \ldots,\ \alpha_{35}-1,\ \ldots)\\
&\qquad+\epsilon'^*\cdot p_3 B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-2,\ \ldots,\ \alpha_{35}-1,\ \ldots)]\\
&+(\epsilon\cdot\epsilon'^*/2\alpha')[B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-1,\ \ldots)-B_6(\alpha_{13}-1,\ \alpha_{14}-1,\ \alpha_{15}-2,\ \ldots)]\,,
\end{aligned}
\tag{B3}
$$

which leads directly to the equation of Table I.

2. Diagram C

Following the same steps as in part A of this appendix, but omitting the words this time, we have

$$\begin{aligned}
B_8 = -\int_0^1 & du_{13}du_{14}du_{15}du_{23}du_{16}u_{13}^{x_{13}}u_{14}^{x_{14}}u_{15}^{x_{15}}u_{23}^{x_{23}}u_{16}^{x_{16}}(1-u_{13})^{x_{24}}(1-u_{14})^{x_{45}} \\
& \times(1-u_{15})^{x_{56}}(1-u_{16})^{x_{67}}(1-u_{23})^{x_{12}}(1-u_{13}u_{23})^{x_{37}-x_{27}-x_{12}-1}(1-u_{13}u_{14})^{x_{35}-x_{34}-x_{45}-1} \\
& \times(1-u_{14}u_{15})^{x_{46}-x_{45}-x_{56}-1}(1-u_{15}u_{16})^{x_{57}-x_{56}-x_{67}-1}(1-u_{13}u_{14}u_{15})^{x_{36}-x_{35}+x_{45}-x_{46}} \\
& \times(1-u_{14}u_{15}u_{16})^{x_{47}-x_{46}+x_{56}-x_{57}}(1-u_{13}u_{14}u_{15}u_{16})^{x_{37}-x_{36}+x_{46}-x_{47}}\{1-u_{13}[u_{23}(1-u_{14})+u_{14}]\}^{x_{25}-x_{24}+x_{34}-x_{35}} \\
& \times\{1-u_{13}[u_{23}(1-u_{14}u_{15})+u_{14}u_{15}]\}^{x_{26}-x_{25}+x_{35}-x_{36}}\{1-u_{13}[u_{23}(1-u_{14}u_{15}u_{16})+u_{14}u_{15}u_{16}]\}^{x_{27}-x_{26}+x_{36}-x_{37}} \\
= -\sum_{i,j,k,l,m}\sum_{i',j',k',l'} & \gamma(i,x_{12})\gamma(j,x_{37}-x_{27}-x_{12}-1)\gamma(k,x_{25}-x_{24}+x_{34}-x_{35})\gamma(l,x_{26}-x_{25}+x_{35}-x_{36}) \\
& \times\gamma(m,x_{27}-x_{26}+x_{36}-x_{37})\gamma(i',x_{67})\gamma(j',x_{57}-x_{56}-x_{67}-1)\gamma(k',x_{47}-x_{46}+x_{56}-x_{57}+m) \\
& \times\gamma(l',x_{46}-x_{47}+x_{27}-x_{26}-m)(x_{23}+1+i+j+k+l+m)^{-1}(x_{16}+1+i'+j'+k'+l')^{-1} \\
& \times B_6(x_{13}+j+k+l+m+l',x_{14}+k'+l',x_{45}+j'+k'+l',x_{24},x_{45}+k,x_{56},x_{25},x_{46}+k+l,x_{26})
\end{aligned} \tag{B4}$$

and

$$\begin{aligned}
(1/\alpha'^2)C = & -\epsilon\cdot p_1[\epsilon'^*\cdot p_6B_6(\alpha_{13},\alpha_{14},\alpha_{15},\alpha_{24},\alpha_{45},\alpha_{56},\alpha_{25},\alpha_{46},\alpha_{26})+\epsilon'^*\cdot p_5B_6(\ldots,\alpha_{15}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot p_4B_6(\ldots,\alpha_{14}-1,\alpha_{15}-1,\ldots)+\epsilon'^*\cdot pB_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)] \\
& -\epsilon\cdot p'[\epsilon'^*\cdot p_6B_6(\alpha_{13}-1,\ldots)+\epsilon'^*\cdot p_5B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot p_4B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)+\epsilon'^*\cdot pB_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots)] \\
& -\epsilon\cdot p_5[\epsilon'^*\cdot p_6B_6(\alpha_{13}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{46}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot p_5B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{46}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot p_4B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{46}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot pB_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{46}-1,\ldots)] \\
& -\epsilon\cdot p_6[\epsilon'^*\cdot p_6B_6(\alpha_{13}-1,\ldots,\alpha_{46}-1,\ldots)+\epsilon'^*\cdot p_5B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{46}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot p_4B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots,\alpha_{46}-1,\ldots) \\
& \qquad +\epsilon'^*\cdot pB_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots,\alpha_{46}-1,\ldots)] \\
& +(\epsilon\cdot\epsilon'^*/2\alpha')[B_6(\alpha_{13}-1,\alpha_{14}-1,\alpha_{15}-1,\ldots)-B_6(\alpha_{13}-2,\alpha_{14}-1,\alpha_{15}-1,\ldots)],
\end{aligned} \tag{B5}$$

whence we obtain the equation of Table I.

3. Diagram D

$$\begin{aligned}
B_8 = \int_0^1 & du_{13}du_{23}du_{14}du_{15}du_{67}u_{13}^{x_{13}}u_{14}^{x_{14}}u_{15}^{x_{15}}u_{67}^{x_{67}}u_{23}^{x_{23}}(1-u_{13})^{x_{24}}(1-u_{14})^{x_{45}}(1-u_{15})^{x_{57}} \\
& \times(1-u_{67})^{x_{16}}(1-u_{23})^{x_{12}}(1-u_{13}u_{23})^{x_{37}-x_{27}-x_{12}-1}(1-u_{15}u_{67})^{x_{26}-x_{27}-x_{16}-1}(1-u_{13}u_{14})^{x_{35}-x_{34}-x_{45}-1} \\
& \times(1-u_{14}u_{15})^{x_{46}-x_{45}-x_{56}-1}(1-u_{13}u_{14}u_{15})^{x_{36}-x_{35}+x_{45}-x_{46}}\{1-u_{13}[u_{23}(1-u_{14})+u_{14}]\}^{x_{25}+x_{34}-x_{24}-x_{35}} \\
& \times\{1-u_{15}[u_{67}(1-u_{14})+u_{14}]\}^{x_{47}-x_{46}+x_{56}-x_{57}}\{1-u_{13}[u_{23}(1-u_{14}u_{15})+u_{14}u_{15}]\}^{x_{26}-x_{25}+x_{35}-x_{36}} \\
& \times\{1-u_{15}[u_{67}(1-u_{13}u_{14})+u_{13}u_{14}]\}^{x_{37}-x_{36}+x_{46}-x_{47}} \\
& \times[(1-u_{13}u_{23})(1-u_{15}u_{67})-u_{13}u_{14}u_{15}(1-u_{23})(1-u_{67})]^{x_{27}-x_{26}+x_{36}-x_{37}} \\
= \sum_{i,j,k,l,m}\sum_{i',j',k',l'} & \gamma(i,x_{12})\gamma(i',x_{16})\gamma(j,x_{37}-x_{27}-x_{12}-1)\gamma(j',x_{26}-x_{27}-x_{16}-1) \\
& \times\gamma(k,x_{25}+x_{34}-x_{24}-x_{35})\gamma(k',x_{47}+x_{56}-x_{46}-x_{57}+m)
\end{aligned}$$

$$\times\gamma(l, x_{26}+x_{35}-x_{25}-x_{36})\gamma(l', x_{46}+x_{27}-x_{47}-x_{26}-m)$$
$$\times\gamma(m, x_{27}+x_{36}-x_{26}-x_{37})(x_{23}+1+i+j+k+l+m)^{-1}(x_{67}+1+i'+k'+j'+l')^{-1}$$
$$\times B_6(x_{13}+j+k+l+m, x_{14}, x_{15}+j'+k'+l', x_{24}, x_{45}+k+k', x_{57}, x_{25}+k'+l', x_{47}+k+l+m, x_{27}) \quad \text{(B6)}$$

and

$$\begin{aligned}(1/\alpha'^2)D = &-\epsilon\cdot p_1[\epsilon'^*\cdot p_8 B_6(\alpha_{13}, \alpha_{14}, \alpha_{15}, \alpha_{24}, \alpha_{45}, \alpha_{57}, \alpha_{25}, \alpha_{47}, \alpha_{27}) + \epsilon'^*\cdot p_1 B_6(\ldots,\alpha_{15}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p_4 B_6(\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{25}-1,\ldots)+\epsilon'^*\cdot p B_6(\ldots,\alpha_{15}-1,\ldots,\alpha_{25}-1,\ldots)]\\
&-\epsilon\cdot p_8[\epsilon'^*\cdot p_8 B_6(\alpha_{13}-1,\ldots)+\epsilon'^*\cdot p_1 B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p_4 B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{25}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{25}-1,\ldots)]\\
&-\epsilon\cdot p_5[\epsilon'^*\cdot p_8 B_6(\alpha_{13}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{47}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p_1 B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{47}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p_4 B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-2,\ldots,\alpha_{25}-1,\alpha_{47}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{25}-1,\alpha_{47}-1,\ldots)]\\
&-\epsilon\cdot p'[\epsilon'^*\cdot p_8 B_6(\alpha_{13}-1,\ldots,\alpha_{47}-1,\ldots)+\epsilon'^*\cdot p_1 B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{47}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p_4 B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{25}-1,\alpha_{47}-1,\ldots)\\
&\quad+\epsilon'^*\cdot p B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{25}-1,\alpha_{47}-1,\ldots)]\\
&+(\epsilon\cdot\epsilon'^*/2\alpha')[B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{45}-1,\ldots,\alpha_{25}-1,\alpha_{47}-1,\ldots)\\
&\quad-B_6(\alpha_{13}-1,\ldots,\alpha_{15}-1,\ldots,\alpha_{25}-1,\alpha_{47}-1,\ldots)],\end{aligned} \quad \text{(B7)}$$

which leads to D of Table I.

---

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