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Spectroscopic Consequences of Exact Duality for Baryons*

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Those backward meson-baryon scattering and (baryon + antibaryon → two mesons) processes which are exotic in the direct channel are assumed to be dominated at high energy by the exchange of baryon trajectories whose imaginary parts cancel exactly. A solution to these constraints in accord with the lowest-lying baryon states is obtained; the residue ratios are found to vary along the trajectory. Properties of a $J^P = \frac{7}{2}^+$ baryon octet in the mass range 1950–2200 MeV, predicted by these constraints, are examined. For example, one expects (a) a $J^P = \frac{7}{2}^+$ $N(1950)$ with $\Gamma(\pi N) \simeq 13$ MeV and $\Gamma(\pi\Delta) \simeq 80$ MeV, whose effects might also be visible in $\pi N \to K\Sigma$, and (b) a $J^P = \frac{7}{2}^+$ $\Lambda(2100)$, with $\Gamma(\Sigma\pi) \simeq 22$ MeV and $\Gamma(\Sigma(1385)\pi) \simeq 60$ MeV, whose $\bar{K}N$ coupling should be very small. A further prediction is that of a G_{39} πN resonance, $\Delta(2200)$ ($J^P = \frac{9}{2}^-$), whose effects could be visible in backward $\pi^+ p$ scattering.

I. INTRODUCTION

The "matching" of low-energy and high-energy descriptions of elementary particle scattering has made use in the past few years of the principle of "duality,"[1] which states that these two descriptions are complementary to one another and possess an intermediate-energy regime in which both are valid. An amplitude which lacks resonant imaginary parts at low energies may then be expected to lack them at high energies as well (aside from the special case of elastic processes, in which diffraction plays a role).[2] In the high-energy regime, an exchange picture seems most economical. The imaginary parts provided by various exchanges must then cancel one another in channels where no low-energy resonances are expected, if duality is to hold.

The self-consistency of this scheme has been the subject of much debate. It was pointed out quite early[3,4] that imaginary parts of $\bar{b}b \to \bar{b}b$ amplitudes (b = baryon) could not be dominated by octet and singlet exchange at high energies and simultaneously be confined to octet and singlet contributions in the direct channel.

One solution to the baryon-antibaryon problem, suggested by the author,[3] was the possible existence of exotic mesons ($qq\bar{q}\bar{q}$ in the quark model, as opposed to $q\bar{q}$ for the usual singlet and octet mesons).[5] Such mesons would couple only to $\bar{b}b$ (not to pairs of ordinary mesons) and are not yet ruled out by experiment.

Another solution which has gained widespread favor has been the rejection of *any* $\bar{b}b$ channels (and, indeed, of *any* channels with too high a threshold) as unreliable.[6] In this approach, known as "broken duality," one rejects constraints on baryon trajectories arising from $\bar{b}b \to MM$ ($M = 0^-$ meson) while keeping those related to backward meson-baryon scattering, $Mb \to bM$.

In this paper we would like to point out some consequences of an "exact duality" approach for baryon exchange. The feasibility of such an approach has been demonstrated some time ago.[7] The solution to the duality constraints in Ref. 7 incorporates – by construction – the baryon spectrum of the harmonic-oscillator quark model.[8] In terms of multiplets of SU(6)×O(3), the following states lie on the leading trajectory:

$$\begin{aligned} &\underline{56},\ L=0,\\ &\underline{70},\ L=1,\\ &\underline{56} \text{ and } \underline{70},\ L=2,\\ &\underline{70} \text{ and } \underline{56},\ L=3. \end{aligned} \tag{1}$$

The fact that Eq. (1) would provide an acceptable solution to duality constraints was conjectured quite early.[9] Our contribution will be to provide a simplified means of solving the constraints consistent with Eq. (1) and to use the predictions thus obtained to suggest some worthwhile tests of this spectrum.

At the moment, the data are still probably consistent with a simpler spectrum[6,10]

$$\underline{56},\ \text{even } L \leftrightarrow \underline{70},\ \text{odd } L. \tag{2}$$

If it is imposed on the baryons, one cannot obtain a consistent solution for $Mb \to bM$ and $\bar{b}b \to MM$. This was the primary reason for rejecting some of the constraints. The latter process was singled out as possibly unreliable because of the apparent difficulties[3,4] in $\bar{b}b \to \bar{b}b$.

One would thus like to distinguish experimentally between the spectrum (1) and the simpler version (2). The existence of a $\underline{70}$, $L=2$ multiplet degenerate with 56, $L=2$ would be the clearest indication in favor of Eq. (1). A 56, $L=3$ degenerate with the known 70, $L=3$ would also favor the harmonic-oscillator spectrum.

We present and solve the constraint equations in Sec. II. The most prominent SU(3) multiplet expected in 70, $L=2$ but not in 56, $L=2$ is an octet with $J^P=\frac{7}{2}^+$. We shall discuss properties of this octet in detail in Sec. III. The states in question probably lie in the mass range 1950–2200 MeV.

In addition (Sec. IV) we discuss the prediction of a G_{39} pion-nucleon resonance, Δ(2200 MeV, $J^P=\frac{9}{2}^-$), whose effects could be visible in backward pion-nucleon scattering. Such a resonance would be a natural candidate for a 56, $L=3$ member.

Our discussion and conclusions are presented in Sec. V. The Appendix contains an alternative treatment of the constraint equations which allows application of the model in Ref. 7 to partial-width predictions. The results are very close to those of Secs. III and IV.

II. CONSTRAINT EQUATIONS

A. Treatment of Spin

We shall be concerned with the following processes:

$$\begin{aligned} &MB \to M'B' \quad (s),\\ &M\overline{M} \to \overline{B}B' \quad (t),\\ &M\overline{B}' \to \overline{B}M' \quad (u); \end{aligned} \tag{3}$$

$$\begin{aligned} &MB \to M'D \quad (s),\\ &M\overline{M}' \to \overline{B}D \quad (t),\\ &M\overline{D} \to \overline{B}M' \quad (u); \end{aligned} \tag{4}$$

and

$$\begin{aligned} &MD \to M'D' \quad (s),\\ &M\overline{M}' \to \overline{D}D' \quad (t),\\ &M\overline{D}' \to \overline{D}M' \quad (u), \end{aligned} \tag{5}$$

where M, M' denote mesons; B, B' denote elements of the baryon octet; and D, D' denote elements of the baryon decimet.

The definitions of s, t, and u channels conform to the discussion of Ref. 11. The Regge limits will be taken as $t\to\infty$ or $u\to\infty$, so that the Regge trajectory is always exchanged in the s channel.

The process (3) has two independent helicity amplitudes for each J; processes (4) and (5) have four and six, respectively. This may be seen directly using the constraints of parity and time-reversal invariance.[12] The partial-wave representation allows a convenient interpretation of these amplitudes. Denote a helicity state by $|J\lambda\rangle$ and a partial-wave state by $|J;l\rangle$. Then

$$|J\lambda\rangle=\sum_l (S\lambda l0|J\lambda)|J;l\rangle\,, \tag{6}$$

where S is the baryon spin ($\frac{1}{2}$ or $\frac{3}{2}$). One may form states of definite natural parity σ ($\sigma=\pm1$):

$$|J\lambda\rangle_\sigma=\tfrac{1}{2}[\,|J\lambda\rangle+\sigma(-)^{S+1/2}|J-\lambda\rangle]\,. \tag{7}$$

Using the property

$$(S-\lambda l0|J-\lambda)=(-)^{J-l-S}(S\lambda l0|J\lambda) \tag{8}$$

and the definitions of signature τ,

$$\tau=(-)^{J-1/2}, \tag{9}$$

and parity $P^{(l)}$,[13]

$$P^{(l)}=-(-)^l\,, \tag{10}$$

one finds

$$|J\lambda\rangle_\sigma=\sum_l[\tfrac{1}{2}(1+\sigma\tau P^{(l)})](S\lambda l0|J\lambda)|J;l\rangle\,. \tag{11}$$

The cases of interest to us and the partial waves which contribute are shown in Table I. The projection operator $\frac{1}{2}[1+\sigma\tau P^{(l)}]$ singles out those partial waves with $\tau P^{(l)}=\sigma$. One may thus separate out two sequences:

$$\sigma=+1:\ J^P=\tfrac{1}{2}^+,\ \tfrac{3}{2}^-,\ \tfrac{5}{2}^+,\ \dots\,, \tag{12}$$

$$\sigma=-1:\ J^P=\tfrac{1}{2}^-,\ \tfrac{3}{2}^+,\ \tfrac{5}{2}^-,\ \dots\,. \tag{13}$$

In the case of spin $\frac{1}{2}$, the two helicity amplitudes can be expressed in terms of $\sigma=\pm1$ contributions, with no further degrees of freedom. For MD states, the fact that each helicity state of definite σ receives contributions from two partial waves accounts for the extra degrees of freedom. In $MB\to MD$, the four helicity amplitudes may be reexpressed in terms of the following four transitions:

$$\begin{array}{ccc} & MB & MD \end{array}$$

$$\sigma=+1:\ l=J+\tfrac{1}{2}\to\begin{cases} l=J-\tfrac{3}{2}\\ l=J+\tfrac{1}{2}\,,\end{cases} \tag{14}$$

$$\sigma=-1:\ l=J-\tfrac{1}{2}\to\begin{cases} l=J-\tfrac{1}{2}\\ l=J+\tfrac{1}{2}\,.\end{cases} \tag{15}$$

TABLE I. Meson-baryon helicity states $|J\lambda\rangle_\sigma$ of definite natural parity $\sigma=\tau P$.

Baryon spin $\frac{1}{2}$ ($\lambda=\frac{1}{2}$):	
$\lvert J\lambda\rangle_\pm=\frac{1}{2}[\lvert J\lambda\rangle\mp\lvert J-\lambda\rangle]$,	$l=J\pm\frac{1}{2}$
Baryon spin $\frac{3}{2}$ ($\lambda=\frac{1}{2},\ \frac{3}{2}$):	
$\lvert J\lambda\rangle_\pm=\frac{1}{2}[\lvert J\lambda\rangle\pm\lvert J-\lambda\rangle]$,	$l=J-\frac{3}{2},\ J+\frac{1}{2}$ / $l=J-\frac{1}{2},\ J+\frac{3}{2}$

In $MD \to MD$, there are six transitions:

$$\begin{array}{lll} & MD & M'D' \\ \sigma=+1: & \left\{\begin{array}{l} l=J-\frac{3}{2} \to l=J-\frac{3}{2} \\ \qquad\qquad\; \to l=J+\frac{1}{2} \\ l=J+\frac{1}{2} \to l=J+\frac{1}{2}, \end{array}\right. & (16) \end{array}$$

$$\sigma=-1: \quad \left\{\begin{array}{l} l=J-\frac{1}{2} \to l=J-\frac{1}{2} \\ \qquad\qquad\; \to l=J+\frac{3}{2} \\ l=J+\frac{3}{2} \to l=J+\frac{3}{2}. \end{array}\right. \qquad (17)$$

Any resonance decays to MD via two partial waves in principle. One may view the duality constraints as acting separately on each helicity amplitude, and hence on each partial wave. As an example, we shall be considering the $\pi\Delta$ decays of $\Delta(1950)$ (the F_{37} resonance) and a predicted $N(1950)$ (an F_{17} πN resonance), both with $J^P=\frac{7}{2}^+$. The constraints will predict

$$\Gamma[N(1950)\to\Delta\pi]=\tfrac{4}{5}\,\Gamma[\Delta(1950)\to\Delta\pi] \qquad (18)$$

separately for F-wave and H-wave contributions.

In the present work we shall treat only $\sigma=-1$ constraints, which involve fewer trajectories and are correspondingly cleaner. The highest-lying $\sigma=-1$ trajectories, moreover, lead the highest $\sigma=+1$ trajectories by at least half a unit. If the sequence (1) is found to be preferred over (2), we believe it will be on the basis of $\sigma=-1$ states.

B. A Simple Example: $\pi\Delta$ Scattering

Consider the process in Fig. 1(a):

$$\pi^+\Delta^0\to\pi^-\Delta^{++} \quad (s \text{ channel}). \qquad (19)$$

The t channel of this reaction is exotic, and thus for $t\to\infty$ the imaginary parts due to s-channel Regge trajectories must cancel one another:

$$\tfrac{1}{2}^{(+)}+\tfrac{1}{2}^{(-)}-(\tfrac{8}{5})(\tfrac{3}{2}^{(+)}+\tfrac{3}{2}^{(-)})=0. \qquad (20)$$

The residue functions are labeled by their s-channel isospin and signature τ. A further constraint comes from the reaction in Fig. 1(b):

$$\pi^-\Delta^{++}\to\pi^-\Delta^{++}, \qquad (21)$$

whose u channel is exotic. As $u\to\infty$, the contributions of positive- and negative-signature trajectories are opposite in sign,[6] leading to

$$\tfrac{1}{2}^{(+)}-\tfrac{1}{2}^{(-)}+(\tfrac{4}{5})(\tfrac{3}{2}^{(+)}-\tfrac{3}{2}^{(-)})=0. \qquad (22)$$

The spectrum used in Eq. (2) of Ref. 6 implies that for $\sigma=-1$ trajectories, $\frac{1}{2}^{(-)}=\frac{3}{2}^{(+)}=0$. This is indeed consistent with the observed states

$$\begin{array}{lll} \Delta(1236): & m^2=1.53, & J^P=\frac{3}{2}^+, \\ N(1670): & m^2=2.79, & J^P=\frac{5}{2}^-, \\ \Delta(1950): & m^2=3.80, & J^P=\frac{7}{2}^+, \text{ etc.,} \end{array} \qquad (23)$$

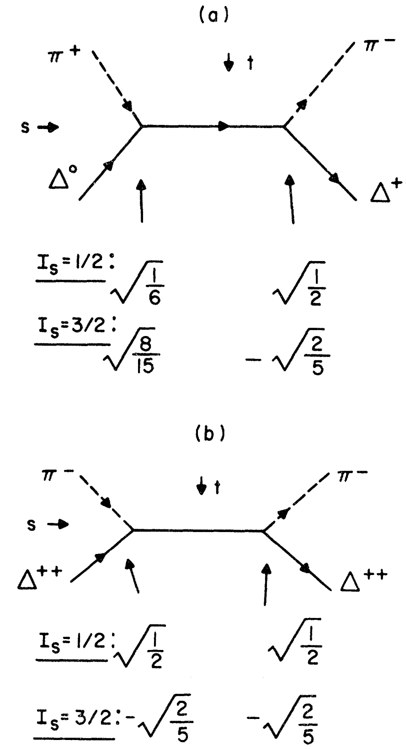

FIG. 1. Diagrams for $\pi\Delta\to\pi\Delta$. (a) Exotic in t channel. (b) Exotic in u channel. Numbers below vertices correspond to s-channel isospin Clebsch-Gordan coefficients.

which seem to lie on a pair of degenerate trajectories[14,15]:

$$\alpha_{\Delta_\delta}(s)=\alpha_{N_\beta}(s)=0.15+0.88s \qquad (24)$$

On the other hand, with such a spectrum, one cannot solve Eqs. (20) and (22) simultaneously: The first implies $\frac{1}{2}^{(+)}=(\frac{8}{5})(\frac{3}{2}^{(-)})$, while the second implies $\frac{1}{2}^{(+)}=(\frac{4}{5})(\frac{3}{2}^{(-)})$. This relative factor of 2 was first noted in Ref. 7.

If one tries to solve Eqs. (20) and (22) with *three* nonzero amplitudes, one encounters the following difficulty. All these amplitudes must be proportional to one another. At least one of these amplitudes must vanish at either the mass of $\Delta(1236)$ or of $N(1670)$, since these two states are each nondegenerate. But then the physical particle pole will be absent as well.

The simultaneous solution of Eqs. (20) and (22) then requires both $I=\frac{1}{2}$ and $\frac{3}{2}$ amplitudes of both signatures. Since these are constrained by only two equations, they need not be proportional to one another. There is thus room to impose the following conditions, based on experiment[16]:

$\Delta(1236)$ has no $N(\frac{3}{2}^+)$ companion

$$\Rightarrow \tfrac{1}{2}^{(-)}=0 \text{ at } \alpha_{\Delta_\delta}=\tfrac{3}{2}; \qquad (25)$$

TABLE II. Residue functions for $\pi\Delta$ amplitudes, labeled by isospin and signature.

$$\tfrac{3}{2}^{(+)}/\tfrac{3}{2}^{(-)}=\tfrac{1}{3}(L_- -1)$$
$$\tfrac{1}{2}^{(+)}/\tfrac{3}{2}^{(-)}=\tfrac{2}{15}(L_- +8)$$
$$\tfrac{1}{2}^{(-)}/\tfrac{3}{2}^{(-)}=\tfrac{2}{5}L_-$$

$N(1670)$ has no $\Delta(\frac{5}{2}^-)$ companion

$$\Rightarrow \tfrac{3}{2}^{(+)}=0 \text{ at } \alpha_{\Delta_\delta}=\tfrac{5}{2}\,. \quad (26)$$

These conditions can be used to our advantage.

It will be convenient to define

$$L_-\equiv\alpha_{\Delta_\delta}-\tfrac{3}{2}, \quad (27)$$

which corresponds in a quark model to the total quark orbital angular momentum for the leading $\sigma=-$ states. (These have quark spin $S_q=\frac{3}{2}$ and $J=L_-+S_q$.) One may then solve Eqs. (20) and (22) at the two points $L_-=0$ and 1. One finds that the ratios of residue functions are *different at the two points*, as they must be in view of Eqs. (25) and (26). We shall interpolate *linearly* between these two points, expressing all residues in terms of $\frac{3}{2}^{(-)}$ [which corresponds to the $\Delta(1236)$]. The result is shown in Table II.

Evaluating the ratios $\frac{1}{2}^{(-)}/\frac{3}{2}^{(-)}$ at $L_-=2$, which corresponds to physical $J^P=\frac{7}{2}^+$ particle poles in both amplitudes, one obtains the prediction

$$\Gamma[N(1950,\tfrac{7}{2}^+)\to\Delta\pi]=\tfrac{4}{5}\Gamma[\Delta(1950,\tfrac{7}{2}^+)\to\Delta\pi], \quad (18)$$

which will be discussed further in Sec. III. At $L_-=3$, one finds

$$\Gamma[N(2200,\tfrac{9}{2}^-)\to\Delta\pi]=\tfrac{11}{5}\Gamma[\Delta(2200,\tfrac{9}{2}^-)\to\Delta\pi], \quad (28)$$

which will be of interest in Sec. IV.

A model with precise functional form for the residue functions has been constructed,[7] and some of its consequences discussed.[17] As we shall show in the Appendix, the model reproduces the result of Table II exactly for $L_-=2$, and is very close to it for $L_-=3$. The result (18) also follows in the nonrelativistic quark model,[18] and is not expected[19] to be affected by relativistic extensions.[20]

The *aficionado* of $SU(6)_W$ may calculate all residue ratios between $\underline{70}$ and $\underline{56}$, $L=2$ decays from Eq. (18) and between $\underline{56}$ and $\underline{70}$, $L=3$ decays from Eq. (28). The objections raised in Ref. 19 to the use of $SU(6)_W$ will not apply to calculations of MB or MD decays of *leading* $L=2$ states (they will be purely F-wave) or *leading* $L=3$ states (G-wave decays). The solution presented in the next subsection is more general in that the scales of MB and MD decay widths are allowed to differ.

C. $Mb\to M'b'$ Solutions for $\sigma=-1$ Trajectories

We use the SU(3) crossing matrices of Ref. 11 to construct solutions obeying duality. The SU(3) amplitudes are referred to a meson-first sign convention.[21]

A number or letter will denote SU(3) quantum numbers of s-channel residues, and a superscript will indicate signature. The letters (s, a) will stand for octets coupled (symmetrically, antisymmetrically) to an MB vertex. In $MB\to M'B'$, time-reversal invariance dictates $sa=as$.

The constraints for processes (3)–(5) are summarized in Tables III–V. In solving these constraints we use the leading $\sigma=-1$ SU(3) multiplets of Eq. (1), with $S_q=\frac{3}{2}$ and $J=L_-+S_q$:

$$\begin{array}{cccc} L_-=0 & L_-=1 & L_-=2 & L_-=3 \\ \underline{10}(\tfrac{3}{2}^+)\leftrightarrow & \underline{8}(\tfrac{5}{2}^-)\leftrightarrow & \left\{\begin{matrix}\underline{10}(\tfrac{7}{2}^+)\\ \underline{8}(\tfrac{7}{2}^+)\end{matrix}\right\}\leftrightarrow & \left\{\begin{matrix}\underline{8}(\tfrac{9}{2}^-)\\ \underline{10}(\tfrac{9}{2}^-)\end{matrix}\right\}\leftrightarrow\cdots\,. \end{array} \quad (29)$$

The conditions imposed in solving the constraints equations are

$$L_-=0:\ \text{no } \underline{8}^-, \quad (30)$$

$$L_-=1:\ \text{no } \underline{10}^+, \quad (31)$$

$$\text{Universal } f/d \text{ for } \underline{8}\to MB. \quad (32)$$

The last is reasonable since all octets on the leading $\sigma=-1$ trajectory arise from the same $SU(6)_W$ multiplet ($\underline{70}$) and have the same S_q ($\frac{3}{2}$).

As in Sec. II B, we solve the equations separately at $L_-=0$ and 1, and then interpolate linearly be-

TABLE III. Constraints on $MB\to M'B'$ amplitudes arising from suppression of imaginary parts in exotic channels as t or $u\to\infty$.

No $\underline{10}_t$ or $\underline{10}_t^*$:	$\frac{1}{8}(\underline{1}^+ + \underline{1}^-)-\frac{2}{5}(ss^+ + ss^-)+\frac{1}{4}(\underline{10}^+ + \underline{10}^-)=0$
No $\underline{27}_t$:	$\frac{1}{8}(\underline{1}^+ + \underline{1}^-)+\frac{1}{5}(ss^+ + ss^-)-\frac{1}{3}(aa^+ + aa^-)-\frac{1}{12}(\underline{10}^+ + \underline{10}^-)=0$
No $\underline{10}_u$:	$\frac{1}{8}(\underline{1}^+ - \underline{1}^-)-\frac{2}{\sqrt{5}}(as^+ - as^-)-\frac{2}{5}(ss^+ - ss^-)-\frac{1}{4}(\underline{10}^+ - \underline{10}^-)=0$
No $\underline{27}_u$:	$\frac{1}{8}(\underline{1}^+ - \underline{1}^-)+\frac{1}{5}(ss^+ - ss^-)+\frac{1}{3}(aa^+ - aa^-)+\frac{1}{12}(\underline{10}^+ - \underline{10}^-)=0$

TABLE IV. Constraints on $MB \to M'D'$ amplitudes.

$$\text{No } \underline{10}_t:\ \tfrac{1}{5}\sqrt{2}\,(s^+ + s^-) + \tfrac{1}{5}\sqrt{10}\,(a^+ + a^-) - \tfrac{1}{2}(\underline{10}^+ + \underline{10}^-) = 0$$

$$\text{No } \underline{27}_t:\ \tfrac{2}{5}(s^+ + s^-) - \tfrac{2}{15}\sqrt{5}\,(a^+ + a^-) - \tfrac{1}{6}\sqrt{2}\,(\underline{10}^+ + \underline{10}^-) = 0$$

$$\text{No } \underline{27}_u:\ \tfrac{2}{5}(s^+ - s^-) + \tfrac{2}{15}\sqrt{5}\,(a^+ - a^-) + \tfrac{1}{6}\sqrt{2}\,(\underline{10}^+ - \underline{10}^-) = 0$$

tween these points. The results, shown in Table VI, have the following interesting properties:

(1) Factorization. Every ratio has the property that

$$(MB \to MD)^2 = (MB \to MB)(MD \to MD). \tag{33}$$

This comes from the fact that all the leading $\sigma = -1$ trajectories ($S_q = \frac{3}{2}$) are either decimets from $\underline{56}$ or octets from $\underline{70}$. For the leading $\sigma = +1$ trajectories, such a relation would not hold in general.

(2) Zeros. The ratios satisfy Eqs. (30) and (31).

(3) Value of f/d. The universal f/d value turns out to equal the $SU(6)_W$ value, $-\frac{1}{3}$.[22]

(4) Weak L_- dependence of $\underline{8}^+/\underline{10}^-$. The two trajectories taken in the "broken duality" solution to have an L_--independent ratio have a ratio here which is only weakly dependent on L_-. The zero at $L_- = -8$ is not present in the model of Ref. 7, and represents only an artifact of our linear extrapolation.

The next two sections apply Table VI to some experimental tests.

III. THE $\frac{7}{2}^+$ OCTET

The equations of Table VI may be evaluated at $L_- = 2$ to give the ratios of octet to decimet couplings of MB and MD:

$MB \to MB$:

$$ss^-/\underline{10}^- = \tfrac{5}{16}$$

$$aa^-/ss^- = \tfrac{1}{5}, \tag{34}$$

$$as^-/ss^- = 1/\sqrt{5}\ ;$$

$MB \to MD$:

$$s^-/\underline{10}^- = 5/8\sqrt{2}\,, \tag{35}$$

$$a^-/s^- = 1/\sqrt{5}\ ,$$

$MD \to MD$:

$$\underline{8}^-/\underline{10}^- = \tfrac{5}{8}\ . \tag{36}$$

We take as inputs typical widths for a $\frac{7}{2}^+$ decimet

TABLE V. Constraints on $MD \to M'D'$ amplitudes.

$$\text{No } \underline{27}_t:\quad (\underline{8}^+ + \underline{8}^-) - \tfrac{5}{4}(\underline{10}^+ + \underline{10}^-) = 0$$

$$\text{No } \underline{27}_u \text{ or } 35_u:\quad (\underline{8}^+ - \underline{8}^-) + \tfrac{5}{8}(\underline{10}^+ - \underline{10}^-) = 0$$

TABLE VI. Linear interpolation of duality constraints for $\sigma = -1$ trajectories. The residue $\underline{10}^-$ is used as a reference since it corresponds to the lowest physical state.

$MB \to M'B'$:

$$\underline{10}^+/\underline{10}^- = \tfrac{1}{3}(L^- - 1)$$

$$ss^+/\underline{10}^- = \tfrac{5}{96}(L^- + 8)$$

$$ss^-/\underline{10}^- = \tfrac{5}{32}L^-$$

$$aa^\pm/ss^\pm = \tfrac{1}{5}$$

$$as^\pm/ss^\pm = 1/\sqrt{5} \quad \longleftrightarrow f/d = -\tfrac{1}{3}$$

$MB \to M'D'$:

$$\underline{10}^+/\underline{10}^- = \tfrac{1}{3}(L^- - 1)$$

$$s^+/\underline{10}^- = 5(L^- + 8)/48\sqrt{2}$$

$$s^-/\underline{10}^- = 5L^-/16\sqrt{2}$$

$$a^\pm/s^\pm = 1/\sqrt{5} \longleftrightarrow f/d = -\tfrac{1}{3}$$

$MD \to M'D'$:

$$\underline{10}^+/\underline{10}^- = \tfrac{1}{3}(L^- - 1)$$

$$\underline{8}^+/\underline{10}^- = \tfrac{5}{48}(L_- + 8)$$

$$\underline{8}^-/\underline{10}^- = \tfrac{5}{16}L_-$$

member into MB and MD

$$\Gamma[\Delta(1950) \to N\pi] \equiv \Gamma_1 \quad (\text{c.m. 3-momentum } p_1), \tag{37}$$

$$\Gamma[\Delta(1950) \to \Delta\pi] \equiv \Gamma_2 \quad (\text{c.m. 3-momentum } p_2). \tag{38}$$

Experimentally both these numbers are about 100 MeV.[16] We shall express all $\Gamma(R \to MB)$ as percentages of Γ_1, and all $\Gamma(R \to MD)$ as percentages of Γ_2, so that these values also reflect, approximately, widths in MeV.

In $SU(6)_W$, Γ_1 and Γ_2 are related. We shall not use such a relation here. Experimentally Γ_2 seems at present to be at least a factor of 3 larger than the value predicted by $SU(6)_W$ on the basis of Γ_1.[19]

The decimet states will be assumed to follow an equal-spacing rule, and the octet states to have a mass-splitting pattern like that of the $\frac{5}{2}^-$ octet. We then take for the masses

$\underline{10}$	$\underline{8}$	
$\Delta(1950)$	$N(1950)$	
$\Sigma_{10}(2030)$	$\Sigma_8(2030)$	(39)
$\Xi_{10}(2110)$	$\Lambda(2100)$	
$\Omega(2190)$	$\Xi_8(2200)$	

in order to have definite values when applying barrier factor corrections to partial widths. The partial widths Γ are determined in terms of Γ_1 or Γ_2 by (p = c.m. 3-momentum)

$$\Gamma/\Gamma_i = (C/C_i)^2(p/p_i)^7(M_i/M)^2 \quad (i=1 \text{ or } 2). \tag{40}$$

Here C/C_i refers to the product of an SU(3) isoscalar factor ratio and an amplitude ratio based on Eq. (34) or (36):

$R\in \underline{8}$:

$$C(R\to MB)/C_1 = (\tfrac{5}{16})^{1/2}\left[\begin{pmatrix}8 & 8 & 8\\ R & M & B\end{pmatrix}_s + \frac{1}{\sqrt{5}}\begin{pmatrix}8 & 8 & 8\\ R & M & B\end{pmatrix}_a\right]\Big/\begin{pmatrix}10 & 8 & 8\\ \Delta & \pi & N\end{pmatrix}, \tag{41}$$

$$C(R\to MD)/C_2 = \begin{pmatrix}8 & 8 & 10\\ R & M & D\end{pmatrix}(\tfrac{5}{8})^{1/2}\Big/\begin{pmatrix}10 & 8 & 10\\ \Delta & \pi & \Delta\end{pmatrix}, \tag{42}$$

$R\in \underline{10}$:

$$C(R\to MB)/C_1 = \begin{pmatrix}10 & 8 & 8\\ R & M & B\end{pmatrix}\Big/\begin{pmatrix}10 & 8 & 8\\ \Delta & \pi & N\end{pmatrix}, \tag{43}$$

$$C(R\to MD)/C_2 = \begin{pmatrix}10 & 8 & 8\\ R & M & D\end{pmatrix}\Big/\begin{pmatrix}10 & 8 & 10\\ \Delta & \pi & \Delta\end{pmatrix}. \tag{44}$$

The terms on the right-hand side in Eqs. (41)–(44) are isoscalar factors with the meson-first phase convention.[21] R denotes the decaying resonance.

Note that Eq. (40) incorporates a zero-radius barrier factor appropriate for *F waves*. This is correct for MB decays; for MD decays, one might have had (in principle) H waves as well. Experimentally the H-wave contribution is small,[23] and we take it to be zero.

The calculated ratios Γ/Γ_1 and Γ/Γ_2 are shown in Tables VII and VIII, respectively. For each resonance R we show the modes expected to be most easily seen. The sign of C/C_i is of interest when discussing interference experiments. We

TABLE VII. Predicted partial widths $\Gamma(R\to MB)$ as percentages of $\Gamma_1 = \Gamma[\Delta(1950)\to N\pi]$, for various resonances R with $J^P = \frac{7}{2}^+$.

Resonance R	Mode	$(C/C_1)^2$	Sgn(C/C_1)	Γ/Γ_1 (pred.) (%)	Γ_{exp} [a] (MeV)
$\Delta(1950)$	πN	1	+	100 (def.)	~100
	$K\Sigma$	1	−	3.5	~4
$N(1950)$	πN	$\frac{1}{8}$	−	13	~25 [b]
	$K\Sigma$	$\frac{1}{2}$	+	1.8	···
	ηN	$\frac{1}{8}$	−	3.4	···
$\Sigma_{10}(2030)$	$\bar{K}N$	$\frac{1}{3}$	+	14	~30
	$\pi\Lambda$	$\frac{1}{2}$	+	31	~30
	$\pi\Sigma$	$\frac{1}{3}$	+	12	~5
$\Sigma_8(2030)$	$\bar{K}N$	$\frac{1}{3}$	−	14	···
	$\pi\Lambda$	$\frac{1}{8}$	+	7.7	···
	$\pi\Sigma$	$\frac{1}{12}$	+	2.9	···
$\Lambda_8(2100)$	$\bar{K}N$	0	(0)	0	···
	$\pi\Sigma$	$\frac{3}{8}$	−	22	···
	$\eta\Lambda$	$\frac{1}{8}$	−	2.9	···
$\Xi_{10}(2110)$	$\pi\Xi$	$\frac{1}{2}$	+	14	···
	$\bar{K}\Lambda$	$\frac{1}{2}$	+	18	···
	$\bar{K}\Sigma$	$\frac{1}{2}$	+	9.3	···
$\Xi_8(2200)$	$\pi\Xi$	$\frac{1}{2}$	+	21	···
	$\bar{K}\Lambda$	$\frac{1}{8}$	−	7.8	···
	$\bar{K}\Sigma$	$\frac{1}{8}$	−	3.7	···
$\Omega(2190)$	$\bar{K}\Xi$	2	+	25	···

[a] From Ref. 16.

[b] From Ref. 24.

TABLE VIII. Predicted partial widths $\Gamma(R \to MD)$ as percentages of $\Gamma_2 \equiv \Gamma[\Delta(1950) \to \Delta\pi]$.

Resonance R	Mode	$(C/C_2)^2$	Sgn(C/C_2)	Γ/Γ_2 (pred.) (%)	$\Gamma_{\rm exp}$ [a] (MeV)
$\Delta(1950)$	$\pi\Delta$	1	+	100 (def.)	~100
	KY_1^*	$\frac{2}{5}$	+	4.4×10^{-4}	b
$N(1950)$	$\pi\Delta$	$\frac{4}{5}$	−	80	...
$\Sigma_{10}(2030)$	πY_1^*	$\frac{8}{15}$	+	29	...
	$\bar{K}\Delta$	$\frac{8}{15}$	+	17	...
$\Sigma_8(2030)$	πY_1^*	$\frac{2}{15}$	+	7.2	...
	$\bar{K}\Delta$	$\frac{8}{15}$	−	17	...
$\Lambda_8(2100)$	πY_1^*	$\frac{3}{5}$	−	61	...
$\Xi_{10}(2110)$	$\pi\Xi^*$	$\frac{1}{5}$	+	5.4	...
	$\bar{K}Y_1^*$	$\frac{4}{5}$	+	10	...
$\Xi_8(2200)$	$\pi\Xi^*$	$\frac{1}{5}$	+	3.6	...
	$\bar{K}Y_1^*$	$\frac{1}{5}$	−	13	...
$\Omega(2190)$	$\bar{K}\Xi^*$	$\frac{4}{5}$	+	8.5	...

[a] From Ref. 16.
[b] See text: We regard the claim[16] for a large value as spurious.

would like to point out several interesting features of the tables.

(a) Smallness of $\Gamma[N(1950)\to\pi N]$. In the prediction

$$\frac{\Gamma[N(1950)\to\pi N]}{\Gamma[\Delta(1950)\to\pi N]}=\frac{1}{8}, \tag{45}$$

centrifugal-barrier factors cancel out. This result is thus expected to be quite reliable. It has been obtained previously in the harmonic-oscillator quark model.[18] On the other hand, such a model also predicts

$$\frac{\tilde{\Gamma}[\Delta(1950)\to\pi\Delta]}{\tilde{\Gamma}[\Delta(1950)\to\pi N]}=\frac{15}{8}, \tag{46}$$

where $\tilde{\Gamma}$ is Γ divided by a suitable barrier factor. This is the $SU(6)_W$ prediction to which we referred earlier, and which is not implied in the present discussion

The smallness of the πN partial width of $N(1950)$ is expected to be reflected in a small elasticity for this state, since the $\pi\Delta$ partial width of this state is probably around 80 MeV. In $\sigma_T(\pi N)_{I=1/2}$, there is certainly no bump at this mass, which lies right between the prominent $N(1688;\frac{5}{2}^+)$ and $N(2190;\frac{7}{2}^-)$ resonances. Nonetheless, a recent phase-shift analysis[24] continues to suggest the *possible* existence of an F_{17} πN resonance with mass around 2 GeV. The parameters of this resonance are not inconsistent with Eq. (45). If anything, the experimental πN coupling is somewhat larger than that predicted in Eq. (45):

$$\Gamma_{\rm exp}[F_{17}(2000)\to\pi N]\simeq 30 \text{ MeV}. \tag{47}$$

Such a large value, however, would begin to be detectable as a bump in $\sigma_T(\pi N)_{I=1/2}$.[25]

(b) Large $\pi\Delta$ *mode of* $N(1950)$. The prediction (18), entailing (if $\Gamma[\Delta(1950)\to\pi\Delta]\simeq 100$ MeV)

$$\Gamma[N(1950)\to\pi\Delta]\simeq 80 \text{ MeV}, \tag{48}$$

implies that this state should show up in phase-shift analyses of $\pi N\to\pi\Delta$. Referring to the signs in Tables VII and VIII, we expect amplitudes for

$$\pi[I_3^{(1)}]N[I_3^{(N)}]\to\pi[I_3^{(2)}]\Delta[I_3^{(\Delta)}]$$

to be proportional to

$$A(\pi N\to\pi\Delta)=(1I_3^{(1)}\tfrac{1}{2}I_3^{(N)}|\tfrac{3}{2}I_3)(1I_3^{(2)}\tfrac{3}{2}I_3^{(\Delta)}|\tfrac{3}{2}I_3)m[m^2-s-im\Gamma_\Delta]^{-1}$$
$$+\frac{1}{\sqrt{10}}(1I_3^{(1)}\tfrac{1}{2}I_3^{(N)}|\tfrac{1}{2}I_3)(1I_3^{(2)}\tfrac{3}{2}I_3^{(\Delta)}|\tfrac{1}{2}I_3)m[m^2-s-im\Gamma_N]^{-1}, \tag{49}$$

where $I_3 \equiv I_3^{(1)} + I_3^{(N)} = I_3^{(2)} + I_3^{(\Delta)}$, $m=1950$ MeV, and Γ_Δ, Γ_N are total widths. The purely $I=\frac{3}{2}$ contribution has been measured rather well in the reaction $\pi^+p \to \pi^0\Delta^{++}$,[23] corresponding to a value at resonance of

$$A(\pi^+p \to \pi^0\Delta^{++})|_{\rm res} = -i(\tfrac{3}{5})^{1/2}/\Gamma_\Delta\ . \qquad (50)$$

Equation (39) implies that we should expect, at the resonance peak,

$$A(\pi^-p \to \pi^+\Delta^-)|_{\rm res} = i(\tfrac{2}{15})^{1/2}\left(\frac{1}{\Gamma_\Delta} - \frac{1}{2\Gamma_N}\right), \qquad (51)$$

i.e., *strong destructive interference* in $\pi^-p \to \pi^+\Delta^-$. If $\Gamma_\Delta \simeq 2\Gamma_N$, as suggested by Tables VII and VIII, this interference could even lead to complete suppression of the $\Delta(1950)$ bump in this reaction. In the absence of $N(1950)$, one would expect the resonance contributions in reactions (50) and (51) to be in the ratio of 9 to 2, respectively.

In the reaction $\pi^-p \to \pi^-\Delta^+$, one would expect constructive interference between $\Delta(1950)$ and $N(1950)$:

$$A(\pi^-p \to \pi^-\Delta^+)|_{\rm res} = -\tfrac{2}{3}i(\tfrac{2}{5})^{1/2}\left(\frac{1}{\Gamma_\Delta} + \frac{1}{4\Gamma_N}\right)\ . \qquad (52)$$

We see that the effects of this constructive interference will be rather mild. Equation (49) implies some deviation from a single Breit-Wigner shape for $\pi^-p \to \pi^\pm\Delta^\mp$ around 1950 MeV as long as $\Gamma_N \neq \Gamma_\Delta$.

(c) Detection of $N(1950)$ *in* $\pi N \to K\Sigma$. A similar discussion leads to the predictions

$$A(\pi^+p \to K^+\Sigma^+)|_{\rm res} = i/\Gamma_\Delta\ , \qquad (53)$$

$$A(\pi^-p \to K^+\Sigma^-)|_{\rm res} = \tfrac{1}{3}i[1/\Gamma_\Delta - 1/2\Gamma_N]\ , \qquad (54)$$

$$A(\pi^-p \to K^0\Sigma^0)|_{\rm res} = (\tfrac{1}{3}i\sqrt{2})[1/\Gamma_\Delta + 1/4\Gamma_N]\ . \qquad (55)$$

The presence of the $\Delta(1950)$ in reaction (53) is well known.[26] Note the similarity of this pattern to Eqs. (50)–(52). Experimentally there seems to be little evidence for a bump at 1950 MeV in *either* $\pi^-p \to K^+\Sigma^-$ or $\pi^-p \to K^0\Sigma^0$.[27] Phase-shift analyses of these reactions would be extremely interesting.

Note that in Eqs. (51) and (54), the suppression of the $\Delta(1950)$ peak is correlated with exotic t-channel exchange.

(d) Smallness of $\Gamma[\Lambda_8(2100) \to \bar{K}N]$; *alternative decays*. We have predicted the existence of a $J^P = \frac{7}{2}^+$ Λ resonance whose $\bar{K}N$ coupling vanishes identically since $f/d = -\frac{1}{3}$. Just as in the case of $\Lambda(1830, \frac{5}{2}^-)$,[16] one might expect some experimental deviation from this value, allowing a small but nonzero $\bar{K}N$ coupling. The dominant decay modes should be $\pi\Sigma$ and πY_1^*:

$$\Gamma[\Lambda_8(2100, \tfrac{7}{2}^+) \to \pi\Sigma] \simeq 22 \text{ MeV}, \qquad (56)$$

$$\Gamma[\Lambda_8(2100, \tfrac{7}{2}^+) \to \pi Y_1^*] \simeq 60 \text{ MeV}. \qquad (57)$$

An F_{07} resonance in $\bar{K}N \to \pi\Sigma$, of mass 2015 MeV, appears in one analysis.[28] This would be an acceptable candidate for the state predicted here.

(e) $\Sigma(2030)$ *decays*.

By comparing signs for $\Sigma_8(2030)$ and $\Sigma_{10}(2030)$ couplings in Tables VII and VIII, one sees that the two states will interfere constructively in $\bar{K}N \to (\bar{K}N, \bar{K}\Delta)$ but destructively in $\bar{K}N \to (\pi\Lambda, \pi\Sigma$, and $\pi Y_1^*)$. In every case where destructive interference is predicted, duality graphs[29] predict a net cancellation of imaginary parts as $s \to \infty$, t fixed. The two $\Sigma(2030)$ states thus are expected to participate in *local* averaging. (For $\bar{K}N \to N\bar{K}$, $s \to \infty$, u fixed, the corresponding averaging requires the additional contribution of negative-parity s-channel states.) This provides a check that our signs are correct.

Experimentally the $\pi\Sigma/\bar{K}N$ ratio for $\Sigma(2030)$ seems to be lower[16] than one would expect from assigning $\Sigma(2030)$ to Σ_{10}. However, the $\pi\Lambda/\bar{K}N$ ratio is consistent with the decimet assignment. No mixing or interference effects between Σ_{10} and Σ_8 can alter the predicted $\pi\Lambda/\pi\Sigma$ ratio, which seems to underestimate the observed value.[30]

Because of the possibility of mixing as well as interference, we hesitate to make a full analysis at present. At the very least, one can expect the resonance shapes to be different in the two cases $\bar{K}N \to \bar{K}N$ and $\bar{K}N \to \pi(\Lambda$ or $\Sigma)$.

(f) Ξ *decays*. The dominant modes depend on the SU(3) representation, as one can see from Tables VII and VIII.

(g) Some SU(3) *remarks*. The Regge recurrence of the $\Omega^-(1670)$ is predicted to have an appreciable $\bar{K}\Xi$ partial width. The $\Delta(1950) \to KY_1^*$ mode is unlikely to be as large (~3 MeV) as claimed in Ref. 16. Probably the bump in this channel comes from the P-wave decay of the P_{31} or F_{35} resonance, both of which lie nearby.[16]

There are thus several interesting tests for whether the leading $\frac{7}{2}^+$ baryons near 2 GeV consist of just a decimet or of a decimet and an octet.

IV. $\Delta(2200, \frac{9}{2}^-)$.

The Regge recurrence of the $D_{15}(1670, \frac{5}{2}^-)$ πN resonance should be degenerate with an $I=\frac{3}{2}$ state if the spectrum (1) is correct. We can use duality to estimate the couplings of this state.

It is necessary first of all to confirm the existence of the state $G_{19}(2200)$ predicted as the recurrence of the $D_{15}(1670)$ on the basis of Eq. (24). Such a state would contribute to the dip in backward π^-p scattering at $p_L = 2.15$ GeV/c (Ref. 31) and to the bump in $\sigma_T(\pi^-p)$ now ascribed purely to the $G_{17}(2190)$ resonance.

When extrapolated to $L_- = 3$, Table VI predicts

$$\frac{\Gamma[N(2200)\to\pi N]}{\Gamma[\Delta(2200)\to\pi N]}=\frac{11}{32} \qquad (58)$$

as well as Eq. (28).

In contrast with the situation at $J^P=\frac{7}{2}^+$, it is the Δ which has the bigger πN partial width at $\frac{9}{2}^-$, by a factor of 3. The $\pi\Delta$ channel is relatively useless in looking for the $\Delta(2200)$; the elastic channel is preferable.

A $\Delta(2200, \frac{9}{2}^-)$ would show up most directly as a deep dip in backward π^+p elastic scattering. The existence of this dip has been known for a long time,[32] but it was usually ascribed to the "blank space" between the peaks due to $\Delta(1950, \frac{7}{2}^+)$ and $\Delta(2420, \frac{11}{2}^+)$.

One can make an order-of-magnitude estimate of $\Gamma[N(2200)\to\pi N]$ and $\Gamma[\Delta(2200)\to\pi N]$ by assuming that the common unknown factor in the residue functions depends exponentially on L_-. In this way, by taking the geometric mean of $\Gamma[\Delta(1950)\to\pi N]$ and $\Gamma[\Delta(2420)\to\pi N]$ we obtain an estimate of 10^- at $L_-=3$. The predictions of Table VI at $L_-=3$ then imply

$$\Gamma[N(2200)\to\pi N]=\tfrac{11}{48}\{\Gamma[\Delta(1950)\to\pi N]\}^{1/2} \times\{\Gamma[\Delta(2420)\to\pi N]\}^{1/2}, \qquad (59)$$

$$\Gamma[\Delta(2200)\to\pi N]=\tfrac{2}{3}\{\Gamma[\Delta(1950)\to\pi N]\}^{1/2} \times\{\Gamma[\Delta(2420)\to\pi N]\}^{1/2}. \qquad (60)$$

Barrier factors roughly cancel out in this procedure; they may be treated more exactly by applying (59) and (60) instead to

$$\tilde{\Gamma}\equiv\frac{\Gamma}{(p/p_0)^{2l+1}}\;,$$

where p_0 is any suitable scale factor and p is the magnitude of the c.m. 3-momentum.

The inelasticities of the two predicted $\frac{9}{2}^-$ states may be estimated using $SU(6)_W$, since these particular estimates all refer to decays involving a single partial wave (G wave).[19] The results imply

$$\frac{\Gamma(\Delta\to\Delta\pi)}{\Gamma(\Delta\to N\pi)}\simeq\frac{1}{3} \qquad (61)$$

and

$$\frac{\Gamma(N\to\Delta\pi)}{\Gamma(N\to N\pi)}\simeq 2\,, \qquad (62)$$

when zero-radius barrier factors are used. The $SU(6)_W$ ratio

$$\frac{\Gamma(\Delta\to\Delta\pi)}{\Gamma(\Delta\to N\pi)}\Big/\frac{\Gamma(N\to\Delta\pi)}{\Gamma(N\to N\pi)}=\frac{5}{32} \qquad (63)$$

is independent of barrier factors and indicates that the "leakage" of the G_{39} resonance $\Delta(2200)$ into $\pi\Delta$ should be considerably less than that of the $G_{19}\,N(2200)$.

One might expect the G_{39} to have some easily observed decay modes besides πN and $\pi\Delta$. Some time ago an enhancement at $p_L=2.26$ GeV/c was noted[33] in the cross section for forming a $\pi N(1670)$ system in the $I=\frac{3}{2}$ πN channel. The exact nature of the $N(1670)$ was not specified, but it appeared to be produced predominantly in the forward direction, indicating baryon-exchange or direct-channel effects. Perhaps the predicted G_{39} state is related to this effect.

V. DISCUSSION AND CONCLUSIONS

We have shown in some detail the existence of a consistent solution to *all* duality constraints for baryon exchange. This does not solve all problems for duality; for instance, the elastic baryon-antibaryon system remains a puzzle. The difficulty is most easily seen in $\bar{\Delta}$-Δ scattering: Both s and t channels must have only $I=0$ or 1, with $I=2$ and 3 suppressed. This cannot be done without the $I=0$ and 1 contributions vanishing as well. The alternatives are as follows:

(a) Exotic mesons exist,[3] coupled only to baryon-antibaryon final states, or

(b) duality does not hold for nondiffractive $\bar{b}b\to\bar{b}b$ amplitudes; their saturation by resonances is a poor approximation.

Very likely (b) is true. Nonetheless, one is not thereby required to reject the usefulness of $\bar{b}b\to MM$ constraints. Indeed, duality diagrams[29] show quite clearly that without exotic mesons, duality will fail for $\bar{b}b\to\bar{b}b$. At the same time, these diagrams hint that a dual solution should exist for $\bar{b}b\to MM$. Moreover, such a solution should have properties in common with the quark model. We reiterate that this solution has already been constructed in Ref. 7, and all we have done here is to divest it of some of its seeming model dependence and present some experimental implications.

It should be pointed out that the saturation of $\bar{b}b$ imaginary parts by resonances is still an unproven assumption. Indeed, both $\bar{b}b\to MM$ (Ref. 34) and $\bar{b}b\to\bar{b}b$ systems seem to have remarkably few prominent resonances. However, the possibility of a large number of overlapping states, perhaps even with similar quantum numbers, is not excluded by the data.

We have not discussed two very important areas: The $\sigma=+1$ constraints and backward meson-baryon scattering.

One can obtain spectroscopic predictions for $\sigma=+1$ from those for $\sigma=-1$ states using $SU(6)_W$, or its relaxed version presented in Ref. 19. We have not found tests of the existence of $\underline{70}$, $L=2$ and $\underline{56}$, $L=3$ multiplets, based on this approach, which are as clean-cut as those already presented. On the other hand, the $\sigma=+1$ constraints are of interest with respect to one prediction they make for backward meson-baryon scattering.

A standing problem confronting any dual theory of baryon exchange [35-37] is the prediction of a trajectory degenerate with the N_α, i.e., the N_γ, whose contribution to backward πN scattering may be expected to fill the dip in the $I=\frac{1}{2}$ exchange amplitude.[38]

We have attempted to solve the exact-duality constraints for $MB \to M'B'$ with a pattern of zeros, f/d ratios, and other $SU(6)_W$ requirements which appear to be the most reasonable for the $\sigma=1$ trajectories. The assumptions may be summarized by saying that of the states with $J=L+\frac{1}{2}$, we expect those with $S_q=\frac{1}{2}$ to be on higher trajectories than those with $S_q=\frac{3}{2}$.[20] On the other hand, we ignore differences among different SU(3) representations with $J=L+\frac{1}{2}$, $S_q=\frac{1}{2}$. A corollary of this is that $\underline{10}^\pm/\underline{1}^\pm=\frac{1}{18}$ all along the trajectory, since both of these will come only from $\underline{70}$ with the above assumption.

As before, a linear interpolation of the constraints is made, with the amplitude ss^+ serving as a reference. If this is done, one finds, for $\pi N \to N\pi$,

$$\frac{\beta_{N_\gamma}}{\beta_{N_\alpha}}=\frac{0.21+0.24L_+}{1+0.1L_+} . \tag{64}$$

This expression vanishes very close to $L_+=-1$, the position of the dip in $d\sigma/du(\pi^+p\to p\pi^+)$. Such a zero is not expected in general in other N-exchange processes, since the amplitudes ss, as, and aa turn out to violate factorizability rather badly:

$$\begin{aligned} ss^-/ss^+&=0.34L_+-0.17,\\ as^-/ss^+&=-0.10L_+-0.30,\\ aa^-/ss^+&=0.32L_++0.57 . \end{aligned} \tag{65}$$

One can construct models, however, similar to that in Ref. 7, based on the sequence $\underline{8}^+ \to \underline{1}^-, \underline{8}^- \to \underline{8}^+, \underline{1}^+, \ldots$, for which the vanishing of *all* octet amplitudes at $L_+=-1$ occurs simply as a result of suppressing the $\underline{1}^+$ at $L_+=0$. (The construction of such a model for $\sigma=-1$ constraints is shown in the Appendix.) Hence all one can say regarding the result (64) is that it is an interesting example of how constraints which vary along the trajectory can suit the needs of backward meson-baryon scattering fits. On the other hand, the effects of Regge cuts in such fits is of course a matter of speculation.[37]

The choice of $\underline{10}^-$ as a reference amplitude for the solutions was made arbitrarily, since it corresponds to the best-known resonances. A more general parametrization of both positive and negative signature decimet and octet amplitudes may be made.[39] In this case all amplitudes are taken as polynomials in L_- of the lowest possible order consistent with the constraints. The general forms may be constructed easily based on Tables III–V and on the zeros we have assumed. There are further points at which all amplitudes must vanish, of course: For negative integer L_- the signature factors

$$(\pm 1-e^{-i\pi L_-})/\sin\pi L_- \equiv \xi(\tau)$$

have poles which must be canceled by an over-all multiplicative factor such as $1/\Gamma(L_-+1)$.

The general form of solutions in Ref. 39 is then

$$\begin{aligned} \underline{10}^- &= A(1+\rho L_-),\\ \underline{10}^+ &= \tfrac{1}{3}A(L_--1),\\ \underline{8}_i^- &= 3AB_iL_-(1+\rho),\\ \underline{8}_i^+ &= AB_i[8+(1+9\rho)L_-], \end{aligned} \tag{66}$$

with

$$\rho \geq 0,$$

where A is an over-all factor with zeros at $L_-=-1, -2, \ldots$, and $\underline{8}_i^\pm$ refers to *any* of the octet amplitudes in Tables III–V, whose choice will determine the value of the constant B_i. In $MD \to MD$, $B=\frac{5}{48}$; in $MD \to MB$, $B_s=\sqrt{5}\,B_a=\frac{5}{48\sqrt{2}}$; in $MB \to MB$, $B_{ss}=\sqrt{5}\,B_{sa}=5B_{aa}=\frac{5}{96}$.

In discussing backward meson-nucleon scattering, it was found[39] that the choice $\rho=0$, made here and in our Louvain lecture,[40] did not permit a satisfactory fit to the data. On the other hand, the case $\rho \simeq 1$ has some merit, which we discuss briefly.

From the double zero near $\alpha_{N_\alpha}=\frac{1}{2}$ of the interference between $I=\frac{1}{2}$ and $I=\frac{3}{2}$ exchange in backward πN scattering,[38] it appears that the imaginary part of the $I=\frac{3}{2}$ amplitude vanishes at this point. This can be arranged if $\underline{10}^- - \underline{10}^+ \simeq 0$ there. (The negative sign comes from the opposite relative parity of the two trajectories.) With $\rho=0$, $\underline{10}^-$ and $\underline{10}^+$ have opposite sign for $L_-<1$, but if one takes $\rho\simeq 1$ the desired cancellation can indeed be achieved.[39]

The choice $\rho \simeq 1$ does not by itself solve the problem[36] of the relative phase of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ exchange amplitudes at 180°. The most naive application of $N_\alpha - \Delta_\delta$ exchange to $\pi N \to N\pi$, without any zeros in residue functions between particle poles and $u \simeq 0$, seems to give the correct relative phase (roughly 60°).[38] It does not seem to be possible, with any choice of ρ close to unity, to reproduce this phase in the present solution with N_α and $\Delta_\delta + \Delta_\beta$ alone. These difficulties are alluded to in Ref. 39.[41] Moreover, whereas $\sigma = +1$ decimet contributions are not *required* in solving $\sigma = +1$ constraints, they may be present. This would alter the choice $\rho \simeq 1$.

With the choice $\rho \neq 0$, several of our conclusions must be modified. The estimates of Sec. III for partial widths of the $\frac{7}{2}^+$ octet are based on ratios such as

$$\{\underline{8}^-/\underline{10}^-\}_{\rho\neq 0, L_-=2} = \frac{1+\rho}{1+2\rho}\{\underline{8}^-/\underline{10}^-\}_{\rho=0, L_-=2}\ , \quad (67)$$

so that all our predicted octet partial widths must be multiplied by $(1+\rho)/(1+2\rho)$, or $\frac{2}{3}$ when $\rho = 1$.

Similar ratios of interest when $L_- = 3$ are

$$\{\underline{8}^+/\underline{10}^-\}_{\rho\neq 0, L_-=3} = \frac{11+27\rho}{11+33\rho}\{\underline{8}^+/\underline{10}^-\}_{\rho=0, L_-=3} \quad (68)$$

and

$$\{\underline{10}^+/\underline{10}^-\}_{\rho\neq 0, L_-=3} = \frac{1}{1+3\rho}\{\underline{10}^+/\underline{10}^-\}_{\rho=0, L_-=3}\ . \quad (69)$$

We thus expect the predictions of $\Gamma[N(2200) \to \pi N, \pi\Delta]$ in Sec. IV to be fairly reliable:

$$\Gamma[N(2200) \to N\pi] \simeq \frac{1+2.45\rho}{(1+2\rho)^{1/2}(1+4\rho)^{1/2}}\ (11\ \text{MeV}) \simeq 10\ \text{MeV} \quad \text{for } \rho \simeq 1, \quad (70)$$

where we have taken $\Gamma[\Delta(1950, 2420) \to \pi N] = (100, 30)$ MeV, respectively, in applying Eq. (59), and, using $SU(6)_W$ as mentioned,

$$\Gamma[N(2200) \to \Delta\pi] \simeq \frac{1+2.45\rho}{(1+2\rho)^{1/2}(1+4\rho)^{1/2}}\ (22\ \text{MeV}) \simeq 20\ \text{MeV} \quad \text{for } \rho \simeq 1. \quad (71)$$

As noted, the condition $\rho \simeq 1$ in Eqs. (70) and (71) follows only from applying the form (66) to backward meson-baryon scattering,[39] and is not a necessary feature of the constraint equations.

Similar estimates for $\Delta(2200)$ decays are much more sensitive to the value of ρ:

$$\Gamma[\Delta(2200) \to N\pi] \simeq \frac{1}{(1+2\rho)^{1/2}(1+4\rho)^{1/2}}\ (32\ \text{MeV}) \simeq 8\ \text{MeV} \quad \text{for } \rho \simeq 1, \quad (72)$$

and, again using $SU(6)_W$,

$$\Gamma[\Delta(2200) \to \Delta\pi] \simeq \frac{1}{(1+2\rho)^{1/2}(1+4\rho)^{1/2}}\ (11\ \text{MeV}) \simeq 3\ \text{MeV} \quad \text{for } \rho \simeq 1\ . \quad (73)$$

Recent attempts[42] to fit new data[43] on backward elastic π^+p scattering near $E_{\text{c.m.}} = 2.2$ GeV seem to require *some* negative-parity resonance near this mass. The cross section at 180° depends only on Γ_{tot} and $(J+\frac{1}{2})\Gamma_{\pi N}/\Gamma_{\text{tot}}$. One cannot extract $\Gamma_{\pi N}$ without an assumption about the spin, which we predict should be $\frac{9}{2}$.

As this manuscript was in preparation, preliminary data on $\pi N \to \pi\pi N$ were brought to our attention[44] which suggest that the partial width $\Gamma_2 \equiv \Gamma[\Delta(1950) \to \pi\Delta] \simeq 100$ MeV quoted in Ref. 16 may be an overestimate by as much as a factor of 3 or 4, as a result of a sizable ρN content of the $\pi\pi N$

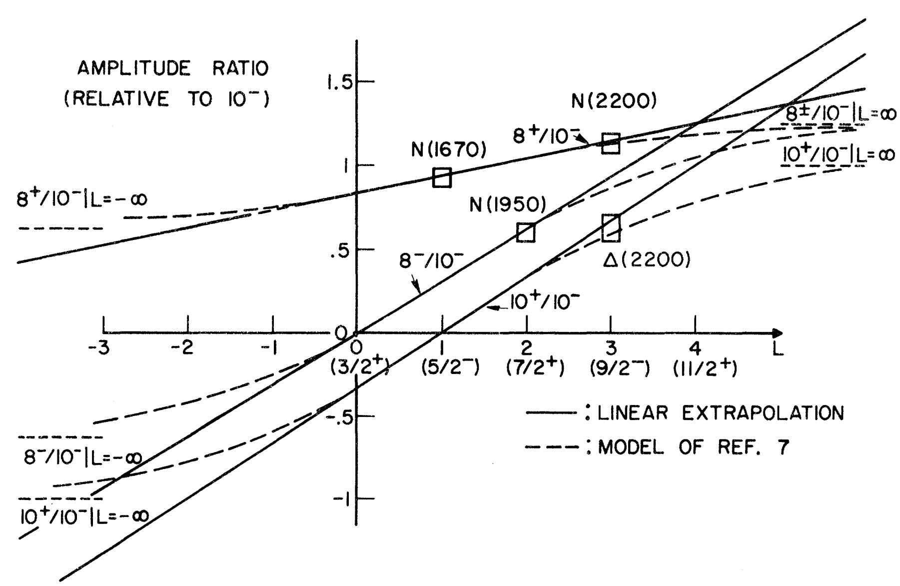

FIG. 2. Comparison of linear and quark-model (Ref. 7) interpolations of duality constraints for $MD \to M'D'$ residue ratios.

mode. A smaller value would certainly be more in accord with $SU(6)_W$, making the estimates in Secs. IV and V more believable. It would reduce, correspondingly, all the other partial-width predictions quoted here for $\frac{7}{2}^+$ states into MD. The percentages in Table VIII would, of course, remain the same.

To summarize, the spectroscopic predictions of exact duality for baryons turn out to be quite rich in the mass region around 2 GeV. We believe the testing of these predictions to be one of the most fruitful uses to which phase-shift analyses could be put in the near future.

Note added. The present work utilizes a collection of leading $\sigma=-1$ trajectories which has been discussed before.[6,45] These involve octets and decimets of both signatures. It should be stressed, however, that the feasibility of relating such a solution to the harmonic oscillator spectrum (29) or even to any physically reasonable spectrum has not generally been accepted in the literature. This is because of a supposed difficulty in imposing Eqs. (30) and (31). A simple example may illustrate the source of confusion.

Consider the relation $(\underline{10}^+)/(\underline{10}^-)=\frac{1}{3}(L_- -1)$ in Table VI. This ratio becomes negative for $L_-<1$. (In the notation of Ref. 6 this appears to be a difficulty since both $\underline{10}^+$ and $\underline{10}^-$ are expressed as sums of squares of factorized couplings.) The actual residue of a *pole* in an elastic amplitude must not become negative, of course, but this will not occur in the present case.

The behavior of residues in Table VI for $L_-<0$ should be mentioned briefly. As stated just above Eq. (66), a spectrum with no particles for $L_-<0$ is desired; it can be achieved by fiat. Hence no problems with positivity arise in this region. Figure 2 then shows that no positivity problems will arise at *any* particle poles.

The spin interpretation of residue functions at unphysical points should be clarified. The relations we obtain for given σ refer to l_{initial} and l_{final}, which are definite functions of J, as illustrated in Eqs. (14)–(17). (For $MB\to MB$, the relations are

$$\sigma=\mp 1:\quad l_{\text{initial}}=l_{\text{final}}=J\pm\tfrac{1}{2}\,.)$$

The parity is defined with respect to l, and hence alternates along the trajectory. Thus, for example, while $\underline{10}^-$ refers to $P=+$ at the physical points $L_-=0$ and 2, it refers to $P=-$ at $L_-=1$ ($J^P=\frac{5}{2}^-$) from the standpoint of spin relations. The signature label $\tau=-$ should thus *not* be interpreted as requiring $P=+$ all along the trajectory. This label only indicates at what point a given residue function corresponds to poles.

The consequence of discussing $\sigma=+1$ and $\sigma=-1$ constraints separately is that we are unable to make statements such as those of Ref. 46 regarding local duality among s-channel resonances. The connection of our work with attempts to prepare the ground for explicit dual models is thus unclear. In the context of such models, incidentally, we have sidestepped the very important problem of parity-doubled baryon states,[47] treated in Refs. 39, 48, and 49. The question of whether parity doublets are canceled by explicit functions with an infinite number of zeros[39,48] or by fixed cuts in the angular momentum plane[49] remains obscure, and must be solved before explicit dual models for meson-baryon scattering can be constructed.

The importance of looking for the states mentioned above has also been stressed[50] as a test of the harmonic-oscillator quark model, as contrasted, e.g., with the simpler level scheme in Eq. (2).[6,10,51]

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APPENDIX: ALTERNATIVE METHOD OF SOLUTION; A MODEL AMPLITUDE

We introduce here an alternative method of solving the constraint equations,[52] which allows use of the model of Ref. 7.

The method involves identifying each exotic amplitude with a function having poles in the two crossed channels. When discussing s-channel Regge poles, we shall be concerned only with functions having poles in s and u (exotic t) or s and t (exotic u). Such functions have been called "nexus functions." The Veneziano amplitude[53] is an example of such a function.

Consider first the process $MD\to M'D'$. We set

$$A_t(27)\equiv\tilde{A}(su)=\sum_{s\text{-ch reps}}X_{ts}A_s\ ,\qquad\text{(A1)}$$

$$A_u(27)\equiv\tilde{B}(st)=\sum_{s\text{-ch reps}}X_{us}A_s\ ,\qquad\text{(A2)}$$

$$A_u(\overline{35})\equiv\tilde{C}(st)=\sum_{s\text{-ch reps}}X_{us}A_s\ ,\qquad\text{(A3)}$$

where the elements of the crossing matrices X_{ts} and X_{us} are given in Ref. 11. Specifically,

$$\tilde{A}(su)=\tfrac{2}{15}\sqrt{7}\,A_s(8)-\tfrac{1}{6}\sqrt{7}\,A_s(10),\qquad\text{(A4)}$$

$$\tilde{B}(st)=\tfrac{2}{15}A_s(8)+\tfrac{1}{12}A_s(10)\ ,\qquad\text{(A5)}$$

$$\tilde{C}(st)=3\tilde{B}(st)\,.\qquad\text{(A6)}$$

With a change of normalization, this reads as follows.

$MD \to MD$:

$$A_s(10) = \tfrac{1}{3}[B_1(st) + 2A_1(su)] \;, \qquad \text{(A7)}$$

$$A_s(8) = \tfrac{5}{12}[B_1(st) - A_1(su)] \;. \qquad \text{(A8)}$$

Similarly one can construct solutions to $MB \to MB$ and $MB \to MD$ involving only leading octets and decimets, as follows.

$MB \to MB$:

$$A_s(10) = \tfrac{1}{3}[B_2(st) + 2A_2(su)] \;, \qquad \text{(A9)}$$

$$A_s(8_{ss}) = \tfrac{5}{24}[B_2(st) - A_2(su)] = \sqrt{5}\,A_s(8_{as}) = 5A_s(8_{aa}) \,. \qquad \text{(A10)}$$

$MB \to MD$:

$$A_s(10) = \tfrac{1}{3}[B_3(st) + 2A_3(su)] \qquad \text{(A11)}$$

$$A_s(8_s) = 5[B_3(st) - A_3(su)]/12\sqrt{2} = \sqrt{5}\,A_s(8_a). \qquad \text{(A12)}$$

Here A_s applies to an entire amplitude, not just to its imaginary part.

The functions $A_i(su)$ and $B_i(st)$ may be chosen so as to eliminate the appropriate contributions at $L=0$ and 1. In Ref. 7 these functions are constructed explicitly from (spinless) quark lines and correspond to the two topologically distinct duality graphs for meson-baryon scattering:

$$A_i(su) = \lambda_i \int_0^1 dx\, x^{-\alpha_s - 1}(1-x)^{-\alpha_u - 1} \times [1 - x(1-x)]^{(\alpha_s + \alpha_u - c)/2} \;, \qquad \text{(A13)}$$

$$B_i(st) = \mu_i \int_0^1 dx\, x^{-\alpha_s - 1}(1-x)^{-\alpha_t - 1} . \qquad \text{(A14)}$$

Here we shall take α_s to refer to L_-, and $\alpha_{t,u}$ are t- and u-channel trajectories.

For $\sigma = -1$ trajectories we want no pole in $A_s(8)$ at $\alpha_s = 0$ and no leading pole in $A_s(10)$ at $\alpha_s = 1$. It can be verified that $\lambda_i = \mu_i$ ensures this.

The leading power of α_t in $A(su)$ and $B(st)$ is given near a pole $\alpha_s = L$ $(L = 0, 1, \ldots)$ by

$$A_i(su) \to \frac{\lambda_i}{L!}\,\frac{(-\frac{1}{2}\alpha_t)^L}{L - \alpha_s} \;, \qquad \text{(A15)}$$

$$B_i(st) \to \frac{\mu_i}{L!}\,\frac{(\alpha_t)^L}{L - \alpha_s} \;. \qquad \text{(A16)}$$

These correspond to the highest spin resonances at $L = \alpha_s$. In Eq. (A15) we have used the fact that at any value of s, $\alpha_u = -\alpha_t + \text{const}$. Factorization at $\alpha_s = 0$ requires $\lambda_3^2 = \lambda_1\lambda_2$ and $\mu_3^2 = \mu_1\mu_2$. The relative scale of λ_1 and λ_2 is arbitrary but will be specified in $SU(6)_W$.

At each pole in α_s, one can separate out positive- and negative-signature contributions using, for any function $f(st)$,

$$f(st) = f^{(+)}(s,t) + f^{(-)}(s,t) , \qquad \text{(A17)}$$

where

$$f^{(\pm)}(s,t) \equiv \tfrac{1}{2}[f(s,t) \pm f(s,-t)] \;. \qquad \text{(A18)}$$

One then finds, at integral values of L, relations such as

$MD \to MD$:

$$\frac{10^+}{10^-} = \frac{2^{L-1} - 1}{2^{L-1} + 1} \;, \qquad \text{(A19)}$$

$$\frac{8^+}{10^-} = \frac{5(2^L + 1)}{4(2^L + 2)} \;, \qquad \text{(A20)}$$

$$\frac{8^-}{10^-} = \frac{5(2^L - 1)}{4(2^L + 2)} \;, \qquad \text{(A21)}$$

which (it may be verified) satisfy the original constraints of Table V for all L. Moreover, these ratios agree with the linear interpolation of the constraints not only at $L=0$ and 1 (where they must) but *also at* $L=2$. At $L=3$, use of Eqs. (A19)–(A21) would replace the number $\frac{11}{5}$, obtained in Eq. (28), by $\frac{12}{5}$. The linear interpolation of Table VI in fact fits Eqs. (A19)–(A21) fairly well over a considerable range of L, as shown in Fig. 2.

The solutions for $MB \to MB$ and $MB \to MD$ differ from Eqs. (A19)–(A21) only in common scale factors, which bear the same ratio as in Table VI.

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