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Some Consequences of Nonet Symmetry*

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Attention is drawn to the existence of certain similarities between predictions of the asymptotic nonet-symmetry model and the quark model. In particular, the electromagnetic decays of vector and pseudoscalar mesons and the decoupling of the ϕ meson from the nucleons are discussed. We find slight differences of prediction for certain electromagnetic decay rates which would have implications in choosing between the so-called quadratic and linear mass mixing formulas for pseudoscalar mesons.

SOME CONSEQUENCES OF NONET SYMMETRY

In a recent paper,[1] we used the hypothesis of asymptotic nonet symmetry, together with the first and modified second spectral-function sum rules of Weinberg,[2] to derive certain results. Among these were the standard nonet result[3] $m_\omega = m_\rho$ together with a derivation of the ideal mixing angle for the vector-meson nonet.

In addition, we derived the following relation between the dimensionless photon–vector-meson coupling constants:

$$\frac{1}{{f_\rho}^2}:\frac{1}{{f_\omega}^2}:\frac{1}{{f_\phi}^2}=9:1:2\left(\frac{m_\omega}{m_\phi}\right)^2 . \tag{1}$$

This is a modification of the usual quark-model prediction which would reduce to that prediction in the SU(3)-symmetric limit.

Before proceeding further, we introduce the mixing angles θ_V and θ_P for the vector and pseudoscalar mesons. We suppose that the physical particles ϕ, ω, η, and X are linear superpositions of pure singlet and octet states

$$|\phi\rangle = \cos\theta_V\,|\phi_8\rangle - \sin\theta_V\,|\omega_0\rangle , \tag{2a}$$

$$|\omega\rangle = -\cos\theta_V\,|\omega_0\rangle - \sin\theta_V\,|\phi_8\rangle \tag{2b}$$

and

$$|\eta\rangle = \cos\theta_P|\eta_8\rangle - \sin\theta_P|X_0\rangle , \tag{3a}$$

$$|X\rangle = \cos\theta_P|X_0\rangle + \sin\theta_P|\eta_8\rangle . \tag{3b}$$

We take the vector mixing angle θ_V to be the ideal angle[1] ($\tan\theta_V = 1/\sqrt{2}$) and consider two possibilities for θ_P:

(1) θ_P is determined by a quadratic Gell-Mann–Okubo mass formula; $\theta_P(\text{quadratic}) = -10°$.

(2) θ_P is determined by a linear Gell-Mann–Okubo mass formula; $\theta_P(\text{linear}) = -23°$.

In this note, we would like to draw attention to certain similarities between predictions made by the asymptotic nonet-symmetry model and the quark model.

ELECTROMAGNETIC DECAYS

In order to compare our model with the normal quark model, we shall first compare our predictions for the electromagnetic decays of vector and pseudoscalar mesons with the predictions of the quark model.[4]

We shall use the vector-dominance model (VDM) of Gell-Mann, Sharp, and Wagner,[5] together with the assumption of SU(3)-symmetric coupling at the V-P-V vertex, in order to calculate the electromagnetic decay rates. For the photon–vector-meson coupling constants, we use the values given in Eq. (1). It is then a simple matter to calculate the various coupling constants for the electromagnetic decays $V \to P\gamma$, $P \to V\gamma$. Using a standard notation[5,6] for the various coupling constants we find

$$\begin{aligned}
g_{\omega\pi\gamma} &= g_{\omega\rho\pi}/f_\rho = G\,,\\
g_{\rho\pi\gamma} &= -\tfrac{1}{3}G\,,\\
g_{\phi\pi\gamma} &= 0\,,\\
g_{\omega\eta\gamma} &= \tfrac{1}{3}\sin(\theta_C - \theta_P)G\,,\\
g_{\rho\eta\gamma} &= -\sin(\theta_C - \theta_P)G\,,\\
g_{\phi\eta\gamma} &= \frac{2}{3}\frac{m_\omega}{m_\phi}\cos(\theta_C - \theta_P)G\,,\\
g_{X\omega\gamma} &= \tfrac{1}{3}\cos(\theta_C - \theta_P)G\,,\\
g_{X\rho\gamma} &= -\cos(\theta_C - \theta_P)G\,,\\
g_{\phi X\gamma} &= -\frac{2}{3}\frac{m_\omega}{m_\phi}\sin(\theta_C - \theta_P)G\,,
\end{aligned} \tag{4}$$

where θ_C is the ideal mixing angle ($\tan\theta_C = 1/\sqrt{2}$).

The above results are in agreement with the predictions of the nonrelativistic quark model[4] *except* for our estimation of the coupling constants $g_{\phi\eta\gamma}$ and $g_{X\phi\gamma}$. Our predictions differ from those of the quark model by a factor of (m_ω/m_ϕ). Had we assumed nonet symmetry together with exact SU(3) symmetry ($m_\omega = m_\phi$), then we would have *exactly* reproduced the quark-model predictions.[7]

It is interesting and reassuring that we have started with different assumptions and finished up with results that are in almost complete agreement with the quark-model results.

We shall not quote decay rates for all the processes listed above since, for the most part, they are identical with the quark-model predictions and may be found in the review article of Morpurgo.[4] However, we shall compare some of our predictions with the quark model because, where there are differences, a comparison of the models is useful with regard to the question of η-X mixing.

Since the decay rate for the best-known decay, $\omega \to \pi^0\gamma$, is uncertain (the quoted decay rate seems to have decreased over the past two years from about 1.1 MeV to 0.9 MeV), we choose to fix the value at 1.1 MeV. This enables us to directly compare our results with those of Morpurgo, who uses the same input value for $\Gamma(\omega \to \pi^0\gamma)$. It is an easy matter to adjust the various absolute rates for a different input value of $\Gamma(\omega \to \pi^0\gamma)$. One simply multiplies the various absolute rates by a constant normalizing factor. The relative rates, however, are in no way affected by our input choice for $\Gamma(\omega \to \pi^0\gamma)$.

From the choice

$$\Gamma(\omega \to \pi^0\gamma) = 1.1\ \text{MeV}\,, \tag{5}$$

we may deduce that

$$g_{\omega\pi^0\gamma} = 0.387/m_{\pi^0}\,. \tag{6}$$

Using the relation

$$g_{\omega\pi^0\gamma} = g_{\omega\rho\pi}/f_\rho\,, \tag{4}$$

together with the Orsay result[8]

$$f_\rho^{\,2}/4\pi = 2.54 \pm 0.23\,, \tag{7}$$

we have

$$g_{\omega\rho\pi}^{\;2}/4\pi = 21 \pm 2\ \text{GeV}^{-2}\,. \tag{8}$$

The results of our calculation are displayed in Table I.[9-12] We have assumed $m_\rho = m_\omega = 784$ MeV. We have used the phase-space factors of Baracca and Bramon[6] in order to estimate the decay rate for $\Gamma(\eta \to \pi^+\pi^-\gamma)$. The decay rates $\pi^0 \to 2\gamma$, $\eta \to 2\gamma$, and $X \to 2\gamma$ have been evaluated by means of the Gell-Mann–Sharp–Wagner model[5,6] taking account of η-X mixing and using our photon–vector-meson coupling constants [Eq. (1)]. Where applicable, decay rates have been evaluated assuming both quadratic mixing ($\theta_P = -10°$) and linear mixing ($\theta_P = -23°$).

It can be seen that in those processes where the ϕ-γ junction enters (namely $\phi \to \eta\gamma$, $\eta \to \gamma\gamma$, $X \to \gamma\gamma$) we make quite different predictions from the quark model.[13] In particular our predictions for the $\phi \to \eta\gamma$ and $X \to \gamma\gamma$ rates are rather lower than those of the quark model, while our prediction for the $\eta \to \gamma\gamma$ rate is rather higher than that of the quark model. It is clear, then, that while we have constructed a model for electromagnetic decay which

TABLE I. Various absolute and relative electromagnetic decay rates are shown. In estimating the experimental decay rates $\Gamma(\phi \to \pi^0\gamma)$ and $\Gamma(\phi \to \eta\gamma)$ we use[9] $\Gamma(\phi \to \text{all}) = 4.4 \pm 0.3$ MeV. For the decay rate $\Gamma(\phi \to \eta\gamma)$, we quote two apparently incompatible experimental values deduced from Refs. 10 and 11.

Quantity	Nonet model		Quark model (Ref. 4)		Experiment
	Quadratic mixing	Linear mixing	Quadratic mixing	Linear mixing	
$\Gamma(\omega \to \pi^0\gamma)$	1.1 MeV (input)	1.1 MeV	1.1 MeV (input)	1.1 MeV	0.9 ± 0.1 MeV (Ref. 9)
$\Gamma(\phi \to \pi^0\gamma)$	0	0	0	0	11 ± 4 keV (Ref. 10)
$\Gamma(\phi \to \eta\gamma)$	125 keV	70 keV	210 keV	115 keV	92 ± 28 keV (Ref. 10) 320 ± 80 keV (Ref. 11)
$\Gamma(\eta \to \pi^+\pi^-\gamma)$	140 eV	202 eV	130 eV (Ref. 13)	185 eV (Ref. 13)	129 ± 38 keV (Ref. 9)
$\Gamma(X \to \rho\gamma)$	155 keV	85 keV	155 keV	85 keV	...
$\Gamma(\pi^0 + \gamma\gamma)$	11.8 eV	11.8 eV	11 eV	11 eV	11.7 ± 0.7 eV (Ref. 12)
$\Gamma(\eta \to \gamma\gamma)$	0.765 keV	1.28 keV	0.480 keV	1 keV	1.00 ± 0.22 keV (Ref. 9)
$\Gamma(X \to \gamma\gamma)$	8.0 keV	5.3 keV	10 keV	6 keV	...
$\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma)$	0.183	0.158	0.271	0.185	0.129 ± 0.005 (Ref. 9)
$\Gamma(X \to \gamma\gamma)/\Gamma(X \to \rho\gamma)$	0.052	0.062	0.065	0.071	0.06 ± 0.01 (Ref. 9)

is very close to the quark model, it makes quite different predictions for a subgroup of those decays.

At the present time the quality of the relevant data is not very good. Certainly it is not good enough to distinguish between these two models for electromagnetic decay. In any case, the main purpose of this note is to draw attention to the similarity of the models rather than to put forward a competing model.

We close this section with the following remarks:

(1) While the quark model favors linear mixing in order to agree with the experimental decay rate for $\eta \to \gamma\gamma$, the nonet model does not favor one form of mixing rather than another in order to fit this rate.

(2) The experimental branching ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma)$ should be a more accurate test of the η-X mixing because it is more precisely known than either of the two absolute rates. The quark-model predictions do not agree for either quadratic or linear mixing. The nonet model is in agreement with the experimental value assuming linear mixing.

(3) Both models make predictions for $\Gamma(X \to \gamma\gamma)/\Gamma(X \to \rho\gamma)$ which are compatible with experiment whether one assumes linear or quadratic mixing.

(4) Unfortunately, the two experiments[10,11] which have measured the decay rate $\Gamma(\phi \to \eta\gamma)$ are in complete disagreement with one another and can shed no light on the question of η-X mixing.

In summary, the two models make essentially the same predictions for electromagnetic decays except in those decays which test the amount of η-X mixing. More accurate experimental data on these decays will be interesting both from the point of view of yielding information on the amount of η-X mixing and from the point of view of testing the nonet model against the quark model, where we expect differences to occur.

NUCLEON-ϕ-MESON COUPLING

We now turn our attention to the question of the electric and magnetic coupling of the ϕ meson to the nucleons. We define the electric and magnetic coupling constants $g^{(1)}_{\phi NN}$ and $g^{(2)}_{\phi NN}$ of the ϕ meson to the nucleons through

$$i\bar{u}_N(p_2)\left(g^{(1)}_{\phi NN}\gamma_\mu + \frac{1}{2M}\, g^{(2)}_{\phi NN}\sigma_{\mu\nu} q_\nu\right) u_N(p_1) = \lim_{q^2 \to -m_\phi^2} (q^2 + m_\phi^2)\left(\frac{E_1 E_2}{M^2}\right)^{1/2} \langle N(p_2)|\,\phi_\mu\,|N(p_1)\rangle\,, \qquad (9)$$

where

$$q_\mu = (p_2 - p_1)_\mu ,$$

and the notation is obvious. We may similarly define the electric and magnetic coupling constants $g^{(1)}_{\omega NN}$ and $g^{(2)}_{\omega NN}$ of the ω meson to the nucleons.

We next recall that the standard VDM definition of the hypercharge and baryonic charge currents is given by[14]

$$J^Y_\mu = \frac{1}{f_Y}(m_\phi{}^2 \cos\theta_Y\, \phi_\mu - m_\omega{}^2 \sin\theta_Y\, \omega_\mu) ,$$
$$J^B_\mu = \frac{1}{f_B}(m_\omega{}^2 \cos\theta_B\, \omega_\mu + m_\phi{}^2 \sin\theta_B\, \phi_\mu) . \tag{10}$$

J^Y_μ and J^B_μ are related to the usual nonet of vector currents V^α_μ through

$$J^Y_\mu = (\tfrac{2}{3})^{1/2} V^8_\mu ,$$
$$J^B_\mu = (\tfrac{2}{3})^{1/2} V^0_\mu . \tag{11}$$

In Ref. 1, we showed that the validity of the first Weinberg sum rules[2] led to the current mixing condition[15]

$$(m_\phi/m_\omega)\tan\theta_B = \tan\theta = (m_\omega/m_\phi)\tan\theta_Y . \tag{12}$$

This condition, together with the asymptotic nonet-symmetry assumption of Ref. 1, enables us to write the hypercharge and baryonic currents [Eq. (10)] in the simpler forms

$$J^Y_\mu = \frac{2}{\sqrt{3}}\,\frac{m_\rho}{f_\rho}\,(m_\phi \cos\theta\, \phi_\mu - m_\omega \sin\theta\, \omega_\mu) ,$$
$$J^B_\mu = -(\tfrac{2}{3})^{1/2}\,\frac{m_\rho}{f_\rho}\,(m_\omega \cos\theta\, \omega_\mu + m_\phi \sin\theta\, \phi_\mu) . \tag{13}$$

Using these definitions, together with the requirement that at zero momentum transfer

$$\langle N(p)|\, J^Y_0 |N(p)\rangle = \tfrac{1}{2}\langle N(p)|\, J^B_0 |N(p)\rangle , \tag{14}$$

gives the relation

$$\sqrt{2}\left(\frac{\cos\theta}{m_\phi}\, g^{(1)}_{NN\phi} - \frac{\sin\theta}{m_\omega}\, g^{(1)}_{NN\omega}\right) = -\left(\frac{\cos\theta}{m_\omega}\, g^{(1)}_{NN\omega} + \frac{\sin\theta}{m_\phi}\, g^{(1)}_{NN\phi}\right), \tag{15}$$

holding between the ω and ϕ electric coupling constants.

Insertion of the ideal mixing angle for θ into the relation Eq. (15) yields the simple result that

$$g^{(1)}_{\phi NN} = 0 .$$

We have made the usual assumption that there is no change in the electric coupling of the ϕ meson to the nucleons as we move away from the ϕ meson pole, i.e., the electric coupling form factor is a constant. (The same assumption is made for the ω meson.)

In order to deduce information about the magnetic coupling constants $g^{(2)}_{\phi NN}$ and $g^{(2)}_{\omega NN}$, we need to make some specific assumption about the relative strengths of the F and D magnetic coupling of the vector mesons to the nucleons. We assume therefore the SU(6) value,[16] $D/F = \frac{3}{2}$, which ensures that the ratio of the proton to the neutron magnetic moments is

$$\mu_p/\mu_n = -\tfrac{2}{3} ,$$

in good agreement with experiment.

At zero momentum transfer it is possible to define both an isoscalar hypercharge magnetic moment μ_Y and an isosinglet baryonic charge magnetic moment μ_B. If we use the definition Eq. (13) for the hypercharge and baryonic charge currents, we may relate μ_Y and μ_B to the magnetic coupling constants $g^{(2)}_{\phi NN}$ and $g^{(2)}_{\omega NN}$ as follows:

$$\mu_Y = \frac{1}{2M}\left[1 + \frac{2}{\sqrt{3}}\,\frac{m_\rho}{f_\rho}\left(\frac{\cos\theta}{m_\phi}\, g^{(2)}_{\phi NN} - \frac{\sin\theta}{m_\omega}\, g^{(2)}_{\omega NN}\right)\right] , \tag{16}$$

$$\mu_B = \frac{1}{2M}\left[1 - (\tfrac{2}{3})^{1/2}\,\frac{m_\rho}{f_\rho}\left(\frac{\cos\theta}{m_\omega}\, g^{(2)}_{\omega NN} + \frac{\sin\theta}{m_\phi}\, g^{(2)}_{\phi NN}\right)\right] . \tag{17}$$

Assuming that the magnetic coupling of the vector mesons to the nucleons has the SU(6) value[16] $D/F = \frac{3}{2}$, we have the simple relation

$$\mu_Y = \mu_B . \tag{18}$$

If we now insert the ideal mixing angle into Eqs. (16) and (17), the equality expressed in Eq. (18) leads to the result

$$g^{(2)}_{\phi NN} = 0 . \tag{19}$$

(As before, we assume that the magnetic coupling form factors for both the ϕ and ω mesons remain constant as we move away from their respective poles.)

This means that the ϕ meson is completely decoupled from the nucleon. It is interesting to note that if we argue conversely, the decoupling of the ϕ meson from the nucleons implies that $g^{(2)}_{\phi NN}$ vanishes. In the nonet-symmetry model with an ideal mixing angle, this implies that the D/F ratio is $\frac{3}{2}$ for the magnetic coupling. This in turn implies that the ratio of the proton to neutron magnetic moment is given by

$$\mu_p/\mu_n = -\tfrac{2}{3} ,$$

which is the SU(6) prediction. The implication of this seems to be that the success of this prediction is very closely connected with the decoupling of the ϕ meson from the nucleons.

The observation that the ϕ meson is completely decoupled from the nucleons is not new.[17] Within the framework of the quark model, it is very simply understood. Here one considers the nucleons to be made up entirely of nonstrange quarks whereas the ϕ meson is made up of a strange quark-antiquark pair. Such a composition prevents any interaction between the nucleons and the ϕ meson.

Our deduction, therefore, that the ϕ meson is completely decoupled from the nucleons is in perfect agreement with the quark model (and, of course, with experiment to the present level of accuracy).

In conclusion, then, we have compared the asymptotic nonet-symmetry model[1] with the quark model and found that they make very similar predictions in the areas that we have discussed. To this extent, the asymptotic nonet-symmetry model may be regarded as an alternative model to the quark model in these areas. In this sense it may be regarded as a useful model because the quark model, although extremely valuable, suffers from certain embarrassments (e.g., nonrelativistic formulation).

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