Tests for Neutral Currents in Weak Pion Production

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Bounds for neutral currents are derived for the inclusive pion production on isoscalar nuclei in the simple Weinberg and the Glashow-Iliopoulos-Maiani models. The bounds involve cross sections induced by neutrinos, antineutrinos, or combination thereof. Final-state interactions are included in the results. For isolated proton targets a bound is obtained in both models independent of any additional assumptions. All the quantities occurring in the bounds are in principle measurable. For those processes where data are not yet available, estimates are presented by allowing a wide range of variation for the unknown quantities.

I. INTRODUCTION

Several papers have appeared in the literature recently which attempt to place lower bounds on cross sections that would involve first-order weakly coupled neutral currents in the Weinberg model[1] of weak and electromagnetic interactions. The cross-section ratios investigated include the elastic lepton interactions[2] $\nu_e + e \to \nu_e + e$ and $\nu_\mu + e \to \nu_\mu + e$, the elastic semileptonic interactions[3] $\nu_\mu + N \to \nu_\mu + N$, inelastic weak pion production[3,4] $\nu_\mu + N \to \nu_\mu + \pi + N$, and the inclusive reaction[5,6] $\nu_\mu + N \to \nu_\mu +$ anything.

The weak-pion-production bounds are of immediate interest to the experimentalist, but the bounds obtained invoke the assumptions of $\Delta(1236)$ dominance[7] and neglect the interaction of outgoing pions with other nucleons in a complex nucleus. The purpose of this paper is to deduce bounds for weak pion production that are more directly applicable to present experiments.

The relevant term in the effective Lagrangian is

$$\mathcal{L} = \frac{G}{\sqrt{2}}\left[\bar{\mu}\gamma_\alpha(1+\gamma_5)\nu(J^1_\alpha + iJ^2_\alpha) + \text{H.c.} + \bar{\nu}\gamma_\alpha(1+\gamma_5)\nu J^{(0)}_\alpha\right], \tag{1}$$

$$J^{(0)}_\lambda = A^3_\lambda + (1 - 2\sin^2\theta_W)V^3_\lambda + J^s_\lambda, \tag{2}$$

where $J^i = (V^i + A^i)$ is an isospin component of the usual $V-A$ current, θ_W is the Weinberg angle,[1,2] and J^s is an isoscalar current. This form holds for the simple Weinberg model without strange particles, and also for the Glashow-Iliopoulos-Maiani[8] (GIM) version. In Eq. (1) and in our results we have set $\cos\theta_C = 1$; as in other papers our results are valid in the $m_\mu = 0$ limit.

In Sec. II we derive general inequalities that hold for models with an effective Lagrangian of the form given by (1) and (2). Numerical estimates are made in Sec. III which allow a comparison with experimental data. In Sec. IV we give some additional inequalities which hold only for the simple Weinberg model in which $J^s_\lambda = -2\sin^2\theta_W \times (J^{\text{em}}_\lambda - V^3_\lambda)$, J^{em}_λ being the electromagnetic current.

II. CROSS-SECTION RATIO INEQUALITIES

Bounds for weak pion production will first be derived for a complex nuclear target with $I=0$. The use of an $I=0$ target makes it possible by combining π^+ and π^- production to eliminate the isoscalar-isovector interference terms for the neutral and electromagnetic currents. The results are derived directly for nuclear targets without assuming neutrino scattering from individual nucleons; final-state interactions other than those of electromagnetic origin are automatically included.

We define the cross sections

$$\sigma^a_- = \sigma(\nu_\mu + N \to \mu^- + \pi^a + x), \tag{3a}$$

$$\sigma^a_0 = \sigma(\nu_\mu + N \to \nu_\mu + \pi^a + x), \tag{3b}$$

$$\sigma^a_e = \sigma(e + N \to e + \pi^a + x), \tag{3c}$$

$$\sigma^{\text{ch}} = \sigma^+ + \sigma^-, \quad \sigma^{\text{tot}} = \sigma^{\text{ch}} + \sigma^0, \tag{3d}$$

$$V^a_{\text{em}} = \frac{G^2}{\pi}\frac{1}{4\pi\alpha^2}\int q^4 \frac{d\sigma^a_e}{dq^2}\,dq^2, \tag{3e}$$

where N is an $I=0$ nucleus and x is any possible final state defined by the type of particle involved, without discrimination for particular charge states. Thus we may consider the inclusive reaction where we sum over all possible states x or we may limit ourselves to states x with zero or a limited number of additional pions.

To make an isospin analysis we define $U^{(I)}_-$, $U^{(I)}_0$, and $U_{\rm em}^{(I)}$ as the contributions of the isovector current to σ_-, σ_0, and $V_{\rm em}$, respectively, where $I = (0, 1, 2)$ is the isospin of x. Similarly $S_0^{(1)}$ and $M_0^{(1)}$ represent the isoscalar and isoscalar-isovector interference contributions, respectively, to σ_0, which occur only for $I = 1$. The isospin analysis gives

$$\sigma_0^{\pm} = \tfrac{1}{3}S_0^{(1)} \mp 2(\tfrac{1}{6})^{1/2}M_0^{(1)} + \tfrac{1}{2}U_0^{(1)} + \tfrac{3}{10}U_0^{(2)} , \qquad (4a)$$

$$\sigma_0^{0} = \tfrac{1}{3}S^{(1)} + U_0^{(0)} + \tfrac{2}{5}U_0^{(2)} , \qquad (4b)$$

$$\sigma_-^{+} = U_-^{(0)} + \tfrac{1}{2}U_-^{(1)} + \tfrac{1}{10}U_-^{(2)} , \qquad (4c)$$

$$\sigma_-^{-} = \tfrac{3}{5}U_-^{(2)} , \qquad (4d)$$

$$\sigma_-^{0} = \tfrac{1}{2}U_-^{(1)} + \tfrac{3}{10}U_-^{(2)} . \qquad (4e)$$

From Eqs. (4a) and (4b) we need only the inequalities

$$\tfrac{1}{2}\sigma_0^{\rm ch} \geq \tfrac{1}{2}U_0^{(1)} + \tfrac{3}{10}U_0^{(2)} , \qquad (5a)$$

$$\sigma_0^{0} \geq U_0^{(0)} + \tfrac{2}{5}U_0^{(2)} . \qquad (5b)$$

Similarly

$$\tfrac{1}{2}V_{\rm em}^{\rm ch} \geq \tfrac{1}{2}U_{\rm em}^{(1)} + \tfrac{3}{10}U_{\rm em}^{(2)} , \qquad (5c)$$

$$V_{\rm em}^{0} \geq U_{\rm em}^{(0)} + \tfrac{2}{5}U_{\rm em}^{(2)} . \qquad (5d)$$

Finally we note that Eq. (8b) of Ref. 6 can be applied separately for each value of I:

$$U_0^{(I)} \geq \tfrac{1}{2}[(U_-^{(I)})^{1/2} - 2\sin^2\theta_W (U_{\rm em}^{(I)})^{1/2}]^2 . \qquad (6)$$

Combining Eqs. (4), (5), and (6) we obtain our major results[9]

$$R_1 \equiv \frac{\sigma_0^{\rm ch}}{2\sigma_-^0} \geq \tfrac{1}{2}\left[1 - 2\sin^2\theta_W \left(\frac{V_{\rm em}^{\rm ch}}{2\sigma_-^0}\right)^{1/2}\right]^2 , \qquad (7a)$$

$$R_2 \equiv \frac{\sigma_0^0}{\sigma_-^{\rm ch} - \sigma_-^0} \geq \tfrac{1}{2}\left[1 - 2\sin^2\theta_W \left(\frac{V_{\rm em}^0}{\sigma_-^{\rm ch} - \sigma_-^0}\right)^{1/2}\right]^2 . \qquad (7b)$$

A somewhat more general result can be obtained[9] in terms of two weighting factors $\alpha, \beta \geq 0$:

$$\frac{\alpha\sigma_0^{\rm ch} + \beta\sigma_0^0}{2\alpha\sigma_-^0 + \beta(\sigma_-^{\rm ch} - \sigma_-^0)} \geq \tfrac{1}{2}\left[1 - 2\sin^2\theta_W \left(\frac{\alpha V_{\rm em}^{\rm ch} + \beta V_{\rm em}^0}{2\alpha\sigma_-^0 + \beta(\sigma_-^{\rm ch} - \sigma_-^0)}\right)^{1/2}\right]^2 . \qquad (7c)$$

If the right-hand sides of Eqs. (7a) and (7b) can be directly evaluated then a weighted sum of these two equations gives a stronger result than Eq. (7c). However, Eq. (7c) may be useful in special cases; in particular, for $\alpha = \beta = 1$ we obtain a total cross-section ratio

$$R \equiv \frac{\sigma_0^{\rm tot}}{\sigma_-^{\rm tot}} \geq \tfrac{1}{2}\left[1 - 2\sin^2\theta_W \left(\frac{V_{\rm em}^{\rm tot}}{\sigma_-^{\rm tot}}\right)^{1/2}\right]^2 ,$$

which is the result previously obtained in Ref. 6.

If we define the charge-to-neutral ratio as $r \equiv \sigma_-^{\rm ch}/\sigma_-^0$, which is necessarily greater than or equal to unity, Eqs. (7a) and (7b) can be rewritten in more useful forms:

$$R_3 \equiv \frac{\sigma_0^{\rm ch}}{2\sigma_-^{\rm ch}} \geq \tfrac{1}{2}\left[r^{-1/2} - 2\sin^2\theta_W \left(\frac{V_{\rm em}^{\rm ch}}{2\sigma_-^{\rm ch}}\right)^{1/2}\right]^2 , \qquad (8a)$$

$$R_4 \equiv \frac{\sigma_0^0}{2\sigma_-^0} \geq \tfrac{1}{4}\left[(r-1)^{1/2} - 2\sin^2\theta_W \left(\frac{V_{\rm em}^0}{\sigma_-^0}\right)^{1/2}\right]^2 . \qquad (8b)$$

All the above results may also be derived for the case in which N is a free nucleon provided one averages over neutron and proton targets. In particular the results may be applied to the exclusive process of single-pion production with the help of the following replacements:

$$\sigma_-^{\rm ch} \to \sigma(\nu + p \to \mu^- + \pi^+ + p) + \sigma(\nu + n \to \mu^- + \pi^+ + n) , \qquad (9a)$$

$$\sigma_-^0 \to \sigma(\nu + n \to \mu^- + \pi^0 + p) , \qquad (9b)$$

$$\sigma_0^{\rm ch} \to \sigma(\nu + p \to \nu + \pi^+ + n) + \sigma(\nu + n \to \nu + \pi^- + p) , \qquad (9c)$$

$$\sigma_0^0 \to \sigma(\nu + p \to \nu + \pi^0 + p) + \sigma(\nu + n \to \nu + \pi^0 + n) , \qquad (9d)$$

$$V_{\rm em}^{\rm ch} \to V_{\rm em}(e + p \to e + \pi^+ + n) + V_{\rm em}(e + n \to e + \pi^- + p), \qquad (9e)$$

$$V_{\rm em}^0 \to V_{\rm em}(e + p \to e + \pi^0 + p) + V_{\rm em}(e + n \to e + \pi^0 + n) . \qquad (9f)$$

As in Ref. 6, these results may be extended to antineutrino reactions. We define

$$\sigma_+^a = \sigma(\bar\nu_\mu + N \to \mu^+ + \pi^a + x) , \qquad (10a)$$

$$\bar\sigma_0^a = \sigma(\bar\nu_\mu + N \to \bar\nu_\mu + \pi^a + x) , \qquad (10b)$$

$$\Sigma^a = \sigma_+^a + \sigma_-^a , \qquad (10c)$$

$$\Sigma_0^a = \sigma_0^a + \bar\sigma_0^a . \qquad (10d)$$

The previous results, Eqs. (7) and (8), hold with the replacements $\sigma_-^a \to \sigma_+^a$ and $\sigma_0^a \to \bar\sigma_0^a$. Analogous to Eq. (18) of Ref. 6 we find

$$\bar R_1 \equiv \frac{\Sigma_0^{\rm ch}}{2\Sigma^0} \geq \tfrac{1}{2}\left(1 - 8\sin^2\theta_W(1 - \sin^2\theta_W)\frac{V_{\rm em}^{\rm ch}}{2\Sigma^0}\right) , \qquad (11a)$$

$$\bar{R}_2 \equiv \frac{\Sigma_0^0}{\Sigma^{ch} - \Sigma^0} \geq \frac{1}{2}\left(1 - 8\sin^2\theta_W(1-\sin^2\theta_W)\frac{V_{em}^0}{\Sigma^{ch} - \Sigma^0}\right). \quad (11b)$$

Analogous to Eq. (20) of Ref. 6 we find

$$R_1 \geq \frac{1}{2}(1-\sin^2\theta_W) \times\left[1 + \frac{\sin^2\theta_W}{B_1(1-\sin^2\theta_W)} - 4\sin^2\theta_W\left(\frac{V_{em}^{ch}}{2\sigma_-^0}\right)\right], \quad (12a)$$

$$R_2 \geq \frac{1}{2}(1-\sin^2\theta_W) \times\left[1 + \frac{\sin^2\theta_W}{B_2(1-\sin^2\theta_W)} - 4\sin^2\theta_W\left(\frac{V_{em}^0}{\sigma_-^{ch} - \sigma_-^0}\right)\right], \quad (12b)$$

where B_1 and B_2 are the experimental upper limits:

$$\frac{\sigma_-^0}{\sigma_+^0} \leq B_1,$$

$$\frac{\sigma_-^{ch} - \sigma_-^0}{\sigma_+^{ch} - \sigma_+^0} \leq B_2.$$

We would also like to be able to use data on isolated proton targets. From Eq. (31) of Ref. 6 we have

$$\frac{\sigma_0(\Delta^+)}{\sigma_-(\Delta^{++})} \geq \frac{1}{3}\left[1 - 2\sin^2\theta_W\left(\frac{V_{em}(\Delta^+)}{\frac{2}{3}\sigma_-(\Delta^{++})}\right)^{1/2}\right]^2, \quad (13)$$

where

$$\sigma_0(\Delta^+) = \sigma(\nu + p \to \nu + \Delta^+),$$

$$\sigma_e(\Delta^+) = \sigma(e + p \to e + \Delta^+),$$

$$\sigma_-(\Delta^{++}) = \sigma(\nu + p \to \mu^- + p + \pi^+).$$

This involves no assumptions provided Δ simply means an $I = \frac{3}{2}$ final state. However, we are interested in

$$R_5 \equiv \frac{\sigma(\nu + p \to \nu + p + \pi^0)}{\sigma_-(\Delta^{++})},$$

$$R_6 \equiv \frac{\sigma(\nu + p \to \nu + n + \pi^+)}{\sigma_-(\Delta^{++})}.$$

By using only isospin properties of the final states we obtain

$$R_5 + R_6 = \frac{\sigma(\nu + p \to \nu + \Delta^+) + \sigma(\nu + p \to \nu + N'^+)}{\sigma_-(\Delta^{++})} \geq \frac{1}{3}\left[1 - 2\sin^2\theta_W\left(\frac{V_{em}(\Delta^+)}{\frac{2}{3}\sigma_-(\Delta^{++})}\right)^{1/2}\right]^2, \quad (14)$$

where N' is an $I = \frac{1}{2}$ final state. Note that the $\frac{1}{2}$-$\frac{3}{2}$ interference terms cancel out and that the N' term was dropped and Eq. (13) used to obtain the inequality above. This result is true in both models of Refs. 1 and 8, independent of any dynamical assumptions.

To obtain limits on R_5 and R_6 separately we need to make a further assumption. In Sec. IV we consider the case of the simple Weinberg model. Here we make the assumption that there is no interference between $I = \frac{1}{2}$ and $I = \frac{3}{2}$ final states since interference is expected to be small on the average over the resonance region. In this case any $I = \frac{1}{2}$ contribution can only increase both $p\pi^0$ and $n\pi^+$, so that the $I = \frac{3}{2}$ contributions can be used to obtain lower limits:

$$R_5 \geq \frac{2}{9}\left[1 - 2\sin^2\theta_W\left(\frac{V_{em}(\Delta^+)}{\frac{2}{3}\sigma_-(\Delta^{++})}\right)^{1/2}\right]^2, \quad (15a)$$

$$R_6 \geq \frac{1}{9}\left[1 - 2\sin^2\theta_W\left(\frac{V_{em}(\Delta^+)}{\frac{2}{3}\sigma_-(\Delta^{++})}\right)^{1/2}\right]^2. \quad (15b)$$

III. NUMERICAL ESTIMATES

In order to obtain numerical estimates for the bounds derived in Sec. II, one needs data for both the electromagnetic and weak charged-current reactions on the same $I = 0$ nucleus. The charged-current neutrino cross sections can be obtained from the total inelastic cross section[10] measurement at 2 GeV

$$\sigma_-^0 + \sigma_-^{ch} = 1.0 \times 10^{-38}\ \text{cm}^2/\text{nucleon}, \quad (16a)$$

along with the ratio

$$r = \sigma_-^{ch}/\sigma_-^0 \simeq 2.0 \quad (16b)$$

determined by the CERN neutrino group using the Gargamelle chamber.[11] The latter value is in substantial agreement with the result

$$\sigma_-^+/\sigma_-^0 = 2.3 \pm 0.9 \quad (16c)$$

from the old Freon experiment,[12] whereas Δ dominance requires the value $r = 5$. Evidence is present for final-state strong interactions.

For the corresponding electromagnetic interactions, no data exist for the same heavy nuclei. In its place, we use the data of Galster *et al.*[13] on hydrogen to estimate

$$V_{em}(e + p \to e + \Delta^+) = 0.156 \times 10^{-38}\ \text{cm}^2 \quad (17a)$$

at 2 GeV and obtain V_{em}^0 and V_{em}^{ch} for a complex nucleus by assuming that

$$V_{em}^0/V_{em}^{ch} = 1.5 \quad (17b)$$

and that there is at most 25% incoherent background. The above imply

$$V_{em}^{ch} = 0.156 \times 1.25 \times \frac{1.0}{2.5} \times 10^{-38}\ \text{cm}^2/\text{nucleon} = 0.078 \times 10^{-38}\ \text{cm}^2/\text{nucleon}, \quad (18a)$$

TABLE I. Lower bounds on the ratios R_i as functions of the ratio $r=\sigma_-^{ch}/\sigma_-^0$ for (a) V_{em} given by Eqs. (18) and (b) V_{em} twice the values in Eqs. (18).

	R_1		R_3		R_4	
r	a	b	a	b	a	b
5	0.19	0.10	0.04	0.02	0.44	0.28
4	0.21	0.13	0.05	0.03	0.32	0.19
3	0.23	0.15	0.08	0.05	0.19	0.10
2	0.26	0.19	0.13	0.09	0.07	0.03

$$V_{em}^0=0.117\times10^{-38}\ \text{cm}^2/\text{nucleon}. \tag{18b}$$

We shall later provide a generous allowance for this uncertainty.

Finally we note that the angle θ_W has been bounded by $\sin^2\theta_W \lesssim 0.33$ in Ref. 2, where one standard deviation in the experimental data for $\bar{\nu}_e+e\to\bar{\nu}_e+e$ was allowed. To be conservative in what follows, we shall take

$$\sin^2\theta_W \lesssim 0.40\,, \tag{19}$$

which corresponds to[2]

$$\frac{\sigma(\bar{\nu}_e+e\to\bar{\nu}_e+e)_{\text{expt}}}{\sigma(\bar{\nu}_e+e\to\bar{\nu}_e+e)_{V-A}}\lesssim 3\,.$$

Several of the bounds listed in Sec. II are of particular interest, and we discuss them in some detail. The ratio $R_4=\sigma_0^0/2\sigma_-^0$ has been bounded both by the Columbia group,[14] who obtained

$$R_4\leqslant 0.14\ (90\%\ \text{c.l.})\,, \tag{20a}$$

and by the Gargamelle group,[11] who obtained

$$R_4\leqslant 0.11\ (90\%\ \text{c.l.})\,. \tag{20b}$$

In Table I we estimate the lower bound for this ratio for values of r ranging from 5 down to 2. It is clear that the experimental results in (20) contradict the predictions of the model only if $r\gtrsim 3$.

A similar ratio, $R_3=\sigma_0^{ch}/2\sigma_-^{ch}$, will probably be measured in the near future. The ratio, $R_1=\sigma_0^{ch}/2\sigma_-^0=rR_3$, is of interest because it is rather insensitive to the ratio r and is fairly large. The lower bounds on the ratios R_1 and R_2 given in (12a) and (12b) cannot be estimated until sufficient data for the antineutrino reactions are obtained which will determine B_1 and B_2.

There is also an experimental bound for

$$R_5+R_6\leqslant 0.31\ (90\%\ \text{c.l.}) \tag{21}$$

from a recent Argonne experiment.[15] All quantities on the right-hand side of Eq. (14) are measurable. Using the new Argonne result

$$\sigma(\nu+p\to\mu^-+p+\pi^+)=(0.78\pm0.16)\times10^{-38}\ \text{cm}^2 \tag{22}$$

we obtain

$$R_5+R_6\geqslant 0.10\,. \tag{23}$$

This bound holds in both models and it is independent of any additional dynamical assumptions.

IV. WEINBERG MODEL

In the simple Weinberg model $J_\lambda^s=-2\sin^2\theta_W(1/\sqrt{3})V_\lambda^8$, leading to

$$J_\lambda^{(0)}=J_\lambda^3-2\sin^2\theta_W J_\lambda^{em}\,. \tag{24}$$

It now follows that for any states $|\alpha\rangle$ and $|\beta\rangle$

$$|\langle\beta|J_\lambda^{(0)}|\alpha\rangle|^2\geqslant(|\langle\beta|J_\lambda^3|\alpha\rangle|-2\sin^2\theta_W|\langle\beta|J_\lambda^{em}|\alpha\rangle|)^2\,, \tag{25}$$

with the isoscalar-isovector interference being completely absorbed in the electromagnetic matrix element. The advantage of (25) is apparent in that it allows us to state bounds for the following ratios which do not average over the final charge states of the pion or over protons and neutrons in the target:

$$R_6\equiv\frac{\sigma(\nu+p\to\nu+n+\pi^+)}{\sigma(\nu+p\to\mu^-+p+\pi^+)}$$

$$\geqslant\tfrac{1}{2}\left[\left(\frac{\sigma(\nu+n\to\mu^-+p+\pi^0)}{\sigma(\nu+p\to\mu^-+p+\pi^+)}\right)^{1/2}-2\sin^2\theta_W\left(\frac{V_{em}(e+p\to e+n+\pi^+)}{\sigma(\nu+p\to\mu^-+p+\pi^+)}\right)^{1/2}\right]^2, \tag{26a}$$

$$R_7\equiv\frac{\sigma_0^-}{\sigma_-^0}\geqslant\tfrac{1}{2}\left[1-2\sin^2\theta_W\left(\frac{V_{em}^-}{\sigma_-^0}\right)^{1/2}\right]^2. \tag{26b}$$

One could obtain a similar result for R_5; however, it is more useful to consider the sum R_5+R_6 because, as was discussed in the previous section, it can be bounded by Eq. (14) in both the Weinberg and the Glashow-Iliopoulos-Maiani models.

The last ratio is appealing from the experimental point of view because σ_0^- has a clear signal; i.e., a π^- always converts in the chamber so that the only ambiguity arises in confusing μ^- with π^-. Thus such bounds are rather safe.

Numerical estimates can be obtained using the results of the previous section. The ratio R_7 is numerically equal to that of R_1 given in Table I. There is already an experimental bound for the ratio

$$R_6 \leq 0.16 \text{ (90\% c.l.)} \tag{27}$$

from an old CERN experiment.[16] Since data for $\sigma(\nu n \to \mu^- p \pi^0)$ occurring in Eq. (26a) are not available, we assume that the isospin amplitudes are incoherent and get

$$R_6 \geq 0.03\,. \tag{28}$$

V. SUMMARY

We have derived bounds for weak pion production relevant to present and future experiments. The bounds R_1 through R_4 follow both in the simple Weinberg and GIM models for neutrinos incident on isospin-zero targets. Similar relations hold when one averages over protons and neutrons, as indicated by the substitutions given in Eq. (9). When both neutrino and antineutrino cross sections are available, they can be combined to give the results stated in Eqs. (11) and (12).

For isolated proton targets, one can derive the bound (14) for the sum $R_5 + R_6$, which also holds in both Weinberg and GIM models and is independent of any additional dynamical assumptions. Bounds stated in Eq. (15) for R_5 and R_6 separately require either $I = \frac{3}{2}$ dominance or incoherence of the isospin amplitudes. However, in the specific Weinberg model in which the neutral current is given by Eq. (24), one can obtain theoretical bounds for R_6 and R_7 as given in Eq. (26) without any additional assumptions.

Comparisons between the present upper experimental bounds and lower theoretical bounds were discussed in Secs. III and IV. The present experimental limit of 0.31 for $R_5 + R_6$ from the Argonne group is well above the estimated lower limit of 0.10. This test for weak pion production in hydrogen is very clean, and better statistics should be obtained in the near future.

The comparisons in complex nuclei depend critically on the charged/neutral pion-production ratio observed in the charged-current reactions, as seen from Table I. Moreover, the neutrino reactions on complex nuclei have not been carried out with isoscalar targets, although the use of heavy liquid Freon is probably a good approximation. This situation should improve when propane is used in the large Gargamelle chamber at CERN. In order to make the comparisons more accurate, we also require data for inclusive pion electroproduction in heavy nuclei. To the extent that these large uncertainties are included in Table I, we see that the present experimental upper limits on R_4 are still compatible with the theoretical predictions based on the models of Refs. 1 and 8.

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The lower value used for this cross section results in lower and more conservative bounds on the cross section ratios quoted in our paper.

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Higher-Order Corrections to Semileptonic Weak Processes*

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Using a strong formulation of lepton-hadron universality, higher-order corrections to semileptonic weak processes are related to the high-energy behavior of lepton-lepton weak-scattering amplitudes. The "weak cutoff" estimates of ~ 4 GeV from the observed K_L-K_S mass difference and ~ 16 GeV from the absence of the decay $K_L \to \mu^+\mu^-$ are interpreted as upper bounds on the center-of-mass energy at which cross sections for pure leptonic processes deviate significantly from the low-energy phenomenological predictions. A phenomenology for describing similar deviations in high-energy neutrino-nucleon reactions is developed. A sum rule relating such deviations is derived, as well as estimates of their order of magnitude using the parton model.

I. INTRODUCTION

It cannot be questioned that the structure of weak interactions as we know them today will be modified at high energy. Modifications will certainly be important at center-of-mass energies of about 600 GeV, for which the present lowest-order theory would violate unitarity,[1,2] and it is probable that they are important at considerably lower energies.[3] Most likely, new classes of states, such as intermediate bosons, will be produced at some characteristic center-of-mass energy Λ, which we may hope is within range of the present generation of accelerators. But in any case, the presence of important modifications at higher energy requires, through the dispersion relations, small modifications of the present picture at lower energy. The purpose of this paper is to study such modifications, mainly to semileptonic weak processes. The method of attack generalizes the approach used in a previous paper,[4] hereafter called I, in which pure lepton-lepton scattering amplitudes were studied from an S-matrix point of view. Here, as in I, we make the following assumptions:

(1) Electromagnetic effects can be neglected.

(2) Lepton masses can be neglected.

(3) The low-energy limit of the lepton-lepton scattering amplitude takes the conventionally assumed charged current-current form.

(4) The full amplitudes exhibit the SU(2) symmetry present in the low-energy (i.e., few-GeV) region, in which the SU(2) multiplets are (e, μ) and (ν_e, ν_μ) doublets. In addition, the amplitudes are symmetric under the interchange

$$\begin{pmatrix} e \\ \mu \end{pmatrix} \leftrightarrow \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}.$$

As a consequence of these assumptions, it was shown in I that all two-body lepton-lepton scattering amplitudes may be described by three helicity amplitudes, $A(s,t)$, $B(s,t)$, and $C(s,t)$. These amplitudes were parametrized as a double power series in s and t, plus small unitarity corrections, which are easily computed from Mandelstam's iteration method. Thus, for s and t large enough that lepton mass can be neglected but small enough that the series converges, we may write

$$\begin{aligned} A(s,t) &= \alpha_{10}Gs + \alpha_{20}(Gs)^2 + \alpha_{11}G^2st + \cdots, \\ B(s,t) &= \beta_{10}Gs + \beta_{20}(Gs)^2 + \beta_{11}G^2st + \cdots, \\ C(s,t) &= \gamma_{10}Gs + \gamma_{20}(Gs)^2 + \gamma_{11}G^2st + \cdots, \end{aligned} \tag{1.1}$$

where $\alpha_{10} = 4\sqrt{2}$. We have neglected the unitarity corrections in (1.1) because at attainable energies they are quite small. However, the corrections proportional to $\alpha_{ij}, \beta_{ij}, \gamma_{ij}$ are not necessarily so small. Other than α_{10}, these coefficients are poorly determined from present limits on lepton-lepton processes.

These amplitudes (1.1) may be represented by an