(1973), and unpublished data (private communication).

[26]W. D. Shephard, in *Proceedings of the International Conference on Inclusive Reactions, at Davis University of California, Davis, 1972,* edited by R. L. Lander (Univ. of Calif., Davis, Press, 1972), p. 272.

[27]Comparison between different experiments is always made difficult by possible systematic errors.

[28]There are data available from the ISR at different energies, but at energies too high to be useful for determining energy dependence, given the precision of these measurements. Data from NAL on $pp \to \gamma X$ [J. Pilcher *et al.*, Harvard report, 1972 (unpublished)] indicate that $(p \to \gamma | p)$ is essentially energy-independent between 30 and 200 GeV/c.

[29]See also Chan Hong-Mo *et al.*, Ref. 13.

[30]E. L. Berger, M. Jacob, and R. Slansky, Phys. Rev. D 6, 2580 (1972).

PHYSICAL REVIEW D VOLUME 7, NUMBER 7 1 APRIL 1973

Adler Sum Rule and the νp-νn Behavior

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(Received 15 December 1972)

It is shown that making just the minimal dynamical assumptions of the scaling-limit Nachtmann convex positivity domains based on the quark-parton model (or the equivalent light-cone approach), Regge asymptotics, rate of convergence based on the longitudinal coherence length (ω_0 for 90% saturation $\lesssim 50$), and the constraints of currently known electroproduction and neutrino data, the Adler inelastic sum rule can be satisfied with *tantalizingly* large $F_2^{\nu n}/F_2^{\nu p}$ ratio at low and intermediate values of ω ($\lesssim 5$). The positivity domain near threshold imposes, however, an increasingly severe strain on attempts to saturate the sum rule. Experimental implications for $\sigma_{\nu n,\nu p}$ and $\sigma_{\nu n}/\sigma_{\nu p}$ are also discussed. Finally, the theoretical basis for the Adler sum rule is briefly reappraised.

I. INTRODUCTION AND PRELIMINARIES

It has always been recognized that the foundation of many theoretical superstructures (e.g., parton ideas and the light-cone algebra) depends intimately on the validity of the Adler sum rule[1] for inelastic lepton-hadron processes. The present author in collaboration with Bjorken[2] and Sakurai and Thacker[3] recently raised some questions concerning the validity of the sum rule. It was pointed out that the sum rule is not saturated unless we allow remarkably slow convergence of the Adler integral (with ω reaching to several hundreds for even 90% saturation) *or* the ratio $F_2^{\nu p}/F_2^{\nu n}$ must be taken $\ll 1$ for $1 \leqslant \omega \lesssim 5$ for a fairly rapid rate of convergence. The latter option was regarded by us as not reasonable (perhaps even bizarre) in terms of physical expectations, since among other things model calculations[4] which incorporate the sum rule do not have this feature but instead require the ultra-slow convergence of the former option.

More recently, Adler and Treiman[5] pointed out that the implied belief that large $F_2^{\nu n}/F_2^{\nu p}$ (or $F^{\nu p}/F^{\nu n} \ll 1$) is unreasonable can be discarded because the basic quark-parton–light-cone approach[6,7] taken in conjunction with electroproduction data *requires* that $F_2^{\nu n}/F_2^{\nu p}$ *must* be large for $x = 1/\omega$ near unity. Furthermore, *specific* models[4] which allow $F^{\nu p}/F^{\nu n}$ and F^{en}/F^{ep} to have comparable values near threshold ($x=1$) of $\frac{1}{2}$ and $\frac{2}{3}$, respectively, and saturate the sum rule extremely slowly, are already in disagreement with data. Likewise, model calculations[3] which assume that the functional forms chosen satisfy $F^{en}/F^{ep} = F^{\nu p}/F^{\nu n}$ are in contradiction with expectations from the quark-parton–light-cone approach supplemented with electroproduction data.

The purpose of this paper is to follow through the suggestion of Adler and Treiman[5] that the most reasonable strategy for obtaining saturation of the sum rule is to exploit just the bare minimum assumptions of scaling-limit positivity conditions (the Nachtmann domains) for the structure functions based on the quark-parton model (or the equivalent light-cone approach), Regge asymptotics for the relevant structure functions, *and* the constraints of currently known electroproduction and neutrino data. To this list we add also the assumption that the rate of convergence of the sum rule should be in accord with expectations based on the longitudinal-coherence-length argument.

Our conclusion is that the Adler inelastic sum rule can be satisfied on the above basis with *tantalizingly* large $F_2^{\nu n}/F_2^{\nu p}$ ratio at low and intermediate values of ω. In fact the functional form $B_{LC}(x)$ proposed by Sakurai[3] does satisfy these assumptions in a suitably modified framework of discussion. However, the Nachtmann positivity domain near threshold imposes an increasingly severe strain on attempts to saturate the sum rule. This is largely attributable to the fact that because the quark-parton–light-cone inequalities are on the border-line of being saturated[8] and hence on the borderline of breaking positivity almost everywhere it is being tested, it follows that attempts to saturate the sum rule on this basis must reflect the same inbuilt instability. Indeed only a narrow corridor of solutions within the $B_{LC}(x)$ framework is acceptable and hence we may argue that they represent a not unreasonable approximation to the real saturation picture – which of course remains to be tested in the future. The $B_{LC}(x)$-type solutions predict that the ratio of total cross sections $\sigma_{\nu n}/\sigma_{\nu p}$ can be as large as 3, and make predictions on the individual cross sections $\sigma_{\nu n,\nu p}$, a result likely to be of some interest to planned νp bubble-chamber work at the National Accelerator Laboratory (NAL).

In Sec. II we discuss in some details the assumptions which underlie our strategy for obtaining saturation of the sum rule; Sec. III discusses the range of solutions and their stability. Experimental implications are spelled out in Sec. IV, while in Sec. V we appraise briefly the theoretical structure of the sum rule.

II. ASSUMPTIONS

We analyze the Adler sum rule on the basis of the following propositions:

A. Structure-Function Inequalities: The Nachtmann Domains

It has been known for some time[9] that sum rules and other relations between the structure functions can be obtained without further assumptions once we decide what the constituents of the nucleon are. These relations are very interesting because they depend directly on the spin and internal quantum numbers of the particles (or partons) in the underlying field theory which is chosen. Of particular relevance to us here are the relations for $\Delta Y=0$ reactions in the scaling region (assuming scale invariance) for quarklike constituents. These are

$$2F_1^{e,\nu,\bar\nu}=\omega F_2^{e,\nu,\bar\nu}\,, \tag{1}$$

$$12(F_1^{ep}-F_1^{en})=F_3^{\nu p}-F_3^{\nu n}\,, \tag{2}$$

$$-x(F_3^{\nu p}+F_3^{\nu n})\leq F_2^{\nu p}+F_2^{\nu n}\leq \tfrac{18}{5}(F_2^{ep}+F_2^{en})\,, \tag{3}$$

or in integrated form

$$-\int_0^1 x(F_3^{\nu p}+F_3^{\nu n})dx\leq\int_0^1 (F_2^{\nu p}+F_2^{\nu n})dx \leq \tfrac{18}{5}\int_0^1 (F_2^{ep}+F_2^{en})dx\approx 1\,, \tag{3$'$}$$

and the sum rules

$$\int_0^1 dx(F_3^{\nu p}+F_3^{\nu n})=-6\,, \tag{4}$$

$$\tfrac{1}{2}\int_0^1 \frac{dx}{x}(F_2^{\nu n}-F_2^{\nu p})=1\,. \tag{5}$$

It is of course recognized that some of these relations are of much more generality than just for quarklike constituents. For instance, Eq. (1) is valid generally for elementary spin-$\frac{1}{2}$ fields, while the subject of our discussion, the famous Adler sum rule (5), is expected to be true in all reputable models. In discussing these relations in the scaling limit we are of course indulging in the hope that there is a region where the incident lepton energy E is large enough for the various relations to apply but still small enough for conventional weak interactions to hold.[10]

Recently it was noted[8] that the quark-parton inequalities, when analyzed with current electroproduction and neutrino (antineutrino) data appear to be satisfied remarkably close to the saturation limit where inequalities are replaced by equalities and we are near the boundary of the positivity domain for the structure functions. Assuming equalities in (3′) and (3) suggests that at least crudely the Gross-Llewellyn Smith quark-parton sum rule (4) is not in disagreement with present trends in data. Again many authors[6,7] have called attention to the fact that experimental indication[11] on F^{en}/F^{ep} may be pressing close to the lower limit of the Nachtmann inequality for quark partons[6] (as $x\to 1$)

$$4\geq F^{en}/F^{ep}\geq\tfrac{1}{4}\,. \tag{6}$$

In attempting to saturate the Adler sum rule in the quark-parton–light-cone framework, it is very important that our solutions lie within the convex positivity domains of Nachtmann.[12] These constraints on the $F^{\nu n,\nu p}$ structure functions exploit the maximum information obtainable from electroproduction data in terms of both the ratio

$$y(x)=F^{en}(x)/F^{ep}(x) \tag{7}$$

and that from F_2^{ep} data alone. In Fig. 1, the positivity domains (shaded areas) in the (η,ξ) plane where

$$\eta=\frac{F_2^{\nu p}}{F_2^{ep}}\quad\text{and}\quad \xi=\frac{F_2^{\nu n}}{F_2^{ep}}\,, \tag{8}$$

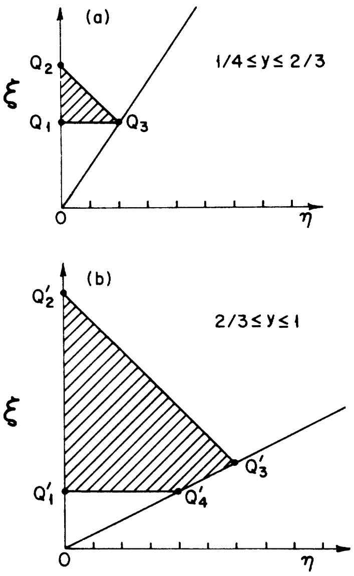

FIG. 1. (a) The schematic convex positivity domain (shaded area) in the (η, ξ) plane for a typical y in range $\frac{1}{4} \le y \le \frac{2}{3}$. Here $y = F_2^{en}/F_2^{ep}$, $\eta = F_2^{\nu p}/F_2^{ep}$, $\xi = F_2^{\nu n}/F_2^{ep}$, and $Q_i(\eta, \xi)$ are given in Table I. (b) The schematic convex positivity domain (shaded area) in the (η, ξ) plane for a typical y in range $\frac{2}{3} \le y \le 1$ with $Q_i'(\eta, \xi)$ tabulated in Table I. Other notations are the same as in (a).

are exhibited for the ranges $\frac{1}{4} \le y \le \frac{2}{3}$ [Fig. 1(a)] and $\frac{2}{3} \le y \le 1$ [Fig. 1(b)], respectively. The end points of the positivity domains $Q_i(\eta, \xi)$ $(i = 1, 2, 3)$ and $Q_i'(\eta, \xi)$ $(i = 1, 2, 3, 4)$ are tabulated in Table I in terms of the y variable. Note that Q_1Q_3 and $Q_1'Q_4'$ are parallel to the η axis while Q_2Q_3 and $Q_2'Q_3'$ make 45° with the η axis. For the ratio $F_2^{\nu p}/F_2^{\nu n} = \eta/\xi$ we obtain a much less restrictive positivity domain (area above lines $0Q_3$ and $0Q_4'Q_3'$ in Fig. 1) since information on F_2^{ep} itself is not used here. The latter domain leads to the Paschos inequalities[13]

$$0 \le R(x) = \frac{F_2^{\nu p}(x)}{F_2^{\nu n}(x)} \begin{cases} \le \dfrac{8}{5} \dfrac{[y(x) - \frac{1}{4}]}{[1 - y(x)]}, & \frac{1}{4} \le y(x) \le \frac{2}{3}, \\ \le 2, & \frac{2}{3} \le y(x) \le 1. \end{cases} \tag{9}$$

The electroproduction data are introduced by assuming that for $x \ge \frac{1}{2}$, the form of F_2^{ep} can be fitted by a single cubic which for convenience we take to be of form

$$F_2^{ep}(x) = c(1 - x^2)^3, \quad c \sim 0.4 \ \text{for} \ x \ge \tfrac{1}{2}. \tag{10}$$

This gives a reasonably good eyeball fit to the data.[14] The accumulating data on electroproduction n-p ratio[11] suggest that $y(x)$ can be parametrized by the following form:

TABLE I. Tabulation of $Q_i(\eta, \xi)$ and $Q_i'(\eta, \xi)$, where (η, ξ) is expressed as a function of $y = F^{en}/F^{ep}$.

	$\eta = F_2^{\nu p}/F_2^{ep}$	$\xi = F_2^{\nu n}/F_2^{ep}$
Q_1	0	$6(1-y)$
Q_2	0	$\frac{18}{5}(1+y)$
Q_3	$\frac{48}{5}(y - \frac{1}{4})$	$6(1-y)$
Q_1'	0	$6(1-y)$
Q_2'	0	$\frac{18}{5}(1+y)$
Q_3'	$\frac{12}{5}(1+y)$	$\frac{6}{5}(1+y)$
Q_4'	$12(1-y)$	$6(1-y)$

$$\begin{aligned} y(x) &= \tfrac{1}{4}(4 - 3x) \quad \text{for} \quad 0 \le x \le 0.8, \\ y(x) &= \tfrac{57}{25} - \tfrac{79}{20}x + 2x^2 \quad \text{for} \quad 0.8 \le x \le 1. \end{aligned} \tag{11}$$

This assumes that $y(x)$ extrapolates to 0.33 at $x = 1$ which is in accord with the data trend. Note that using Eqs. (9) and (11) alone we can already conclude that basic quark-parton ideas *require* large $F_2^{\nu n}/F_2^{\nu p}$ at x near unity, a point first stressed by Paschos.[13] The physically allowed region for $R(x)$ versus x using (9) and (11) is depicted in Fig. 2.

B. Regge Asymptotics

For the inaccessible region of very small x (or large ω) we resort to Regge theory, where typically the parametrization for $F_2^{\nu n} - F_2^{\nu p}$ is $\sim x^{1-\alpha}$ as $x \to 0$; here $\alpha = \frac{1}{2}$ for the principal trajectory and we allow for the possibility of low-lying "daughter" trajectory contributions as well with say $\alpha = -\frac{1}{2}$.

C. Convergence of Sum Rule Based on the Longitudinal Coherence Length

Arguments based on longitudinal coherence length[15,16] strongly suggest that F_2 at high ω (or $x = 1/\omega$ small) reveals features reminiscent of meson-induced reactions; in particular, diffractive contributions should dominate over nondiffractive contributions at high ω. According to this viewpoint for a suitably large ω_1 we expect $F_2^{\nu n}(\omega) = F_2^{\bar{\nu} p}(\omega) \simeq F_2^{\nu p}(\omega)$ for $\omega > \omega_1$ as the distinction between $\bar{\nu}$ and ν disappears as incident probe. This in turn suggests that the Adler sum rule (5) should be saturated for $1 \ge x \ge x_1$.

Delicacy is evidently involved in estimating x_1. The most optimistic estimate,[2] one based on the longitudinal distance over which the process is incoherent,[15] would suggest that ω_1 can be as small as 5 to 10. On the other hand, the quark-parton approach would favor a somewhat slower rate of convergence. Nachtmann[12,6] showed that assuming Eqs. (1), (4), and (5),

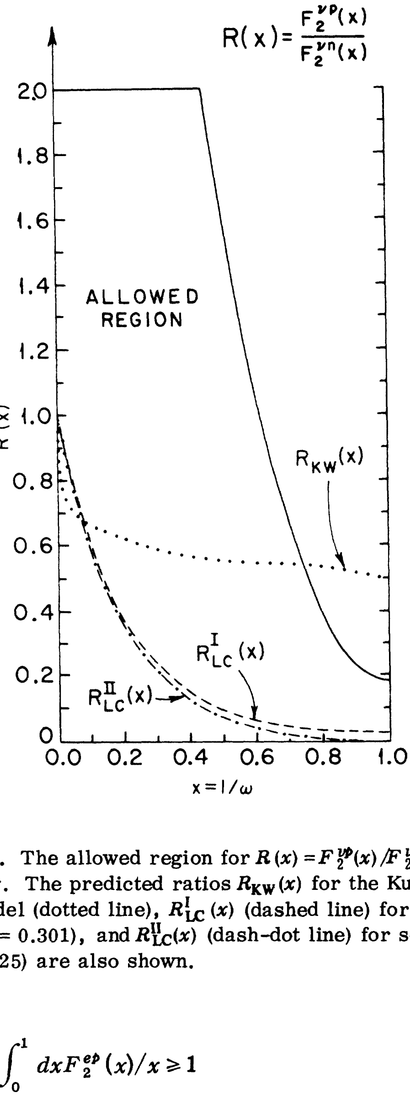

FIG. 2. The allowed region for $R(x)=F_2^{\nu p}(x)/F_2^{\nu n}(x)$ versus x. The predicted ratios $R_{KW}(x)$ for the Kuti-Weisskopf model (dotted line), $R^{I}_{LC}(x)$ (dashed line) for solution I ($\rho=0.301$), and $R^{II}_{LC}(x)$ (dash-dot line) for solution II ($\rho=0.25$) are also shown.

$$I_1=\int_0^1 dx F_2^{ep}(x)/x \geq 1$$

as a positivity condition results. Taking empirical data that

$$\int_{1/12}^1 dx F_2^{ep}(x)/x = 0.58 ,$$

we have

$$I_1' = \int_{x_1}^{1/12} [F_2^{ep}(x)/x]\,dx \geq 0.42 .$$

For x away from threshold $F_2^{ep}(x) \sim 0.3$ to 0.35, hence $I_1' \geq 0.42$ leads to $\omega_1 \geq 40.45$ [for $F_2^{ep}(x)=0.3$] and $\omega_1 \geq 49.4$ [for $F_2^{ep}(x)=0.35$]. A similar argument has been presented by Llewellyn Smith[17] who pointed out that from Eqs. (3) and (4) plus SLAC data[14]

$$\left|\int_0^{1/12} (F_3^{\nu p}+F_3^{\nu n})\,dx\right| \geq 2.4 , \qquad (12)$$

i.e., F_3 (which has no diffractive component) must still be substantial for $\omega > 12$. Of course one may argue that perhaps the saturation of the Adler sum rule (5) is faster than that for the Gross-Llewellyn Smith sum rule (4). However, given also the constraint of Regge asymptotics (which again favors slow convergence), it does appear that *if we take the whole quark structure seriously,* the very rapid convergence option appears to be ruled out.

On the other hand there is no evidence that *very* slow convergence of the Adler sum rule with ω_0 for 90% saturation of the order of several hundred can be a reasonable expectation either.[18] In fact specific dynamical models[4] based on valence quarks plus sea (a unitary singlet), duality, and exchange degeneracy etc., which do yield ultra-slow convergence,[2,3] appear to be in serious contradiction with data. These models all share common features like relations of type[19]

$$F_2^{\nu n} - F_2^{\nu p} = 6(F_2^{ep} - F_2^{en})$$

which together with Eq. (5) leads to the experimentally unsatisfactory quark charge sum rule[3]

$$\int_0^1 [(F_2^{ep} - F_2^{en})/x]\,dx = \tfrac{1}{3} . \qquad (13)$$

They also lead to $y(1)=\frac{2}{3}$ which is in disagreement with data.[20] In Fig. 2 we have plotted $R_{KW}(x)$ for the Kuti-Weisskopf model[4]; it is seen that even for the νp-νn ratio there is a violation of the allowed positivity region for $x \geq 0.75$.

For definiteness, we shall assume that the "coherence-length"-type argument proposed by Langacker and Suzuki[16] is valid. This has been successful in relating the pion-nucleon cross section at laboratory energy ν to the corresponding F_2 for electroproduction at ω, in terms of a correspondence (for our purpose)

$$\left.\frac{\sigma(\pi^- p)-\sigma(\pi^+ p)}{\sigma(\pi^- p)+\sigma(\pi^+ p)}\right|_\nu = \left.\frac{F_2^{\nu n}-F_2^{\nu p}}{F_2^{\nu n}+F_2^{\nu p}}\right|_\omega , \qquad (14)$$

where ν and ω are assumed to be related via

$$\nu = (\omega-1)b^2/2m , \quad b^2 = 0.24\ \mathrm{GeV}^2 . \qquad (15)$$

The correspondence suggests[3] that functional forms for $F_2^{\nu p,\nu n}$ satisfying Adler sum rule (5) should achieve 90% saturation for $\omega_0 \sim 50$ consistent with trends for well-tested $Q^2 \cong 0$ sum rules.[21] Such a value for ω_0 is not inconsistent with quark-parton expectations as discussed above.

D. Constraints of Neutrino Cross-Section Data

Our form for the structure function must satisfy the constraint imposed by ν and $\bar{\nu}$ total cross sections at high energy. Using Eq. (1), we expect

$$\tfrac{1}{2}\{[\sigma_{\nu p}(E)+\sigma_{\nu n}(E)]+[\sigma_{\bar{\nu} p}(E)+\sigma_{\bar{\nu} n}(E)]\}=(2G^2ME/3\pi)\int_0^1 (F_2^{\nu p}+F_2^{\nu n})dx\ . \tag{16}$$

The recent data[22] suggest that the left-hand side $=\frac{4}{3}[G^2ME/\pi](0.47\pm 0.07)$.

III. SOLUTIONS AND THEIR STABILITY

A functional form has in fact already been proposed[3] which can be adapted to satisfy all the required conditions (Secs. II A–II D). The form for the difference of neutrino structure functions is

$$B_{\rm LC}(x)=\tfrac{1}{2}(F_2^{\nu n}-F_2^{\nu p}) = \rho B(x;\tfrac{1}{2},3)+(1-\rho)B(x;-\tfrac{1}{2},3)\ , \tag{17}$$

where $B_{\rm LC}(x)$ is given by the following expression and normalization:

$$B(x;\alpha,\beta)=[\Gamma(2+\beta-\alpha)/\Gamma(1-\alpha)\Gamma(1+\beta)]\,x^{1-\alpha}(1-x)^{\beta}\,,$$

$$\int_0^1 [B(x;\alpha,\beta)/x]\,dx=1\,. \tag{18}$$

For the sum we assume the phenomenological form suggested by Myatt and Perkins[23] on the basis of their neutrino data, namely,

$$\tfrac{1}{2}(F_2^{\nu n}+F_2^{\nu p})=a(1-x^2)^3\ . \tag{19}$$

This functional form also fits the electroproduction data reasonably well. For $a=1.095$, Eq. (19) leads to

$$\int_0^1 (F_2^{\nu n}+F_2^{\nu p})dx=1\ ,$$

corresponding to the saturation limit of quark-parton inequality (3′). Evidently $B_{\rm LC}(x)$ satisfies sum rule (5).

The most severe test to be imposed on our chosen functional forms (17) and (19) is that the parameters (ρ, a) must be so chosen that they lead to values for η and ξ which lie within the Nachtmann domains (shaded areas in Fig. 1) throughout the range of values for $0\le x\le 1$ [or the corresponding range of y determined from Eq. (11)] with $F_2^{ep}(x)$ taken from electroproduction data [the sensitive range $x\ge\frac{1}{2}$ is assumed to be of cubic form given by Eq. (10)]. In other words, the lines

$$\begin{aligned}\xi-\eta&=2B_{\rm LC}(x)/F_2^{ep}(x)\ ,\\ \xi+\eta&=2a(1-x^2)^3/F_2^{ep}(x)\end{aligned} \tag{20}$$

must intersect within these domains. Our conclusion is that for

$$0.25\le\rho\le 0.301,\quad a\cong 0.958\ , \tag{21}$$

the condition is satisfied. Note that $a=0.958$ corresponds to 87.5% saturation of inequality (3′). Use of (19) and (21) in (16) leads to good agreement with recent neutrino and antineutrino data. Regge asymptotic behavior plus a contribution from a low-lying daughter trajectory ($\alpha=\frac{1}{2}$ and $-\frac{1}{2}$, respectively) is maintained in the form $B_{\rm LC}(x)$ as evident from Eq. (17) and Eq. (18). Convergence of the sum rule is in accord with the expectations from longitudinal distance; e.g., for 90% saturation of sum rule[3] $\omega_0=53.22$ ($\rho=0.301$) and $\omega_0=41.82$ ($\rho=0.25$).[24] The calculated ratio

$$R_{\rm LC}^{\rm I}(x)=\left.\frac{F_2^{\nu p}(x)}{F_2^{\nu n}(x)}\right|_{\rho=0.301}\quad\text{and}\quad R_{\rm LC}^{\rm II}(x)=\left.\frac{F_2^{\nu p}(x)}{F_2^{\nu n}(x)}\right|_{\rho=0.25} \tag{22}$$

from Eqs. (17)–(19) and (21) are shown in Fig. 2. As emphasized earlier, the ratio νp-νn does not impose a strong constraint on positivity at least in comparison with the Nachtmann domains.

The stability[25] of our range of solutions (21) is governed by the fact that in terms of our input, the intersection (η,ξ) of (20) moves increasingly close to Q_2 of Fig. 1(a) as x approaches unity. This places a strain on maintaining positivity especially for solutions near $\rho=0.25$ for x in range $0.9\le x\le 1$. The situation is easily aggravated if near $x=1$ one of the following conspiring circumstances were to occur: (i) The parameter a in Eq. (19) is such that $a>0.958$; (ii) the parameter c in Eq. (10) is much smaller than 0.4; (iii) if the ratio y were found experimentally to converge even closer to the lower bound of $\frac{1}{4}$. On the other side of the coin (iv) the parameter a cannot be much smaller than 0.958 without running foul of the positivity criterion that $F_2^{\nu n}+F_2^{\nu p}\ge F_2^{\nu n}-F_2^{\nu p}$. The possibilities (i) and (iii) are clearly related to the near saturation of the quark-parton–light-cone inequalities [(3), (3′), and (6)] and affect adversely the stability of solutions to the Adler sum rule throughout the range $0.25\le\rho\le 0.301$ for x near unity. It seems unlikely to us that a solution can be found within our chosen framework with ρ outside the range (0.25, 0.301) and a of order 0.958. Note that in terms of the Langacker-Suzuki correspondence, Eq. (14), $\rho=0.25$ and $\rho=0.301$ already need rather large ω (=100 and 150, respectively) to effect the correspondence via (15). Finally we remark that the fact (η,ξ) is very close to Q_2 near $x=1$, necessitate that $R_{\rm LC}^{\rm I}$ and $R_{\rm LC}^{\rm II}$ must both be very small here (cf. Fig. 2); the solutions I ($\rho=0.301$) and II ($\rho=0.25$) are in fact remarkably similar in magnitude and shape except near $x=1$

which augur well for a unique situation. It need scarcely be emphasized that the tightness reflected in the narrow corridor of solutions possible, merely underlies the fact that the constraints, Secs. II A–II D, while mutually compatible, impose nevertheless very stringent criteria on the manner in which the sum rule can be saturated.

IV. EXPERIMENTAL IMPLICATIONS

Apart from the obvious testable predictions like the forms for $F_2^{\nu n}$ and $F_2^{\nu p}$ (Fig. 3) which allow literally $F_2^{\nu n}/F_2^{\nu p}$ to be of order 30 at threshold to a still respectable 2.7 at $\omega=5$, Bjorken[26] has stressed the importance of their predictions concerning the total cross sections $\sigma_{\nu p,\nu n}$. This is because the initial NAL neutrino work will emphasize bubble-chamber work on a hydrogen target. Hence predictions concerning the total cross section $\sigma_{\nu p}$ is likely to be of interest in terms of calibration of counting rates for neutrino events. It is known that in the scaling limit

$$\sigma_{\nu n,\nu p}=C_{\nu n,\nu p}(G^2ME/\pi)\ , \tag{23}$$

where

$$C_{\nu n,\nu p}=\int_0^1(\tfrac{1}{2}F_2^{\nu n,\nu p}+\tfrac{1}{3}xF_1^{\nu n,\nu p}-\tfrac{1}{3}xF_3^{\nu n,\nu p})dx\ . \tag{24}$$

The Callan-Gross relation (1) connects xF_1 and F_2, while in a three-quark picture (without, say,

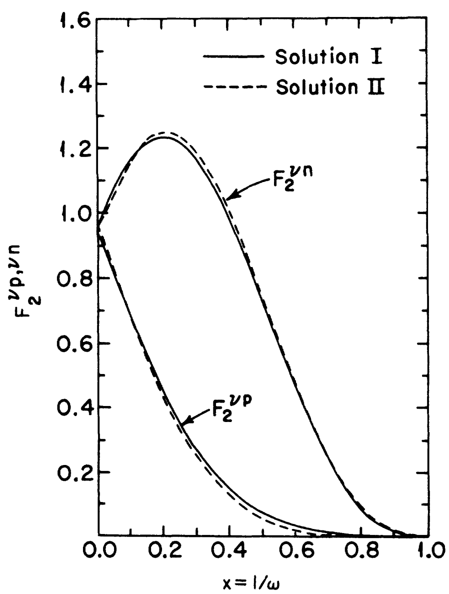

FIG. 3. Typical behaviors of $F_2^{\nu n}(x)$ and $F_2^{\nu p}(x)$ versus x calculated from $B_{LC}(x)$ are shown. Solution I ($\rho=0.301$) is given by solid lines while solution II ($\rho=0.25$) is given by dashed lines.

a background "sea" of $q\bar{q}$ pairs) for which there is some indirect support from neutrino data,[8] we expect

$$F_3^{\nu n,\nu p}=-(1/x)F_2^{\nu n,\nu p}$$

$$[\text{and also } F_3^{\bar{\nu}n,\bar{\nu}p}=+(1/x)F_2^{\bar{\nu}n,\bar{\nu}p}]\ .$$

Inclusion of the $q\bar{q}$ sea suppresses $|F_3|$ relative to F_2, though it is always the case that $\sigma_\nu>\sigma_{\bar{\nu}}$, hence it is reasonable to assume

$$-F_2^{\nu n,\nu p}\leqslant xF_3^{\nu n,\nu p}\leqslant 0\ . \tag{25}$$

Our solutions I and II give the following values for the integrals:

$$\int_0^1[F_2^{\nu n},F_2^{\nu p}]dx=[0.6616,\ 0.2144]\quad \text{for solution I}\ ,$$

$$\int_0^1[F_2^{\nu n},F_2^{\nu p}]dx=[0.670,\ 0.206]\quad \text{for solution II}. \tag{26}$$

From Eqs. (1) and (23)–(26), we have the following range of possible values for $\sigma^{\mathrm{I},\mathrm{II}}_{\nu n,\nu p}$:

$$\begin{aligned}\sigma^{\mathrm{I}}_{\nu n}&=(G^2ME/\pi)[0.66,0.44]\,,\\ \sigma^{\mathrm{I}}_{\nu p}&=(G^2ME/\pi)[0.214,0.143]\ ,\\ \sigma^{\mathrm{II}}_{\nu n}&=(G^2ME/\pi)[0.67,0.45]\,,\\ \sigma^{\mathrm{II}}_{\nu p}&=(G^2ME/\pi)[0.206,0.137]\ .\end{aligned} \tag{27}$$

Solutions I and II also yield for the ratio of cross sections the following values [exact at end points of (25)]:

$$\frac{\sigma^{\mathrm{I}}_{\nu n}}{\sigma^{\mathrm{I}}_{\nu p}}=3.08,\quad \frac{\sigma^{\mathrm{II}}_{\nu n}}{\sigma^{\mathrm{II}}_{\nu p}}=3.25\ . \tag{28}$$

This is substantially larger than the predictions of specific models[4] which give typically values around 1.7 and 1.8. It is also larger than the preliminary measurement[23] which suggests a value 1.5 ± 0.3, though this result is currently under revision.

V. DISCUSSIONS

Much of the theoretical criticism of the Adler sum rule is directed towards the discrete assumption that certain amplitudes are assumed to satisfy unsubtracted dispersion relations. More precisely, one assumes that the $J=1$ fixed pole required by the divergence condition[27]

$$(q\cdot P)A+q^2B_2=D+(\text{a real constant}) \tag{29}$$

appears in the A amplitude (coefficient of $P_\mu P_\nu$) but not in B_2 (coefficient of $q_\mu P_\nu+q_\nu P_\mu$) and[28] D – even though all three are assumed to satisfy unsubtracted dispersion relations. This is not unreasonable since Dashen[29] pointed out that $J=1$ is indeed a nonsense fixed pole in A. However, $J=1$ is a sense point with right signature for B_2 and D leading not to a fixed pole but Kronecker δ's[30] in angular momentum for weak processes. Such nonanalytic pieces can then lead to the breakdown of Regge analysis. In fact, perturbation theory tells us that the Reggeology of hadron amplitudes involving hadrons plus currents ought to be the same thing as that for hadrons alone except for *fixed poles at nonsense points*. Of course if the unsubtracted assumption itself is questioned, then we would have to proceed directly from Eq. (29) to test the current density algebra. Determination of ReA, ReB_2, etc. will then require colliding-beam ($\nu l^{\pm}$) facilities. Fortunately, we may be spared such a horror if the present analysis of the Adler sum rule (5) should be correct.

In conclusion, we must reiterate that the genesis of the Adler sum rule (5) is more general than quark partons or even spin-$\frac{1}{2}$ fields in general. For instance the Callan-Gross relation (1) has a simple connection with σ_S, to wit $\sigma_S=0$ implies Eq. (1). However, if for instance future SLAC data were to say that the ratio $\sigma_S/\sigma_T \neq -q^2/\nu^2$ (which vanishes in the scaling limit as required by spin-$\frac{1}{2}$ constituents) but rather tends to a finite constant instead, then quark-parton theory does not represent the complete story. This does not necessarily invalidate the Adler sum rule, since in principle the sum rule is sufficiently universal to encompass situations like a mixture of spin-$\frac{1}{2}$ and spin-0 partons or perhaps even a two-component picture.[31]

ACKNOWLEDGMENTS

I would like to thank S. L. Adler and O. Nachtmann both for very helpful suggestions and for their critical readings of the manuscript. Discussions and communications with J. D. Bjorken, C. G. Callan, R. Dashen, D. Gross, J. Kogut, C. H. Llewellyn Smith, J. J. Sakurai, and S. B. Treiman are also gratefully acknowledged. Finally I would like to thank Dr. Carl Kaysen for the hospitality of the Institute for Advanced Study during which this work was performed.

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[20]In the Kuti-Weisskopf model (Ref. 4), replacement of the valence neutron quark distribution $\sqrt{x}\,(dx/x)$ inside the proton by $(1-x)\sqrt{x}\,(dx/x)$ can salvage the $y(1)\sim\frac{1}{4}$ ratio, however, the difficulty with Eq. (13) remains.

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