PHYSICAL REVIEW D VOLUME 7, NUMBER 6 15 MARCH 1973

Fixed Poles and Operator Schwinger Terms

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(Received 24 April 1972)

By assuming that it is possible to go from the Regge limit ($\nu \to \infty$, q^2 fixed) to the Bjorken-Johnson-Low limit ($q_0 \to \infty$, $|\vec{q}|$ fixed) by making $q^2 \to q_0{}^2$ in the frame $\vec{p}=0$ in the Regge limit, a connection between fixed-pole residues and operator Schwinger terms is established. Comments are made on the electromagnetic mass-shift divergence problem and the Cheng-Tung conjecture regarding the polynomial structure of fixed-pole residues.

I. INTRODUCTION

Recently[1] there has been some interest in trying to establish the presence of a fixed $J=0$ pole in the virtual forward spin-averaged Compton scattering amplitude[2] $T^*_{\mu\nu}(\nu, q^2)$ off a proton. It has been argued that if one makes assumptions regarding the behavior of the Regge residues as a function of q^2 then the recent electroproduction experimental data[3] suggest that a fixed $J=0$ pole is present in $T^*_{\mu\nu}(\nu, q^2)$. An interesting theoretical point that has been noted by several authors is the connection which seems to exist between the residue of the fixed $J=0$ pole, $R_2(q^2)$, and the presence of operator (or q-number) Schwinger terms.[4] Some time back Cheng and Tung[5] conjectured that the residue function of any fixed J pole should have a polynomial structure in the variable q^2. In view of this a puzzling feature of the electroproduction data is the fact that $R_2(q^2)/q^2$ at $q^2=0$ and $q^2=\infty$ (spacelike) are both finite and have opposite signs.[6] Although the difference in sign has not been completely established it does cast doubt on the Cheng-Tung conjecture. In this paper we would like to establish the connection between operator Schwinger terms and fixed $J=0$ pole residues in a very direct way by assuming it is possible to go from the Regge ($q_0 \to \infty$, q^2 fixed) to the Bjorken-Johnson-Low (BJL)[7] ($q_0 \to \infty$, $q^2 \to q_0{}^2$) limit. This assumption fixes the behavior of $R_i(q^2)$ ($i=1, 2$) as $q^2 \to \infty$. Using this information the contribution from the fixed-pole terms to the mass-shift divergence problem[8] may be determined. We also comment on the Cheng-Tung conjecture and then summarize our results in the Conclusion.

II. PRELIMINARIES

We start by collecting results needed for our discussion. If we define $h_{\mu\nu}(x, p)$ to be equal to $\langle p|[J_\mu(x), J_\nu(0)]|p\rangle$, $J_\mu(x)$ being the Heisenberg electromagnetic current operator and $|p\rangle$ being a single-hadron state of momentum p, then, assuming $h_{\mu\nu}(x, p)\delta(x_0)$ is well defined[9] and contains at most one derivative of a δ function, the most general form for $h_{\mu\nu}(x, p)\delta(x_0)$ consistent with general symmetry requirements like translation invariance, TCP, etc. is[10]

$$\delta(x_0)h_{00}(x, p)=0\,,$$

$$\delta(x_0)h_{0i}(x, p)=i[C(p_0)\partial_i+D(p_0)p_i(\partial\cdot p-\partial_0 p_0)]\delta^4(x)\,, \tag{1}$$

$$\delta(x_0)h_{ij}(x, p)=i[D(p_0)(\partial_j p_i+\partial_i p_j)+C'(p_0)g_{ij}(\partial\cdot p-\partial_0 p_0)-D'(p_0)p_i p_j(\partial\cdot p-\partial_0 p_0)]\delta^4(x)\,,$$

where $C(p_0)$ and $D(p_0)$ are arbitrary real functions of p_0 and the prime denotes differentiation with respect to p_0.

Next we show that if one assumes the set of Eqs. (1) then

$$T^*_{\mu\nu}(\nu, q^2)=T_{\mu\nu}(\nu, q^2)+C(p_0)(g_{\mu\nu}-g_{\mu 0}g_{\nu 0})+D(p_0)(p_\mu-p_0 g_{\mu 0})(p_\nu-p_0 g_{\nu 0}) \tag{2}$$

is covariant and gauge-invariant, where

$$T_{\mu\nu}(\nu, q^2)=i\int d^4x\, e^{iq\cdot x}\theta(x_0)\langle p|[J_\mu(x), J_\nu(0)]|p\rangle\ . \tag{3}$$

Although this result has been obtained earlier by Creutz and Sen,[11] we will briefly sketch the proof, as this result is important in our subsequent discussions. Since $T^*_{\mu\nu}(\nu, q^2)$ is covariant and gauge-invariant we can write

$$T^*_{\mu\nu}(\nu, q^2) = T_1^*(\nu, q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + T_2^*(\nu, q^2)\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right), \tag{4}$$

while for $T_{\mu\nu}(\nu, q^2)$ we have to write

$$T_{\mu\nu}(\nu, q^2, n) = T_1 g_{\mu\nu} + T_2 p_\mu p_\nu + T_3 q_\mu q_\nu + T_4 n_\mu n_\nu + T_5(p_\mu q_\nu + p_\nu q_\mu) + T_6(p_\mu n_\nu + p_\nu n_\mu) + T_7(q_\mu n_\nu + q_\nu n_\mu), \tag{5}$$

where $n_\mu = g_{\mu 0}$. Here T_i $(i = 1, \ldots, 7)$ are in general functions of q^2, ν, $q\cdot n$, $p\cdot n$. Following Bjorken[12] we now assume that $T^*_{\mu\nu}$ and $T_{\mu\nu}$ considered as analytic functions of q_0 have the same absorptive parts so that they differ at most by polynomials in q_0. This implies that T_1, T_2 differ from T_1^*, T_2^* by polynomials in q_0, while T_4, T_6, and T_7 are polynomials in q_0. From the set of equations (1) and the definition of $T_{\mu\nu}$, and using current conservation, it is easy to show that

$$\begin{aligned}
T_3 &= -\frac{1}{q^2} T_1 + T_2 \frac{(p\cdot q)^2}{q^4} - \frac{1}{q^2} C(p_0) - \frac{(p\cdot q)^2}{q^4} D(p_0),\\
T_4 &= C(p_0) + p_0{}^2 D(p_0),\\
T_5 &= -T_2 \frac{p\cdot q}{q^2} + \frac{p\cdot q}{q^2} D(p_0),\\
T_6 &= -D(p_0) p_0,\\
T_7 &= 0.
\end{aligned} \tag{6}$$

Therefore we have

$$\begin{aligned}
S_{\mu\nu} &= T_{\mu\nu} - T^*_{\mu\nu}\\
&= \{T_1 - T_1^*\}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + (T_2 - T_2^*)\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) + C(p_0)\left(-\frac{1}{q^2} q_\mu q_\nu + g_{\mu 0} g_{\nu 0}\right)\\
&\quad + D(p_0)\left(-\frac{(p\cdot q)^2}{q^4} q_\mu q_\nu + p_0{}^2 g_{\mu 0} g_{\nu 0} - p_0(p_\mu g_{\nu 0} + p_\nu g_{\mu 0}) + \frac{p\cdot q}{q^2}(p_\mu q_\nu - p_\nu q_\mu)\right).
\end{aligned}$$

This difference $S_{\mu\nu}$ is sometimes called the seagull term. For this difference to be polynomial in q_0 we must have

$$\begin{aligned}
T_2 - T_2^* &= D(p_0) + q^2 P_2(q_0),\\
T_1 - T_1^* &= -C(p_0) + q^2 P_1(q_0) + (p\cdot q)^2 P_2(q_0),
\end{aligned}$$

where P_1 and P_2 are arbitrary polynomials in q_0 which can depend on $(p\cdot n)$ and $\vec{q}$. Thus

$$\begin{aligned}
T^*_{\mu\nu} &= T_{\mu\nu} + C(p_0)(g_{\mu\nu} - g_{\mu 0} g_{\nu 0}) + D(p_0)(p_\mu - p_0 g_{\mu 0})(p_\nu - p_0 g_{\nu 0}) + P_1(q_0)(q^2 g_{\mu\nu} - q_\mu q_\nu)\\
&\quad + P_2(q_0)[(p\cdot q)^2 g_{\mu\nu} + q^2 p_\mu p_\nu - (p\cdot q)(p_\mu q_\nu + p_\nu q_\mu)].
\end{aligned}$$

In order that $T^*_{\mu\nu}$ be covariant we must have $T^*_{\mu\nu}$ be independent of n_μ, with $n^2 = 1$. From this requirement it is possible to show that P_1 and P_2 do not depend on n_μ. They are therefore covariant functions of q^2 and $p\cdot q$. Thus it is possible to define a "minimal" covariant and gauge-invariant $T^*_{\mu\nu}$ by dropping P_1 and P_2. This $T^*_{\mu\nu}$ is given by

$$T^*_{\mu\nu} = T_{\mu\nu} + C(p_0)(g_{\mu\nu} - g_{\mu 0} g_{\nu 0}) + D(p_0)(p_\mu - p_0 g_{\mu 0})(p_\nu - p_0 g_{\nu 0}),$$

which is the result we had set out to establish. We note that for this "minimal" $T^*_{\mu\nu}$

$$\begin{aligned}
T_2^* &= T_2 + D(p_0),\\
T_1^* &= T_1 + C(p_0).
\end{aligned} \tag{7}$$

From now on we will assume that this "minimal" $T^*_{\mu\nu}$ is the physical $T^*_{\mu\nu}$ which describes Compton scattering.

III. CONNECTION BETWEEN FIXED-POLE RESIDUES AND OPERATOR SCHWINGER TERMS

We start by recalling that a fixed $J=0$ pole in the amplitude $T_i^*(\nu, q^2)$ $(i=1,2)$ with residue $R_i(q^2)$ means that[13]

$$\lim_R T_2^*(\nu, q^2) = \sum_{\alpha_i} \beta_2(q^2, \alpha_i)\nu^{\alpha_i - 2} + \frac{R_2(q^2)}{\nu^2}\,,$$
$$\lim_R T_1^*(\nu, q^2) = \sum_{\alpha_i} \beta_1(q^2, \alpha_i)\nu^{\alpha_i} + R_1(q^2)\,, \tag{8}$$

where $\lim_R$ stands for the Regge limit $\nu \to \infty$, q^2 fixed, and we have absorbed the signature factors in the $\beta_i(q^2, \alpha)$'s. Next we prove that

$$\lim_{\text{BJL}} T_1^*(\nu, q^2) \sim C(p_0)\,,$$
$$\lim_{\text{BJL}} T_2^*(\nu, q^2) \sim D(p_0)\,. \tag{9}$$

The proof is trivial. We have only to note that

$$\lim_{\text{BJL}} T_{0i}^* = q_0 q_i \lim_{\text{BJL}} \frac{T_1^*(\nu, q^2)}{q^2} + q_0 p_i (\vec{p}\cdot\vec{q}) \lim_{\text{BJL}} \frac{T_2(q^2, \nu)}{q^2}\,,$$

where we have called the limit $q_0 \to \infty$, $|\vec{q}|$ fixed, the BJL limit and then used the connection between $\lim_{\text{BJL}} T_{0i}^*$ and the equal-time commutator $\langle p|[J_0(x), J_i(0)]|p\rangle \delta(x_0)$ displayed in Eq. (1).

If we now assume[14] that it is possible to go from the R limit ($q_0 \to \infty$, q^2 fixed) to the BJL limit ($q_0 \to \infty$, $|\vec{q}|$ fixed) in the frame $\vec{p}=0$ by making $q^2 \to \infty$ in the R limit, then we can relate the functions $C(m)$ and $D(m)$, which are due to the presence of operator Schwinger terms, to the asymptotic ($q^2 \to \infty$) form of the fixed-pole residue functions $R_i(q^2)$ $(i=1,2)$.

To see this we note that from (7) and (9)

$$\lim_{\text{BJL}} T_i(\nu, q^2) = 0\,, \quad i=1,2 \tag{10}$$

while from (7) and (8)

$$\lim_R T_1(\nu, q^2) = \sum_i \beta_1(q^2, \alpha_i)\nu^{\alpha_i} + R_1(q^2) - C(p_0)$$

and

$$\lim_R T_2(\nu, q^2) = \sum_i \beta_2(q^2, \alpha_i)\nu^{\alpha_i - 2} + \frac{R_2(q^2)}{\nu^2} - D(p_0)\,. \tag{11}$$

In order that (11) and (10) be compatible under our assumption it follows that

$$\lim_{q^2\to\infty} R_i(q^2) = C(p_0)|_{p_0=m}\,,$$
$$\lim_{q^2\to\infty\,(\text{like }\nu^2)} \lim_{\nu\to\infty} \frac{R_2(q^2)}{\nu^2} = D(p_0)|_{p_0=m}\,. \tag{12}$$

These equations would require $R_1(q^2)$ to behave like a constant for large q^2, while $R_2(q^2)$ should behave like $q^2 D(m)$ for q^2 large. In getting the set of equations (12) we have also assumed that $\beta_i(q^2, \alpha_i)$ $(i=1,2)$ fall off as $q^2 \to \infty$.[14] The second equation in (12) has been obtained before, using different methods, by a number of authors.[4,15]

Since we now know the asymptotic behavior of the fixed-pole residues, we can use this information to determine the contribution these fixed-pole terms make to the electromagnetic-mass-shift divergence problem. Within the framework of the Cottingham formula[16] for the electromagnetic mass shift, the fixed-pole contribution is given by[17]

$$\Delta M^{\text{fixed-pole}} = -\frac{3}{8}\int_0^\infty dq^2 \left[q^2 R_1(q^2) + \frac{\gamma_L^{(1)}(q^2)}{\gamma_L^{(1)}(q^2) - \gamma_T^{(1)}(q^2)} R_2(q^2) \right]\,,$$

where

$$\frac{\gamma_L^{(1)}(q^2)}{\gamma_T^{(1)}(q^2)} = \lim_{\nu\to\infty} \frac{\sigma_L^{(1)}(q^2, \nu)}{\sigma_T^{(1)}(q^2, \nu)}\,,$$

$\sigma_L(q^2, \nu)$ and $\sigma_T(q^2, \nu)$ being the total photoabsorption cross sections for longitudinal and transverse photons, respectively.[18] Now it is consistent with experimental data to set $\sigma_L(\omega) = \sigma_L(q^2, \omega) = 0$ for $\nu, q^2 \to \infty$, ν/q^2 fixed. Also we have related $R_1(q^2)$ to $C(m)$, and $C(m)$ can be shown[19] to be proportional to $\int_0^2 \sigma_L(\omega) f(\omega) d\omega$ and hence is experimentally equal to zero. Thus $M^{\text{fixed-pole}}$ with our assumptions is an at most logarithmically divergent object. It is thus possible for the logarithmic divergences coming from the fixed poles and from the scaling region to the mass-shift problem to cancel, leaving a finite expression for the mass shift. All this is, of course, very speculative, since we do not, at present, have a way of calculating $R_i(q^2)$ in a realistic manner.

IV. THE CHENG-TUNG CONJECTURE

We finally turn to the Cheng-Tung conjecture and begin by briefly reviewing the central part of their argument for suggesting a polynomial structure for the fixed-pole residue function $R(q^2)$. The starting point of their argument is to assume that $R(q^2)$ satisfies a dispersion relation in q^2 of the form

$$R(q^2) = \sum_{i=1}^{N} (q^2)^{n-1} R_n + \frac{(q^2)^N}{\pi} \int \frac{dq'^2}{(q'^2)^N} \frac{\text{Im}R(q'^2)}{q'^2 - q^2}\,. \tag{13}$$

$R(q^2)$ can be related to a matrix element of a product of two currents,[20] and hence the right-hand side of Eq. (13) can be graphically represented as in Fig. 1. A cross at the end of a photon line indicates possible subtraction in q^2 (contact terms). Figure 1(a) corresponds to the subtraction term in the equation for $R(q^2)$, while Figs. 1(b)–1(d) all

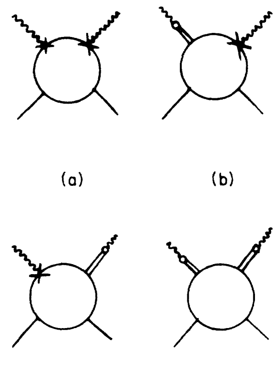

FIG. 1. Contributions to $\mathrm{Im}R(q^2)$.

contribute to the dispersion integral. The contributions from Figs. 1(b) and 1(c) are proportional to the residue function of a fixed J pole for a photoproduction amplitude; that from Fig. 1(d), for a hadron-hadron scattering amplitude. Cheng and Tung now state that (i) pure hadron-hadron amplitudes cannot have fixed poles at right-signature points and (ii) there is theoretical[21] and experimental evidence[22] against the presence of fixed poles for photoproduction off hadrons. From this it thus follows that

$$R(q^2)=\sum_{n=1}^{N}(q^2)^{n-1}R_n\ .$$

We now point out that even if one accepts statement (ii) of Cheng and Tung their statement (i) can be questioned. In fact the idea that right-signature fixed poles are possible in spite of unitarity has been discussed in detail in the literature.[23] We will therefore only quote the conclusion of these investigations relevant to our discussion. It is found that square-root branch points occur in each helicity amplitude at sense-nonsense points and there are fixed branch cuts running along the real J axis from σ_T-1 to $-\sigma_T$, where $\sigma_T=\max\{\sigma_1+\sigma_3,\sigma_2+\sigma_4\}$, the σ's representing the spins in the s-channel process $(1+2\to 3+4)$. Since the $d^J_{\lambda\lambda'}$'s have complementary branch points, these cuts do not contribute to the asymptotic behavior of the amplitude. They could however permit the existence of fixed poles at nonsense right-signature points with $J<\sigma_T-1$. In the special case of a spin-1, spin-0 scattering process a fixed pole at $J=0$ is thus allowed. Thus $R(q^2)$ could have a dispersive part, and hence a difference of sign in $R_2(q^2)/q^2$ at $q^2=0$ and $q^2=\infty$ could occur.

V. CONCLUSION

We have found that if operator Schwinger terms are present and if one can go from the Regge to the BJL limit, then fixed poles must be present. Furthermore, the asymptotic behavior of the fixed-pole residue function in q^2 is completely determined by the operator Schwinger terms. Using this information we have shown that the divergence which appears in the Cottingham formula for electromagnetic mass shifts from the fixed-pole term is logarithmic. We have speculated that this fixed-pole divergence might cancel the "scaling region" divergence, leaving a finite expression for the mass shift.

We have also pointed out that unitarity does not always rule out fixed poles in hadron-hadron scattering amplitudes. In particular there is no "unitarity argument" to prevent $R_2(q^2)$ from having an imaginary part. This weakens the basis of the Cheng-Tung conjecture, and the presence of $\mathrm{Im}R(q^2)$ can accommodate a change in sign of $R_2(q^2)/q^2$ at $q^2=0$ and at $q^2=\infty$.

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*Work supported by a grant from the National Science Council.

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[2]The variable ν is defined as $p\cdot q$, where p is the four-momentum of the hadron while q is the four-momentum of the photon.

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[9]By well defined we mean as a distribution with respect to the space of all infinitely differentiable test functions of bounded support. Also when we write $[J_\mu(x),J_\nu(0)]$ we mean $[J_\mu(x),J_\nu(0)]-\langle 0\,|\,[J_\mu(x),J_\nu(0)]\,|\,0\rangle$, so that only operator Schwinger terms appear in our discussions.

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[12]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[13]For a general discussion of fixed poles, Zee's review article (Ref. 6) is recommended.

[14]The data analysis in Ref. 3 assumed that

$$\lim_{q^2\to\infty} \beta_i(q^2,\alpha) \to (1/q^2)^{\alpha_i - n},$$

n being so adjusted that the combinations $\mathrm{Im}T_1^*$ and $\nu\mathrm{Im}T_2^*$ scale in the region $\nu\to\infty$, $q^2\to\infty$, ν/q^2 fixed. This kind of assumption – some sort of "asymptotic smoothness" [see R. Brandt, Phys. Rev. 187, 2192 (1969)], of being able to go from the Regge limit ($\nu\to\infty$, q^2 fixed) to the scaling region ($\nu\to\infty$, q^2, ν/q^2 fixed) – has been made by a number of authors [see for instance H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969); H. Harari, Phys. Rev. Letters 22, 1078 (1969)]. Our assumption is in this spirit. [See also G. Altarelli and H. R. Rubenstein, Phys. Rev. 187, 2111 (1969).]

[15]Note that we do not require $R(q^2)$ to have a polynomial structure.

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[20]For details regarding the connection of $R(q^2)$ to matrix elements of the product of two currents and other matters, we refer the reader to Cheng and Tung's original paper (Ref. 5).

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Energy Eigenvalues for Charged Particles in a Homogeneous Magnetic Field – An Application of the Foldy-Wouthuysen Transformation*

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(Received 17 November 1972)

We find that the equation of motion for spin-$\frac{1}{2}$ and spin-1 particles with an anomalous magnetic moment in a homogeneous magnetic field can be diagonalized by applying the Foldy-Wouthuysen transformation. The energy eigenvalues are then easily obtained by observation.

The traditional method for obtaining the energy eigenvalues of a system is to solve the eigenvalue equations.[1-3] This method becomes increasingly complicated when anomalous-magnetic-moment couplings are introduced and higher-spin particles are involved.[2,3] It was only recently that the energy eigenvalues for the motion of a Dirac particle and a spin-1 particle with an anomalous magnetic moment in a homogeneous magnetic field were calculated.[2-5] A simpler method for obtaining the eigenvalues of a system without solving the equation of motion was proposed by Tsai and Yildiz.[4,5] They observed that, even though the second-order form of the eigenvalue equation is not diagonal, it can be diagonalized by going to the fourth-order form.

The purpose of this paper is to present an even simpler method, by applying the Foldy-Wouthuysen transformation,[6-8] to obtain the energy eigenvalues of the spin-$\frac{1}{2}$ and spin-1 systems with anomalous-magnetic-moment couplings in a homogeneous magnetic field. The transformation method of Foldy and Wouthuysen is well known in its application to the reduction of a relativistic equation to the nonrelativistic form. One of its virtues is that the Dirac equation for a free particle and for a particle moving in a homogeneous magnetic field can be diagonalized by this transformation.[6,7,9] The extension of this feature to the cases when anomalous-magnetic-moment couplings are introduced and higher-spin particles are involved enables us to obtain the energy eigenvalues easily.

For the spin-$\frac{1}{2}$ system, the eigenvalue equation is