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Pionic and Electromagnetic Production of η in a Model of Higher Baryon Couplings

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An s-channel analysis of η-production processes induced by pions and photons on protons is made in the momentum region 2.9–13.3 GeV/c. The analysis is carried out in a model of higher baryon couplings, which is characterized by a relativistic extension of a broken $SU_6 \otimes O_3$ with partial symmetry scheme for $\bar{B}B_LP$ and $\bar{B}B_LV$ couplings in a unified fashion. In this high-energy extension of the model we incorporate seven baryon trajectories, viz., $N_\alpha(938)$, $N_\beta(1530)$, $N_\beta'(1715)$, $N_\beta''(1675)$, $N_\gamma(1520)$, $N_\gamma'(1675)$, and $N_\delta(1860)$, for which we carry out piecewise summations over the contributions of the particles involved in the corresponding towers, formally up to infinity. The predictions of the theory are compared with experiment with respect to the (i) total and differential cross sections and the recoil neutron polarization for the process $\pi^-p \to \eta n$, and (ii) differential cross sections for the process $\gamma p \to \eta p$ at several incident energies. In general the agreement with experiment is rather good up to ~ 10 GeV/c. The reasons for the success up to this momentum, as well as the limitations to extension beyond this value, are discussed in the context of contemporary ideas.

I. INTRODUCTION

Because of the peculiar quantum numbers of the η meson which forbid its couplings to several meson pairs, a study of the production of this particle in πN and γN processes provides a unique opportunity to examine the working of duality in a relatively clean fashion. From the point of view of Regge theory, η production in πN and γN collisions is supposed to be dominated by the exchange of a Reggeized A_2 meson (and perhaps also ρ and ω mesons in the second case).[1] A direct-channel resonance study of these processes offers a neat alternative to the mechanism of Regge exchanges (which are believed to dominate the processes) and thus furnishes some test of the duality principle through a direct comparison with experiment. The theoretical significance of the simulation of A_2 exchange via direct-channel resonances stems from the standard belief in the crucial role of the former in producing a mechanism for electromagnetic (e.m.) masses[2] via the so-called tadpole diagrams.[3] Thus a physically viable simulation of the A_2-exchange effect by the direct-channel resonances offers a nontrivial alternative mechanism for the evaluation of e.m. masses in the original "Feynman-Speisman"[4] spirit where the form factors at the vertices provide the necessary extrapolations off the mass shell, rather than in the "dispersion spirit" where the resonances provide only the "pole" contributions.

One of the motivations of this paper is to study a few selected processes involving η production up to sufficiently high energies via direct-channel resonances with a view to learn experimentally about the simulation of A_2-exchange effects. A second motivation is to examine, in relation to these processes, the working of a model of higher baryon couplings which has already been used for several other reactions in recent times, viz., pion-nucleon scattering,[5] photoproduction,[6] and vector-meson production,[7] mostly in the resonance region. In particular, the process $\gamma N \to \eta N$ which has been investigated in the "resonance region"[6] is characterized by a comparative "flatness" of the total and differential cross sections, with little structure in energy (except the threshold peak) and angle, indicating dominance of the lower-spin resonances (N_β, N_δ). This is in sharp contrast to pion photoproduction which requires most of its contributions to come from the higher-spin resonances ($N_\alpha, N_\gamma, \Delta_\delta$). The couplings of these lower-spin resonances are in turn governed mainly by the "quark recoil" contribution rather than via the "direct" term in the primitive quark-meson (or photon) coupling, as was emphasized recently.[6] Among additional ingredients to which the results are sensitive are the mixing angles between (i) the quartet (q) and doublet (d) states corresponding to a specified J value for the resonance [e.g., $S_{11}(1530)$ versus $S'_{11}(1715)$], and (ii) the SU_3 octet and singlet $I=0$ members of the P-meson nonet. Finally, the solution to the problem of heavy-meson enhancement (a vital requirement for the η meson) lies partly in the structure of the recoil couplings and partly in the effect of the Van-Royen–Weisskopf[8] factor associated with $\eta\bar{N}N^*$ coupling.

In this paper we shall present the results of our study of the two specific processes $\pi N \to \eta N$ and $\gamma N \to \eta N$ up to energies extending far beyond the resonance region and well into the Regge domain. This idea is really not as ambitious as it would appear in the first instance, since the basic assumptions of the model do provide enough formal machinery for such an extrapolation, thus retrieving it from a simple Breit-Wigner formalism applicable only to the resonance region. Specifically the extension beyond the resonance region is warranted by (i) the availability of couplings of arbitrarily high-spin resonances via the general structure of the form factor and the "Regge universality" condition on the "reduced" coupling constant and, hence, by (ii) the facility of summation over entire towers of resonances via the unified coupling structures for $\bar{B}B_L(P, V)$ which the model provides. The question still remains as to how far the comparison of such a framework with experiment can be regarded as physically meaningful. For we recognize that despite the above facilities of extension certain other ingredients such as contributions of successive daughter trajectories should become progressively more important at higher energies in terms of the concept of central waves ($J_{\max} \propto \sqrt{s}$), which seems to have experimental support.[9] Therefore, the question of applicability of our model to higher energies is essentially a quantitative one rather than one of principle. We believe that the simplest answer to this quest lies in a conscious comparison with experiment in relation to different processes as a function of energy. This should hopefully give a fairly clear idea as to the energy region beyond which the model breaks down. It is essentially in this spirit that we shall examine the processes of pionic production and photoproduction of η mesons off nucleons and try to infer the limitations on the high-energy applicability of this model, though we would *a priori* expect the latter to extend well beyond the resonance region because of the presence of several important "high-energy" ingredients mentioned in the beginning of the paragraph.

The coupling structures for the above processes have been spelled out in detail in some recent communications.[6,10,11] We shall freely draw from these papers except for places where marginal modifications (e.g., form factors) are involved. Explanation of these as well as some specific details bearing on η production are summarized in Sec. II. Section III describes the construction of amplitudes for η production in πN and γN collisions and discusses the method of summation over whole towers of resonances in relation to the problem of the extrapolation of the form factors in the energy variable. Sections IV and V describe the results for η production in πN and γN collisions, respectively, in relation to experiments as well as to A_2-exchange models. Section VI summarizes our main conclusions, especially the limitation on the high-energy extension of this model.

II. η COUPLINGS

In this section we shall summarize some of the main points dealing with some specific features of the η couplings. For both the processes $\pi^- p \to \eta n$ and $\gamma p \to \eta p$, the quantum numbers restrict the intermediate states in a "formation experiment" in the direct channel only to the N-type resonances. The N-type trajectories that are expected to contribute are the following[12]:

$$N_\alpha(938, \tfrac{1}{2}^+), \quad N_\beta''(1675, \tfrac{5}{2}^-), \quad N_\gamma(1515, \tfrac{3}{2}^-), \quad N_\gamma'(1675, \tfrac{3}{2}^-), \quad N_\beta(1525, \tfrac{1}{2}^-), \quad N_\beta'(1715, \tfrac{1}{2}^-), \quad N_\delta(1860, \tfrac{3}{2}^+). \tag{2.1}$$

Here the leading particle masses as well as J^P assignments are bracketed for each trajectory. At this stage of inadequate experimental knowledge the Regge recurrences of all these trajectories are mere postulates, except for the trajectory N_γ, where three more recurrences are experimentally known, viz., $N(2190, \frac{7}{2}^-)$, $N(2650, \frac{11}{2}^-)$, and $N(3030, \frac{15}{2}^-)$. As the particles in a particular set of angular momentum rotational bands have the same quantum numbers as well as parity, we have only to specify the relations between the spin J and the corresponding complex masses (M_J, Γ_J) of the successive resonances to be able to specify a trajectory completely. Now, the spin of the nth resonance in the ith trajectory is given by $J_n^{(i)} = J_0^{(i)} + 2n$, where $J_0^{(i)}$ is the spin of the leading particle of the corresponding trajectory. Once $J_0^{(i)}$ is known, the Chew-Frautschi relation for the masses together with a similar empirical relation for the widths is adequate for a complete empirical "determination" of $M_J^{(i)}$ and $\Gamma_J^{(i)}$ in terms of $J^{(i)}$.

TABLE I. List of the towers considered, together with the complex masses and the spins of the leading particles.

Trajectories	Complex masses of the leading particle (M_0, Γ_0) in GeV	Spin of the leading particle, J_0
N_α	(0.938, 0)	$\frac{1}{2}$
N_β	(1525, 0.127)	$\frac{1}{2}$
N'_β	(1.715, 0.320)	$\frac{1}{2}$
N''_β	(1.675, 0.240)	$\frac{5}{2}$
N_γ	(1.515, 0.105)	$\frac{3}{2}$
N'_γ	(1.675, 0.101)	$\frac{3}{2}$
N_δ	(1.860, 0.335)	$\frac{3}{2}$

To specify the mass dependence of the different resonances on their spins, we assume the simplest linear relation of the form

$$M_J^{(i)2} = a^{-1}(J^{(i)} - J_0^{(i)}) + M_0^{(i)2}, \tag{2.2}$$

where $a = 1\ \mathrm{GeV}^{-2}$, and $M_0^{(i)}$, $J_0^{(i)}$ are the mass and spin, respectively, of the leading (lowest-spin) member of a particular family (i). As to the dependence of the widths on the spins, we again postulate an empirical, but numerically accurate, relation of the form[13]

$$\Gamma_J^{(i)} = \Gamma_0^{(i)} + b(M_J^{(i)} - M_0^{(i)}), \tag{2.3}$$

where $b \approx 0.2$ is (hopefully) a universal constant applicable to all the families (i) considered, while $\Gamma_0^{(i)}$ and $M_0^{(i)}$ are the corresponding values for the lowest members. The relations (2.2) and (2.3) are reasonably well satisfied for the N_γ trajectory for which three candidates are available. For the other trajectories these relations are to be regarded as the defining relations for the masses and widths of their successive Regge recurrences, and will be utilized in Sec. III for evaluating the summation over the amplitudes arising from the resonances of arbitrarily large masses and spins. Table I summarizes the values of M_0, Γ_0, and J_0 for the different nucleon trajectories listed in Eq. (2.1).

For a realistic specification of couplings to these trajectories, consideration of the mixtures between spin-doublet (d) and spin-quartet (q) states in the quark model is essential. For the N_β trajectory, we define the mutually orthogonal mixed states in terms of a parameter θ_S as

$$\begin{aligned} N_\beta(1525) &= N_\beta^q \cos\theta_S + N_\beta^d \sin\theta_S, \\ N_\beta(1715) &= -N_\beta^q \sin\theta_S + N_\beta^d \cos\theta_S, \end{aligned} \tag{2.4}$$

where mixing of the two corresponding trajectories is implied. The value of θ_S used in MM was 63°, but we shall also explore a possibility of its "determination" through a direct comparison with the total cross section of the process $\pi^- p \to \eta n$. For the two N_γ trajectories the mixing is not so important, so we take $N(1515)$ and $N_\gamma'(1675)$ as N^d and N^q states, respectively, with negligible error. Similarly, within our model no mixing

TABLE II. The $\bar{B}B_L(\pi, \eta, \gamma)$ couplings for the different towers considered. The $SU_6 \otimes O_3$ factors without SU_2 (isospin) factors are displayed. The quantities C_ρ and C_ω are em_ρ^2/g_ρ and $em_\rho^3/3m_\omega g_\rho$, respectively, where $g_\rho^2 = 4\pi \times 3.6$.

Trajectories $N(J^P)$	$\bar{B}B_L(\pi, \eta)$ couplings: Direct	$\bar{B}B_L(\pi, \eta)$ couplings: Recoil	$\bar{B}B_L\gamma$ couplings: Direct	$\bar{B}B_L\gamma$ couplings: Recoil
$N_\alpha(\frac{1}{2}^+)$	$\left(\frac{5}{\sqrt{3}}, \left(\frac{2}{3}\right)^{1/2}\sin\delta + \frac{1}{\sqrt{3}}\cos\delta\right)$		$A\left(\frac{5}{\sqrt{3}}C_\rho + \sqrt{3}C_\omega\right) - B\sqrt{3}(C_\rho + 3C_\omega)$	$-\left(\frac{L}{2L+1}\right)^{1/2} E\left(\frac{5}{\sqrt{3}}C_\rho + \sqrt{3}C_\omega\right)$
$N_\beta^q(\frac{1}{2}^-)$	$\cdots$	$\left(\frac{2}{L+1}\right)^{1/2}(1, \sqrt{2}\sin\delta + \cos\delta)$	$\cdots$	$\cdots$
$N_{\beta'}^d(\frac{1}{2}^-)$	$\cdots$	$\left(\left(\frac{8}{3}\right)^{1/2}, \frac{2}{\sqrt{3}}\sin\delta + \left(\frac{2}{3}\right)^{1/2}\cos\delta\right)$	$\cdots$	$-\left(\frac{L+1}{2L+1}\right) C\left(\left(\frac{8}{3}\right)^{1/2}C_\rho + \sqrt{6}C_\omega\right) + \left(\frac{L-1}{2L+1}\right) D\left(\left(\frac{8}{3}\right)^{1/2}C_\rho + \sqrt{6}C_\omega\right)$
$N_{\beta''}^q(\frac{5}{2}^-)$	$(1, \sqrt{2}\sin\delta + \cos\delta)$	$\cdots$	$\cdots$	$\cdots$
$N_\gamma^d(\frac{3}{2}^-)$	$\left(\left(\frac{8}{3}\right)^{1/2}, \frac{2}{\sqrt{3}}\sin\delta + \left(\frac{2}{3}\right)^{1/2}\cos\delta\right)$	$\cdots$	$A\left(\left(\frac{8}{3}\right)^{1/2}C_\rho + \sqrt{6}C_\omega\right) - \sqrt{6}BC_\rho$	$-\left(\frac{L}{2L+1}\right)^{1/2} E\left(\left(\frac{8}{3}\right)^{1/2}C_\rho + \sqrt{6}C_\omega\right)$
$N_{\gamma'}^q(\frac{3}{2}^-)$	$\left(\frac{L+1}{2L+1}\right)^{1/2}(1, \sqrt{2}\sin\delta + \cos\delta)$	$\cdots$	$\cdots$	$\cdots$
$N_\delta(\frac{3}{2}^+)$	$\cdots$	$\left(\frac{5}{\sqrt{3}}, \left(\frac{2}{3}\right)^{1/2}\sin\delta + \frac{1}{\sqrt{3}}\cos\delta\right)$	$\cdots$	$\left(\frac{L-1}{2L+1}\right) D\left(\frac{5}{\sqrt{3}}C_\rho + \sqrt{3}C_\omega\right) - \left(\frac{L+1}{2L+1}\right) C\left(\frac{5}{\sqrt{3}}C_\rho + \sqrt{3}C_\omega\right)$

need be considered for N_α, N_δ, and N_β'' as long as the possibility of mixing between different supermultiplets [(56, $2L^+$) and (70, $(2L+1)^-$)] is ignored. Finally, for the physical η meson we consider the singlet-octet mixing and make for the η meson the state

$$\eta = \eta_1 \sin\delta + \eta_8 \cos\delta, \tag{2.5}$$

where $\delta = 10.5°$.[14]

A detailed account of the coupling scheme has been given in some recent papers.[6,10,11] The coupling structures $\bar{N}N^*(\eta, \pi)$ and $\bar{N}N^*\gamma$ can be evaluated as in these papers under the general assumption that the $(L+1)$ and $(L-1)$ wave couplings are generated via the "direct" and the "recoil" terms, respectively (at the $\bar{Q}MQ$ level). The structures (A)–(E) of the $\bar{N}N^*\gamma$ couplings are as given in the preceding paper,[15] while those for $\bar{N}N^*\eta$ are as follows:

$$\bar{\psi}i\gamma_5 i\not{q}\, q_{\mu_1}\cdots q_{\mu_L}\psi^{L+1/2}_{\mu_1\cdots\mu_L}\eta \qquad (L+1)\text{ wave}$$

and (2.6)

$$\bar{\psi}q_{\mu_2}\cdots q_{\mu_L}\psi^{L-1/2}_{\mu_2\cdots\mu_L}\eta \qquad (L-1)\text{ wave},$$

in the same notation as MM. The SU_6 Clebsch-Gordan coefficients for the vertices $\bar{N}N^*(\pi, \eta)$ and $\bar{N}N^*\gamma$ are summarized in Table II.

As to the structure of the form factor we use the same form as given in the preceding paper [Eq. (2.5)], the Van Royen–Weisskopf factor $(m_\eta/m_\pi)^{1/2}$ being essential for this coupling, while its role for $\bar{N}N^*\gamma$ is at best regarded as an open question for which a considerable amount of empirical evidence in the "resonance" region of energies have already been found.[6,7] The problem of extrapolation of the form factor off the mass shell is taken up in Sec. III in connection with summation over contributions from an infinite sequence of resonances.

III. CONSTRUCTION OF AMPLITUDES FOR η-MESON PRODUCTION IN πN AND γN COLLISIONS

A. Process $\pi^- p \to \eta n$

Taking account of the ingredients described in the last section, the contribution to the T matrix of the process arising from a given resonance of spin J and mass M belonging to any one of the possible trajectories (in the notation of A) is

$$\bar{u}_{p'}T_J u_p = -\sqrt{\tfrac{2}{3}}\, G_\eta G_\pi \bar{u}_{p'}\Gamma^{(S)\dagger}\frac{-i\not{P} + M_J}{s - M_J^2 + iM_J\Gamma_J} \times \Theta^{q_1\cdots q_{J-1/2}}_{q'_1\cdots q'_{J-1/2}}(J)\Gamma^{(S)}u_p, \tag{3.1}$$

where

$\Gamma^{(S)} = 1, i\gamma_5 i\not{q}$ for $S = 1, 2$.

The quantities G_π and G_η represent the vertex functions for $\bar{B}B_L\pi$ and $\bar{B}B_L\eta$ vertices, respectively (including the SU_6 Clebsch-Gordan coefficient given by Table II and the appropriate form factor, while the Clebsch-Gordan coefficients of SU_2 are explicitly shown). The other details including the propagator for spin J are given in A in the same notation. From Eq. (3.1) it is easy to identify the contribution of a resonance of given spin to the invariant amplitudes A' and B according to the relations

$$T_J = -A_J + i\left(\frac{\not{q}+\not{q}'}{2}\right) B_J\,, \tag{3.2}$$

$$A'_J = A_J + \frac{\nu + t/4m}{1 - t/4m^2}\, B_J\,, \tag{3.3}$$

while the total invariant amplitudes[16] are defined as

$$\begin{aligned} A' &= \sum_{(i)}\sum_J A'^{(i)}_J\,, \\ B &= \sum_{(i)}\sum_J B^{(i)}_J\,. \end{aligned} \tag{3.4}$$

The remaining details of the calculation are exactly the same as those in A.

B. The Process $\gamma p \to \eta p$

The kinematics of this process is essentially the same as that of MM1 and MM2. We therefore indicate the additional notational problems involved as a result of the inclusion of a variety of different trajectories listed in Sec. II. As an example, we show the contribution to the invariant amplitudes $[A(\alpha), \alpha = 1, \ldots, 4]$ arising from a N'_β type of particle (algebraically the most complicated one) via the T matrix, which is expressed (in the notation of MM2) as

$$\bar{u}_{p'} T_J u_p = -\frac{i}{\sqrt{3}} G_\eta G_\gamma\, \bar{u}_{p'} \frac{-i\not{P} + M_J}{s - M_J^2 + iM_J\Gamma_J}$$

$$\times\, \Theta^{\mu,k,\ldots,k_{J-1/2-1}}_{q,\ldots,q_{J-1/2}}(J)\epsilon_{\mu\nu\sigma\rho} k_\nu \epsilon_\sigma \gamma_\rho u_p \tag{3.5}$$

$(i = \beta')$, where G_η is the same as in the expression (3.1) and G_γ is the corresponding expression for the photon coupling expressible as a product of the form factor and the SU_6 Clebsch-Gordan coefficient as given in Table II under the vector-dominance assumption. The remaining details of Eq. (3.5) arising out of the photon coupling structure of the type D are the same as in MM1. We now rewrite Eq. (3.5) in the standard form as[17]

$$\begin{aligned} \bar{u}_{p'} T^{(i)}_J u_p = \bar{u}_{p'}\gamma_5[&iA^{(i)}_J(1)\not{\epsilon} + A^{(i)}_J(2)\not{k}\not{\epsilon} \\ &+ iA^{(i)}_J(3)q\cdot\epsilon\not{k} + A^{(i)}_J(4)q\cdot\epsilon]u_p\,, \end{aligned} \tag{3.6}$$

from which the contributions of a given resonance of spin J and type (i) to these invariant amplitudes are directly read off. The total invariant amplitudes $A(\alpha)$ as in the previous case are given by

$$A(\alpha) = \sum_{(i)}\sum_J A^{(i)}_J(\alpha) \qquad (\alpha = 1, \ldots, 4). \tag{3.7}$$

To facilitate an explicit algebraic summation over an infinite number of the resonances, it is essential to bring out the J dependence of the different invariant amplitudes arising from each trajectory type. This problem was much less serious in the earlier applications[5-7] where summation over a few low-lying resonances are involved, but assumes greater importance in the present investigation where a desire to go to higher energies must necessarily be accompanied by a preparedness to sum over an arbitrarily large number (formally infinite) of resonances of different types. To illustrate the technique we outline in the Appendix the method of treatment for the J-dependent part of the amplitude. The other ingredients needed for the evaluation of the various amplitudes are (i) the J dependence of the mass and widths of the resonance and (ii) extrapolation of the form factor so as to make it formally applicable at sufficiently high incident energies. For the former, we take the prescription outlined in Sec. II [Eqs. (2.2) and (2.3)]. As to the extrapolation of the form factor the simplest recipe which was used in the earlier applications was $M^2 \to s$, a prescription we also propose in the present application. This extrapolation must also be made in the numerator of the Feyman propagator through the following replacement[18] in the basic "building block":

$$\theta_{\mu\nu} = \delta_{\mu\nu} + \frac{p_\mu p_\nu}{M^2} \to \delta_{\mu\nu} + \frac{p_\mu p_\nu}{s}\,. \tag{3.8}$$

Finally, the factor $J^{1/4}$ in the form factor, which causes a formal divergence as the summation over J is carried out, must be replaced by a numerically equivalent expression which is free from this trouble. For this purpose, we first use the observed linear relation between J and M_J^2 for a given trajectory, and then make the replacement $M_J^2 \to s$, e.g., for the N_γ trajectory the over-all result of this manipulation is expressed by

$$2S_F(M_L m)^{1/2} J^{1/4} \to 2S_F\sqrt{s}\,. \tag{3.9}$$

As to the question of gauge invariance in the photon couplings, the problem is mostly a formal one in this case, where the incident photon is real.

Indeed, the gauge modifications of the various couplings, which are listed in Eq. (2.4) of the preceding paper,[15] show that the contributions from the extra terms vanish identically for the coupling of a real photon.[19]

IV. PIONIC PRODUCTION OF η MESON

A. Total Cross Section near the Resonance Region

In this section, we shall describe results mostly of those aspects of the $\pi^- p \to \eta n$ process which we believe have a bearing on our basic physical motivations for this investigation: an experimentally visible test of the duality principle in terms of a concrete model of s-channel resonances. Since the working of the present model in the resonance region has already been investigated for several related processes,[5-7] we shall be interested here not so much in a detailed fit to the process $\pi^- p \to \eta n$ in the resonance region as in the working of the model at higher energies, overlapping with the Regge region. We recognize that the investigations in the resonance region have also been covered extensively, using various methods, such as (i) partial-wave analysis,[20] (ii) the role of several resonances using the Breit-Wigner formalism,[21] (iii) multichannel formalism (K matrix, etc.),[22] and (iv) miscellaneous dynamical methods.[23] All these methods more or less agree on the physical mechanism for the most conspicuous experimental feature of a threshold peak in the total cross section for $\pi^- p \to \eta n$: the threshold dominance of the $S_{11}(1525)$ resonance. A second, less conspicuous, feature is the dip around $\sqrt{s} \sim 1.675$ GeV which none of the above investigations seems to have explained so far. Apart from these structures, a study of the total cross section does not seem to provide any particularly sensitive test of a model. We give only one result for the reaction $\pi^- p \to \eta n$ in the resonance region: the energy dependence of the total cross section, as a device to explore the sensitivity to the mixing angle θ_S [Eq. (2.4)] for $S_{11}(1525)$, considering the key role of this resonance in the understanding of the resonance peak. This should help in keeping this parameter out of the way for purposes of (more detailed) investigation of the high-energy region. Figure 1 shows the total cross section for two different values of θ_S, together with the experimental points,[24] in the resonance region covering the range from threshold to $\sqrt{s} \sim 1.9$ GeV. The sensitivity to θ_S is quite marked, and $\theta_S \approx 55°$ seems to fit the data somewhat better than the value $\theta_S \approx 63°$ which had been found from a comparison of[25] the ηN and πN decay modes of the physical $S_{11}(1525)$ and $S'_{11}(1715)$. The dip around $\sqrt{s} \sim 1.675$ GeV however does not come about in our model.

B. Differential Cross Section

We now come to the high-energy region for which the natural candidate for comparison is $d\sigma/dt$ rather than σ_T. Most of the contemporary analyses in these energies are in terms of Regge (A_2 exchange) mechanisms – using different models such as simple A_2 exchange,[26] absorption cor-

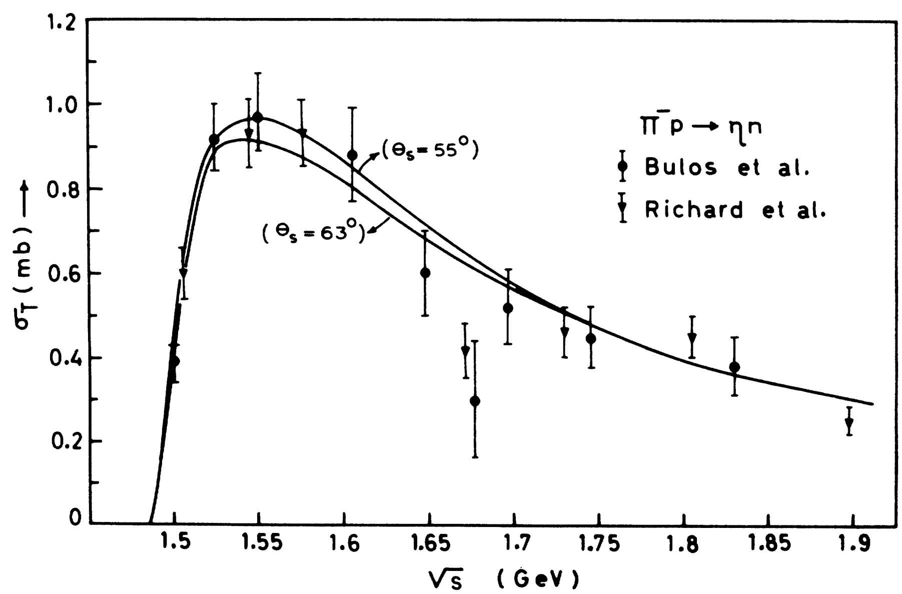

FIG. 1. Total cross section (σ_T) for two values of the mixing angle, $\theta_S = 63°$ and $\theta_S = 55°$, against $\sqrt{s}$ for the process $\pi^- p \to \eta n$. The experimental points are from Ref. 23.

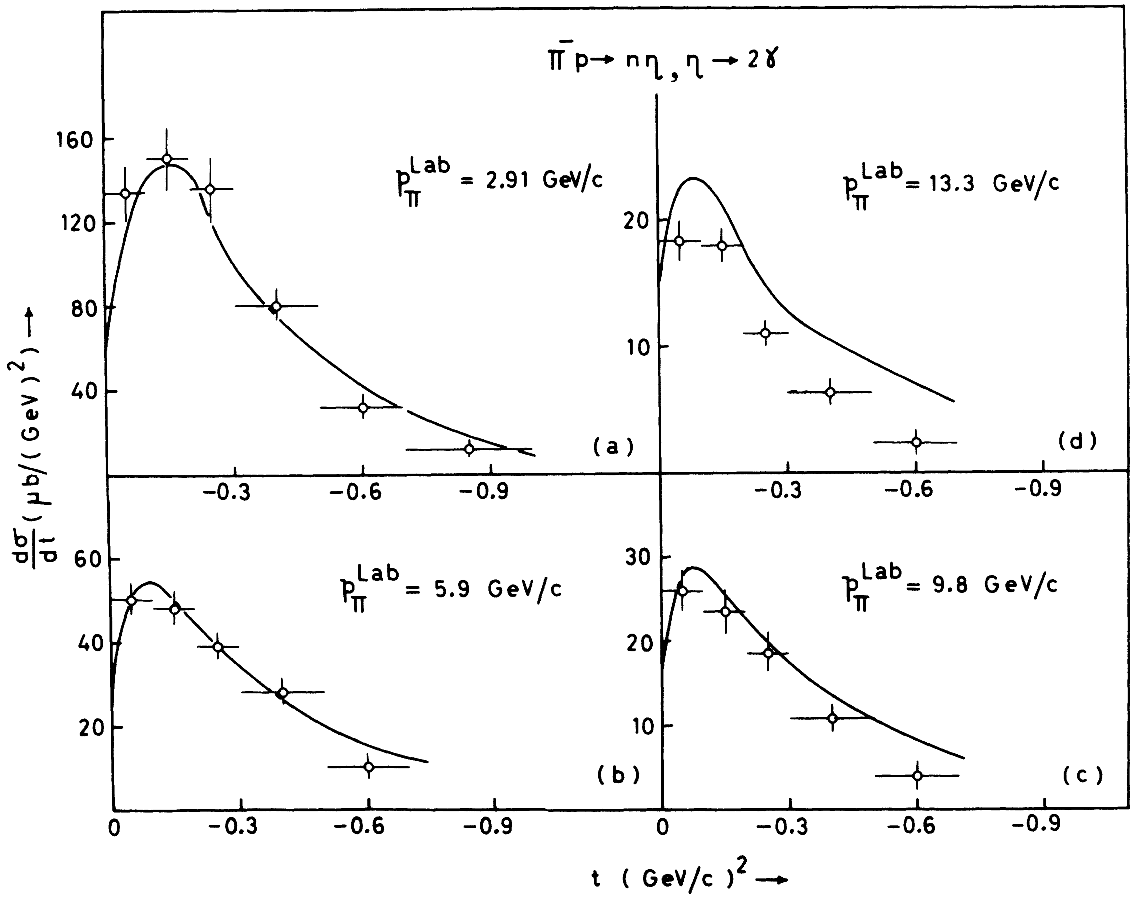

FIG. 2. Differential cross section for the process $\pi^- p \to \eta n$ as a function of t, at four different pion laboratory momenta, (a) 2.91 GeV/c, (b) 5.9 GeV/c, (c) 9.8 GeV/c, (d) 13.3 GeV/c. The data points are from Ref. 31.

rection,[27] two-Reggeon exchanges,[28] deduction from πN scattering via SU_3 considerations, and so on.[29] Attempts to apply the resonance model in the duality spirit, as distinct from interference[30] and Veneziano[31] models, do not seem to have found much favor with this process so far.

Our results, which can at most be compared only with those of Regge-pole models, are shown in Figs. 2(a)–2(d), together with experimental data in the (pion laboratory) momentum range, 2.91 GeV/c to 13.3 GeV/c.[32] These are the results of inclusion of all the towers of resonances listed in Eq. (2.1). Of these the dominant role is found to be that of the trajectories N_β, N_α, N_γ, and N_γ'. As to the other (higher-mass) trajectories $P_{11}(1470)$ and $P_{11}'(1750)$ our calculations indicate that they make relatively small contributions (<10%), so these have not been taken seriously in this investigation. We note also that *both* the Weisskopf factor and the q-d mixing angle play a crucial role in producing the fits to the data.

The agreement seems to be quite good over the entire range (2.91 to 9.4 GeV/c), but the model shows signs of breaking down beyond ~10 GeV/c since the curve (2d) corresponding to $p_\pi^{lab} = 13.3$ GeV/c already starts showing a poor fit, except for small t. We therefore regard the energy of ~10.0 GeV/c as an empirical limit of validity of our model, which is still a considerable way beyond the range of applicability of the more orthodox resonance models.[19]

C. Polarization of the Recoil Neutron

In this subsection we record the results on the recoil neutron polarization as a function of p_π^{lab} for for the process $\pi^- p \to \eta n$ (Fig. 3). Considering the inaccuracy of the data points,[33] the fit is as satisfactory as can be expected. The point which seems to emerge clearly from our investigation is that the recoil neutron polarization is definitely *greater than zero*, thus signifying that in our model there is more than a mere simulation of a single A_2 Regge pole. In our calculation we have taken into consideration all the trajectories listed in Sec. II,

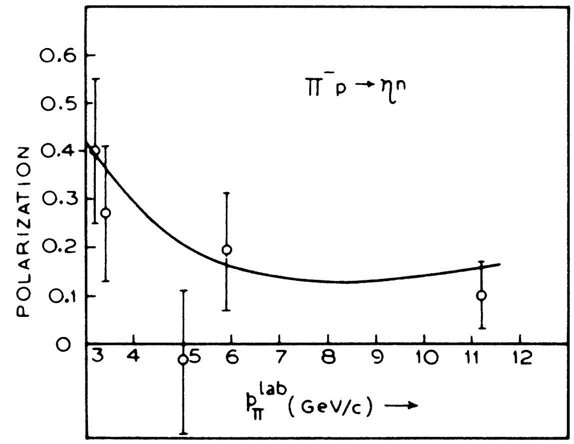

FIG. 3. Recoil neutron polarization of the process $\pi^- p \rightarrow \eta n$ at $t \approx 0.2$ GeV^2, as a function of pion laboratory momentum, compared with experimental points given in Ref. 32.

thus offering enough scope for interference in principle. The reproduction of the observed features of the polarization data would therefore seem to suggest that this interference among the N trajectories is presumably in the right direction.

The above results on $d\sigma/dt$ and polarization can be compared with other high-energy approaches in a somewhat indirect fashion via duality arguments. The peculiar quantum numbers of η would seem to suggest that only A_2 exchange would be allowed *a priori*. Yet most Regge approaches have required some additional ingredients such as absorption,[27] double-Regge exchange,[28] and interference with other (s-channel) amplitudes in order to produce the observed features of η production. Apparently, something more than A_2 exchange must be invoked before a quantitative agreement with experiment may be expected. Therefore, if duality is to be believed as a basic language, the apparent success of our s-channel model up to momenta as high as ~10 GeV/c would indicate that the inclusion of the resonances already implies something more than A_2 exchange, perhaps cuts, in which case the concept of duality may well need some sort of extension.[34]

V. PHOTOPRODUCTION OF η MESON

In keeping with our general philosophy enunciated for the case of $\pi^- p \rightarrow \eta n$, we give the results for the process $\gamma p \rightarrow \eta p$ also for moderate to high energies, noting that the resonance region for this process was covered in a recent analysis along similar lines.[6] Our results for $d\sigma/dt$ at the incident photon energies 4.0, 5.5, 6.5, and 8.0 GeV are shown in Figs. 4(a)–4(d), along with the ex-

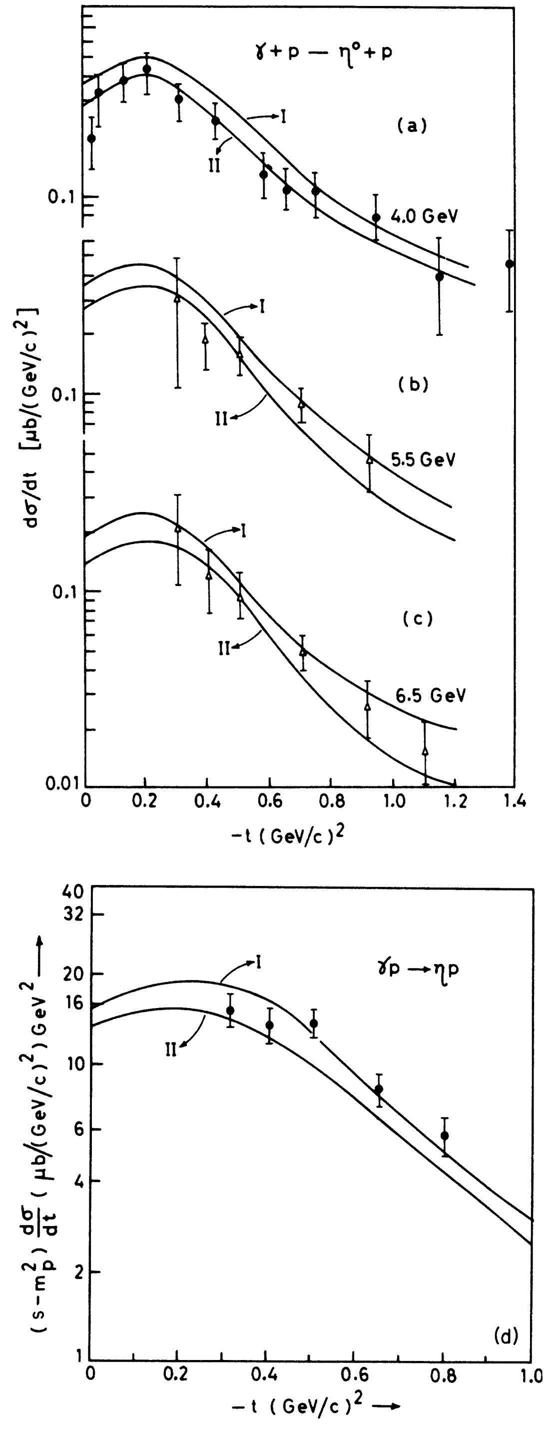

FIG. 4. Differential cross section for the process $\gamma p \rightarrow \eta p$ as a function of t at different photon energies, (a) 4.0 GeV, (b) 5.5 GeV, (c) 6.5 GeV. Curves I and II correspond to extrapolation $M^2 \rightarrow W^2$ and $M^2 \rightarrow W_m{}^2$, respectively. (d) Differential cross section for the process $\gamma p \rightarrow \eta p$ as a function of t, at photon energy $E_\gamma = 8.0$ GeV. Curves I and II correspond to extrapolation $M^2 \rightarrow W^2$ and $M^2 \rightarrow W_m{}^2$, respectively.

perimental points.[35] The ingredients used are the same as in the earlier section in respect of both the form factor and the towers of N resonances considered. The dominant contributions to the process are the towers N_α and N_γ, while the contributions from the other towers which are much smaller can be classified (in decreasing order of magnitude) as N_β, N'_β, and N_δ. For ease of comparison between the two processes $\pi^- p \to \eta n$ and $\gamma p \to \eta p$ we reproduce in Table III the relative contributions to these two processes arising from the different towers. The explanation for the difference in the relative importance of the contributions from different towers lies partly in the role of q-d mixing for the N_β states and partly in the effect of the Moorhouse section rule[36] which forbids γp coupling (not πp) to N_q states.

For completeness we record the relative roles that the different couplings (A)–(E) have played in producing the above results on photoproduction of the η meson. Indeed, the most important roles have been those of (A), (B), and (E) which govern the couplings to the N_α and N_γ towers. The effects of type (C) and type (D) are appreciably less significant. The comparable roles of (A) and (B) on the one hand and (E) on the other in this process would seem to suggest that *both* the "direct" and the "recoil" terms in the $\bar{Q}MQ$ coupling are essential for an understanding of the process $\gamma p \to \eta p$.

As to the distinction between the extrapolations of M^2 in terms of W^2 and $W_m{}^2$, the results which were already anticipated in the preceding paper[15] show the difference to be small, the W curve lying only slightly higher than the W_m curve. The experimental points for all the photon energies involved (4.0, 5.5, 6.5, and 8 GeV) seem roughly to be bounded by the curves W and W_m which lie fairly close to each other. The slight lowering of the W_m curves with respect to the W curves in these diagrams is to be contrasted with a considerable amplification of the same effect in electroproduction (see the preceding paper). This is a result of the large extrapolation from the spacelike value of the photon mass to its (timelike) value on the ρ meson mass shell.

While we have not been able to extend the comparison of this process with the data beyond 8 GeV, it is unlikely that our model would be valid much beyond this value, or, say, beyond ~10 GeV as the results of $\pi^- p \to \eta n$ and electroproduction (SM) seem to indicate. A Reggeized model (with absorption corrections, etc.), on the other hand, has *in principle* no bar to its high-energy validity. However, the applications of such models to date[37] do not seem to have gone much beyond the energies considered here.

Comparison of recoil polarization or asymmetry in the process $\gamma p \to \eta p$ has not been possible due to nonavailability of reliable data.

TABLE III. The relative importance of the different towers that contribute to the processes $\pi^- p \to \eta n$ and $\gamma p \to \eta p$. They are arranged in decreasing order of their contributions to the two processes as we go from top to bottom. The gap signifies a major break in the magnitude of the contributions.

Process $\pi^- p \to \eta n$	Process $\gamma p \to \eta p$
N_β	N_α
N_α	N_γ
N_γ	
N'_γ	N_β
	N'_β
N'_β	N_δ
N''_β	
N_δ	

VI. SUMMARY AND CONCLUSION

In this paper we have considered yet another example in a series of recent applications of our relativistic $SU_6 \otimes O_3$ model of resonance couplings, viz., the processes of η production by pionic and photonic excitations, which in the t channel are characterized essentially by Regge pole(s) (A_2 for the former and ρ and ω in addition for the latter) together with other possible (nonpole) mechanisms. The calculations have been done in the over-all spirit of duality with a view to find some semiquantitative limits on the extent to which the s-channel towers (listed in Sec. II) do or do not suffice as an alternative to the Regge mechanisms for the two processes. Our agreement with experiment on the nonzero recoil-neutron polarization in $\pi^- p \to \eta n$ suggests that the s-channel resonances simulate more than a mere A_2 exchange. The same mechanism seems to work also for the $\gamma p \to \eta p$ process as the fits to the data suggest. We have also found that the effect of the Van Royen–Weisskopf factor as well as the singlet-octet mixing of the η meson are quite important for bringing about the details of agreement with the data. The model however seems to break down beyond about 10 GeV/c incident laboratory momentum.

Since the agreement of our results depends vitally on a *simultaneous* extrapolation $M^2 \to s$ *both* in the form factor and in the structure of the propagator for the high-spin resonances, we consider this ingredient as a most significant factor for whatever success has been achieved in this model, especially the fits at as high a momentum as ~10 GeV/c. This momentum is appreciably higher than one would, at first sight, dare to apply an orthodox resonance model to. It is not much of a surprise therefore that results of other related models such as the relativistic quark model are not available for comparison up to such energies. However, we strongly believe that a Gaussian form factor (such as the one employed in the relativistic quark model) would produce too sharp a fall with energy to be compatible with the actual trend. In a way the inverse power of energy characterizing our form factor is somewhat reminiscent of Veneziano features, but for the lack of an important ingredient of the latter, viz., a sequence of daughter trajectories which are progressively more important at higher energies. The numerical results show that the price of this inadequacy is a progressive breaking down of this model beyond about 10 GeV/c. This momentum is however well within the Regge region (for which the results of traditional Regge models are available) and as such our recipe for off-shell extrapolation probably has some value as a practical means of bridging a difficult gap between the resonance and Regge domains.

APPENDIX

Here we derive the expressions for the $X_J^{(i)}$ ($i=1,\ldots,6$) which carry the J dependence of the amplitude via the projection operator. We consider the D-type coupling of the $\bar{B}B_L V$ vertex for the process $\gamma p \to \eta p$. The contracted projection operator which can be read off from the expression for the T matrix (Eq. 3.5) is of the form

$$\Theta^{\mu,k,\ldots,k_{J-1/2-1}}_{q,\ldots,q_{J-1/2}}(J)\epsilon_{\mu\nu\sigma\rho}k_\nu\epsilon_\sigma\gamma_\rho\,, \tag{A1}$$

which can be written as

$$C_L\sum_{m=0}^{[L/2]} C(L,m)\gamma_\nu\frac{\overleftarrow{\partial}}{\partial q_\nu}\,\mathcal{g}_q{}^m\phi^S\mathcal{g}_k{}^m\frac{\overleftarrow{\partial}}{\partial k_\lambda}\gamma_\lambda\frac{\overleftarrow{\partial}}{\partial k_\mu}\,\hat{\sigma}_\mu\,. \tag{A2}$$

The various quantities are given by

$$\begin{aligned}
&L=J+\tfrac{1}{2}\,,\\
&C_L=\frac{\Gamma(L+1)\Gamma(L+1)}{L(L-1)\Gamma(2L+2)}\,,\\
&C(L,m)=\frac{(-)^m\Gamma(2L-2m+1)}{\Gamma(m+1)\Gamma(L-m+1)\Gamma(L-2m+1)}\,,\\
&\mathcal{g}_q=q_\mu\theta_{\mu\nu}q_\nu\,,\\
&\mathcal{g}_k=k_\mu\theta_{\mu\nu}k_\nu\,,\\
&\phi=q_\mu\theta_{\mu\nu}k_\nu\,,\\
&\hat{\sigma}_\mu=\epsilon_{\mu\nu\sigma\rho}k_\nu\epsilon_\sigma\gamma_\rho\,,\\
&S=L-2m\,,\\
&[L/2]=\text{integer part of } L/2\,.
\end{aligned}$$

Performing the straightforward differentiation, followed by the contraction, keeping in mind that $\hat{\sigma}_\mu$ does not commute with the γ matrices, we can at once cast (A2) in a form where the J dependence is explicit:

$$\begin{aligned}
&[X_J^{(1)}(\gamma_\mu\theta_{\mu\nu}q_\nu)(\gamma_\lambda\theta_{\lambda\sigma}k_\sigma)+X_J^{(2)}]\Omega_1\\
&\quad+[X_J^{(3)}(\gamma_\mu\theta_{\mu\nu}q_\nu)(\gamma_\lambda\theta_{\lambda\sigma}k_\sigma)+X_J^{(4)}]\Omega_2\\
&\qquad+[X_J^{(5)}(\gamma_\mu\theta_{\mu\nu}q_\nu)+X_J^{(6)}(\gamma_\lambda\theta_{\lambda\sigma}k_\sigma)]\Omega_3\,,
\end{aligned}$$

where

$$\begin{aligned}
&\Omega_1=q_\lambda\theta_{\lambda\mu}\hat{\sigma}_\mu\,,\\
&\Omega_2=k_\lambda\theta_{\lambda\mu}\hat{\sigma}_\mu\,,\\
&\Omega_3=\gamma_\lambda\theta_{\lambda\mu}\hat{\sigma}_\mu\,,
\end{aligned}$$

and the $X_J^{(i)}$'s are

$$\begin{aligned}
X_J^{(1)}&=\bar{\Sigma}[4m^2S\mathcal{g}_q{}^{m-1}\phi^{S-1}\mathcal{g}_k{}^{m-1}-S(S-1)(S-2)\mathcal{g}_q{}^m\phi^{S-3}\mathcal{g}_k{}^m]\,,\\
X_J^{(2)}&=\bar{\Sigma}[2mS(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^m+S(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^m(\gamma_\mu\theta_{\mu\nu}\gamma_\nu)\\
&\qquad+2S(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^m+S(S-1)(S-2)\mathcal{g}_q{}^m\phi^{S-3}\mathcal{g}_k{}^m\Delta+4mS\mathcal{g}_q{}^m\phi^{S-1}\mathcal{g}_k{}^{m-1}+2mS(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^m]\,,\\
X_J^{(3)}&=\bar{\Sigma}[8m^2(m-1)\mathcal{g}_q{}^{m-1}\phi^S\mathcal{g}_k{}^{m-2}-2mS(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^{m-1}]\,,\\
X_J^{(4)}&=\bar{\Sigma}[4m^2S\mathcal{g}_q{}^m\phi^{S-1}\mathcal{g}_k{}^{m-1}+2mS\mathcal{g}_q{}^m\phi^{S-1}\mathcal{g}_k{}^{m-1}(\gamma_\mu\theta_{\mu\nu}\gamma_\nu)\\
&\qquad+2mS(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^{m-1}\Delta+4mS(m-1)\mathcal{g}_q{}^m\phi^{S-1}\mathcal{g}_k{}^{m-1}]\,,\\
X_J^{(5)}&=\bar{\Sigma}[4m^2\mathcal{g}_q{}^{m-1}\phi^S\mathcal{g}_k{}^{m-1}-S(S-1)\mathcal{g}_q{}^m\phi^{S-2}\mathcal{g}_k{}^m-2mS\mathcal{g}_q{}^m\phi^{S-1}\mathcal{g}_k{}^{m-1}]\,,\\
X_J^{(6)}&=\bar{\Sigma}\,2mS\mathcal{g}_q{}^m\phi^{S-1}\mathcal{g}_k{}^{m-1}\,,
\end{aligned}$$

and

$$\bar{\Sigma}=C_L\sum_{m=0}^{[L/2]}C(L,m)\,.$$

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