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Comment on the Absence of Radiative Corrections to the Anomaly of the Axial-Vector Current

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The absence of radiative corrections to the Schwinger-Bell-Jackiw-Adler anomaly of the axial-vector–current Ward-Takahashi identity is demonstrated using normal-product methods.

Adler and Bardeen[1] have given convincing, but not completely rigorous, arguments that the coefficient r in the Ward-Takahashi identity of the axial-vector current in (say) quantum electrodynamics (QED),

$$\partial^\mu\langle 0|Tj_{\mu 5}(x)X|0\rangle = 2iM\langle 0|Tj_5(x)X|0\rangle + r\langle 0|TF_{\mu\nu}\tilde{F}^{\mu\nu}(x)X|0\rangle - \sum_i[\delta(x-x_i)\gamma^5_{x_i} + \delta(x-y_i)\gamma^{5T}_{y_i}]\langle 0|TX|0\rangle \ , \quad (1)$$

has no radiative corrections to the second-order (triangle-graph) contribution. Here X represents any product of the basic fields,

$$\prod_{i=1}^{N}\psi(x_i)\prod_{j=1}^{N}\bar{\psi}(y_i)\prod_{k=1}^{L}A_{\mu_k}(z_k) \ .$$

The principal line of reasoning in Ref. 1 is that the higher-order contributions to r vanish for a theory in which the photon lines are regulated in a gauge-invariant manner, and that this property should persist when the regulator is removed. The Adler-Bardeen claim has been further supported by a number of explicit computations[1,2] showing that

$$r^{(4)} = 0 \ .$$

A more systematic approach to the problem is provided by normal-product methods in Zimmermann's formulation of renormalized perturbation theory.[3] As we shall see below, the normal-product algorithm allows one to derive in a straightforward manner certain Callan-Symanzik equations[4] and Ward-Takahashi identities from which the vanishing of $r - r^{(2)}$ follows easily. The advantage of this approach is that regulators are completely avoided, the finiteness (and often the gauge invariance) of vertex functions being guaranteed by the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction scheme.[3]

In the massive vector-meson model (massive QED) with effective Lagrangian[4]

$$\mathcal{L}_{\rm EFF} = (1+d)\tfrac{1}{2}i\bar{\psi}\gamma^\mu\overleftrightarrow{\partial}_\mu\psi - (M-c)\bar{\psi}\psi - \tfrac{1}{2}(1-b)\partial_\mu A_\nu\partial^\mu A^\nu + \tfrac{1}{2}(m^2+a)A_\mu A^\mu + (e+f)\bar{\psi}\gamma_\mu\psi A^\mu + \tfrac{1}{2}\left(1-b-\frac{m^2+a}{m_0^2}\right)(\partial_\mu A^\mu)^2 \quad (2)$$

we have shown[5] that the Schwinger-Bell-Jackiw-Adler anomaly[6] may be expressed as

$$(1+d-s)\partial^\mu\langle 0|TN_3[\bar{\psi}\gamma_\mu\gamma^5\psi](x)X|0\rangle = 2i(M-c)\langle 0|TN_3[\bar{\psi}\gamma^5\psi](x)X|0\rangle + r\langle 0|TN_4[F_{\mu\nu}\tilde{F}^{\mu\nu}](x)X|0\rangle - \sum_i[\delta(x-x_i)\gamma^5_{x_i} + \delta(x-y_i)\gamma^{5T}_{y_i}]\langle 0|TX|0\rangle \ , \quad (3)$$

with

$$r = i\frac{M-c}{96}\epsilon^{\mu\kappa\rho\sigma}\left(\frac{\partial}{\partial p^\mu}\frac{\partial}{\partial q^\kappa}\langle 0|TN_3[\bar\psi\gamma^5\psi](0)\tilde A_\rho(p)\tilde A_\sigma(q)\rangle^{\rm PROP}\right)_{p=q=0}, \tag{4}$$

where $N_{\delta_a}[\;]$ represents a normal product of basic fields and their derivatives with dimension $d_a \le \delta_a$, and the superscript PROP indicates that only one-particle irreducible diagrams are included. As a first step in the derivation of a Callan-Symanzik equation for r, let us employ the method of differential vertex operations[7] (DVO's) to obtain Callan-Symanzik equations for a vertex function $\Gamma^{(2N,L)}$ (sum over contributions from one-particle irreducible diagrams with $2N$ external fermion lines and L external meson lines). As in the case of the A^4 model[7] and the massive Thirring model,[8] we simply observe that the *eight* functions

$$\begin{aligned} &\frac{\partial}{\partial m^2}\Gamma^{(2N,L)}, \quad \frac{\partial}{\partial M}\Gamma^{(2N,L)}, \quad \frac{\partial}{\partial m_0^{\,2}}\Gamma^{(2N,L)}, \quad \frac{\partial}{\partial e}\Gamma^{(2N,L)}, \\ &N\Gamma^{(2N,L)}, \quad L\Gamma^{(2N,L)}, \quad \Delta_0\Gamma^{(2N,L)}, \quad \Delta_0'\Gamma^{(2N,L)}, \end{aligned} \tag{5}$$

where Δ_0 and Δ_0' are the two scalar DVO's of degree three, which we write symbolically as

$$\Delta_0 = \tfrac{1}{2}i\int d^4x\, N_3[A_\mu A^\mu](x), \quad \Delta_0' = i\int d^4x\, N_3[\bar\psi\psi](x),$$

are all linear combinations of the *six* linearly independent $\Delta_i\Gamma^{(2N,L)}$, where, symbolically,

$$\begin{aligned} &\Delta_1 = \tfrac{1}{2}i\int N_4[A_\mu A^\mu](x)\,d^4x, && \Delta_4 = -\int N_4[\bar\psi\gamma^\mu\partial_\mu\psi](x)\,d^4x, \\ &\Delta_2 = \tfrac{1}{2}i\int N_4[\partial_\mu A_\nu\partial^\mu A^\nu](x)\,d^4x, && \Delta_5 = i\int N_4[\bar\psi\gamma^\mu\psi A_\mu](x)\,d^4x, \\ &\Delta_3 = i\int N_4[\bar\psi\psi](x)\,d^4x, && \Delta_6 = \tfrac{1}{2}i\int N_4[(\partial_\mu A^\mu)^2](x)\,d^4x, \end{aligned} \tag{6}$$

and, in each case, the coefficients of the various Δ_i may be determined by application of the normal-product normalization conditions at the origin in momentum space. In particular,

$$\begin{aligned} \frac{\partial}{\partial s}\Gamma^{(2N,L)} &= \left[\frac{\partial}{\partial s}(a+m^2)\Delta_1 + \frac{\partial}{\partial s}(b-1)\Delta_2 + \frac{\partial}{\partial s}(c-M)\Delta_3 + \frac{\partial}{\partial s}(1+d)\Delta_4 \right. \\ &\quad \left. + \frac{\partial}{\partial s}(e+f)\Delta_5 + \frac{\partial}{\partial s}\left(1-b-\frac{m^2+a}{m_0^{\,2}}\right)\Delta_6\right]\Gamma^{(2N,L)} \quad \text{for } s = m^2,\ M,\ m_0^{\,2},\ e, \\ N\Gamma^{(2N,L)} &= (c-M)\Delta_3 + (1+d)\Delta_4 + (e+f)\Delta_5, \\ L\Gamma^{(2N,L)} &= 2(m^2+a)\Delta_1 + 2(b-1)\Delta_2 + (e+f)\Delta_5 + 2\left(1-b-\frac{m^2+a}{m_0^{\,2}}\right)\Delta_6, \\ \Delta_0\Gamma^{(2N,L)} &= \sum_{i=1}^{6} s_i\Delta_i, \quad s_1 = 1, \quad s_3 = 0, \\ \Delta_0'\Gamma^{(2N,L)} &= \sum_{i=1}^{6} t_i\Delta_i, \quad t_1 = 0, \quad t_3 = 1. \end{aligned} \tag{7}$$

Consequently there are two independent linear relations among the quantities (5), which we may take to be

$$(D - 2N\gamma - L\delta)\Gamma^{(2N,L)} = \Delta\Gamma^{(2N,L)}, \tag{8}$$

$$(D' - 2N\gamma' - L\delta')\Gamma^{(2N,L)} = \Delta'\Gamma^{(2N,L)}, \tag{9}$$

where

$$D = m\frac{\partial}{\partial m} + M\frac{\partial}{\partial M} + \alpha m_0\frac{\partial}{\partial m_0} + \beta\frac{\partial}{\partial e},$$

$$D' = m\frac{\partial}{\partial m} + \alpha' m_0\frac{\partial}{\partial m_0} + \beta'\frac{\partial}{\partial e},$$

$$\Delta = \rho\Delta_0 + \sigma\Delta_0',$$

$$\Delta' = \rho'\Delta_0 + \sigma'\Delta_0'.$$

The coefficients α, β, γ, etc. (α', β', γ', etc.) may be determined as perturbation series in e by equating to zero the coefficient of each $\Delta_i\Gamma^{(2N,L)}$ in Eq. (8) [Eq. (9)]. In particular,

$$\begin{aligned}
&(D-2\delta)(a+m^2)=\rho\ ,\\
&(D-2\delta)(b-1)=\rho s_2+\sigma t_2\ ,\\
&(D-2\gamma)(c-M)=\sigma\ ,\\
&(D-2\gamma)(1+d)=\rho s_4+\sigma t_4\ ,\\
&(D-2\gamma-\delta)(e+f)=\rho s_5+\sigma t_5\ ,\\
&(D-2\delta)\left(1-b-\frac{m^2+a}{m_0^{\,2}}\right)=\rho s_6+\sigma t_6\ .
\end{aligned} \tag{10}$$

As discussed in Ref. 5, the vector-current Ward-Takahashi identity and the imposition of mass-shell, intermediate, or other appropriate normalization conditions lead to the identity $f=de$, and similar arguments show that

$$s_5=es_4\ ,$$
$$t_5=et_4\ .$$

Thus, the fourth and fifth equations in (10) lead to the following simple relation between the coefficients in (8):

$$\beta=e\delta\ , \tag{11}$$

with a similar relation between β' and δ' in (9).

Equation (8) is the Callan-Symanzik equation[4] for the model and may be used to study the asymptotic behavior of vertex functions in the region where all momenta become large simultaneously, whereas Eq. (9) controls the $m\to 0$ limit of the theory, allowing one to prove the smooth passage to QED of the gluon model with intermediate normalization. In the present note we shall need only the first of these equations.

From Eq. (8) one may proceed without difficulty to Callan-Symanzik equations for vertex functions involving normal products as well as basic fields. In particular, for

$$\Gamma_5^{(2N,L)}(p,p_1\cdots p_N,q_1\cdots q_N,k_1\cdots k_L)=\left\langle 0\left|TN_3[\bar\psi\gamma^5\psi](0)\prod_i^N\tilde\psi(p_i)\prod_j^N\tilde{\bar\psi}(q_j)\prod_l^L\tilde A_{\mu_l}(k_l)\right|0\right\rangle^{\text{PROP}}$$

the analog of (7) is

$$\begin{aligned}
\frac{\partial}{\partial s}\Gamma_5^{(2N,L)}&=\left[\frac{\partial}{\partial s}(a+m^2)\Delta_1+\frac{\partial}{\partial s}(b-1)\Delta_2+\frac{\partial}{\partial s}(c-M)\Delta_3+\frac{\partial}{\partial s}(1+d)\Delta_4\right.\\
&\left.\quad+\frac{\partial}{\partial s}(e+f)\Delta_5+\frac{\partial}{\partial s}\left(1-b-\frac{m^2+a}{m_0^{\,2}}\right)\Delta_6\right]\Gamma_5^{(2N,L)}\ ,\quad \text{for } s=m^2,M,m_0^{\,2},e\ ,\\
L\Gamma_5^{(2N,L)}&=\left[2(m^2+a)\Delta_1+2(b-1)\Delta_2+(e+f)\Delta_5+2\left(1-b-\frac{m^2+a}{m_0^{\,2}}\right)\Delta_6\right]\Gamma_5^{(2N,L)}\ ,\\
(N-1)\Gamma_5^{(2N,L)}&=[(c-M)\Delta_3+(1+d)\Delta_4+(e+f)\Delta_5]\Gamma_5^{(2N,L)}\ ,\\
\Delta_0\Gamma_5^{(2N,L)}&=\sum_{i=1}^{6}s_i\Delta_i+u_0\Gamma_5^{(2N,L)}\ ,\\
\Delta_0'\Gamma_5^{(2N,L)}&=\sum_{i=1}^{6}t_i\Delta_i+u_0'\Gamma_5^{(2N,L)}\ ,
\end{aligned} \tag{12}$$

where

$$u_0\gamma^5=-i\Delta_0\Gamma_5^{(2,0)}(0,0)\ ,$$
$$u_0'\gamma^5=-i\Delta_0'\Gamma_5^{(2,0)}(0,0)\ .$$

The additional terms on the right-hand side of the last two relations of (12) arise from the fact that a subgraph containing two external fermion lines, the $N_3[\bar\psi\gamma^5\psi]$ vertex, and one of the DVO vertices Δ_0 or Δ_0' has no over-all subtraction in the BPHZ prescription, whereas a subgraph with the identical structure but with DVO vertex Δ_1 or Δ_3 has one subtraction at the origin in momentum space. From (10) and (12) one obtains the Callan-Symanzik equation for $\Gamma_5^{(2N,L)}$,

$$[D-(N-2)\gamma-L\delta]\Gamma_5^{(2N,L)}=(\Delta-u)\Gamma_5^{(2N,L)}\ , \tag{13}$$

with

$$u=\rho u_0+\sigma u_0'\ .$$

Referring to the definition (4) of r, we see that (13) and the Callan-Symanzik equation for $(c-M)$ given in (10) leads directly to the following relation:

$$(D-2\delta)r=\frac{\sigma}{c-M}r+(\Delta-u)r\ , \tag{14}$$

where

$$\Delta r=i\frac{M-c}{96}\,\epsilon^{\mu\kappa\rho\sigma}\left(\frac{\partial}{\partial p_\mu}\,\frac{\partial}{\partial q_\kappa}\,\Delta\Gamma^{(0,2)}_{5\rho\sigma}(p,q)\right)_{p=q=0}\ .$$

In order to simplify the right-hand side of (14), we employ the Ward-Takahashi identity

$$-(1+d-s)(p+q)^\mu\Delta\Gamma^{(0,2)}_{\mu 5\rho\sigma}(p,q)=2(M-c)\Delta\Gamma^{(0,2)}_{5\rho\sigma}(p,q)+[2(c-M)u-2\sigma]\Gamma^{(0,2)}_{5\rho\sigma}(p,q)+2r(p+q)^\mu\Delta\Gamma^{(0,2)}_{A\tilde{F}\mu\rho\sigma}(p,q)\ , \tag{15}$$

with

$$\Delta\Gamma^{(2N,L)}_{\mu 5}=\int d^4x\langle 0|T(\rho N_3[A_\mu A^\mu](x)+\sigma N_3[\bar\psi\psi](x))\,N_3[\bar\psi\gamma_\mu\gamma^5\psi](0)X|0\rangle^{\rm PROP}\ ,$$

$$\Delta\Gamma^{(2N,L)}_{A\tilde{F}\mu}=\int d^4x\langle 0|T(\rho N_3[A_\mu A^\mu](x)+\sigma N_3[\bar\psi\psi](x))\,N_3[A^\nu\tilde{F}_{\mu\nu}](0)X|0\rangle^{\rm PROP}\ ,$$

which may be derived by the same normal-product algorithm which led to (3) in Ref. 5. The origin of the term proportional to $u\Gamma^{(0,2)}_{5\rho\sigma}$ on the right-hand side is similar to that of the "extra" term in (13). Evaluating the second-order Taylor-series coefficient of each term in (15), one obtains

$$2[\sigma-(M-c)(\Delta-u)]r=\frac{-i}{96}\,\epsilon^{\mu\kappa\rho\sigma}\left(\frac{\partial}{\partial p^\mu}\,\frac{\partial}{\partial q^\kappa}(p+q)^\nu[(1+d-s)\Delta\Gamma^{(0,2)}_{\nu 5\rho\sigma}(p,q)+2r\Delta\Gamma^{(0,2)}_{A\tilde{F}\nu\rho\sigma}(p,q)]\right)_{p=q=0}\ . \tag{16}$$

$\Delta\Gamma^{(0,2)}_{\nu 5\rho\sigma}(p,q)$ and $\Delta\Gamma^{(0,2)}_{A\tilde{F}\mu\rho\sigma}(p,q)$ are necessarily transverse in the external momenta p and q, i.e.,

$$p^\rho\Delta\Gamma^{(0,2)}_{\nu 5\rho\sigma}(p,q)=0=q^\sigma\Delta\Gamma^{(0,2)}_{\nu 5\rho\sigma}(p,q)\ ,$$
$$p^\rho\Delta\Gamma^{(0,2)}_{A\tilde{F}\mu\rho\sigma}(p,q)=0=q^\sigma\Delta\Gamma^{(0,2)}_{A\tilde{F}\mu\rho\sigma}(p,q)\ , \tag{17}$$

and hence have expansions

$$\Delta\Gamma_{\nu 5\rho\sigma}(p,q)=\epsilon_{\rho\sigma\alpha\beta}\,p^\alpha q^\beta(p+q)_\nu H(p^2,q^2,pq)\ ,$$
$$\Delta\Gamma_{A\tilde{F}_{\nu\rho\sigma}}(p,q)=\epsilon_{\rho\sigma\alpha\beta}\,p^\alpha q^\beta(p+q)_\nu G(p^2,q^2,pq)\ , \tag{18}$$

where G and H are regular at the origin. Thus the right-hand side of (16) vanishes, so that

$$(D-2\delta)r=\left(\beta\frac{\partial}{\partial e}-2\delta\right)r=\frac{\beta}{e}\left(e\frac{\partial}{\partial e}-2\right)r=0\ . \tag{19}$$

Here we have used the fact[5] that r is independent of m_0, as well as the relation (11). Since β is nonvanishing in third order,[9] we obtain, finally,

$$r-r^{(2)}=0\ . \tag{20}$$

The m_0 independence of r, which played a crucial role in our derivation of (20), was obtained in Ref. 5 with the aid of the formula

$$\frac{\partial}{\partial m_0^{\,2}}\langle 0|TX|0\rangle=\tfrac{1}{2}i\frac{m^2+a}{m_0^{\,4}}\int\langle 0|T:(\partial_\mu A^\mu)^2:(x)X|0\rangle\ . \tag{21}$$

Unfortunately the Green's function of the Wick product appearing on the right-hand side was not defined sufficiently precisely, and a brief clarification is in order here. The confusion on this point arises from the fact that the usual Green's functions of the Wick product[10]

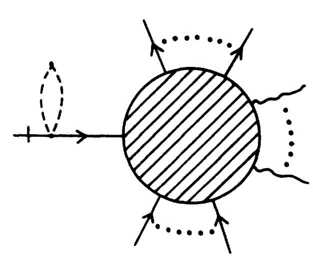

FIG. 1. Diagrams responsible for the divergence of $\langle 0|T\,\partial_\mu A^\mu(x+\xi)\,\partial_\nu A^\nu(x-\xi)X|0\rangle$ when $\xi\to 0$. The loop consisting of two mass-m_0 propagators (dashed lines) arises after two applications of the Ward-Takahashi identity for $\partial_\mu A^\mu$.

$$\langle 0|T:\partial^\mu A_\mu(x+\xi)\partial^\nu A_\nu(x-\xi):X|0\rangle=\langle 0|T\partial_\mu A^\mu(x+\xi)\partial_\nu A^\nu(x-\xi)X|0\rangle-\langle 0|T\partial_\mu A^\mu(x+\xi)\partial_\nu A^\nu(x-\xi)|0\rangle\langle 0|TX|0\rangle \tag{22}$$

are logarithmically divergent for $\xi\to 0$, thanks to the presence of the graphs (after applying the Ward-Takahashi identity for both $\partial_\mu A^\mu$ factors) shown in Fig. 1. Finite time-ordered functions for $:(\partial_\mu A^\mu)^2:$ may be defined by subtracting off the divergent part of the offending diagrams:

$$\left\langle 0\left|T:(\partial_\mu A^\mu)^2:(x)\prod_i^N\psi(x_i)\prod_j^N\bar\psi(y_j)\prod_k^L A_{\mu_k}(z_k)\right|0\right\rangle$$
$$=\lim_{\xi\to 0}\left(\langle 0|T:\partial_\mu A^\mu(x+\xi)\partial_\nu A^\nu(x-\xi):X|0\rangle-\omega(\xi^2)\sum_{i=1}^N[\delta(x-x_i)+\delta(x-y_i)]\langle 0|TX|0\rangle\right), \tag{23}$$

where

$$\omega(\xi^2)=-e^2\left(\frac{m_0^{\,2}}{m^2+a}\right)^2\Delta_F(\xi,m_0^{\,2})^2 .$$

To see that the DVO $\frac{1}{2}i\int d^4x:(\partial_\mu A^\mu)^2:(x)$ may be expressed as a linear combination of the scalar DVO's of degree four (this is important for our discussion of gauge invariance in Ref. 5), we may apply the Zimmermann identity[10] expressing $:\partial_\mu A^\mu(x+\xi)\partial_\nu A^\nu(x-\xi):$ in terms of N_4 normal products, as well as the counting identity[7]

$$-NG^{(2N,L)}=[(c-M)\Delta_3+(1+d)\Delta_4+(e+f)\Delta_5]G^{(2N,L)} , \tag{24}$$

to obtain

$$\tfrac{1}{2}i\int\langle 0|\hat T:(\partial_\mu A^\mu)^2:(x)X|0\rangle d^4x=\lim_{\xi\to 0}\left(\tfrac{1}{2}i\int d^4x\langle 0|T:\partial_\mu A^\mu(x+\xi)\partial_\nu A^\nu(x-\xi):X|0\rangle-iN\omega(\xi^2)\langle 0|TX|0\rangle\right)$$
$$=\lim_{\xi\to 0}\left(\tfrac{1}{2}i\int d^4x\langle 0|TN_4[\partial_\mu A^\mu(x+\xi)\partial_\nu A^\nu(x-\xi)]X|0\rangle+\sum_{i=1}^5 r_i(\xi^2)\Delta_i G^{(2N,L)}\right)$$
$$=\Delta_6 G^{(2N,L)}+\sum_{i=1}^5 r_i\,\Delta_i G^{(2N,L)} . \tag{25}$$

Formulas for the limiting coefficients r_i, obtained by application of the normalization conditions for DVO's, are listed in Eq. (2.22) of Ref. 5.

Our proof of Eq. (20) using Callan-Symanzik equations and Ward-Takahashi identities was inspired by the success of similar methods in the proof of asymptotic scale invariance in the massive Thirring model.[8] Recently we have learned that Zee[11] has arrived at the Adler-Bardeen result using arguments which also exploit the Callan-Symanzik equations and Ward-Takahashi identities. The advantage of our derivation, in contrast to Zee's, is that cutoffs (and consequently the rather delicate questions concerning their removal) are completely avoided.

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