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Quantum Field Theories in the Infinite-Momentum Frame III. Quantization of Coupled Spin-One Fields*

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Light-front quantization of spin-one fields coupled to a conserved or nonconserved current constructed from a Dirac field is studied. It is shown that an operator phase transformation must be performed on the Dirac field in order to maintain simple canonical commutation relations and a simple Hamiltonian. In this formulation quantum electrodynamics emerges as the zero-mass limit of the massive gluon model. Lorentz invariance of the vector-gluon model is explicitly verified. Vacuum expectation values of operator products and Green's functions are studied and spectral sum rules are derived. The general structure of the current commutators on a light front is *formally* not altered by the interactions. Feynman's parton model for deep-inelastic electron scattering is derived from canonical light-front current commutation relations. The structure function in the Bjorken scaling limit is related to the p^+ distribution of the constituents of the hadron target in any frame of reference.

I. INTRODUCTION

In this third of a series of papers devoted to the study of quantum field theories in an infinite-momentum frame,[1] we consider the quantization of spin-one fields coupled to a Dirac field. Interacting spin-one fields possess several new features not shared by scalar and Dirac fields studied earlier.[1] In particular, the canonical commutation relations are modified by the presence of interactions. The commonly adopted procedure of imposing the commutation relations obtained from free-field theories even in the presence of interactions does not work in this case, although it does in the cases of coupled scalar and Dirac fields.

If light-front quantization is to be claimed as an alternative to the conventional equal-time quantization, it should also be applicable to physical systems involving spin-one fields. Recently, Soper[2] has succeeded in formulating a theory of a vector field coupled to a conserved current (the gluon model) in a special gauge with the introduction of an additional scalar field. In the present paper a general procedure for quantization of interacting spin-one fields in light-front coordinates will be described. Schwinger's action principle[3] will be used to identify the canonical variables and to deduce their commutation relations. The formalism is applied to several models involving spin-one fields. The familiar gluon model of a massive vector field coupled to a conserved current constructed from a Dirac field is discussed without introducing any superfluous scalar field. It will be shown that in this model the Dirac field which satisfies simple canonical commutation relations is not the one which appears in the usual covariant Lagrangian. Rather, it is multiplied by a phase factor involving a line integral of the vector field. Quantum electrodynamics (QED) is obtained as the limit of this model as the mass of the vector field approaches zero. It recovers automatically the theory of Kogut and Soper[4] in the so-called "infinite-momentum gauge." A brief discussion is given to the gluon model of Gross and Treiman,[5] in which a negative-metric scalar field is introduced. An attractive feature of this particular model is that the vector field which appears in the phase transformation of the Dirac field mentioned above is commutative with itself on a light front. We also discuss the quantization of a spin-one field coupled to a nonconserved axial-vector current. This axial-vector model is included to demonstrate the flexibility of our technique of quantization for vector fields coupled to a conserved or nonconserved current. It will be illustrated[6] in low-order examples that familiar results are reproduced in this nonrenormalizable axial-vector model.

Feynman's parton model[7] for deep-inelastic electron scattering, previously derived from a cutoff field theory,[8] is rederived from light-front formulation under the assumption that Bjorken's scaling limit[9] exists. Scalar current is used as an example. The structure function in the Bjorken limit is related to the p^+ distribution of the charged constituents of the hadron target in any reference frame. This parton picture follows if the bilocal operators which appear in the structure function are products of two local fields separated by light-like distances. Other topics discussed in this paper are the Lorentz invariance of the vector-gluon model, Green's functions and spectral sum rules, the existence of Schwinger terms,[10] and current commutators.

II. FREE MASSIVE SPIN-ONE FIELD

The free-field theory of massive spin-one particles will be considered in this section. This theory will set the basis for comparison when interactions are introduced. Besides, many interesting aspects already manifest themselves in this simple situation.

The Lagrange function for a free vector field B_μ of mass m in the action integral[11]

$$W_{12}=\int_{x_2^+}^{x_1^+} d^4x\,\mathcal{L} \tag{2.1}$$

is

$$\mathcal{L}=-\tfrac{1}{2}B^{\mu\nu}(\partial_\mu B_\nu-\partial_\nu B_\mu)+\tfrac{1}{4}B^{\mu\nu}B_{\mu\nu}+\tfrac{1}{2}m^2B^\mu B_\mu\,. \tag{2.2}$$

This is not written in the standard form proposed by Schwinger.[3] To obtain the standard form in which gradients of $B_{\mu\nu}$ also appear symmetrically, the first term in (2.2) is replaced by

$$\begin{aligned}-\tfrac{1}{2}B^{\mu\nu}&(\partial_\mu B_\nu-\partial_\nu B_\mu)\\ &\to-\tfrac{1}{4}[B^{\mu\nu}(\partial_\mu B_\nu-\partial_\nu B_\mu)-2B_\nu\partial_\mu B^{\mu\nu}]\\ &=-\tfrac{1}{2}B^{\mu\nu}(\partial_\mu B_\nu-\partial_\nu B_\mu)+\tfrac{1}{2}\partial_\mu(B_\nu B^{\mu\nu})\,.\end{aligned} \tag{2.3}$$

It differs from (2.2) by a total divergence which can only affect the generator in the action principle. To simplify the notation and writing, we will adopt (2.2) and keep in mind that the generator must be symmetrized with respect to all field operators which appear in it. The field equations are

$$\begin{aligned}&B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu\,,\\ &-\partial_\nu B^{\mu\nu}+m^2B^\mu=0\,.\end{aligned} \tag{2.4}$$

The generator is identified from the surface terms in the variation of the action integral

$$\delta W_{12}=G(x_1^+)-G(x_2^+)\,. \tag{2.5}$$

It is

$$G=-\tfrac{1}{2}\int dx^-d^2x(\tfrac{1}{2}B^{+-}\delta B^+-B^{+i}\delta B^i)\,, \tag{2.6}$$

where

$$\begin{aligned}&x^\pm=x^0\pm x^3,\quad x^i=(x^1,x^2),\\ &B^\pm=B^0\pm B^3,\quad B^i=(B^1,B^2),\\ &\text{etc.}\end{aligned} \tag{2.7}$$

The variations δB^+ and δB^i are postulated to be commuting c numbers in accordance with the Bose statistics of integer-spin particles.[3] The field equations (2.4) can be written in the noncovariant form

$$B^{+-}=\partial^+B^--\partial^-B^+, \tag{2.8a}$$

$$B^{+i}=\partial^+B^i-\partial^iB^+, \tag{2.8b}$$

$$B^{-i}=\partial^-B^i-\partial^iB^-, \tag{2.8c}$$

$$B^{ij}=\partial^iB^j-\partial^jB^i, \tag{2.8d}$$

$$\tfrac{1}{2}\partial^-B^{+-}+\partial^iB^{-i}+m^2B^-=0\,, \tag{2.8e}$$

$$-\tfrac{1}{2}\partial^+ B^{+-} + \partial^i B^{+i} + m^2 B^+ = 0\,, \qquad (2.8f)$$

$$\tfrac{1}{2}\partial^- B^{+i} + \tfrac{1}{2}\partial^+ B^{-i} + \partial^j B^{ij} + m^2 B^i = 0\,, \qquad (2.8g)$$

where we define

$$\partial^\pm = 2\frac{\partial}{\partial x^\mp} \qquad (2.9)$$

and

$$\partial_i = -\partial^i = \frac{\partial}{\partial x^i}\,. \qquad (2.10)$$

It follows from (2.8b) and (2.8f) that[12]

$$B^i(x) = \tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)[B^{+i}(y) + \partial^i B^+(y)]\,, \qquad (2.11)$$

$$B^{+-}(x) = \tfrac{1}{2}\int dy^- \epsilon(x^- - y^-)[\partial^i B^{+i}(y) + m^2 B^+(y)]\,, \qquad (2.12)$$

where y^μ is specified by $y^+ = x^+$, $y^i = x^i$, and y^-. Combining (2.8a), (2.8b), (2.8c), and (2.8g), we get

$$B^{-i}(x) = \tfrac{1}{8}\int dy^- |x^- - y^-| \{[\delta_{ij}(\vec{\nabla}^2 - m^2) - 2\partial_i\partial_j]B^{+j}(y) - 2m^2\partial^i B^+(y)\}\,. \qquad (2.13)$$

Making use of (2.8c), (2.8f), and (2.8g), we have

$$B^-(x) = \tfrac{1}{4}\int dy^- |x^- - y^-| [\partial^i B^{+i}(y) + \tfrac{1}{2}(\vec{\nabla}^2 + m^2)B^+(y)]\,, \qquad (2.14)$$

where $|x^- - z^-|$ is the product of distributions defined in terms of the Fourier transform in momentum space:

$$|x^- - z^-| = \tfrac{1}{2}\int dy^- \epsilon(x^- - y^-)\epsilon(y^- - z^-)\,. \qquad (2.15)$$

Before we proceed further, let us point out that in the present case of a free massive spin-one field, naive canonical quantization fails completely. According to this scheme $(B^+, -\tfrac{1}{4}B^{+-})$ and $(B^i, \tfrac{1}{2}B^{+i})$ form three pairs of canonical conjugate variables. The commutation relations should then be the following.

$$\begin{aligned} x^+ = y^+:\quad & [B^{+-}(x), B^+(y)] = 4i\delta(x^- - y^-)\delta^2(x - y)\,,\\ & [B^{+k}(x), B^l(y)] = -2i\delta^{kl}\delta(x^- - y^-)\delta^2(x - y)\,, \qquad \text{(False)}\\ & [B^{+-}, B^i] = [B^{+-}, B^{+i}] = [B^+, B^i] = [B^+, B^{+i}] = 0\,. \end{aligned} \qquad (2.16)$$

Using (2.11) and (2.16) we should conclude

$$x^+ = y^+:\quad [B^k(x), B^l(y)] = -\tfrac{1}{2}i\delta^{kl}\epsilon(x^- - y^-)\delta^2(x - y)\,. \quad \text{(False)} \qquad (2.17)$$

Not only the numerical coefficient but also the functional form of (2.17) disagrees with the free-field results. The mistake arises from the fact that although each of B^+, B^{+-}, B^i, and B^{+i} seems to satisfy an equation of motion (i.e., their "time derivatives" appear in the field equations), they are not independent of each other, as they are related by the constraint equations (2.11) and (2.12), for example. Full use must be made of the content of all the constraint equations in (2.8) in order to obtain the correct results. In the following we show how this can be accomplished in Schwinger's approach.

It is seen from (2.11)–(2.14) that we have succeeded in expressing all field operators in terms of three independent variables B^+ and B^{+i}. Substitute (2.12) for B^{+-} and

$$\delta B^i(x) = \tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)[\delta B^{+i}(y) + \partial^i \delta B^+(y)]\,; \qquad (2.18)$$

we get finally

$$G = -\tfrac{1}{8}\int dx^- d^2x\, dy^- \epsilon(x^- - y^-)[B^{+i}(y)\delta B^{+i}(x) + m^2 B^+(y)\delta B^+(x)]\,. \qquad (2.19)$$

No further symmetrization with respect to field operators is necessary since (2.19) is already manifestly so. The canonical equal-x^+ commutation relations follow from the interpretation of G as the generator:

$$\begin{aligned} \tfrac{1}{2}i\delta B^{-i}(x) &= [B^{+i}(x), G]\,,\\ \tfrac{1}{2}i\delta B^+(x) &= [B^+(x), G]\,. \end{aligned} \qquad (2.20)$$

They are the following.

$$x^+=y^+:\quad [B^{+k}(x), B^{+l}(y)] = i\delta^{kl}\partial^+\delta(x^- - y^-)\delta^2(x-y),$$
$$[B^+(x), B^+(y)] = i\frac{1}{m^2}\partial^+\delta(x^- - y^-)\delta^2(x-y),\tag{2.21}$$
$$[B^+, B^{+k}] = 0.$$

Equations (2.11) and (2.21) now imply the following.

$$x^+=y^+:\quad [B^k(x), B^l(y)] = -\tfrac{1}{4}i\epsilon(x^- - y^-)\left(\delta_{kl} - \frac{\partial_k\partial_l}{m^2}\right)\delta^2(x-y),\tag{2.22}$$

which now agrees with the result of free-field theory. The results expressed in (2.21) can also be explicitly verified in a free-field theory.

III. INTERACTING VECTOR FIELDS

In this section light-front quantization of the familiar vector-gluon model as well as the particular version of Gross and Treiman[5] will be discussed; Lorentz invariance of the gluon model will be verified. In the limit as the mass of the vector field approaches zero, this theory consists of two dynamically decoupled systems, one of which is shown to be the quantum electrodynamics formulated in "the infinite-momentum gauge."[4] The other corresponds to the longitudinal degree of freedom of the original vector field. Quantization of an axial-vector field coupled to a nonconserved current is deferred to the next section.

A. Quantization of the Vector-Gluon Model

We start with the usual Lagrange function for a vector field B_μ of mass m coupled to a conserved current constructed from a Dirac field ψ' of mass M:

$$\mathcal{L} = -\tfrac{1}{2}B^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) + \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} + \tfrac{1}{2}m^2 B^\mu B_\mu + \bar{\psi}'(\gamma^\mu \tfrac{1}{2} i \overleftrightarrow{\partial}_\mu - M)\psi' - g\bar{\psi}'\gamma_\mu\psi' B^\mu,\tag{3.1}$$

where a prime is attached to the Dirac field since the unprimed ψ will be reserved for later use. We shall first treat (3.1) as a classical Lagrangian. Some manipulations which ignore operator ordering must be made before well-defined expressions suitable for quantization emerge.

The Lagrange function (3.1) gives the field equations

$$[\gamma^\mu(i\partial_\mu - gB_\mu) - M]\psi' = 0,$$
$$\bar{\psi}'[\gamma^\mu(-i\overleftarrow{\partial}_\mu - gB_\mu) - M] = 0,\tag{3.2}$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$
$$-\partial_\nu B^{\mu\nu} + m^2 B^\mu = j^\mu,$$

where

$$j^\mu = g\bar{\psi}'\gamma^\mu\psi'.\tag{3.3}$$

The generator associated with this system is

$$G = \tfrac{1}{2}\int dx^- d^2x[i(\psi'^{(+)\dagger}\delta\psi'^{(+)} - \delta\psi'^{(+)\dagger}\psi'^{(+)}) - \tfrac{1}{2}B^{+-}\delta B^+ + B^{+i}\delta B^i],\tag{3.4}$$

where, as defined in paper I,

$$\psi'^{(\pm)} \equiv \Lambda^{(\pm)}\psi',\quad \Lambda^{(\pm)} = \tfrac{1}{2}(1 \pm \gamma^0\gamma^3).\tag{3.5}$$

As mentioned in the last section, we cannot quantize the system naively. Following a similar procedure used before, we solve the constraint equations to obtain

$$
\begin{aligned}
B^{i}(x) &= \tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)[B^{+i}(y) + \partial^i B^+(y)]\,,\\
B^{+-}(x) &= \tfrac{1}{2}\int dy^- \epsilon(x^- - y^-)[\partial^i B^{+i}(y) + m^2 B^+(y) - j^+(y)]\,,\\
B^{-i}(x) &= \tfrac{1}{8}\int dy^- |x^- - y^-|[(\vec{\nabla}^2 - m^2)B^{+i}(y) - 2\partial^i\partial^j B^{+j}(y) - 2m^2\partial^i B^+(y) + \partial^i j^+(y) + \partial^+ j^i(y)]\,,\\
B^{-}(x) &= \tfrac{1}{8}\int dy^- |x^- - y^-|[2\partial^i B^{+i}(y) + (\vec{\nabla}^2 + m^2)B^+(y) - j^+(y)]\,.
\end{aligned}
\tag{3.6}
$$

The explicit dependence of the vector fields on the current j^μ is the new feature when (3.6) is compared with (2.11)–(2.14) for the free-field case. The independent field variables B^{+i} and B^+ satisfy the equations of motion

$$
\begin{aligned}
&\partial^- B^+ = -B^{+-} + \partial^+ B^-\,,\\
&\tfrac{1}{2}\partial^- B^{+i} + \tfrac{1}{2}\partial^+ B^{-i} + \partial^j B^{ij} + m^2 B^i = j^i\,.
\end{aligned}
\tag{3.7}
$$

The Dirac field equation can be decomposed into the equation of motion

$$(i\partial^- - gB^-)\psi'^{(+)} = \gamma^0[\gamma_k(i\partial_k - gB_k) + M]\psi'^{(-)} \tag{3.8}$$

and the constraint equation

$$(i\partial^+ - gB^+)\psi'^{(-)} = \gamma^0[\gamma_k(i\partial_k - gB_k) + M]\psi'^{(+)}\,. \tag{3.9}$$

The constraint equation (3.9) cannot be solved for $\psi'^{(-)}$ explicitly since B^+ does not commute with itself on a light front (see discussion below). This turns out to be intimately related to the question of identifying the proper canonical variables. The generator, with the aid of (3.6), reduces to

$$
\begin{aligned}
G = {}&\tfrac{1}{2}\int dx^- d^2x\, i[\psi'^{(+)\dagger}\delta\psi'^{(+)} - \delta\psi'^{(+)\dagger}\psi'^{(+)}]\\
&-\tfrac{1}{8}\int dx^- d^2x\, dy^- \epsilon(x^- - y^-)[B^{+i}(y)\delta B^{+i}(x) + m^2 B^+(y)\delta B^+(x) - j^+(y)\delta B^+(x)]\,.
\end{aligned}
\tag{3.10}
$$

The spin-one part of G differs from the corresponding expression (2.19) for a free field by the term involving j^+. The presence of this term complicates the quantization procedure. However, observe that this term can be removed by an appropriate phase transformation on the Dirac field:

$$
\begin{aligned}
&\psi'^{(+)}(x) = \psi^{(+)}(x)e^{-ig\Lambda(x)}\,,\\
&\psi'^{(-)}(x) = \psi^{(-)}(x)e^{-ig\Lambda(x)}\,,
\end{aligned}
\tag{3.11}
$$

with

$$\Lambda(x) = \tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)B^+(y)\,. \tag{3.12}$$

In terms of the unprimed field ψ, we have

$$i[\psi'^{(+)\dagger}\delta\psi'^{(+)} - \delta\psi'^{(+)\dagger}\psi'] = i[\psi^{(+)\dagger}\delta\psi^{(+)} - \delta\psi^{(+)\dagger}\psi^{(+)}] + \tfrac{1}{4}j^+(x)\int dy^- \epsilon(x^- - y^-)\delta B^+(y)\,. \tag{3.13}$$

The term involving j^+ in (3.13) indeed cancels the corresponding term in (3.10) as promised. The generator is now diagonal:

$$G = \tfrac{1}{2}i\int dx^- d^2x(\psi^{(+)\dagger}\delta\psi^{(+)} - \delta\psi^{(+)\dagger}\psi^{(+)}) - \tfrac{1}{8}\int dx^- d^2x\, dy^- \epsilon(x^- - y^-)[B^{+i}(y)\delta B^{+i}(x) + m^2 B^+(y)\delta B^+(x)]\,. \tag{3.14}$$

Thus the vector field operators satisfy the same commutation relations stated in (2.21) and (2.22) for a free field. The new Dirac field $\psi^{(+)}$ also satisfies the following free-field anticommutation relation.

$$
\begin{aligned}
x^+ = y^+:\quad &\{\psi^{(+)}(x), \psi^{(+)\dagger}(y)\} = \Lambda^{(+)}\delta(x^- - y^-)\delta^2(x - y)\,,\\
&\{\psi^{(+)}, \psi^{(+)}\} = \{\psi^{(+)\dagger}, \psi^{(+)\dagger}\} = 0\,.
\end{aligned}
\tag{3.15}
$$

Under the transformation (3.11) the field equations for the vector field in (3.2) or (3.6) and (3.7) remain unmodified, but the Dirac equations (3.8) and (3.9) now become

$$i(\partial^- - g\bar{B}^-)\psi^{(+)} = \gamma^0[\gamma_k(i\partial_k - g\bar{B}_k) + M]\psi^{(-)},$$
$$i\partial^+\psi^{(-)} = \gamma^0[\gamma_k(i\partial_k - g\bar{B}_k) + M]\psi^{(+)}, \quad (3.16)$$

where

$$\bar{B}^\mu(x) = B^\mu - \partial^\mu\Lambda$$
$$= \tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)B^{+\mu}(y)\,. \quad (3.17)$$

In particular

$$\bar{B}^+ \equiv 0\,. \quad (3.18)$$

The constraint equation in (3.16) can now be solved explicitly, giving

$$\psi^{(-)}(x) = -\tfrac{1}{4}i\int dy^- \epsilon(x^- - y^-)\gamma^0\{\gamma_k[i\partial_k - g\bar{B}_k(y)] + M\}\psi^{(+)}(y)\,. \quad (3.19)$$

The current j^μ is unchanged under (3.11):

$$j^\mu = g\bar{\psi}'\gamma^\mu\psi' = g\bar{\psi}\gamma^\mu\psi\,. \quad (3.20)$$

The transformation (3.11) solves two problems simultaneously. It diagonalizes the generator and at the same time makes the constraint equation (3.9) soluble.

Manipulations leading to (3.14), (3.16), and (3.19) ignore the ordering among operators. But we can now take the field equations (3.6), (3.7), and (3.16), and the commutation relations (2.21), (2.22), and (3.15), as well as the energy-momentum tensor to be given later, as the defining equations of the gluon model. We shall show below that they together define a consistent theory, and we shall show in the following paper[6] that they lead to the same S matrix as the ordinary gluon model.

For later applications we state some equal-x^+ commutation relations which can be derived with the aid of (3.6), (3.17), and (3.19).

$$x^+ = y^+: \quad [B^+(x), B^{+-}(y)] = -2i\delta(x^- - y^-)\delta^2(x-y)\,,$$
$$[B^{+k}(x), B^l(y)] = -i\delta^{kl}\delta(x^- - y^-)\delta^2(x-y)\,,$$
$$[B^+(x), B^k(y)] = \frac{i}{m^2}\delta(x^- - y^-)\partial^k\delta^2(x-y)\,,$$
$$[B^{+k}(x), B^{+-}(y)] = 2i\delta(x^- - y^-)\partial^k\delta^2(x-y)\,, \quad (3.21)$$
$$[B^{+k}(x), \psi^{(-)}(y)] = -\tfrac{1}{4}g\epsilon(x^- - y^-)\delta^2(x-y)\gamma^0\gamma^k\psi^{(+)}(x)\,,$$
$$[\psi^{(+)}(x), B^{+-}(y)] = g\epsilon(x^- - y^-)\delta^2(x-y)\psi^{(+)}(y)\,,$$
$$\{\psi^{(-)}(y), \psi^{(+)}(x)^\dagger\} = \tfrac{1}{4}i\epsilon(x^- - y^-)\gamma^0\{\gamma_k[-i\partial_k - g\bar{B}_k(x)] + M\}\Lambda^{(+)}\delta^2(x-y)\,.$$

From the definition (3.18) one also derives the following.

$$x^+ = y^+: \quad [\bar{B}^k(x), \bar{B}^l(y)] = -\tfrac{1}{4}i\delta_{kl}\epsilon(x^- - y^-)\delta^2(x-y)\,. \quad (3.22)$$

B. Lorentz Invariance

Lorentz invariance of a quantum field theory demands that the ten generators of the Lorentz group constructed from the field operators must satisfy the appropriate commutation relations of the group. The generators are constructed from the symmetric energy-momentum tensor $T_{\mu\nu}$. From (3.1) standard procedure gives

$$T_{\mu\nu} = T_{\nu\mu} = -g_{\mu\nu}\mathcal{L} + B^\lambda_\mu B_{\lambda\nu} + m^2 B_\mu B_\nu + \tfrac{1}{2}\bar{\psi}'[\gamma_\mu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\nu - gB_\nu) + \gamma_\nu(\tfrac{1}{2}i\overleftrightarrow{\partial}_\mu - gB_\mu)]\psi'\,. \quad (3.23)$$

We must express ψ' and $\bar{\psi}'$ in terms of the canonical variables ψ and $\bar{\psi}$ through (3.11). The result is

$$T^{++} = B^{+i}B^{+i} + m^2B^+B^+ + \psi^{(+)\dagger}i\overleftrightarrow{\partial}^+\psi^{(+)}, \quad (3.24a)$$

$$T^{+k} = \tfrac{1}{2}B^{+-}B^{+k} + m^2B^+B^k + m^2B^{+l}B^k_l + \tfrac{1}{2}\bar{\psi}[\gamma^+(\tfrac{1}{2}i\overleftrightarrow{\partial}^k - g\bar{B}^k) + \gamma^k\tfrac{1}{2}i\overleftrightarrow{\partial}^+]\psi\,, \quad (3.24b)$$

$$T^{+-} = \tfrac{1}{4}(B^{+-})^2 + \tfrac{1}{2}B^{kl}B^{kl} + m^2B^kB^k + \bar{\psi}[\gamma_k(\tfrac{1}{2}i\overleftrightarrow{\partial}_k - g\bar{B}_k) + M]\psi\,. \quad (3.24c)$$

The translation operators P_μ and the angular momentum operators $J_{\mu\nu}$ are constructed from $T^{+\mu}$ according to

$$P^\mu = \tfrac{1}{2}\int dx^- d^2x T^{+\mu},$$
$$J^{\mu\nu} = \tfrac{1}{2}\int dx^- d^2x(x^\mu T^{+\nu} - x^\nu T^{+\mu}). \tag{3.25}$$

Since $T^{\mu\nu}$ are conserved, P_μ and $J_{\mu\nu}$ given by (3.25) are the same as those integrated over an equal-time surface:

$$P^\mu = \int d^3x T^{0\mu},$$
$$J^{\mu\nu} = \int d^3x(x^\mu T^{0\nu} - x^\nu T^{0\mu}). \tag{3.26}$$

For practical computations we find it useful to rewrite T^{+k} and T^{+-} as

$$T^{+k} = 2\psi^{(+)\dagger}(\tfrac{1}{2}i\overleftrightarrow{\partial}^k - g\overline{B}^k)\psi^{(+)} - \tfrac{1}{2}\partial^l[\psi^{(+)\dagger}\sigma^{kl}\psi^{(+)}] - \tfrac{1}{4}i\partial^+(\psi^{(+)\dagger}\gamma^0\gamma^k\psi^{(-)} - \psi^{(-)\dagger}\gamma^0\gamma^k\psi^{(+)})$$
$$- \tfrac{1}{2}B^{+-}\partial^k B^+ + B^{+l}\partial^k B^l - \partial^l(B^{+l}B^k) + j^+B^k + \tfrac{1}{2}\partial^+(B^{+-}B^k), \tag{3.27}$$

$$T^{+-} = \tfrac{1}{4}(B^{+-})^2 + \tfrac{1}{2}B^{kl}B^{kl} + m^2B^kB^k + 2\psi^{(+)\dagger}\gamma^0[\gamma^l(\tfrac{1}{2}i\overleftrightarrow{\partial}^l - g\overline{B}^l) + M]\psi^{(-)}$$
$$-\tfrac{1}{2}i\partial^l[\psi^{(+)\dagger}\gamma^0\gamma^l\psi^{(-)} + \psi^{(-)\dagger}\gamma^0\gamma^l\psi^{(+)}] + i\partial^+(\psi^{(-)\dagger}\psi^{(-)}). \tag{3.28}$$

With the help of the commutation relations (2.21), (2.22), (3.14), (3.21), and (3.22) a straightforward but very lengthy calculation gives

$$[B^\mu, P^\nu] = i\partial^\nu B^\mu,$$
$$[B^{\mu\nu}, P^\lambda] = i\partial^\lambda B^{\mu\nu},$$
$$[B^\lambda, J^{\mu\nu}] = i(x^\mu\partial^\nu - x^\nu\partial^\mu)B^\lambda + i(g^{\mu\lambda}B^\nu - g^{\nu\lambda}B^\mu),$$
$$[B^{\mu\nu}, J^{\lambda\kappa}] = i(x^\lambda\partial^\kappa - x^\kappa\partial^\lambda)B^{\mu\nu} + i(g^{\mu\lambda}B^{\kappa\nu} + g^{\nu\lambda}B^{\mu\kappa} - g^{\mu\kappa}B^{\lambda\nu} - g^{\nu\kappa}B^{\mu\lambda}), \tag{3.29}$$
$$[\overline{B}^\lambda, J^{\mu\nu}] = i(x^\mu\partial^\nu - x^\nu\partial^\mu)\overline{B}^\lambda + i(g^{\mu\lambda}\overline{B}^\nu - g^{\nu\lambda}\overline{B}^\mu) - i(g^{+\mu}\partial^\lambda\Phi^\nu - g^{+\nu}\partial^\lambda\Phi^\mu),$$
$$[\psi, P^\mu] = i\partial^\mu\psi,$$
$$[\psi, J^{\mu\nu}] = [i(x^\mu\partial^\nu - x^\nu\partial^\mu) + \tfrac{1}{2}\sigma^{\mu\nu}]\psi - g(g^{+\mu}\Phi^\nu - g^{+\nu}\Phi^\mu)\psi,$$

where

$$\Phi^\mu(x) = \tfrac{1}{4}\int dy^-\epsilon(x^- - y^-)\overline{B}^\mu(y). \tag{3.30}$$

Thus B_μ and $B^{\mu\nu}$ behave as a vector and a second-rank tensor, respectively, under a Lorentz transformation. But $\overline{B}^\mu$ and ψ do not transform simply as a four-vector and spinor, respectively. An additional gauge transformation is accompanied under the operation of J^{-k}, with the gauge function given by (3.30). This gauge transformation is necessary to preserve the condition

$$\overline{B}^+ \equiv 0 \tag{3.31}$$

in all Lorentz frames. The situation here is quite similar to that in quantum electrodynamics quantized in the radiation gauge.

It is not difficult to verify, using (3.29), that $T_{\mu\nu}$, expressed in terms of the canonical fields ψ and $\overline{\psi}$, transforms as a second-rank tensor since $\overline{B}^\mu$ and ψ always appear in gauge-invariant combinations. That is,

$$[T^{\mu\nu}, P^\lambda] = i\partial^\lambda T^{\mu\nu},$$
$$[T^{\lambda\kappa}, J^{\mu\nu}] = i(x^\mu\partial^\nu - x^\nu\partial^\mu)T^{\lambda\kappa} + i(g^{\mu\kappa}T^{\lambda\mu} - g^{\nu\kappa}T^{\lambda\mu} + g^{\lambda\mu}T^{\nu\kappa} - g^{\lambda\nu}T^{\mu\kappa}). \tag{3.32}$$

It follows from the discussion in Appendix A of paper I that (3.32) implies the Lorentz-group commutation relations for P_μ and $J_{\lambda\kappa}$

$$[P^\mu, P^\nu] = 0,$$
$$[P^\mu, J^{\lambda\kappa}] = i(g^{\mu\lambda}P^\kappa - g^{\mu\kappa}P^\lambda), \tag{3.33}$$
$$[J^{\mu\nu}, J^{\lambda\kappa}] = i(g^{\lambda\nu}J^{\mu\kappa} - g^{\lambda\mu}J^{\nu\kappa} - g^{\mu\kappa}J^{\lambda\nu} + g^{\nu\kappa}J^{\lambda\mu}).$$

The same conclusion can also be inferred from the Jacobi identities

$$[\chi, [G_1, G_2]] = [[\chi, G_1], G_2] - [[\chi, G_2], G_1], \tag{3.34}$$

χ being the independent dynamical variables $\psi^{(+)}$, B^+, and B^{+i}, which are assumed to form a complete set of basis operators, and G_1 and G_2 being any two of the ten generators of the group.

C. QED as the Zero-Mass Limit of the Gluon Model

The vector-gluon model is completely specified by the canonical commutations (2.21), (2.22), and (3.15), and the energy and momentum densities (3.24), together with the constraint equations (3.6) and (3.19). Equations of motion, for instance, follow from the Heisenberg relation

$$[\chi, P^\mu] = i\partial^\mu \chi \,, \tag{3.35}$$

χ being any field operator.

We now show that quantum electrodynamics can be obtained as the zero-mass limit of the massive-gluon model. Observe that in the zero-mass limit $m^2 \to 0$, commutators involving B^+ are singular. It is therefore incorrect to set $m^2=0$ naively in the energy and momentum densities (3.24). The field operator which gives finite commutation relations in the limit $m^2 \to 0$ is ξ as defined by

$$B^+ = \frac{1}{m}\,\xi \,. \tag{3.36}$$

It follows from (2.21) that

$$\begin{aligned} x^+ = y^+: \quad & [\xi(x), \xi(y)] = i\partial^+\delta(x^- - y^-)\delta^2(x-y)\,, \\ & [\xi, B^{+-}] = 0\,. \end{aligned} \tag{3.37}$$

In terms of ξ, the energy and momentum densities become in the limit $m^2 \to 0$

$$\begin{aligned} T^{++} &= T^{++}_{\text{em}} + T^{++}_{\xi}\,, \\ T^{+k} &= T^{+k}_{\text{em}} + T^{+k}_{\xi}\,, \\ T^{+-} &= T^{+-}_{\text{em}} + T^{+-}_{\xi}\,, \end{aligned} \tag{3.38}$$

where

$$\begin{aligned} T^{++}_{\text{em}} &= F^{+i}F^{+i} + \psi^{(+)\dagger} i\overleftrightarrow{\partial}^+ \psi^{(+)}\,, \\ T^{+k}_{\text{em}} &= \tfrac{1}{2}F^{+-}F^{+k} + F^{+l}F^{k}_{\ l}\,, \\ &\quad + \tfrac{1}{2}\bar\psi[\gamma^+(\tfrac{1}{2}i\overleftrightarrow{\partial}^k - eA^k) + \gamma^k \tfrac{1}{2} i\overleftrightarrow{\partial}^+]\psi\,, \\ T^{+-}_{\text{em}} &= \tfrac{1}{4}(F^{+-})^2 + \tfrac{1}{2}F^{kl}F_{kl} \\ &\quad + \bar\psi[\gamma_k(\tfrac{1}{2}i\overleftrightarrow{\partial}_k - eA_k) + M]\psi\,, \end{aligned} \tag{3.39}$$

where we have reinstated the notations $F^{\mu\nu}$ and A^μ for the electromagnetic-field tensor and vector potential, respectively, and e is the electric charge of the matter field ψ. Also,

$$\begin{aligned} T^{++}_{\xi} &= \xi^2\,, \\ T^{+k}_{\xi} &= \tfrac{1}{4}\xi \int dy^- \epsilon(x^- - y^-)\partial^k \xi(y)\,, \\ T^{+-}_{\xi} &= \left[\tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)\partial^k\xi(y)\right]^2. \end{aligned} \tag{3.40}$$

The constraint equations are

$$\begin{aligned} A^k(x) &= \tfrac{1}{4}\int dy^- \epsilon(x^- - y^-)F^{+k}(y)\,, \\ F^{+-}(x) &= \tfrac{1}{2}\int dy^- \epsilon(x^- - y^-)[\partial^k F^{+k}(y) - j^+(y)]\,, \\ F^{-k}(x) &= \tfrac{1}{8}\int dy^- |x^- - y^-| [\vec\nabla^2 F^{+k}(y) - 2\partial^k\partial^l F^{+l}(y) \\ &\qquad + \partial^k j^+(y) + \partial^+ j^k(y)]\,, \\ F^{kl} &= \partial^k A^l - \partial^l A^k\,, \\ \psi^{(-)}(x) &= -\tfrac{1}{4} i \int dy^- \epsilon(x^- - y^-) \\ &\qquad \times \gamma^0\{\gamma_k[i\partial_k - eA_k(y)] + M\}\psi^{(+)}(y)\,, \end{aligned} \tag{3.41}$$

and

$$j^\mu = e\bar\psi\gamma^\mu\psi\,. \tag{3.42}$$

The canonical commutation relations are the following.

$$\begin{aligned} & x^+ = y^+: \\ & [F^{+k}(x), F^{+l}(y)] = i\delta^{kl}\partial^+\delta(x^- - y^-)\delta^2(x-y)\,, \\ & \{\psi^{(+)}(x), \psi^{(+)\dagger}(y)\} = \Lambda^{(+)}\delta(x^- - y^-)\delta^2(x-y)\,, \\ & [A^k(x), A^l(y)] = -\tfrac{1}{4}i\delta^{kl}\epsilon(x^- - y^-)\delta^2(x-y)\,, \\ & [\xi(x), \xi(y)] = i\partial^+\delta(x^- - y^-)\delta^2(x-y)\,, \\ & [\xi, A^i] = [\xi, F^{+k}] = [\xi, \psi] = 0\,. \end{aligned} \tag{3.43}$$

The field ξ is seen to be totally decoupled from the electrodynamic system. Its contributions to $T^{+\mu}$ can therefore be subtracted. Since the Lorentz invariance for the combined system has been verified for all values of m^2, it must remain so separately for the electrodynamic system as well as the ξ system in the limit $m^2 \to 0$. Furthermore, quantum electrodynamics so obtained is recognized to coincide with the theory of Kogut and Soper[4] in the so-called "infinite-momentum gauge"

$$A^+ = 0\,, \tag{3.44}$$

which has never appeared in our defining equations for quantum electrodynamics.

D. The Gross-Treiman Model

Since the vector-gluon field is coupled to a conserved current, it is possible to formulate the theory in different fashions yet with the same physical consequences. Recently, Gross and Treiman[5] proposed a particular version of the vector-gluon model which they claim to have a smoother behavior near the light cone.[13] This model contains a scalar field ϕ with negative metric. In this subsection we discuss briefly some consequences of this model as well as illustrate the utility of our quantization technique.

To distinguish the vector field in the Gross-Treiman model from the one in the previous model, it will be denoted by b^μ. The Lagrange function for this model is

$$\mathcal{L}=\bar{\psi}'[\gamma^\mu(i\partial_\mu-gb_\mu)-M]\psi'-\tfrac{1}{2}b^{\mu\nu}(\partial_\mu b_\nu-\partial_\nu b_\mu)+\tfrac{1}{4}b^{\mu\nu}b_{\mu\nu}+\tfrac{1}{2}m^2b_\mu b^\mu-\tfrac{1}{2}m(b_\mu\partial^\mu\phi+\partial^\mu\phi b_\mu)-\tfrac{1}{2}m^2\phi^2. \tag{3.45}$$

The field equations are, in noncovariant notation,

$$\begin{aligned}&(i\partial^- -gb^-)\psi'^{(+)}=\gamma^0[\gamma_k(i\partial_k-gb_k)+M]\psi'^{(-)},\\&(i\partial^+ -gb^+)\psi'^{(-)}=\gamma^0[\gamma_k(i\partial_k-gb_k)+M]\psi'^{(+)},\end{aligned} \tag{3.46}$$

$$\begin{aligned}&b^{+-}=\partial^+b^- -\partial^-b^+,\\&b^{+-}=\partial^+b^i-\partial^ib^+,\\&b^{ij}=\partial^ib^j-\partial^jb^i,\\&\tfrac{1}{2}\partial^-b^{+-}+\partial^ib^{-i}+m^2b^- -m\partial^-\phi=j^-,\\&-\tfrac{1}{2}\partial^+b^{+-}+\partial^ib^{+i}+m^2b^+-m\partial^+\phi=j^+,\\&\tfrac{1}{2}\partial^-b^{+i}+\tfrac{1}{2}\partial^+b^{-i}+\partial^jb^{ij}+m^2b^i-m\partial^i\phi=j^i,\\&\tfrac{1}{2}\partial^+b^-+\tfrac{1}{2}\partial^-b^+-\partial^ib^i=m\phi.\end{aligned} \tag{3.47}$$

Constraint equations can be solved to give, for example,

$$b^i(x)=\tfrac{1}{4}\int dy^-\epsilon(x^- -y^-)[b^{+i}(y)+\partial^ib^+(y)], \tag{3.48}$$

$$b^{+-}(x)=-2m\phi(x)+\tfrac{1}{2}\int dy^-\epsilon(x^- -y^-)[m^2b^+(y)+\partial^ib^{+i}(y)]-j^+(y). \tag{3.49}$$

The expressions for b^- and b^{-i} in terms of b^+ and b^{+i} and $j^\mu=g\bar{\psi}'\gamma^\mu\psi'$ can also be obtained similarly.

The generator associated with (3.45) is

$$G=\int dx^-d^2x[\tfrac{1}{2}i(\psi'^{(+)\dagger}\delta\psi'^{(+)}-\delta\psi'^{(+)\dagger}\psi'^{(+)})-\tfrac{1}{2}(\tfrac{1}{2}b^{+-}\delta b^+-b^{+k}\delta b^k)-\tfrac{1}{2}mb^+\delta\phi]. \tag{3.50}$$

Substituting (3.48) and (3.49) we obtain

$$\begin{aligned}G=&\int dx^-d^2x[\tfrac{1}{2}i(\psi'^{(+)\dagger}\delta\psi'^{(+)}-\delta\psi'^{(+)\dagger}\psi^{(+)\prime})]\\&+\tfrac{1}{8}\int dx^-d^2x\,dy^-\epsilon(x^- -y^-)[m\phi^+(y)\delta b^+(x)-mb^+(x)\delta\phi^+(y)-m^2b^+(y)\delta b^+(x)+b^{+k}(x)\delta b^{+k}(y)+j^+(y)\delta b^+(x)],\end{aligned} \tag{3.51}$$

where

$$\phi^+\equiv\partial^+\phi. \tag{3.52}$$

As before, the term involving j^+ can be removed by a phase transformation on the Dirac field

$$\psi'(x)=\psi(x)\exp\left(-ig\tfrac{1}{4}\int dy^-\epsilon(x^- -y^-)b^+(y)\right). \tag{3.53}$$

In terms of the new field ψ and its variation, G becomes

$$\begin{aligned}G=&\tfrac{1}{2}i\int dx^-d^2x(\psi^{(+)\dagger}\delta\psi^{(+)}-\delta\psi^{(+)\dagger}\delta\psi^{(+)})\\&-\tfrac{1}{8}\int dx^-d^2x\,dy^-\epsilon(x^- -y^-)[m^2\bar{b}^+(y)\delta\bar{b}^+(x)+b^{+k}(y)\delta b^{+k}(x)-\phi^+(y)\delta\phi^+(x)],\end{aligned} \tag{3.54}$$

which is diagonal in canonical variables. The field $\bar{b}^+$ is defined by

$$\bar{b}^+ = b^+ - \frac{1}{m}\phi^+ . \tag{3.55}$$

The canonical commutation relations can now be stated.

$$\begin{aligned}
&x^+ = y^+: \\
&\{\psi^{(+)}(x), \psi^{(+)\dagger}(y)\} = \Lambda^{(+)}\delta(x^- - y^-)\delta^2(x-y), \\
&[b^{+k}(x), b^{+l}(y)] = i\delta^{kl}\partial^+\delta(x^- - y^-)\delta^2(x-y), \\
&[\bar{b}^+(x), \bar{b}^+(y)] = i\frac{1}{m^2}\partial^+\delta(x^- - y^-)\delta^2(x-y), \\
&[\phi^+(x), \phi^+(y)] = -i\partial^+\delta(x^- - y^-)\delta^2(x-y), \\
&[\bar{b}^+, \phi^+] = [\bar{b}^+, b^{+k}] = [b^{+k}, \phi] = 0 .
\end{aligned} \tag{3.56}$$

The negative sign on the right-hand side of the commutator $[\phi^+(x), \phi^+(y)]$ is a consequence of the negative metric associcated with this field. As a result we observe that at $x^+ = y^+$,

$$\begin{aligned}
[b^+(x), b^+(y)] &= [\bar{b}^+(x), \bar{b}^+(y)] + \frac{1}{m^2}[\phi^+(x), \phi^+(y)] \\
&= 0 ,
\end{aligned} \tag{3.57}$$

i.e., b^+ commutes with itself on a light front. This is to be contrasted with the B^+ in the previous model, which does not commute with itself on a light front.[14] This result confirms the assertion made by Gross and Treiman[5] that the vector field can be treated as a c number on the light cone. It is interesting to note that this property follows simply from the action principle.

In terms of the new field ψ the Dirac equation (3.46) becomes

$$\begin{aligned}
&(i\partial^- - g\bar{b}^-)\psi^{(+)} = \gamma^0[\gamma_k(i\partial_k - g\bar{b}_k) + M]\psi^{(-)}, \\
&i\partial^+\psi^{(-)} = \gamma^0[\gamma_k(i\partial_k - g\bar{b}_k) + M]\psi^{(+)},
\end{aligned} \tag{3.58}$$

where

$$\begin{aligned}
&\bar{b}^k(x) = \tfrac{1}{4}\int dy^-\epsilon(x^- - y^-)b^{+k}(y), \\
&\bar{b}^-(x) = \tfrac{1}{4}\int dy^-\epsilon(x^- - y^-)b^{+-}(y).
\end{aligned} \tag{3.59}$$

The second equation in (3.58) can be solved to give

$$\begin{aligned}
\psi^{(-)}(x) = -\tfrac{1}{4}i\int dy^-\epsilon(x^- - y^-) \\
\times\gamma^0\{\gamma_k[i\partial_k - g\bar{b}_k(y)] + M\}\psi^{(+)}(y).
\end{aligned} \tag{3.60}$$

IV. INTERACTING AXIAL-VECTOR FIELD

As another example of our quantization procedure, we consider a spin-one field coupled to an axial-vector current constructed from a Dirac field. Since the axial-vector current, unlike the vector current considered in the last section, is not conserved in general, the quantization scheme of Soper[2] is not directly applicable here. In this section we will show that our scheme is flexible enough to handle this case as well. Needless to say, much of our consideration is formal, as the theory is unrenormalizable. However, we shall illustrate in the following paper[6] that in low-order expansions our formalism reproduces the expected results.

The Lagrange function for an axial-vector field a_μ coupled to an axial-vector current of a Dirac field ψ' is

$$\begin{aligned}
\mathcal{L} = -\tfrac{1}{2}a^{\mu\nu}(\partial_\mu a_\nu - \partial_\nu a_\mu) + \tfrac{1}{4}a^{\mu\nu}a_{\mu\nu} + \tfrac{1}{2}m^2a^\mu a_\mu \\
+ \bar{\psi}'(\gamma^\mu i\partial_\mu - M)\psi' - j_5^\mu a_\mu ,
\end{aligned} \tag{4.1}$$

where

$$j_5^\mu = g\bar{\psi}'\gamma^\mu\gamma_5\psi' . \tag{4.2}$$

The field equations are

$$[\gamma^\mu(i\partial_\mu - g\gamma_5 a_\mu) + M]\psi = 0 \tag{4.3}$$

and its Hermitian conjugate, and

$$\begin{aligned}
&a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu , \\
&-\partial_\nu a^{\mu\nu} + m^2 a^\mu = j_5^\mu .
\end{aligned} \tag{4.4}$$

The generator is

$$\begin{aligned}
G = \tfrac{1}{2}\int dx^- d^2x[i(\psi'^{(+)\dagger}\delta\psi'^{(+)} - \delta\psi'^{(+)\dagger}\psi'^{(+)}) \\
- \tfrac{1}{2}a^{+-}\delta a^+ + a^{+k}\delta a^k] .
\end{aligned} \tag{4.5}$$

All field operators of the axial-vector field can be expressed in terms of three independent variables a^{+k}, a^+ and the axial-vector current j_5^μ. Explicitly we have

$$\begin{aligned}
&a^k(x) = \tfrac{1}{4}\int dy^-\epsilon(x^- - y^-)[a^{+k}(y) + \partial^k a^+(y)], \\
&a^{+-}(x) = \tfrac{1}{2}\int dy^-\epsilon(x^- - y^-)[\partial^k a^{+k}(y) + m^2a^+(y) - j_5^+(y)], \\
&a^-(x) = \tfrac{1}{8}\int dy^-|x^- - y^-|[2\partial^k a^{+k}(y) + (\vec{\nabla}^2 + m^2)a^+(y) - j_5^+(y)] + \frac{1}{4m^2}\int dy^-\epsilon(x^- - y^-)[2gM\bar{\psi}'(y)i\gamma_5\psi'(y)], \\
&a^{-k}(x) = \tfrac{1}{8}\int dy^-|x^- - y^-|[(\vec{\nabla}^2 - m^2)a^{+k}(y) - 2\partial^k\partial^l a^{+l}(y) - 2m^2\partial^k a^+(y) + \partial^k j_5^+(y) + \partial^+ j_5^k(y)] .
\end{aligned} \tag{4.6}$$

These equations differ from (3.6) only in the appearance of the nonvanishing divergence of the axial-vector current

$$\partial_\mu j_5^\mu = 2gM\bar{\psi}'\gamma_5\psi' . \tag{4.7}$$

Upon making the transformation

$$\begin{aligned} \psi'(x) &= \psi(x)\exp[-i\gamma_5 g\lambda(x)] , \\ \lambda(x) &= \tfrac{1}{4}\int dy^-\epsilon(x^- - y^-)a^+(y) \end{aligned} \tag{4.8}$$

and substituting a^{+-} and a^k from (4.6) the generator (4.4) simplifies

$$G = \tfrac{1}{2}i\int dx^- d^2x(\psi^{(+)\dagger}\delta\psi^{(+)} - \delta\psi^{(+)\dagger}\psi^{(+)}) - \tfrac{1}{8}\int dx^- d^2x\, dy^-\epsilon(x^- - y^-)[a^{+k}(y)\delta a^{+k}(x) + m^2 a^+(y)\delta a^+(x)] , \tag{4.9}$$

which is identical in form to (3.14). Thus the canonical commutation relations are the following.

$$\begin{aligned} x^+ = y^+ : \quad & \{\psi^{(+)}(x), \psi^{(+)\dagger}(y)\} = \Lambda^{(+)}\delta(x^- - y^-)\delta^2(x - y) , \\ & [a^{+k}(x), a^{+l}(y)] = i\delta^{kl}\partial^+\delta(x^- - y^-)\delta^2(x - y) , \\ & [a^+(x), a^+(y)] = \frac{i}{m^2}\partial^+\delta(x^- - y^-)\delta^2(x - y) , \\ & [a^+, a^{+i}] = 0 . \end{aligned} \tag{4.10}$$

One then derives

$$x^+ = y^+ : \quad [a^k(x), a^l(y)] = -\tfrac{1}{4}i\epsilon(x^- - y^-)\left(\delta^{kl} - \frac{\partial^k\partial^l}{m^2}\right)\delta^2(x - y) . \tag{4.11}$$

In terms of the new field ψ the Dirac equation becomes

$$\begin{aligned} (i\partial^- - g\gamma_5\bar{a}^-)\psi^{(+)} &= \gamma^0[\gamma_k(i\partial_k - g\gamma_5\bar{a}_k) + MI]\psi^{(-)} , \\ i\partial^+\psi^{(-)} &= \gamma^0[\gamma_k(i\partial_k - g\gamma_5\bar{a}_k) + MI]\psi^{(+)} , \end{aligned} \tag{4.12}$$

where

$$\bar{a}^\mu(x) = \tfrac{1}{4}\int dy^-\epsilon(x^- - y^-)a^{+\mu}(y) \qquad (\bar{a}^+ \equiv 0) \tag{4.13}$$

and

$$\begin{aligned} I(x) &= \exp\left[-\tfrac{1}{2}ig\gamma_5\int dy^-\epsilon(x^- - y^-)a^+(y)\right] \\ &= \exp[-2ig\gamma_5\lambda(x)] . \end{aligned} \tag{4.14}$$

The dependent part $\psi^{(-)}$ is explicitly given by

$$\psi^{(-)}(y) = -\tfrac{1}{4}i\int dy^-\epsilon(x^- - y^-)\gamma^0\{\gamma_k[i\partial_k - g\gamma_5\bar{a}_k(y)] + MI(y)\}\psi^{(+)}(y) . \tag{4.15}$$

The phase factor $I(x)$ appears because γ_5 anticommutes with γ_μ. In terms of ψ we have

$$j_5^\mu = g\bar{\psi}\gamma^\mu\gamma_5\psi \tag{4.16}$$

and

$$\partial_\mu j_5^\mu = 2gM\bar{\psi}i\gamma_5 I\psi . \tag{4.17}$$

The only difference between the quantization of this system and that of the vector-gluon model is that in the axial-vector case the phase transformation (4.8) involves γ_5.

The energy-momentum tensor $T_{\mu\nu}$ of the system can be derived as usual, from which the Lorentz generators P_μ and $J_{\mu\nu}$ can be constructed. For instance we have

$$P^- = \tfrac{1}{2}\int dx^- d^2x T^{+-}$$
$$= \int dx^- d^2x \{\tfrac{1}{8}(a^{+-})^2 + \tfrac{1}{4}(a^{kl})^2 + \tfrac{1}{2}m^2(a^k)^2 + \psi^{(+)\dagger}\gamma^0[(\gamma_k i\partial_k - g\gamma_k\gamma_5\bar{a}_k) + MI]\psi^{(-)}\}. \tag{4.18}$$

With the help of the canonical commutation relations (4.10) and (4.11) as well as the derived equal-x^+ commutators

$$[a^+(x), \lambda(y)] = -\frac{i}{m^2}\delta(x^- - y^-)\delta^2(x-y),$$
$$[a^+(x), \psi^{(-)}(y)] = \frac{M}{2m^2} g\gamma^0\gamma_5 I(x)\psi^{(+)}(x)\epsilon(x^- - y^-)\delta^2(x-y) \tag{4.19}$$

it is straightforward but lengthy and tedious to verify that the Heisenberg equations

$$[\psi^{(+)}, P^-] = i\partial^-\psi^{(+)},$$
$$[a^+, P^-] = i\partial^- a^+, \tag{4.20}$$
$$[a^{+k}, P^-] = i\partial^- a^{+k}$$

reproduce correctly the equations of motion (4.12) and (4.4).

In the following paper[6] we shall show in simple examples that the Hamiltonian (4.18) gives the same S matrix as in the ordinary formulation. In the case of the vector-gluon model where the vector current is conserved, the gradient terms in the propagator for a spin-one particle are ineffective to the S matrix. But in the axial-vector case the axial-vector current is not conserved and consequently the gradient terms in the spin-one propagator are physically significant. The transformation (4.8) changes the structure of these gradient terms. It is therefore instructive to see explicitly how the phase factor $I(x)$, which is absent in the vector-gluon model, supplies the necessary contributions to restore the proper gradient terms in the propagator of spin-one particles.

V. VACUUM EXPECTATION VALUES, GREEN'S FUNCTIONS, AND SPECTRAL SUM RULES

In paper I general properties of vacuum expectation values and Green's functions of scalar and Dirac fields have been studied. In this section we consider some general properties of vacuum expectation values and Green's functions of a vector field. In particular, spectral sum rules will be derived, and the necessary existence of Schwinger terms[10] will be demonstrated.

A standard derivation gives, for a conserved vector field B_μ,

$$\langle 0|B_\mu(x)B_\nu(0)|0\rangle = \int dM^2 \int \frac{d^4p}{(2\pi)^3} e^{-ipx}\delta(p^2 - M^2)\theta(p^+)\rho_{\mu\nu}(M^2), \tag{5.1}$$

with

$$\rho_{\mu\nu}(M^2) = (2\pi)^3\sum_n \delta^4(p-p_n)\langle 0|B_\mu(0)|n\rangle\langle n|B_\nu(0)|0\rangle|_{p^2=M^2}$$
$$\equiv -\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}\right)\rho(M^2), \tag{5.2}$$

which defines the positive-definite spectral function $\rho(M^2)$. The particular tensor structure is appropriate for a conserved four-vector. Thus

$$\langle 0|B_\mu(x)B_\nu(0)|0\rangle = -\int dM^2\left(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{M^2}\right)\Delta^{(+)}(x, M^2)\rho(M^2). \tag{5.3}$$

The vacuum expectation value of the commutator can now be constructed from (5.3). It is

$$\langle 0|[B_\mu(x), B_\nu(0)]|0\rangle = -i\int dM^2\left(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{M^2}\right)\Delta(x, M^2)\rho(M^2). \tag{5.4}$$

The invariant functions are defined by

$$\Delta^{(\pm)}(x,M^2)=\int\frac{d^4p}{(2\pi)^3}e^{\mp ipx}\theta(p^+)\delta(p^2-M^2)\,,$$
$$\Delta(x,M^2)=-i[\Delta^{(+)}(x,M^2)-\Delta^{(-)}(x,M^2)]\,. \tag{5.5}$$

In particular

$$\Delta(x,M^2)|_{x^+=0}=-\tfrac{1}{4}\epsilon(x^-)\delta^2(x)\,, \tag{5.6}$$

which is independent of the mass parameter M^2. If we specialize (5.4) to $x^+=0$ and use (5.6), we get

$$\langle 0|[B^+(x),B^+(0)]|0\rangle=+i\partial^+\delta(x^-)\delta^2(x)\int\frac{dM^2}{M^2}\rho(M^2)\,, \tag{5.7}$$

$$\langle 0|[B^+(x),B^-(0)]|0\rangle=+i\left[\int\frac{dM^2}{M^2}\rho(M^2)+\int dM^2\rho(M^2)\vec\nabla^2\right][\tfrac{1}{4}\epsilon(x^-)\delta^2(x)]\,, \tag{5.8}$$

$$\langle 0|[B^k(x),B^l(0)]|0\rangle=-i\int dM^2\rho(M^2)\left(\delta^k-\frac{\partial^k\partial^l}{M^2}\right)[\tfrac{1}{4}\epsilon(x^-)\delta^2(x)]\,, \tag{5.9}$$

$$\langle 0|[B^+(x),B^k(0)]|0\rangle=i\int\frac{dM^2}{M^2}\rho(M^2)\delta(x^-)\partial^k\delta^2(x)\,. \tag{5.10}$$

Use has been made of the relation

$$\begin{aligned}\partial^+\partial^-\Delta(x,M^2)|_{x^+=0}&=(\vec\nabla^2-M^2)\Delta(x,M^2)|_{x^+=0}\\&=(\vec\nabla^2-M^2)[-\tfrac{1}{4}\epsilon(x^-)\delta^2(x)]\,.\end{aligned} \tag{5.11}$$

All of the right-hand sides of (5.7)–(5.11) cannot vanish since $\rho(M^2)$ is positive definite. These are the various Schwinger terms[10] demanded by Lorentz invariance and positivity. Generalization of these considerations to a nonconserved vector is straightforward.[15]

Our consideration so far is generally valid for any conserved field, such as the electromagnetic current, for example. In particular, we may identify B_μ with the vector-gluon field considered in Sec. III. Since the vector-gluon field is coupled to a conserved current, we have from (3.2)

$$\partial_\mu B^\mu=\frac{1}{m^2}\partial_\mu j^\mu=0\,. \tag{5.12}$$

Comparison of (5.7) and (5.9) with (2.21) then supplies the well-known spectral sum rules[16]

$$\int dM^2\rho(M^2)=1\,,$$
$$\int\frac{dM^2}{M^2}\rho(M^2)=\frac{1}{m^2}\,. \tag{5.13}$$

Since the vector field $\overline{B}_\mu$ defined by (3.17) plays an important role in discussing the S matrix in the gluon model, it is useful to consider the properties of the vacuum expectation values and Green's functions associated with $\overline{B}_\mu$. From (3.17) and (5.3) we construct

$$\begin{aligned}\langle 0|\overline{B}^\mu(x)\overline{B}^\nu(0)|0\rangle&=\left\langle 0\left|B^\mu(x)B^\nu(0)-\frac{\partial^\mu}{\partial^+}B^+(x)B^\nu(0)-\frac{\partial^\nu}{\partial^+}B^\mu(x)B^+(0)+\frac{\partial^\mu\partial^\nu}{\partial^+\partial^+}B^+(x)B^+(0)\right|0\right\rangle\\&=-\int dM^2\rho(M^2)\left(g^{\mu\nu}-g^{\mu+}\frac{\partial^\nu}{\partial^+}-g^{\nu+}\frac{\partial^\mu}{\partial^+}\right)\Delta^{(+)}(x,M^2)\,,\end{aligned} \tag{5.14}$$

where

$$\frac{1}{\partial^+}f(x)=\tfrac{1}{4}\int dy^-\epsilon(x^--y^-)f(y) \tag{5.15}$$

is an integral operator.

From this follows the x^+-ordered product

$$\langle 0|T^+(\overline{B}^\mu(x)\overline{B}^\nu(0))|0\rangle=-i\int dM^2\rho(M^2)\left(g^{\mu\nu}-g^{+\mu}\frac{\partial^\nu}{\partial^+}-g^{+\nu}\frac{\partial_\mu}{\partial^+}\right)\Delta_F(x,M^2)+g^{+\mu}g^{+\nu}\tfrac{1}{4}i|x^-|\delta(x^+)\delta^2(x)\,, \tag{5.16}$$

where T^+ stands for x^+-ordering and

$$\Delta_F(x,M^2)=\int\frac{d^4p}{(2\pi)^4}\,e^{-ipx}\,\frac{1}{p^2-M^2+i\epsilon}\,. \tag{5.17}$$

In arriving at (5.16) use has been made of (5.13) and the relations

$$\partial^-\Delta(x,M^2)|_{x^+=0}=-\tfrac{1}{8}|x^-|(\vec{\nabla}^2-M^2)\delta^2(x)\,, \tag{5.18}$$

which is obtained by integrating (5.11), and

$$\begin{aligned}&\theta(x^+)\partial^-\Delta^{(+)}(x,M^2)+\theta(-x^+)\partial^-\Delta^{(-)}(x,M^2)=\partial^-i\Delta_F(x,M^2)+\tfrac{1}{2}i\delta(x^+)\epsilon(x^-)\delta^2(x)\,,\\&\theta(x^+)(\partial^-)^2\Delta^{(+)}(x,M^2)+\theta(-x^+)(\partial^-)^2\Delta^{(-)}(x,M^2)=(\partial^-)^2i\Delta_F(x,M^2)+\tfrac{1}{2}i\partial^-\delta(x^+)\epsilon(x^-)\delta^2(x)\\&\qquad\qquad+\tfrac{1}{4}i\delta(x^+)|x^-|(\vec{\nabla}^2-m^2)\delta^2(x)\,,\end{aligned} \tag{5.19}$$

which can be readily verified from the definitions of the invariant functions (5.5) and (5.17).

It should be noted that the spectral function $\rho(M^2)$ in (5.16) is a Lorentz scalar. All the noncovariant structure of (5.16) appears explicitly in the gradient terms and the last term. The gradient terms will not affect the S matrix when B_μ is coupled to a conserved current. Only the last term in (5.16) is the genuine noncovariant contribution to the x^+-ordered product, which is seen to be independent of interactions as a result of the sum rules (5.13).

For completeness we mention in passing that a similar situation also exists in the case of a Dirac field. Recall from paper I that

$$\langle 0|\psi(x)\bar{\psi}(0)|0\rangle=-\int dM^2[i\rho_1(M^2)\gamma^\mu\partial_\mu+\rho_2(M^2)]\Delta^{(+)}(x,M^2)\,, \tag{5.20}$$

from which one obtains

$$i\langle 0|T^+(\psi(x)\bar{\psi}(0))|0\rangle=\int dM^2[i\rho_1(M^2)\gamma^\mu\partial_\mu+\rho_2(M^2)]\Delta_F(x,M^2)-\tfrac{1}{4}i\gamma^+\delta(x^+)\epsilon(x^-)\delta^2(x)\,. \tag{5.21}$$

The noncovariant term proportional to γ^+ is also independent of interactions as a result of the sum rule derived in paper I,

$$\int dM^2\rho_1(M^2)=1\,. \tag{5.22}$$

The leading singularities near the light cone for the vacuum expectation values, Green's functions, and commutators of a vector field can be discussed along the same line of reasoning presented in paper I for scalar and Dirac fields. Since the situation is so similar they will not be repeated here.

VI. CURRENT COMMUTATORS, BILOCAL OPERATORS, AND THE PARTON MODEL

Beside the academic interest in the study of quantization on a nonspacelike surface, light-front formulation of quantum field theories finds interesting applications in current algebra,[17] lepton-induced processes,[17] and high-energy scattering processes in an external field.[18] In this section two topics are discussed. In Sec. VI A we discuss the structure of current commutators on a light front, bilocal operators, and deep-inelastic processes. This discussion will make contact with previous work on the subject by Cornwall and Jackiw,[19] Gross and Treiman,[5] and Dicus, Jackiw, and Teplitz.[17] In Sec. VI B we discuss a rederivation of Feynman's parton model[7] for deep-inelastic electron scattering from the light-front formulation. This new derivation assumes the existence of Bjorken's scaling limit and has the virtue that it does not require the use of an infinite momentum frame, or the order-by-order expansion in perturbation series, or the explicit use of a transverse-momentum cutoff. The assumed existence of Bjorken's scaling limit amounts to the cutoff assumption in the previous derivation,[8] at least in a perturbation calculation of any renormalizable field theory, since the structure functions diverge in the Bjorken limit unless a cutoff is imposed.

A. Light-Front Current Commutators, Current Algebra, and Deep-Inelastic Lepton Scattering

Let us assume that strong-interaction dynamics is described by a triplet of Dirac fields coupled to a unitary singlet vector field B_μ described by the Lagrangian (3.1). The commutation relations on a light front among the conserved vector currents

$$j_a^\mu=\bar{\psi}\gamma^\mu\lambda_a\psi,\quad a=0,1,\ldots,8 \tag{6.1}$$

can be evaluated explicitly. For example,

$$x^+=y^-:\quad [\tfrac{1}{2}j_a^+(x), \tfrac{1}{2}j_b^+(y)] = 2if_{abc}\,j_c^+(x)\delta(x^- - y^-)\delta^2(x-y) + i(\partial^+)^2[S_{ab}(x,y)\epsilon(x^- - y^-)\delta^2(x-y)]\,. \tag{6.2}$$

The necessary Schwinger term denoted by S_{ab} is supplied by hand, as canonical manipulation is incapable of producing it.

The commutators $[j_a^+, j_b^-]$ are somewhat more involved. We have

$$x^+=y^+:\quad [j_a^+(x), j_b^-(y)] = [\psi^{(+)\dagger}(x)\lambda_a\psi^{(+)}(x), \psi^{(-)\dagger}(y)\lambda_b\psi^{(-)}(y)]\,. \tag{6.3}$$

Making use of (3.21) and (3.17), we can simplify this equation to finally obtain

$$\begin{aligned}
x^+=y^+:&\\
[j_a^+(x), j_b^-(y)] = &-\tfrac{1}{4}i\epsilon(x^- - y^-)f_{abc}\{\tfrac{1}{2}\partial^+[\bar\psi(x)\gamma^-\lambda_c\psi(y) + \bar\psi(y)\gamma^-\lambda_c\psi(x)]\delta^2(x-y)\\
&\quad -\tfrac{1}{2}[\bar\psi(x^+,\vec y, x^-)\gamma_k\lambda_c\psi(y) + \bar\psi(y)\gamma_k\lambda_c\psi(x^+,\vec y, x^-)\\
&\quad -i\epsilon_{kl}\bar\psi(x^+,\vec y, x^-)\gamma_l\gamma_5\lambda_c\psi(y) + i\epsilon_{kl}\bar\psi(y)\gamma_l\gamma_5\lambda_c\psi(x^+,\vec y,x^-)]\partial_k\delta^2(x-y)\}\\
&+\tfrac{1}{4}\epsilon(x^- - y^-)d_{abc}\{\tfrac{1}{2}\partial^+[\bar\psi(y)\gamma^-\lambda_c\psi(x) - \bar\psi(x)\gamma^-\lambda_c\psi(y)]\delta^2(x-y)\\
&\quad +\tfrac{1}{2}[\bar\psi(x^+,\vec y, x^-)\gamma_k\lambda_c\psi(y) - \bar\psi(y)\gamma_k\lambda_c\psi(x^+,\vec y, x^-) - i\epsilon_{kl}\bar\psi(x^+,\vec y,x^-)\gamma_l\gamma_5\lambda_c\psi(y)\\
&\quad -i\epsilon_{kl}\bar\psi(y)\gamma_l\gamma_5\lambda_c\psi(x^+,\vec y, x^-)]\partial_k\delta^2(x-y)\}\\
&+i\vec\nabla^2[S'_{ab}(x,y)\epsilon(x^- - y^-)\delta^2(x-y]\,.
\end{aligned} \tag{6.4}$$

Again a Schwinger term is supplied by hand. In (6.4) ϵ_{kl} is the antisymmetric tensor in two-dimensional space,

$$\begin{aligned}\epsilon_{12} &= -\epsilon_{21} = 1\,,\\ \epsilon_{11} &= \epsilon_{22} = 0\,.\end{aligned} \tag{6.5}$$

To derive the above result use is made of the identities

$$\lambda_a\lambda_b = if_{abc}\lambda_c + d_{abc}\lambda_c\,, \tag{6.6}$$

$$\gamma^0\gamma_k\gamma^3 = -i\epsilon_{kl}\gamma_l\gamma_5\,. \tag{6.7}$$

It should be noted that (6.2) and (6.4) have a structure independent of interactions which are hidden in the dependent part of the Dirac field ψ. Recall that the Dirac fields ψ and $\bar\psi$ which enter the commutators are not the ones which appear in the original Lagrangian conventionally employed. They are related by an operator phase transformation. Consider first the model of Gross and Treiman[5] in which b^+ commutes with itself on a light front. When the field operators ψ and $\bar\psi$ are expressed in terms of the original Dirac fields ψ' and $\bar\psi'$, using (3.53), the result is the following substitution in (6.4):

$$\bar\psi(x)\Gamma\psi(y) = \bar\psi'(x)\Gamma\exp\left[-ig\tfrac{1}{2}\int_y^x dz^- b^+(z)\right]\psi'(y)\,, \tag{6.8}$$

where the path of integration is a straight line connecting x and y. The exponential factor comes from the combination

$$\exp\left[-ig\tfrac{1}{4}\int dz^-\epsilon(x^- - z^-)b^+(z)\right] \times\exp\left[ig\tfrac{1}{4}\int dz^-\epsilon(y^- - z^-)b^+(z)\right] = \exp\left[-ig\tfrac{1}{2}\int_y^x dz^- b^+(z)\right]. \tag{6.9}$$

The kind of structure (6.9) was obtained by Gross and Treiman.[5] They start with canonical equal-time commutation relations and equations of motion, and sum up the leading singularities contributing to the commutators near the light cone.

In the Gross-Treiman model the commutativity of b^+ on a light front makes the inversion of (3.53) possible to reexpress ψ in terms of ψ'. Similar inversion for (3.11) is impossible since ψ' and B^+ do not commute. Thus in the gluon model without a negative-metric scalar field the original Dirac field loses its physical significance. All physical quantities are expressed in terms of ψ and $\bar\psi$, and the current commutators still possess the structure exhibited in (6.2) and (6.4). In our treatment the canonical Dirac fields ψ and $\bar\psi$ are gauge-independent and their relations to the original ones are given by (3.11) and (3.54). The dependence on the vector field can be explicitly studied. On the other hand, in the work of Cornwall and Jackiw[19] the special gauge $B^+=0$ has to be imposed. As a result the dependence on the vector field cannot be examined.

Application of the light-front current commutators (6.2) and (6.4) to current algebra and deep-inelastic lepton scattering has been extensively studied by Cornwall and Jackiw[19] and by Dicus, Jackiw, and Teplitz.[17] For details the reader is referred to their original papers. It has been shown that current-algebra sum rules of Adler[20] and of Dashen, Gell-Mann, and Fubini,[21] as well as others derived earlier by the infinite-momentum technique, follow from (6.2) and (6.4). One interesting result is that some of these earlier sum rules acquire corrections in this light-front formulation derivation.[17] These corrections remove some inconsistencies noted before. These corrections are due to the presence of the bilocal operators in (6.4) which are totally absent in boosting the equal-time commutators to the infinite-momentum frame.

The sum rules of Adler[20] and of Dashen, Gell-Mann, and Fubini[21] seem to follow from (6.2) immediately without reference to any infinite-momentum limit. Dicus, Jackiw, and Teplitz,[19] and Calucci, Jengo, Furlan, and Rebbi[22] recently have shown that the derivation is actually valid only if there are no fixed poles in the amplitudes for semistrong processes. They argue that this is equivalent to the assumption of superconvergence required in the old derivation. Thus, a question raised earlier in paper I concerning the necessity of unsubtracted dispersion relations and the validity of these sum rules appears to have been answered.

B. Bilocal Operators and Parton Model

Some time ago Feynman's parton model[7] for deep-inelastic electron scattering on a hadron target was derived[8] from a canonical quantum field theory with a transverse-momentum cutoff imposed. This derivation is accomplished by examining each term in the old-fashioned time-ordered perturbation expansion in an infinite-momentum frame. The imposed transverse-momentum cutoff ensures the existence of Bjorken's scaling limit of the structure functions, and it is partly motivated by the properties observed in the high-energy scattering of hadrons. The parton model offers considerable insight for the nature of Bjorken scaling and it provides a vivid intuitive picture for the physical process in the deep-inelastic region. All these can be understood in terms of the so-called "impulse approximation." Earlier derivation is often subject to the criticisms that it is valid only in a certain class of infinite-momentum frames, and that it is derived by keeping only the leading contribution in P, the infinite momentum of the target hadron, of each term in the perturbation series. In this subsection the parton model will be rederived in the light-front formulation. The present derivation does not seem to suffer the drawbacks mentioned above. It is valid in any coordinate frame, the laboratory system for example. Nor does it require an explicit perturbation expansion. The structure functions are now related to the p^+ distribution of the bare constituents of the hadron target. A transverse-momentum cutoff is not *explicitly* needed in the present derivation, but the existence of the Bjorken scaling limit of the structure functions must be assumed.

We shall only illustrate the essential ideas involved by considering a scalar current constructed from a scalar field. Considerations on the complications due to tensor structure, spin of the fundamental fields, and internal symmetries, as well as the question of gauge invariance, will be deferred to another publication. Two methods of derivation will be given. One employs the generalized Bjorken-Johnson-Low theorem as applied to the forward virtual Compton scattering amplitude. The other deals directly with the absorptive part of the forward Compton scattering amplitude.

Consider the spin-averaged forward virtual Compton scattering amplitude for a virtual scalar photon scattered off a hadron target,

$$T(P,q)=(4\pi)^2MP^+\int d^4x\, e^{iqx} i\langle P|T^+(J(x)J(0))|P\rangle + \text{polynomials}\,, \tag{6.10}$$

where P, M are the four-momentum and mass of the hadron, respectively, and q is the four-momentum of the virtual photon, with $Q^2 \equiv -q^2 > 0$. A spin average is understood in (6.10). The polynomial terms are the necessary Schwinger terms to make (6.10) covariant. As $q^- \to \infty$ the generalized Bjorken-Johnson-Low[23] theorem gives

$$T(P,q)\underset{q^-\to\infty}{\sim} \text{polynomials} - \frac{1}{q^-}(4\pi)^2MP^+\int dx^- d^2x\, e^{iq^+x^-/2 - i\vec{q}\cdot\vec{x}}\langle P|[J(x),J(0)]|P\rangle\big|_{x^+=0}\,. \tag{6.11}$$

For spacelike q^2 the amplitude $T(P,q)$ satisfies a dispersion relation

$$T(P,q)=\frac{1}{2\pi}\int d\nu'\,\frac{W(q^2,\nu')}{\nu'-\nu}\,, \tag{6.12}$$

where

$$\nu=\frac{P\cdot q}{M}\,. \tag{6.13}$$

The question of subtraction need not concern us since we are ultimately only interested in the absorptive part of $T(P,q)$. Introducing the scaling variable

$$\omega = \frac{2M\nu}{Q^2}, \quad 1 \leqslant \omega < \infty \tag{6.14}$$

we get

$$T(P,q) = \frac{1}{2\pi}\frac{2M}{Q^2}\int \frac{d\omega'}{\omega'}\frac{\nu' W(q^2,\nu')}{\omega' - \omega}. \tag{6.15}$$

Notice that as $q^- \to \infty$ ω approaches a fixed limit:

$$\omega \xrightarrow[q^- \to \infty]{} -\frac{P^+}{q^+}. \tag{6.16}$$

In the meantime both Q^2 and $M\nu$ tend to infinity. That is, $q^- \to \infty$ with P^+ and q^+ fixed is the Bjorken scaling limit. Let us assume that $\nu W(q^2,\nu)$ approaches a scaling limit as $Q^2, M\nu \to \infty$ with ω fixed. Then

$$\lim_{\substack{Q^2,\nu \to \infty \\ \omega \text{ fixed}}} \nu W(q^2,\nu) = F(\omega). \tag{6.17}$$

In this limit (6.11) and (6.15) together give

$$-\frac{1}{\pi}\int \frac{d\omega'}{\omega'}\frac{F(\omega')}{\omega'-\omega} = -(4\pi)^2 P^+ q^+ \int dx^- d^2x\, e^{iq^+x^-/2 - i\vec{q}\cdot\vec{x}}\langle P|[J(x),J(0)]|P\rangle|_{x^+=0} + \text{polynomials}. \tag{6.18}$$

Let $J(x)$ be given by

$$J(x) = \phi^\dagger(x)\phi(x), \tag{6.19}$$

where the scalar field ϕ satisfies the canonical commutation relation

$$x^+ = 0: \quad [\phi^\dagger(x), \phi(0)] = -\tfrac{1}{4} i\epsilon(x^-)\delta^2(x). \tag{6.20}$$

The current commutator in (6.18) can now be evaluated:

$$[J(x),J(0)] = -\tfrac{1}{4} i\epsilon(x^-)\delta^2(x)[\phi^\dagger(x)\phi(0) + \phi(x)\phi^\dagger(0)], \tag{6.21}$$

where x on the right-hand side of (6.21) is lightlike,

$$x^2 = x^+x^- - \vec{x}^2 = 0. \tag{6.22}$$

Equation (6.18) now becomes

$$\frac{1}{\pi}\int \frac{d\omega'}{\omega'}\frac{F(\omega')}{\omega'-\omega} = -\tfrac{1}{4} i(4\pi)^2 P^+ q^+ \int dx^- e^{iq^+x^-/2}\epsilon(x^-)\langle P|[\phi^\dagger(x)\phi(0) + \phi(x)\phi^\dagger(0)]|P\rangle + \text{polynomials}. \tag{6.23}$$

By Lorentz covariance we have

$$P^+\langle P|\phi^\dagger(x)\phi(0) + \phi(x)\phi^\dagger(0)|P\rangle = f(P\cdot x), \tag{6.24}$$

since $P\cdot x = \frac{1}{2}P^+x^-$ is the only invariant available and P^+ is fixed. Now

$$\epsilon(x^-) = -\frac{i}{\pi}\int dq'^+ \frac{1}{q'^+} e^{iq'^+x^-/2};$$

therefore

$$\frac{1}{\pi}\int \frac{d\omega'}{\omega'}\frac{F(\omega')}{\omega'-\omega} = -4\pi q^+ \int dx^- \int \frac{d\omega'}{\omega'}\frac{\omega}{\omega'-\omega} f(x\cdot P) e^{-i(1/\omega')P\cdot x} + \text{polynomials}. \tag{6.25}$$

Taking the absorptive part on both sides we obtain[24]

$$F(\omega) = 8\pi^2 \int d\eta\, e^{-i(1/\omega)\eta} f(\eta), \tag{6.26}$$

where

$$\eta = P\cdot x = \tfrac{1}{2}P^+x^-. \tag{6.27}$$

The Heisenberg field operators $\phi^\dagger$ and ϕ are related to the corresponding free-field operators in the interaction picture, $\phi_0^\dagger$ and ϕ_0, by the usual U transformation

$$\phi(x) = U^{-1}(x^+)\phi_0(x)U(x^+)\,, \tag{6.28}$$

where

$$U(x^+) = T^+ \exp\left[-i\int_{-\infty}^{x^+} d^4x\, \mathcal{H}_I(x)\right] \tag{6.29}$$

is the "dressing operator." Since all field operators in (6.26) are evaluated at a common value of $x^+=0$, $f(\eta)$ defined by (6.24) becomes

$$f(\eta) = P^+\langle UP|[\phi_0^\dagger(x)\phi_0(0) + \phi_0(x)\phi_0^\dagger(0)]|UP\rangle\,, \tag{6.30}$$

where

$$|UP\rangle = U(0)|P\rangle \tag{6.31}$$

can be expanded in terms of a complete set of bare states,

$$|UP\rangle = \sum_n \int \prod_{i=1}^n \frac{dp_i^+ d^2p_i}{(2p_i^+)^{1/2}}\, \delta\left(P^+ - \sum_i p_i^+\right)\delta^2\left(\vec{\mathrm{P}} - \sum_i \vec{\mathrm{p}}_i\right) f_n(p_1\cdots p_n, P)|p_1\cdots p_n\rangle\,. \tag{6.32}$$

The normalization condition

$$\begin{aligned}\langle UP|UP'\rangle &= \langle P|U^{-1}U|P'\rangle \\ &= \langle P|P'\rangle \\ &= \delta(P^+ - P'^+)\delta^2(\vec{\mathrm{P}} - \vec{\mathrm{P}}')\end{aligned} \tag{6.33}$$

implies

$$\sum_n \int \prod_{i=1}^n \frac{dp_i^+}{2p_i^+}\, d^2p_i |f_n(p_1\cdots p_n, P)|^2 \delta\left(P^+ - \sum_i p_i^+\right)\delta^2\left(\vec{\mathrm{P}} - \sum_i \vec{\mathrm{p}}_i\right) = 1\,. \tag{6.34}$$

Since the free fields $\phi_0^\dagger$ and ϕ_0 are one-body operators we have

$$\begin{aligned} f(\eta) = P^+ \sum_n \sum_i \int \prod_{k=1}^n \frac{dp_k^+}{2p_k^+}\, d^2p_k |f_n(p_1\cdots p_i \cdots p_n, P)|^2 \delta\left(P^+ - \sum_k p_k^+\right) \\ \times \delta^2\left(\vec{\mathrm{P}} - \sum_k \vec{\mathrm{p}}_k\right)\langle p_i|\phi_0^\dagger(x)\phi_0(0) + \phi_0(x)\phi_0^\dagger(0)|p_i\rangle\,, \end{aligned} \tag{6.35}$$

where the summation over i extends to all "charged constituents" associated with the field ϕ_0. The product $\phi_0^\dagger(x)\phi_0(0)$ gets contributions only from particles, and the other, $\phi_0(x)\phi_0^\dagger(0)$, only from antiparticles when $f(\eta)$ is inserted into (6.26) since $1/\omega$ is positive in the physical region. We have

$$\begin{aligned} \langle p|\phi_0^\dagger(x)\phi_0(0)|p\rangle &= \frac{1}{(2\pi)^3}\frac{1}{2p^+}\, e^{+i\eta y}\,, \\ \langle \bar{p}|\phi_0(x)\phi_0^\dagger(0)|\bar{p}\rangle &= \frac{1}{(2\pi)^3}\frac{1}{2\bar{p}^+}\, e^{+i\eta y}\,, \end{aligned} \tag{6.36}$$

where $p_i, \bar{p}_i$ refer to a particle and an antiparticle, respectively, and y is defined by

$$\begin{aligned} p_i^+ &= y_i P^+\,, \\ \bar{p}_i^+ &= y_i P^+\,. \end{aligned} \tag{6.37}$$

Carrying out the η integration gives finally

$$F(\omega) = \omega \sum_n \sum_i \int \frac{dp_k^+}{2p_k^+}\, d^2p_k \delta\left(P^+ - \sum_k p_k^+\right)\delta^2\left(\vec{\mathrm{P}} - \sum_k \vec{\mathrm{p}}_k\right)\delta\left(y_i - \frac{1}{\omega}\right)|f_n(p_1\cdots p_i\cdots p_n, P)|^2\,. \tag{6.38}$$

This is the parton-model expression for the structure function $F(\omega)$ in the scaling limit.[8] It states that the scattered charged constituent has a p^+ given by $(1/\omega)P^+$. It has been pointed out in paper II that the p^+ of every particle in all intermediate states in the x^+-ordered perturbation expansion cannot be negative. Consequently we have

$0 \leqslant y_i \leqslant 1$,

consistent with the kinematic constraint on $1/\omega$ given in (6.14). Equation (6.38) makes it clear that the right-hand side is a function of ω only.

We now summarize the essential conditions for the validity of (6.38) in the present derivation.

(1) Bjorken's scaling limit exists for the structure function νW, so that (6.17) and (6.18) are meaningful. It also means that the Fourier transform of the one-particle matrix element of the bilocal operator (6.26) is finite.[25]

(2) The generalized Bjorken-Johnson-Low theorem[23] is applicable.

(3) The bilocal operator which appears in (6.21) is the product of two local fields.

(4) The U operator (6.29) exists.

We now sketch the second derivation. This derivation differs from the previous one in that instead of using the Bjorken-Johnson-Low theorem the dominance of light-cone singularity is assumed.

We infer from (6.21) that as $x^2 \to 0$ we have[26]

$$\langle P|[J(x),J(0)]|P\rangle \underset{x^2\to 0}{\longrightarrow} i\Delta(x,0)\langle P|[\phi^\dagger(x)\phi(0)+\phi(x)\phi^\dagger(0)]|P\rangle$$

$$\underset{x^2\to 0}{\longrightarrow} -\frac{i}{2\pi}\,\epsilon(x\cdot P)\delta(x^2)\langle P|[\phi^\dagger(x)\phi(0)+\phi(x)\phi^\dagger(0)]|P\rangle, \tag{6.39}$$

as required by causality and relativistic covariance. Along the particular path $x^+=0$, $\vec{x}=0$ (6.21) and (6.39) coincide. It will be assumed that

(1) the bilocal operators in (6.39) have finite matrix elements,[25] and

(2) the light-cone singularity $\Delta(x,0)|_{x^2\to 0}$ in (6.39) dominates in the Bjorken scaling limit.

Under these assumptions many authors[27] have shown that the structure function in the Bjorken limit is related to the one-dimensional Fourier transform of the object

$$P^+\langle P|\phi^\dagger(x)\phi(0)+\phi(x)\phi^\dagger(0)|P\rangle = f(P\cdot x) \tag{6.40}$$

along the light cone. The relation in our case is

$$F(\omega) = 8\pi^2\int d(P\cdot x)\exp\left[-i\,\frac{1}{\omega}\,(P\cdot x)\right]f(P\cdot x)\Big|_{x^2=0}. \tag{6.41}$$

Since (6.41) is covariant the path of integration can be arbitrary as long as it lies on the light cone. The choice

$$x^+=0,\ \vec{x}=0 \tag{6.42}$$

then gives the previous result (6.26).

Thus, we have shown that dominance of light-cone singularity and the fact that bilocal operators are products of two local fields imply the parton picture for deep-inelastic electron scattering. It is then clear that both the parton and light-cone approaches will lead to identical predictions as far as the lepton-hadron scattering is concerned.

A similar conclusion has been reached by Jaffe,[28] who recently made a comparison of the parton and light-cone approaches of the deep-inelastic electron scattering from the other direction starting from the parton model. However, Jaffe emphasized that application of the two approaches to other processes, such as the massive μ-pair production in high-energy proton-proton scattering,[29] will in general lead to different results.

We now comment on the possible effects of the line integral in a gluon model on this derivation of the parton model. Since ψ and $\bar{\psi}$ (or ϕ and $\phi^\dagger$), not ψ' and $\bar{\psi}'$ (or ϕ' and $\phi'^\dagger$), satisfy simple canonical commutation relations and appear in the simple Hamiltonian,[14] the connection between operators in the Heisenberg and interaction picture such as (6.28) should apply to ψ and $\bar{\psi}$ but not to ψ' and $\bar{\psi}'$. Therefore it is evident that no line integral appears in the derivation. These line integrals are present only when one insists on using the field operators ψ' and $\bar{\psi}'$ which appear in the original covariant Lagrangian. As we have remarked before, it is not always a meaningful operation.

VII. DISCUSSION

Light-front quantization of spin-one fields and some related questions are studied in this paper. As we have seen in the previous sections, certain novel features unique to the vector fields appear. Naive canonical quantization fails completely not only because the canonical variables are also the conjugate momenta to themselves, but also because there exist secondary constraints among field variables which seem to obey equations of motion. The former requires an additional factor of $\frac{1}{2}$ to be included in the commutation relations,[30] and the latter totally modifies the commutator structure. It is further complicated by the necessity of an operator phase transformation on the coupled Dirac field in order to maintain simple commutation relations and a simple Hamiltonian. This phase factor is analogous to the eikonal phase acquired by the wave function of a fast particle traversing through an external vector field. It is known that in an external scalar field no such phase accumulation occurs.

Quantization of vector fields is a very important problem, since the only field theory with quantitative success, namely, quantum electrodynamics, belongs to this category. Furthermore, a massive vector field represents the first unusual case in this new formulation. Intuitively speaking, the light-front coordinate system is qualitatively similar to an infinite-momentum frame. Now a fast-moving particle tends to behave like a particle without mass which would have only two transverse polarization states. On the other hand, the mass-shell condition $p^2=m^2$ requires that the third polarization state corresponding to zero helicity be present. A conflict between the two tendencies leads to the unusual situation of the existence of secondary constraints and the necessity of an operator phase transformation on the coupled Dirac field.

Because a light front contains a line tangent to the light cone, valuable information about a current commutator on a light cone can be inferred from the canonical commutation relations on a light front. Many applications of the light-front quantization to the deep-inelastic processes induced by leptons are based on this property. Here we mention only the derivation of Feynman's parton model presented in the last section. A qualification must be pointed out, however. Individual terms in the x^+-ordered perturbation expansion of the S matrix often behave singularly as $p^+\to 0$, although the whole S matrix is regular. Consequently, the U operator which enters the parton model is not without ambiguity. Validity of the parton model assumes that this ambiguity is unimportant. This need not be a bad assumption since the derivation requires only the general properties, such as the singularity structure of the current commutator near the light cone and the unitarity of the U operator as well as the existence of Bjorken's scaling limit.

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[11]For conventions of metric, Dirac matrices, and invariant functions we follow J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965). For notations in light-front coordinates we adopt those in papers I and II. In particular, the nonvanishing elements of the metric tensor $g^{\mu\nu}$ are $g^{+-}=g^{-+}=2$, $g^{11}=g^{22}=-1$; a four-vector x^μ is decomposed into $x^\pm=x^0\pm x^3$ and $\vec{x}=(x^1, x^2)$; the Latin indices i, j, k, l run from 1 to 2. We also use the notations $d^2p=dp^1dp^2$, $d^2x=dx^1dx^2$, and $\delta^2(x)=\delta^2(\vec{x})$.

[12]Unless specified otherwise, if only y^- integration appears in an integral the four-vector y^μ differs from x^μ only in the "minus" component. This convention will be used throughout the paper.

[13]The gluon model formulated in Sec. III A is also smooth after the operator phase transformation on the Dirac field. In fact, the two models give the same structure for the current commutators on a light front (see discussion in Sec. VI).

[14]This is the essential difference between the two models. It implies in particular that the original field $\psi'^{(+)}$ in the Gross-Treiman model also satisfies the same anticommutation relation as $\psi^{(+)}$. However, in terms of the independent variables $\psi'^{(+)}$, b^+, b^{+k}, and

ϕ^+, the energy density T^{+-} or the Hamiltonian will be very complicated since the phase factor in (3.53) appears.

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[24]The derivation given here essentially follows the one given in Ref. 17 for a vector current. Our new contribution is to relate (6.26) to the p^+ distribution of the constituents of the hadron as given in (6.38).

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[26]Equation (6.39) can be regarded as the covariant version of the parton model as formulated by P. V. Landshoff, J. C. Polkinghorne, and R. D. Short [Nucl. Phys. B28, 225 (1971)]. The function $\Delta(x,0)$ describes the free propagation of a particle between x and 0. For more details of this interpretation, see R. Jackiw and R. E. Waltz, Phys. Rev. D 6, 702 (1972).

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Quantum Field Theories in the Infinite-Momentum Frame. IV. Scattering Matrix of Vector and Dirac Fields and Perturbation Theory*

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The scattering matrix of coupled spin-one and Dirac field theories formulated in light-front quantization of the preceding paper is studied. The scattering matrix of the vector-gluon model in this new formulation is shown to give the same predictions as in the equal-time formulation to all orders in perturbation theory. Renormalizability of this model in the new formulation is also established. A further test of the light-front quantization of spin-one fields is discussed by examples of fermion-fermion interaction and virtual as well as real Compton scattering in the axial-vector–gluon model in the lowest-order perturbation theory. A reduction formula for vector particles is derived and the Wick theorem is proved. Peculiarities in the perturbation theory of the light-front formulation are discussed. Finally, a partonlike model for scattering of two energetic particles is proposed which satisfies manifest s-channel unitarity.

I. INTRODUCTION

In this fourth and final paper in a program[1] devoted to the study of quantum field theories in light-front coordinates, we study the properties of the S matrix of the coupled spin-one field theories formulated in the preceding paper,[2] and certain general questions in perturbation theory in this new formulation.

The Hamiltonian and the propagators for Dirac and vector particles involve many noncovariant terms and are much more complicated than the corresponding expressions in the usual equal-time formulation. The problem is further complicated by the operator phase transformation on the Dirac field which is necessary in order to maintain sim-