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Finite n-p Mass Difference in Spontaneously Broken Gauge Theories

Daniel Z. Freedman*
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

and

Wolfgang Kummer†
University of Pennsylvania, Philadelphia, Pennsylvania 19104
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In two examples of spontaneously broken gauge theories in which electromagnetic isodoublet mass differences are finite, the sign of $\Delta m|_{n-p}$ for pointlike protons and neutrons is negative, although the general formula for mass differences in gauge theories allows both signs.

I. INTRODUCTION

Renormalizable gauge-invariant field theories with spontaneously broken symmetry are now being studied intensively in the hope of obtaining a correct unified description of electromagnetic and weak interactions.[1] The fact that intramultiplet mass differences are finite in perturbation expansions and therefore computable in terms of the parameters of the Lagrangian in such theories was noted by ' t Hooft[2] and recently emphasized by Weinberg.[3]

In this paper we investigate fermion isodoublet mass differences in broken-symmetry gauge field theories in order to see whether the "neutron" can be heavier than the "proton" in a second-order calculation. Traditional approaches[4] to the neutron-proton mass-difference problem usually yield an answer with negative sign. Therefore a positive result in gauge field theories, although not realistic because of neglect of strong interactions, would surely spur further research efforts.

We find that the general expression for the mass difference Δm in a class of gauge-field-theory models suggests that the sign is model-dependent, but that in the two models for which explicit calculations are made, the sign is unfortunately negative.

The calculation of Δm in a simple model with $SU(2)\times U(1)$ gauge symmetry and parity-conserving couplings is treated in Sec. II, and a general expression for Δm is presented and discussed in Sec. III. In Sec. IV we calculate Δm in an $SU(2)_L\times SU(2)_R \times U(1)_Y$-invariant model suggested by Weinberg.[3]

II. $SU(2)\times U(1)$ MODEL

It is instructive to begin our study in a model of minimal algebraic complexity. We therefore choose a model incorporating triplet and singlet gauge fields $\vec{A}_\mu$ and B_μ and fermion and scalar doublets

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{2.1}$$

each with $T=\frac{1}{2}$, $Y=1$, and the parity-conserving Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu)^2 - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2$$
$$+\bar{\psi}(i\partial\!\!\!/ - \tfrac{1}{2} g\vec{\tau}\cdot\vec{A} - \tfrac{1}{2}g' B\!\!\!/ - m)\psi + |\partial_\mu \Phi + i\tfrac{1}{2} g\vec{\tau}\cdot\vec{A}_\mu \Phi + i\tfrac{1}{2} g' B_\mu \Phi|^2 - \mu^2 \Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 . \tag{2.2}$$

SU(2)×U(1) invariance permits a mass term with equal proton and neutron masses but does not allow Yukawa couplings. This Lagrangian cannot describe weak interactions but may be thought of as a simplified description of the ρ-meson–photon–nucleon system.

A perturbative solution to this field theory with spontaneous symmetry breaking is obtained by letting ϕ_2 develop a real vacuum expectation value. After a gauge transformation to eliminate redundant scalar field components (U gauge), we can write

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + s(x)\end{pmatrix}, \tag{2.3}$$

with $v = (-\mu^2/\lambda)^{1/2}$, and find the vector-meson mass term

$$+\tfrac{1}{4}v^2[(gA_\mu^3 - g'B_\mu)^2 + g^2(A_\mu^1)^2 + g^2(A_\mu^2)^2]. \tag{2.4}$$

Diagonalization is trivial, and, as in Weinberg's original model,[1] gives us the eigenfields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2), \qquad m_W = \tfrac{1}{2}vg,$$
$$Z_\mu = \frac{1}{(g^2+g'^2)^{1/2}}(gA_\mu^3 - gB_\mu), \qquad m_Z = \tfrac{1}{2}v(g^2+g'^2)^{1/2}, \tag{2.5}$$
$$A_\mu = \frac{1}{(g^2+g'^2)^{1/2}}(g'A_\mu^3 + gB_\mu), \qquad m_A = 0 .$$

The interaction term between nucleon and gauge fields can then be rewritten as

$$-\frac{1}{2\sqrt{2}} gW_\mu^+ \bar{p}\gamma^\mu n + \text{H.c.} - \frac{gg'}{(g^2+g'^2)^{1/2}} A_\mu \bar{p}\gamma^\mu p$$
$$- \frac{g^2 - g'^2}{2(g^2+g'^2)^{1/2}} Z_\mu \bar{p}\gamma^\mu p + \tfrac{1}{2}(g^2+g'^2)^{1/2} Z_\mu \bar{n}\gamma^\mu n . \tag{2.6}$$

At this point we can identify A_μ as the electromagnetic field, and the electric charge of the proton as $e = gg'/(g^2+g'^2)^{1/2}$.

We now turn to the computation of Δm in second-order perturbation theory where the vector-meson loop graphs of Fig. 1 contribute. Following Weinberg,[3] we work directly in U gauge, where we will encounter the curious property that quadratically divergent terms in the self-energy $\Sigma(p\!\!\!/)$ disappear at $p\!\!\!/ = m$, so that mass shifts of the proton and neutron are logarithmically divergent, and the mass difference Δm finite.

For the loop graph of any massive vector, such as Z, we have, ignoring coupling constants for the moment,

$$\Sigma(p\!\!\!/) = -i\int \frac{d^4k}{(2\pi)^4}\gamma_\mu \frac{1}{p\!\!\!/ - k\!\!\!/ - m + i\epsilon}\gamma_\nu \frac{g^{\mu\nu} - (k^\mu k^\nu/m_Z^2)}{k^2 - m_Z^2 + i\epsilon}, \tag{2.7}$$

where we can give meaning to the highly divergent integrals by propagator regulation. The elementary Ward identity can be used to rewrite a factor in the integrand of the $k^\mu k^\nu$ term as

$$k\!\!\!/ \frac{1}{p\!\!\!/ - k\!\!\!/ - m} k\!\!\!/ = [(p\!\!\!/ - k\!\!\!/ - m) - (p\!\!\!/ - m)]\frac{1}{p\!\!\!/ - k\!\!\!/ - m}$$
$$\times [(p\!\!\!/ - k\!\!\!/ - m) - (p\!\!\!/ - m)]$$
$$= -k\!\!\!/ - (p\!\!\!/ - m) + (p\!\!\!/ - m)\frac{1}{p\!\!\!/ - k\!\!\!/ - m}(p\!\!\!/ - m). \tag{2.8}$$

The entire contribution of the $k^\mu k^\nu$ term therefore vanishes at $p\!\!\!/ = m$, and need not be considered further in the calculation of Δm. A more general discussion of the cancellation of such spurious divergences in observable amplitudes has been given by Kummer and Lane.[5]

The $g^{\mu\nu}$ term is responsible for logarithmic-divergent mass shifts of proton and neutron, but according to the general arguments of Weinberg, even this divergence must cancel in the mass difference Δm. To see how this cancellation occurs in our model, one need only refer to the coupling coefficients in the individual loop contributions of Fig. 1. The $W^\pm$ loops do not contribute to Δm, and photon and Z contributions are equal in magnitude and opposite in sign, so we can write

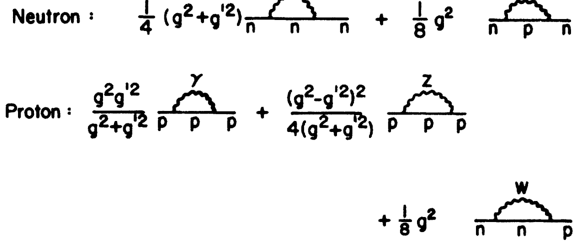

FIG. 1. Feynman diagrams and coupling factors for the neutron and proton self-energy parts in the SU(2) × U(1) model of Sec. II.

$$\Delta m|_{n-p} = \frac{+i}{4m}\frac{g^2g'^2}{g^2+g'^2}\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}(\not p+m)\gamma_\mu(\not p-\not k+m)\gamma^\mu}{(p-k)^2-m^2+i\epsilon}\times\left(\frac{1}{k^2+i\epsilon}-\frac{1}{k^2-m_Z^2+i\epsilon}\right). \tag{2.9}$$

Because of the simple relation between the couplings, the Z particle acts as a regulator in the mass-difference calculation, and the result is finite. The momentum integration is readily performed by standard methods and gives the result

$$\Delta m = -\frac{\alpha m}{2\pi}\int_0^1 dx\,(1+x)\ln\left(1+\frac{1-x}{x^2}\frac{m_Z^2}{m^2}\right), \tag{2.10}$$

which is simple but, unfortunately, manifestly negative.

III. A GENERAL EXPRESSION FOR Δm

In the two models discussed in this paper, Δm has the following properties:

(i) Δm vanishes in zero and first order;

(ii) only vector-meson loops, with the $g_{\mu\nu}$ part of the propagators, contribute to Δm in second order.

In all such models Δm takes the form

$$\Delta m = \frac{ie^2}{4m}\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}(\not p+m)\gamma_\mu(\not p-\not k+m)\gamma^\mu}{(p-k)^2-m^2}\times\left(\frac{1}{k^2+i\epsilon}-\sum_{i=1}^n d_i\frac{1}{k^2-M_i^2}\right), \tag{3.1}$$

where we have separated the photon, which by definition couples with strength e^2 to Δm, from the massive vector mesons, which couple with strength $e^2 d_i$. The d_i and M_i are functions of the coupling constants and vacuum expectation values which appear in the Lagrangian of the model and must be calculated by diagonalization of the vector-meson mass matrix. As discussed quite generally by Weinberg,[3] Δm is finite and we must therefore have the condition

$$\sum_{i=1}^n d_i = 1 \tag{3.2}$$

as a functional identity in any specific model.

The integral may be evaluated using the condition (3.2) to give

$$\Delta m = -\frac{\alpha m}{2\pi}\sum_{i=1}^n d_i J(M_i^2/m^2), \tag{3.3}$$

where

$$\begin{aligned} J(\beta) &= \int_0^1 dx\,(1+x)\ln\left(1+\frac{1-x}{x^2}\beta\right)\\ &= -\tfrac{1}{2}\beta+\tfrac{1}{4}\beta^2\ln\beta-\beta[\tfrac{1}{4}\beta^2-\tfrac{1}{2}\beta-2]\frac{1}{(\beta^2-4\beta)^{1/2}}\ln\left(\frac{[\beta+(\beta^2-4\beta)^{1/2}-2]^2\beta}{[\beta+(\beta^2-4\beta)]^2}\right)\\ &\approx\left(\frac{3}{2}+\frac{2}{\beta}\right)\ln\beta+\frac{3}{4}+O\left(\frac{1}{\beta}\right). \end{aligned} \tag{3.4}$$

The exact expression is correct for all β, but should be rewritten in terms of $\tan^{-1}$ functions for $\beta<4$, while the asymptotic form is accurate to 1% or better for $\beta \geq 4$. Further $J(\beta)$ is a monotonic increasing function for $\beta>0$ and $J(0)=0$.

It is worthwhile to examine the possible sign of Δm on the basis of the expression (3.3) and the constraint (3.2). If, as in the SU(2)×U(1) model of Sec. II, only one massive vector meson couples to Δm, we must have $d_1=1$ and $\Delta m<0$. If there are two or more massive vectors, it is convenient to make the ordering convention $M_1>M_2>\cdots>M_n$, so that $J(M_i^2/m^2)>J(M_k^2/m^2)$ if $i<k$. If we conceive of the d_i and M_i as functionally independent quantities, subject only to the constraint (3.2), then both signs are allowed for Δm. For two massive vectors, Δm will be positive if the following two conditions hold simultaneously:

(a) $d_1 \ll 0$,

(b) $M_1 \gg M_2$.

For $n>2$ similar conditions can be stated qualitatively. Of course, in any model d_i and M_i are functionally related and explicit calculations must be undertaken to see if conditions (a), (b) or their analogs can be satisfied.

IV. $SU(2)_L\times SU(2)_R\times U(1)_Y$ MODEL

In this model, suggested by Weinberg[3] as a reasonable framework for computation of Δm, we have seven gauge fields and chiral nucleon and scalar doublets as follows:

$$\begin{aligned} &\vec{A}^L_\mu,\ \vec{A}^R_\mu,\ B_\mu\\ &\psi_{L,R}=\begin{pmatrix}\tfrac{1}{2}(1\mp\gamma_5)p\\ \tfrac{1}{2}(1\mp\gamma_5)n\end{pmatrix},\\ &T_{L,R}=\tfrac{1}{2},\ T_{R,L}=0,\ Y=1,\\ &\Phi=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},\\ &T_L=\tfrac{1}{2},\ T_R=0,\ Y=1, \end{aligned} \tag{4.1}$$

with conventional gauge transformation properties. In addition there is an additional scalar field $H(x)$ with $T_L = T_R = \frac{1}{2}$, $Y = 0$ for which we use a matrix notation [6]

$$H(x) = \begin{pmatrix} h_{11}(x) & h_{12}(x) \\ h_{21}(x) & h_{22}(x) \end{pmatrix} . \tag{4.2}$$

The gauge property is

$$H(x) \to H'(x) = e^{-i\vec{\tau}\cdot\vec{\Lambda}_L/2} H(x) e^{+i\vec{\tau}\cdot\vec{\Lambda}_R/2} . \tag{4.3}$$

A Hermiticity condition is necessary to ensure equal neutron and proton masses in zero order.[3] We choose

$$H(x) = [\tau_2 H^\dagger(x) \tau_2]^{\mathrm{Tr}} , \tag{4.4}$$

which is compatible with the gauge property and reduces the number of degrees of freedom in the field from eight to four.

The Lagrangian can be written in terms of gauge field covariant tensors $F_{\mu\nu}$ and covariant derivatives

$$\begin{aligned} d_\mu^{L,R} &\equiv \partial_\mu + i g_{L,R} \tfrac{1}{2}\vec{\tau}\cdot\vec{\mathrm{A}}_\mu^{L,R} + i\tfrac{1}{2} g_Y B_\mu , \\ D_\mu H &\equiv \partial_\mu H + i g_L \tfrac{1}{2}\vec{\tau}\cdot\vec{\mathrm{A}}_\mu^L H - i\tfrac{1}{2} g_R H \vec{\tau}\cdot\vec{\mathrm{A}}_\mu^R \end{aligned} \tag{4.5}$$

as

$$\begin{aligned} \mathcal{L} = &-\tfrac{1}{4}\{(\vec{\mathrm{F}}_{\mu\nu}^L)^2 + (\vec{\mathrm{F}}_{\mu\nu}^R)^2 + (F_{\mu\nu}^Y)^2\} + \bar{\psi}_L i\gamma\cdot d^L \psi_L + \bar{\psi}_R i\gamma\cdot d^R\psi_R \\ &+ (d_\mu^L\Phi)^\dagger (d^{L\mu}\Phi) + \tfrac{1}{2}[\mathrm{Tr}(D_\mu H)^\dagger D^\mu H] + f(\bar{\psi}_L H\psi_R + \bar{\psi}_R H^\dagger \psi_L) + V(H,\Phi) . \end{aligned} \tag{4.6}$$

The Higgs meson potential contains five independent field monomials up to quartic order. With no loss in generality, the broken-symmetry solution can be chosen in the form, in U gauge,

$$\begin{aligned} H(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} v + s(x) & 0 \\ 0 & v + s(x) \end{pmatrix} , \\ \Phi(x) &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v' + s'(x) \end{pmatrix} . \end{aligned} \tag{4.7}$$

The terms of $\mathcal{L}$ which are relevant to the computation of Δm in second-order perturbation theory are the vector-boson–nucleon interaction

$$\mathcal{L}_{\bar{\psi}\psi V} = \tfrac{1}{4}\bar{\psi}\{g_L \vec{\tau}\cdot\vec{A\!\!\!/}^L (1-\gamma_5) + g_R \vec{\tau}\cdot\vec{A\!\!\!/}^R(1+\gamma_5) + 2 g_Y B\!\!\!/\}\psi \tag{4.8}$$

and the vector mass term

$$\mathcal{L}_{VV} = \tfrac{1}{8} v^2 (g_L \vec{\mathrm{A}}_\mu^L - g_R \vec{\mathrm{A}}_\mu^R)^2 + \tfrac{1}{8} v'^2 \{ g_L^2 [(A_{\mu 1}^L)^2 + (A_{\mu 2}^L)^2] + (g_L A_{\mu 3}^L - g_Y B_\mu)^2 \} . \tag{4.9}$$

The neutral- and charged-boson mass matrices decouple, although there is mixing within each segment.

The neutral-vector-boson eigenfields are the massless photon A^μ and two massive fields $Z_{1,2}^\mu$ with

$$M_{1,2}^2 = \tfrac{1}{8}\{v^2(g_L^2+g_R^2) + v'^2(g_L^2+g_Y^2) \pm ([v^2(g_L^2+g_R^2)+v'^2(g_L^2+g_Y^2)]^2 - 4v^2v'^2[g_Y^2(g_L^2+g_R^2)+g_L^2 g_R^2])^{1/2}\} , \tag{4.10}$$

which are orthogonally related to $A_3^{\mu L,R}$, B^μ by

$$\begin{pmatrix} A^\mu \\ Z_1^\mu \\ Z_2^\mu \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} A_3^{\mu L} \\ A_3^{\mu R} \\ B^\mu \end{pmatrix} , \tag{4.11}$$

$$\begin{aligned} \sum_k a_{ik} a_{jk} &= \sum_k a_{ki} a_{kj} \\ &= \delta_{ij} . \end{aligned} \tag{4.12}$$

The elements of the matrix a_{ij} are rather complicated functions of g_L, g_R, g_Y and the symmetry-breaking parameters v and v' [Eq. (4.7)]

$$\begin{aligned} a_{ij} &= \frac{1}{n_i} b_{ij} , \\ n_i &= (b_{i1}^2 + b_{i2}^2 + b_{i3}^2)^{1/2} , \\ b_{11} &= g_R g' , \quad b_{12} = g_L g' , \quad b_{13} = g_L g_R , \\ b_{21} &= (v^2 g_R^2 - M_1^2)(v'^2 g'^2 - M_1^2) , \\ b_{22} &= (v'^2 g'^2 - M_1^2) v^2 g_L g_R , \\ b_{23} &= (v^2 g_R^2 - M_1^2) v'^2 g_L g_Y , \\ b_{31} &= (v^2 g_R^2 - M_2^2)(v'^2 g'^2 - M_2^2) , \\ b_{32} &= (v'^2 g'^2 - M_2^2) v^2 g_L g_R , \\ b_{33} &= (v^2 g_R^2 - M_2^2) v'^2 g_L g_Y . \end{aligned} \tag{4.13}$$

We can now calculate the second-order vector-meson loop contribution to Δm. The expression for Δm is simplified by anticipating the cancellations coming from isotopic considerations. Charged vector bosons do not contribute and the nonvanishing contribution involves the off-diagonal neutral vector propagators

$$\langle 0|TA_3^{\mu L}(k)B^\nu(-k)|0\rangle = \langle 0|TB^\nu(k)A_3^{\mu L}(-k)|0\rangle$$

and

$$\langle 0|TA_3^{\mu R}(k)B^\nu(-k)|0\rangle = \langle 0|TB^\nu(k)A_3^{\mu R}(-k)|0\rangle ,$$

where, in terms of the eigenfield propagators,

$$\langle 0|TA_3^{\mu L}(k)B^\nu(-k)|0\rangle = -g^{\mu\nu}\left(\frac{a_{11}a_{13}}{k^2+i\epsilon}+\frac{a_{21}a_{23}}{k^2-M_1^2+i\epsilon}+\frac{a_{31}a_{33}}{k^2-M_2^2+i\epsilon}\right), \tag{4.14}$$

$$\langle 0|TA_3^{\mu R}(k)B^\nu(-k)|0\rangle = -g^{\mu\nu}\left(\frac{a_{12}a_{13}}{k^2+i\epsilon}+\frac{a_{22}a_{23}}{k^2-M_1^2+i\epsilon}+\frac{a_{32}a_{33}}{k^2-M_2^2+i\epsilon}\right).$$

Only the $g^{\mu\nu}$ term need be written, as we have argued in Sec. II. Further, a γ_5 term in the mass matrix can be ignored since it affects Δm only in higher order where fermion field redefinition is necessary.

Incorporating these simplifications we may write

$$\Delta m = \frac{i}{4m}\int\frac{d^4k}{(2\pi)^4}\,\frac{\mathrm{Tr}(\not p+m)\gamma^\mu(\not p-\not k+m)\gamma^\mu}{(p-k)^2-m^2}\,\tfrac{1}{2}g_Y\left\{\frac{a_{13}(g_La_{11}+g_Ra_{12})}{k^2+i\epsilon}+\frac{a_{23}(g_La_{21}+g_Ra_{22})}{k^2-M_1^2+i\epsilon}+\frac{a_{33}(g_La_{31}+g_Ra_{32})}{k^2-M_2^2+i\epsilon}\right\}, \tag{4.15}$$

which is a convergent expression because the matrix a_{ij} is orthogonal.

The electric charge may be identified either as the coefficient of the photon term in (4.15) or in the Lagrangian $\mathcal{L}_{\bar\psi\psi V}$ (4.8) expressed in terms of eigenfields. We find

$$e = g_Lg_Rg_Y/(g_L^2g_R^2+g_L^2g_Y^2+g_R^2g_Y^2)^{1/2}. \tag{4.16}$$

If we factor out e^2, Δm may be written in a form directly comparable with the general expression (3.1) and (3.3) with $n=2$ and

$$d_1 = -\frac{a_{23}(g_La_{21}+g_Ra_{22})}{a_{13}(g_La_{11}+g_Ra_{12})}\,,\qquad d_2 = -\frac{a_{33}(g_La_{31}+g_Ra_{32})}{a_{13}(g_La_{11}+g_Ra_{12})}\,, \tag{4.17}$$

obeying (3.2) in view of the orthogonality conditions (4.12).

Since the d_i and M_i are very complicated functions of g_L, g_R, g_Y, v, and v', we used numerical methods to evaluate Δm. We introduce polar coordinates[7]

$$\begin{aligned} g_L &= g\cos\theta ,\\ g_R &= g\sin\theta\cos\phi, \quad 0<\theta<\pi\\ g_Y &= g\sin\theta\sin\phi, \quad 0<\phi<2\pi\\ v^2 &= c\cos\psi,\quad v'^2 = c\sin\psi, \quad 0<\psi<\tfrac{1}{2}\pi . \end{aligned} \tag{4.18}$$

The dimensionless d_i and the mass ratio M_1^2/M_2^2 are independent of g and c. These parameters can be used to fix $e^2/4\pi = 1/137.04$ and the lowest vector mass M_2 although it is unnecessary for us to do this explicitly.

Recall that a necessary but decidedly insufficient condition for positive Δm is $d_1<0$. In our computer runs, we preassign a value of M_2. The program then generates random values for θ, ϕ, ψ and computes d_1. If $d_1>0$, it stops and generates another random set. If $d_1<0$, the program goes on to compute M_1^2/M_2^2 and Δm, prints the results, and then generates another set of random values.

The results are summarized in Table I where the Δm is computed for a nucleon mass m of 1 GeV. The condition $d_1<0$ is satisfied in about 20% of the random trials. It is clear that Δm varies within a small range of negative values for each preassigned value of M_2, and that this range of values becomes more negative with increasing M_2. Most trials were done at $M_2=2$ GeV because we felt that this was the smallest physically reasonable value for M_2. The 900 nonrandom trials were an unsuccessful attempt to look in what we guessed to be "preferred" regions of the parameter space in the hope of increasing the upper limit on Δm. For 20 trials we printed results for Δm irrespective of the sign of d_1, and it became clear that there is no lower bound on Δm. A value $m=1$ GeV was taken for the zero-order nucleon mass in all of our work.

Our conclusion is that for $M_2=2$ GeV, Δm is bounded from above in the parameter space by -3

TABLE I. Numerical results for the n-p mass difference.

M_2 (GeV)	Number of trials	Number of trials giving $d_1 < 0$	Range of values of Δm (MeV)
0.25	300	61	$-0.64 > \Delta m > -0.81$
1	100	24	$-2.13 > \Delta m > -2.55$
2	900	180	$-3.35 > \Delta m > -4.08$
37	200	40	$-12.7 > \Delta m > -13.5$
2	900 (nonrandom)	56	$-3.41 > \Delta m > -4.09$
2	20	d_1 both signs	$-3.77 > \Delta m > -890.0$

MeV. This upper bound moves toward zero as M_2 decreases, and the 300 trials at the very low value of $M_2 = 0.25$ GeV suggest that the maximum value of Δm is never positive. This last statement is less firm than the others because insufficient numerical work has been done for low values of Δm.

In Sec. III we saw that two conditions, (a) $d_1 \ll 0$ and (b) $M_1 \gg M_2$, are necessary to achieve $\Delta m > 0$. Operationally in the $SU(2)_L \times SU(2)_R \times U(1)_Y$ model it seems possible to satisfy (a) and (b) individually but never simultaneously. For example, with $M_2 = 2$ GeV, one trial found $d_1 = -17.3$ but $M_1 = 2.03$ GeV, and another trial found $M_1 = 46.2$ GeV but $d_1 = -0.00093$. All of our results showed this same pattern.

Before discussing the significance of our study of Δm, we must mention a problem in the application of the $SU(2)_L \times SU(2)_R \times U(1)_Y$ model to leptonic and semileptonic weak interactions. Lepton fields[3] are included by assigning all leptons $T_R = 0$ with other quantum numbers as in Weinberg's 1967 model.[1] The new feature of the present model is that the charged vector eigenstates are mixtures of the fields $(1/\sqrt{2})(A_1 + iA_2)^{L,R}$ of the Lagrangian (4.6) so that in general one must expect a right-handed chirality component at the nucleon vertex in neutron β decay. A calculation of β decay and μ decay gives the surprising result that the left- and right-handed chirality components have the same strength for all values of the parameters and that therefore

$$G_V \equiv 2G_\mu , \qquad G_A/G_V \equiv 0 . \qquad (4.19)$$

The present form of the model therefore violates the experimentally observed universality of μ and β decay, and must be modified. One simple way to modify the model[8] is to introduce a second Higgs scalar doublet with $T_L = 0$, $T_R = \frac{1}{2}$, $Y = +1$. One can show that universality is recovered for a sufficiently large value of the symmetry-breaking constant of this field. We have not considered the effect of this modification on the calculation of Δm.

It is clear that the calculations of Δm presented here are not realistic both because the underlying field theories are unrealistic and because strong interactions have been neglected. Our work gives some feeling for the sign and magnitude of Δm in that class of gauge field theories where the entire contribution up to second order in perturbation theory is from vector-meson loop graphs. We find that two relatively simple models give the wrong sign although the general expression allows both signs. A similar situation was found by Hagiwara and Lee[9] in another type of gauge field theory in which there is a zero-order relation between Δm and coupling constants of πNN system. Although experimental information is rough at present, it seems that the wrong sign of Δm appears when strong-interaction corrections beyond the one-loop approximation are ignored.

In the traditional approach to the mass-difference problem[4] in which strong-interaction corrections are treated using the Cottingham formula, special scaling properties of the electroproduction structure function seem to be necessary to produce a finite Δm, and the sign must also depend upon these structure functions. If the idea of combining the weak and electromagnetic interactions in a gauge field theory with spontaneously broken symmetry is correct, then Δm will be finite without special assumptions about the scaling properties. Our work suggests that the strong-interaction corrections must still be important in determining the sign of Δm.

Added note. An analytic proof that $\Delta m < 0$ for all values of the parameters of the $SU(2)_L \times SU(2)_R \times U(1)_Y$ model has recently been obtained by S.-Y. Pi together with one of us (D.Z.F.).

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†Max Kade Foundation Fellow, on leave of absence from Technical University, Vienna, Austria.

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Singular Cores in the Three-Body Problem. I. Theory*

D. D. Brayshaw
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 12 October 1971; revised manuscript received 5 December 1972)

The three-body formalism for singular cores previously introduced by the author is considered in some detail. A new derivation is presented which clearly demonstrates the uniqueness of this formalism and clarifies its relationship to appropriate boundary conditions on the three-body wave function. It is shown that an auxiliary boundary condition must be imposed to uniquely specify a solution; this leads to an integral equation with a square-integrable kernel. A detailed proof of three-particle unitarity is given for the amplitudes defined by this equation, and explicit formulas are presented for a representative model.

I. INTRODUCTION

In a recent letter,[1] the present author introduced a generalization of the Faddeev formalism to include two-body interactions whose extremely short-range behavior is characterized by a hard core, or by a boundary condition on the wave function (BCM). Using the special properties of the BCM t matrix developed earlier,[2] it was shown that the usual Faddeev equations do not yield a unique solution for such interactions, but that a particular solution can be defined which yields the desired physical properties. In particular, the resultant three-body wave function vanishes whenever any pair of particles are within their respective core radius, while its asymptotic behavior corresponds to a unitary three-particle t matrix. In this paper we give detailed proofs of these assertions, present a new derivation of our equation which clearly demonstrates its uniqueness, and consider in some detail the special case of BCM alone (no external potential). This provides the theoretical groundwork for subsequent articles in this series dealing with the actual solution of our equations for specific models.

The principal motivation for this development is the versatility afforded by being able to utilize this additional class of interactions in the three-body problem. For example, calculations to date in the three-nucleon system with realistic interactions have been almost exclusively restricted to soft-core models, the single exception being the long and difficult variational calculation on the Hamada-Johnston hard core by Delves *et al.*[3] The results of these computations have generated some doubt as to the ability of such models to fit the experimental data. For example, it appears that any soft-core model which fits the two-nucleon phase shifts reasonably well will underbind the triton by about 2 MeV. It has also been suggested that a significant discrepancy exists in the case of the triton charge form factor.[4] Of course, it is quite possible that the source of such disagreement does not lie with the nature of the potential model, but with the neglect of corrections due to three-body forces and relativistic effects, which could well be