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Surface Dominance in Deep-Inelastic Electron Scattering*

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Let a finite part of the scaling curve $\nu W_2(\omega')$ at low ω' be locally dual to a set of resonances obeying a mass formula $M_n{}^2 = a + m_0{}^2 n$. Then it is shown that a simple way of satisfying this condition within the context of several models is that the contributing resonances satisfy the condition $j \sim \sqrt{s}$.

INTRODUCTION

The study of deep-inelastic electron-nucleon scattering[1] has spawned many new paradigms for hadronic physics. Among these is the duality observation of Bloom and Gilman[2]: Let $\nu W_2(\omega')$, $\omega' = (s+q^2)/q^2$, be the scaling curve constructed from data with $\sqrt{s} \geqslant 2$ GeV, $q^2 \geqslant 1$ GeV^2. Here $\sqrt{s}$ is the c.m. energy of the virtual photon and nucleon, and $-q^2$ is the (mass)^2 of the virtual photon. Let $W_2(q^2, M_R{}^2)$ be the structure function for $\sqrt{s}$ in the region of the mass M_R of one of the prominent low-lying resonances, and q^2 not too large. Then to a good approximation, the scaling curve $\nu W_2(\omega')$ for $\omega' \simeq 1 + M_R{}^2/q^2$ provides a reasonable average to $\nu W_2(q^2, M_R{}^2)$, as long as $\omega' \lesssim 4$.

One way of implementing this property is to build the scaling curve itself out of resonance contributions. This has been done by several authors.[3,4] The present work addresses itself to the following problem: If a finite piece of the scaling curve is locally dual to a set of resonances, what is the nature of the dominant intermediate states which contribute? The result of this study *suggests* (but does not prove) the answer: *The angular momentum of the states whose contributions are consistent with scaling obey the constraint $j \sim \sqrt{s}$ at large s.* This is just Harari's condition[5,6] for the peripheral nature of the nondiffractive amplitude in reactions which are not exotic in the direct channel. Such a "surface dominance" may produce some predictions for dips, which we shall deal with elsewhere.

ARGUMENTS FOR THE RESULT

First we write down the polarization tensor [with $J_\mu \equiv J_\mu(0)$]

$$W_{\mu\nu} = (2\pi)^3 \tfrac{1}{2} \sum_{n,\lambda} \delta^4(p+q-P_n) \langle p\lambda | J_\mu | n\rangle \langle n | J_\nu | p\lambda\rangle$$
$$= \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu)$$
$$+ \frac{1}{m^2}\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) W_2(q^2, \nu) . \quad (1)$$

We have used a covariant normalization $\langle p' | p\rangle = (2\pi)^3 (E_p/m)\delta^3(\vec{p}\,' - \vec{p})$, $\bar{u}u = 1$, with m being the nucleon mass.

For a discrete state of mass M_n, spin j, and normality κ to the sum in (1) one makes the replacement

$$\sum_n \to \int \frac{d^3P_n}{(2\pi)^3} \frac{M_n}{E_n} \sum_{\text{spins}}$$
$$= 2M_n \int \frac{d^4P_n}{(2\pi)^3} \delta(P_n{}^2 + M_n{}^2) \sum_{\text{spins}} . \quad (2)$$

So the contribution of this state to $W_{\mu\nu}$ is

$$W_{\mu\nu}^{(n,j,\kappa)} = 2M_n \delta(W^2 - M_n{}^2)$$
$$\times \tfrac{1}{2} \sum_{\lambda\lambda'} \langle p\lambda | J_\mu | p'\lambda'\rangle \langle p'\lambda' | J_\nu | p\lambda\rangle , \quad (3)$$

where $W^2 = -(p+q)^2$, $p' = p + q$.

Let $\epsilon_T \cdot q = 0$, $\epsilon_T^* \cdot \epsilon_T = 1$. We also choose $\epsilon_T \cdot p = 0$, so that we may write

$$W_1^{(n,j,\kappa)} = \epsilon_T^{*\mu} W_{\mu\nu}^{(n,j,\kappa)} \epsilon_T^\nu$$
$$= 2M_n \delta(W^2 - M_n{}^2) \tfrac{1}{2} \sum_{\lambda\lambda'} |\langle p\lambda | J_T | p'\lambda'\rangle|^2 . \quad (4)$$

Define the Lorentz invariant

$$\mathfrak{F}_{nj\kappa}(q^2) = \sum_{\lambda\lambda'} |\langle p\lambda | J_T | p'\lambda'\rangle|^2 . \quad (5)$$

The decay width of the resonance (n, j, κ) into a proton and a *real* transverse photon of polarization ϵ_T is given by

$$\Gamma(nj\kappa \to p\gamma) = \frac{4\pi\alpha}{2j+1} \frac{m}{M_n} \frac{K^*}{2\pi} \mathfrak{F}_{nj\kappa}(0) , \quad (6)$$

where

$$\alpha = e^2/4\pi ,$$
$$k^*(q^2, s) = \tfrac{1}{2}\sqrt{s}\left[1 - \frac{2(m^2 - q^2)}{s} + \left(\frac{m^2 + q^2}{s}\right)^2\right], \quad (7)$$

and

$$K^* = k^*(0, s) .$$

Combining (4)–(6) we find

$$2mW_1^{(n,j,\kappa)}(s,q^2)=\frac{16\pi M_n(j+\frac{1}{2})}{1-m^2/M_n^2}\left[\frac{\Gamma(nj\kappa\to p\gamma)}{4\pi\alpha}\right]\times R_{nj\kappa}(q^2)\,\delta(s-M_n^2)\,, \tag{8}$$

where

$$R_{nj\kappa}(q^2)\equiv\frac{\mathfrak{F}_{nj\kappa}(q^2)}{\mathfrak{F}_{nj\kappa}(0)}\,. \tag{9}$$

Now impose local duality on $W_1^{\text{res}}\equiv\sum_{n,j,\kappa}W_1^{(n,j,\kappa)}$. That is, demand that for fixed q^2

$$\int_{s_a}^{s_b}ds\,W_1^{\text{res}}(s,q^2)=q^2\int_{1+s_a/q^2}^{1+s_b/q^2}d\omega' W_1^{\text{nd}}(\omega')\,, \tag{10}$$

where W_1^{nd} is the nondiffractive piece of W_1. We choose s_a and s_b to enclose a single tower of resonances. For a mass formula

$$M_n^2=a+m_0^2 n\,, \tag{11}$$

choose $s_b-s_a=m_0^2$, $s_a<M_n^2<s_b$. Then the substitution of (8) into (10) gives

$$2mW_1^{\text{nd}}(\omega')=\sum_{j,\kappa}16\pi\frac{M_n}{m_0^2}\frac{j+\frac{1}{2}}{1-m^2/M_n^2}\times\left[\frac{\Gamma(nj\kappa\to p\gamma)}{4\pi\alpha}\right][R_{nj\kappa}(q^2)] \tag{12}$$

for $s=M_n^2$, $\omega'=1+s/q^2$.

What is the ratio in the last bracket? As an example we may consider the excitation of a heavy spin-$\frac{1}{2}$ resonance [e.g., the $P_{11}(1400)$ or the $S_{11}(1535)$]. For these the ratio is

$$R(q^2)=\left[\frac{E^*(q^2)\mp m}{E^*(0)\mp m}\right]^2\left[\frac{G_M(q^2)}{G_M(0)}\right]^2, \tag{13}$$

where E^* is the energy of the proton in the rest frame of the resonance, G_M is the magnetic-transition form factor, and the upper (lower) sign is for P_{11} (S_{11}). In the scaling region ($M_n^2\gg m^2$, $q^2\gg m^2$), we conjecture (along with others[3,7]) that, independent of j and κ,

$$\frac{G_M(q^2)}{G_M(0)}\approx\left(\frac{1}{1+\lambda^2q^2/M_n^2}\right)^2=\left(\frac{\omega'-1}{\omega'-1+\lambda^2}\right)^2. \tag{14}$$

For $M_n^2=m^2$, $\lambda^2\approx1.4$. For simplicity, we take $\lambda^2\approx1$ in general.

Also in the scaling region, the first factor in (13) is $\approx\omega'/(\omega'-1)$, so that there evolves the ansatz

$$R_{nj\kappa}(q^2)\underset{M_n\gg m;\,q^2\gg m}{\approx}\left(1-\frac{1}{\omega'}\right)^3. \tag{15}$$

The dynamics of the form factors presumably suppress the longitudinal cross section.[3] This results in the relation $2mW_1(\omega')=\omega'\nu W_2(\omega')$. Equations (12) and (15) then give our working results,

$$\nu W_2^{\text{nd}}(\omega')\approx\frac{16\pi}{m_0^2}\frac{1}{\omega'}\left(1-\frac{1}{\omega'}\right)^3\sqrt{s}\times\sum_{j,\kappa}(j+\tfrac{1}{2})\left[\frac{\Gamma(nj\kappa\to p\gamma)}{4\pi\alpha}\right]. \tag{16}$$

As an example, to see if the expression makes sense, we may evaluate it at its maximum ($\omega'\simeq4$), and for $\sqrt{s}$ in the region of the $D_{13}(1525)$ and $S_{11}(1535)$ resonances. The sum may be obtained from electroproduction data using the relation

$$\sum_{j\kappa}(j+\tfrac{1}{2})\frac{\Gamma(nj\kappa\to p\gamma)}{4\pi\alpha}=\frac{\Gamma_{\text{tot}}}{4\pi}\frac{K^{*2}\sigma_T^{(\text{res})}(q^2=0,\sqrt{s}=M^*)}{4\pi\alpha}\,, \tag{17}$$

where K^* was defined in Eq. (7). From an extrapolation to $q^2=0$ of the analysis of Clegg,[8] we find the right-hand side of (17) to be ≈0.03, which when inserted into Eq. (16) gives $(\nu W_2^{\text{nd}})_{\max}\approx0.22$. This is in qualitative agreement with experiment when one considers that (a) these resonances are not in the scaling region and (b) some fraction of the scaling curve at $\omega'=4$ consists of a diffractive component which is not dual to the resonances.

With this confidence in our formula (16), we now argue as follows: Equation (16) tells us that agreement with the observed behavior of $\nu W_2(\omega')$ can be obtained if

$$\frac{\sqrt{s}}{m_0^2}\sum_{j,\kappa}(j+\tfrac{1}{2})\frac{\Gamma(nj\kappa\to p\gamma)}{4\pi\alpha}\approx\text{const}\approx0.07\,. \tag{18}$$

The number on the right-hand side of (18) is obtained by normalizing $\nu W_2^{\text{nd}}(\omega')$ to 0.3 at $\omega'=3$. *The condition expressed by Eq. (18) must be independent of s.* We emphasize at this point that the sum in (18) runs over all the resonances in the tower which actually contribute to the nondiffractive part of the structure function.

The reader may object to the $1/\omega'$ behavior of νW_2^{nd} implied by Eqs. (16) and (18), instead of the conventional $1/\omega'^{\frac{1}{2}}$ implied by Regge theory. All we can say at this point is that the model proposed here is not at all meant to describe the Regge region of large ω'; there is absolutely no evidence that Bloom-Gilman duality has any meaning in that region of ω'. If such evidence appears, the procedure of this paper would have to be modified.

As s gets large, we try to satisfy (18) within the context of three models:

(1) Simple 4-point dual structure[9] – resonances with $0<j<n$. This situation has been analyzed by Shapiro[10] in the case of $\pi\pi$ scattering. The resonances with $0<j\lesssim\sqrt{n}$ have fairly equal partial widths, while the ones with $\sqrt{n}\lesssim j<n$ are highly damped due to centrifugal effects. Such a spectrum, when compared with the condition (18),

would imply that

$$\frac{\Gamma(nj\kappa \to p\gamma)}{4\pi\alpha} \sim \frac{1}{s^{3/2}}. \quad (19)$$

This is inconsistent with the energy behavior of the elastic width $\Gamma_{el} \sim s^{-1}$ resulting[10] from the 4-point function which gives rise to this spectrum in the first place. So this spectrum is incompatible with scaling.

(2) N-point dual structure. Chiu, Hermann, and Schwimmer[11] have calculated the density of states for given angular momentum l to be

$$\rho(l) \sim (\sqrt{s})^{-b} e^{a\sqrt{s}} l^2$$

for $0 < l \lesssim \sqrt{s}$, and decreasing exponentially thereafter. Agreement with the condition (18) then would demand a partial-width behavior $\Gamma(nj\kappa \to p\gamma) \sim (\sqrt{s})^{b-5} e^{-a\sqrt{s}}$, which is prohibitively difficult to check for consistency but is not far-fetched.[12] The peaking of $\rho(l)$ at $l \sim \sqrt{s}$ would then imply our result of a surface dominance. On the whole, we are rather loath to invoke the complexity inherent in the N-point dual structure to discuss the behavior of νW_2 without a detailed knowledge of the elastic-width behavior. In this way we come to the simplest alternative.

(3) Spectrum deduced from performing a partial-wave analysis of a finite number of Regge amplitudes. Kugler[13] has found that this results in prominent resonances obeying $j \sim \sqrt{s}$, with elastic widths $\Gamma_{el} \sim s^{-1}$, and total width $\Gamma_{tot} \sim \sqrt{s}$. The lower j's contribute a background; the higher j's are damped.

One can easily see that this behavior is entirely compatible with the condition (18).

If we set $j + \frac{1}{2} \approx \frac{1}{2} r\sqrt{s}$, $r \approx 0.8$ F in (18), and multiply by 2 to account for both normalities, we have a condition

$$\left\langle \frac{\Gamma(nj\kappa \to p\gamma)}{4\pi\alpha} \right\rangle_{av} \simeq \frac{0.018 m_0}{n} \simeq \frac{18 \text{ MeV}}{n}, \quad (20)$$

where $n = M_n{}^2/m_0{}^2$, $m_0 \approx 1$ GeV$/c^2$. This compares to

$$\langle \Gamma(nj\kappa \to p\pi^0) \rangle_{av} \approx \frac{40 \text{ MeV}}{n} \quad (21)$$

for the prominent πN resonances, like the $N^*(1688)$. It should be kept in mind that the width in Eq. (18) is for decay into a single polarization. If summed over both, the agreement with Eq. (19) is even closer.

CONCLUSION

To sum up, we have analyzed the condition that a finite piece of the scaling curve at low ω' is locally dual to a set of resonances obeying the mass formula (11). Examination of this condition within the framework of several current models led to the conclusion that the resonances which contribute in the scaling region must obey the on-shell peripheral condition $j \sim \sqrt{s}$.

It should be stressed that this result is independent of the presence of diffractive pieces in the amplitude or of fixed poles.[14, 15] It depends only on the assumption that a *finite piece* of $\nu W_2(\omega')$ is dual to resonances.

It is perhaps worthwhile to end by remarking that the result of this paper cannot fail to raise the following questions: Why should a highly virtual photon see the proton as an absorbing annulus rather than a disk, and what are the connections between the effect discussed here (as well as the whole picture of duality) and partons? Answers to these are certainly not yet at hand.

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[12]Angular-momentum-barrier arguments would give, for $l \sim \sqrt{s}$, and elastic width [cf. Eq. (7)]

$$\Gamma_{el} \sim \left(\frac{1}{2l+1}\right)\left(\frac{m}{\sqrt{s}}\right)\left(\frac{ek\beta}{2l}\right)^{2l} k$$

into low-mass states. [H. Goldberg, Phys. Rev. Letters 21, 778 (1968).] For $k \simeq \frac{1}{2}\sqrt{s}$, $l \simeq \frac{1}{2} r\sqrt{s}$, this gives $\Gamma_{el} \sim s^{-1/2} e^{-a\sqrt{s}}$, where $e^{-a} = (e\beta/2r)$. If $b = 4$, the value obtained from the statistical bootstrap [S. C. Frautschi and C. J. Hamer, Phys. Rev. D 4, 2125 (1971); Huan Lee, Y. C. Leung, and C. G. Wang, Astron. J. 166, 387 (1971)], this is actually in agreement with the requirement given in the text.

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Inclusive Production of Vector Mesons in the Dual-Resonance Model

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The vector-meson inclusive distribution is considered by making use of the eight-point dual amplitude for which two of the external lines are regarded as the decay products of a single vector meson. We find that the vector-meson vertex is factorizable both in the fragmentation and central regions only for large values of $\kappa = p_\perp^2 + m^2$. Nonfactorizability for low values of κ is due to the correlations between the beam-target system and the decay products. Explicit and yet simple expressions for the vertex function as well as the vector-meson production cross section are given to the leading order in κ for the triple-Regge and pionization limits. They show that the vector-meson decays, in either of the kinematic regions, along the preferred beam direction with a $\cos^2\beta$ dependence, in the rest frame of the vector meson, provided that the intercept of the leading vacuum trajectory takes the value unity. Furthermore, it is found that in the central region the states with helicity ± 1 contribute dominantly to the decay correlations, while in the triple-Regge region the state with helicity zero is favored.

I. INTRODUCTION

In previous publications,[1,2] we have reported on the two-particle inclusive distributions for $a+b \rightarrow 1+2+$anything obtained from the eight-line dual amplitudes. Particular emphasis was given to the limiting behavior and factorization of the distributions[1] in various kinematic regions as well as the two-particle correlations[2] in the central (pionization) region. As the observed particles 1 and 2 can take a wide range of momenta, the invariant mass s_{12} or the relative rapidity y can change from a finite value to infinity. Accordingly, the central distribution, for example, makes a transition from a correlated form to an uncorrelated form thus exhibiting factorization. Such a transition effect is particularly convenient for the study of the two-particle correlation.[2]

On the other hand, it has been noted that[3] if one restricts oneself to the region of finite s_{12} or y where the two particles can form a resonance, the two-particle distributions can be used to examine the distribution of a single outgoing resonance which decays subsequently into the particles 1 and 2. Namely, if the two particles are in the fragmentation region of, say, the target b, one can learn the single resonance distribution in the b fragmentation, whereas if they are in the pionization region, one can study the resonance distribution in the central region. In general, the results obtained in this way contain the decay correlation effects coming from the coupling between the resonance and the decay products even in the narrow-width approximation. To isolate the single-resonance distribution, one thus needs to factor out the vertex function.

In this paper, we would like to carry out the study of the single-resonance distributions within the context of the original dual eight-point amplitude. Although the case of arbitrarily large spin can be worked out, we will confine ourselves in this paper to the case of vector mesons only, which we will refer to as "ρ," since the vector-meson production will be the most interesting process next to the single stable particle production in experiments now under way. Furthermore, the dual-resonance model in its original context has been shown[4] to yield a consistent scheme for constructing the multiparticle amplitude with any number of spinning particles having various decay modes and having explicit decay correlations. For example, starting from an eight-line amplitude B_8 in which all external lines have zero spin, one can construct a six-point amplitude $\overline{B}_6$ with two spin-