CPT Violation and the $K_L \to \mu^+\mu^-$ Puzzle

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(Received 18 April 1972)

The effects of possible CPT violation on the $K_L \to 2\mu$ amplitude are found to be concentrated at the right place to suppress the experimentally violated "conventional" unitarity bound, irrespective of $K_S \to 2\mu$. CPT nonconservation may, therefore, resolve the $K_L \to 2\mu$ puzzle, even if CP violation or the recently proposed new interactions will fail to do this. The possible breaking of CPT invariance is studied systematically and bounds are given on the violating part of the related amplitudes. Only nonconservation of CPT in the absorptive part of the $K_L \to 2\mu$ amplitude (e.g., in $K_L \to 2\gamma$) may be accessible to experiment in the near future. Superweak theory is inconsistent with experiment, under the "conventional" assumptions, even in its CPT-violating version.

The new experimental result[1]

$$B(K_L \to 2\mu) = \left.\frac{\Gamma(K_L \to 2\mu)}{\Gamma(K_L \to \mathrm{all})}\right|_{\exp} \leq 1.9\times 10^{-9} \qquad (1)$$

is in contradiction with the conventional unitarity lower bound, derived under the following assumptions:

(i) validity of quantum electrodynamics for leptons;

(ii) CPT and CP invariance;

(iii) that the absorptive part of the $K_L \to 2\mu$ amplitude is given entirely by the contribution of the on-mass-shell 2γ intermediate state, while the absorptive part of $K_L \to 2\gamma$ is zero; and

(iv) unitarity, i.e.,

$$\begin{aligned}\mathrm{Abs}T_{\alpha\beta} &\equiv \tfrac{1}{2}i(T^\dagger_{\alpha\beta} - T_{\alpha\beta}) = \tfrac{1}{2}\sum_n T_{\alpha n}T^\dagger_{n\beta} \\ &= \tfrac{1}{2}\sum_n T^\dagger_{\alpha n}T_{n\beta}\,. \end{aligned} \qquad (2)$$

We take the T-matrix elements to contain the appropriate phase-space factors, so that the squares of their magnitude give the partial widths. We use also the conventions:

$$\begin{aligned}&CPT|K^0\rangle \equiv |\tilde{K}^0\rangle = |\bar{K}^0\rangle\,,\\ &CPT|2\gamma, CP=\pm 1\rangle \equiv |2\tilde{\gamma}_\pm\rangle = \pm\,|2\gamma_\pm\rangle\,,\\ &CPT|\mu^+\mu^-, CP=\pm1\rangle \equiv |2\tilde{\mu}_\pm\rangle = \pm\,|2\mu_\pm\rangle\end{aligned}$$

for free states, and

$$K_{1,2} \equiv \frac{1}{\sqrt{2}}(K^0 \pm \bar{K}^0).$$

Using these assumptions, we have

$$\begin{aligned}\mathrm{Abs}\langle 2\mu_+|T|K_L\rangle &= \tfrac{1}{2}\sum_n \langle 2\tilde{\mu}_+|T|\tilde{n}\rangle^*\langle n|T|K_L\rangle\\ &= \lambda\beta\langle 2\gamma_+|T|K_L\rangle\,,\end{aligned}$$

$$\begin{aligned}\mathrm{Abs}\langle 2\mu_-|T|K_L\rangle &= \tfrac{1}{2}\sum_n \langle 2\tilde{\mu}_-|T|\tilde{n}\rangle^*\langle n|T|K_L\rangle\\ &= \lambda\langle 2\gamma_-|T|K_L\rangle\,,\end{aligned} \qquad (3)$$

where explicit calculations[2] give

$$\begin{aligned}&\lambda^2 = |\langle 2\mu_-|T|2\gamma\rangle|^2 \simeq 1.2\times 10^{-5}\,,\\ &\beta^2\lambda^2 = |\langle 2\mu_+|T|2\gamma\rangle|^2 \simeq 0.8\times\lambda^2\,.\end{aligned} \qquad (4)$$

Taking into account[3]

$$B(K_L \to 2\gamma) = (5.6\pm 0.5)\times 10^{-4}\,, \qquad (5)$$

(3) and (4) yield

$$B(K_L \to 2\mu)|_{\text{theory}} \geq \lambda^2\beta(K_L \to 2\gamma) \simeq 6\times 10^{-9}\,, \qquad (6)$$

in obvious contradiction with (1).

One way to look for a resolution of this puzzle is to relax the requirement of CP invariance. In this case Christ and Lee[4] derived the following inequalities:

$$\begin{aligned}&\mathrm{Re}\,\epsilon\,\Gamma^{1/2}(K_S \to 2\mu) \geq \beta\lambda\Gamma^{1/2}(K_L \to 2\gamma) - \Gamma^{1/2}(K_L \to 2\mu)\,,\\ &\mathrm{Re}\,\epsilon\,\Gamma^{1/2}(K_S \to 2\mu) \leq \lambda\Gamma^{1/2}(K_L \to 2\gamma) + \Gamma^{1/2}(K_L \to 2\mu)\,,\end{aligned} \qquad (7)$$

where $\epsilon \simeq 2\times 10^{-3}e^{i\pi/4}$. These inequalities imply, using (1), (4), and (5) (see Ref. 5),

$$5\times 10^{-7} \lesssim B(K_S \to 2\mu)|_{\text{theory}} \lesssim 1\times 10^{-5}\,,$$

while conventional, CP-conserving estimates[6] would give

$$B(K_S \to 2\mu) \sim 10^{-10} - 10^{-11}\,.$$

The present experimental upper bound is[3]

$$B(K_S \to 2\mu)|_{\exp} \leq 7\times 10^{-6}\,,$$

and more sensitive experiments are now in progress.[7]

This means that the breakdown of the conventional unitarity bound is due to CP violation only if $\Gamma(K_S \to 2\mu)$ is largely enhanced (probably by a big CP-violating mechanism). If one disregards such an enhancement while neglecting terms proportional to ϵ, the unitarity bound cannot be lowered by more than 18%.[8] In particular, the superweak theory[9] of CP violation (which assumes that the decay amplitudes are CP-invariant) is in contradiction with experiment, under the assumptions referred to above.[5,10]

A possible way out, without requiring a big CP violation, is to relax assumption (iii). Practically, this requires that $\mathrm{Abs}(K_L \to 2\mu)$ receives an unexpected large contribution from the 3π intermediate state,[8,11-13] or intermediate states involving yet undetected particles.[12,14-16] The number of such possibilities is quite limited as they have to satisfy severe restrictions, and this may be tested experimentally.

We study here the effect of possible CPT violation.[17] This may be of special interest if big enhancements of $(K_S \to 2\mu)$ or the above-mentioned contributions to $\mathrm{Abs}(K_L \to 2\mu)$ are found to contradict experiment.

If CPT is conserved,

$$T_{\tilde{\alpha}\tilde{\beta}} = T_{\beta\alpha}.$$

In the general case we denote therefore

$$T_{\tilde{\alpha}\tilde{\beta}} \equiv T_{\beta\alpha} + \tilde{T}_{\alpha\beta}, \tag{8}$$

where

$$\tilde{T}_{\tilde{\alpha}\tilde{\beta}} + \tilde{T}_{\beta\alpha} \equiv 0.$$

Also,

$$K_L \simeq K_2 + \epsilon_L K_1, \quad K_S \simeq K_1 + \epsilon_S K_2,$$

and we neglect consistently terms proportional to $|\epsilon_{L,S}|^2$. Experimentally,[17]

$$0.35\times 10^{-3} \lesssim \mathrm{Re}\epsilon_L \lesssim 8\times 10^{-3}.$$

Using (2) with (8), we get

$$\mathrm{Abs}T_{\alpha\beta} = (\mathrm{Abs}T_{\alpha\beta})_{CPT} - \tfrac{1}{2}i\tilde{T}^*_{\tilde{\alpha}\tilde{\beta}} = (\Sigma_{\alpha\beta})_{CPT} - \tilde{\Sigma}_{\alpha\beta}, \tag{9}$$

where

$$(\mathrm{Abs}T_{\alpha\beta})_{CPT} = \tfrac{1}{2}i(T^*_{\tilde{\alpha}\tilde{\beta}} - T_{\alpha\beta}) = (\Sigma_{\alpha\beta})_{CPT} - R_{\alpha\beta}, \tag{10}$$

$$(\Sigma_{\alpha\beta})_{CPT} = \tfrac{1}{2}\sum_n T_{\alpha n}T^*_{\tilde{n}\tilde{\beta}} = \tfrac{1}{2}\sum_n T_{\tilde{\beta}\tilde{n}}T^*_{n\alpha}, \tag{11}$$

$$\tilde{\Sigma}_{\alpha\beta} = \tfrac{1}{2}\sum_n T_{\alpha n}\tilde{T}^*_{n\beta} = \tfrac{1}{2}\sum_n \tilde{T}_{\beta n}T^*_{n\alpha}, \tag{12}$$

and

$$R_{\alpha\beta} = \tilde{\Sigma}_{\alpha\beta} - \tfrac{1}{2}i\tilde{T}^*_{\tilde{\alpha}\beta}. \tag{13}$$

Similarly,

$$\begin{aligned} \mathrm{Disp}T_{\alpha\beta} &= (\mathrm{Disp}T_{\alpha\beta})_{CPT} - \tfrac{1}{2}\tilde{T}^*_{\tilde{\alpha}\beta}, \\ (\mathrm{Disp}T_{\alpha\beta})_{CPT} &= \tfrac{1}{2}(T^*_{\tilde{\alpha}\tilde{\beta}} + T_{\alpha\beta}). \end{aligned} \tag{14}$$

In particular, (3) gives

$$\begin{aligned} (\mathrm{Abs}T_{+L})_{CPT} &= \lambda\beta\Gamma^{1/2}(K_L \to 2\gamma_+) - R_{+L}, \\ (\mathrm{Abs}T_{-L})_{CPT} &= \lambda\Gamma^{1/2}(K_L \to 2\gamma_-) - R_{-L}, \end{aligned} \tag{15}$$

where

$$A_{\pm\alpha} \equiv \langle 2\mu_\pm|A|K_\alpha\rangle.$$

Hence,

$$\begin{aligned} \lambda^2\beta^2\Gamma(K_L \to 2\gamma) &\le |(\mathrm{Abs}T_{+L})_{CPT} + R_{+L}|^2 + |(\mathrm{Abs}T_{-L})_{CPT} + R_{-L}|^2 \\ &\le \lambda^2\Gamma(K_L \to 2\gamma) \end{aligned} \tag{16}$$

or

$$\begin{aligned} \lambda^2\beta^2\Gamma(K_L \to 2\gamma) - R_L &\le |\mathrm{Abs}T_{+L}|^2_{CPT} + |\mathrm{Abs}T_{-L}|^2_{CPT} \\ &\le \lambda^2\Gamma(K_L \to 2\gamma) - R_L, \end{aligned} \tag{17}$$

where

$$\begin{aligned} R_L &= |R_{+L}|^2 + |R_{-L}|^2 + 2\,\mathrm{Re}[(\mathrm{Abs}T_{+L})_{CPT}R^*_{+L} + (\mathrm{Abs}T_{-L})_{CPT}R^*_{-L}] \\ &= -|R_{+L}|^2 - |R_{-L}|^2 + 2\lambda\beta\Gamma(K_L \to 2\gamma_+)\,\mathrm{Re}(R_{+L}) + 2\lambda\Gamma(K_L \to 2\gamma_-)\,\mathrm{Re}(R_{-L}). \end{aligned} \tag{18}$$

Following Christ and Lee's[4] derivation of the inequalities (7) (in the version of Oakes[10]), we get

$$(\mathrm{Abs}T_{+L})_{CPT} + i\,\mathrm{Re}(T_{+L}) = i\,\mathrm{Re}\epsilon_L T_{+1}, \tag{19a}$$

$$(\mathrm{Abs}T_{-L})_{CPT} - \mathrm{Im}(T_{-L}) = -i\,\mathrm{Re}\epsilon_L T_{-1}. \tag{19b}$$

The corresponding triangle inequalities give

$$\begin{aligned} \Gamma(K_L \to 2\mu) &\ge |\mathrm{Re}\epsilon_L|T_{+1}| - |\mathrm{Abs}T_{+L}|_{CPT}|^2 + |\mathrm{Re}\epsilon_L|T_{-1}| - |\mathrm{Abs}T_{-L}|_{CPT}|^2 \\ &\ge [|\mathrm{Abs}T_{+L}|_{CPT} + |\mathrm{Abs}T_{-L}|_{CPT} - \mathrm{Re}\epsilon_L\Gamma^{1/2}(K_S \to 2\mu)]^2, \end{aligned} \tag{20a}$$

and in the very same way

$$\Gamma(K_L \to 2\mu) \geq [|\mathrm{Disp}T_{+L}|_{CPT} + |\mathrm{Disp}T_{-L}|_{CPT} - \mathrm{Im}\epsilon_L \Gamma^{1/2}(K_S \to 2\mu)]^2 . \tag{20b}$$

Taking now (17) into account, we get

$$\mathrm{Re}\epsilon_L \Gamma^{1/2}(K_S \to 2\mu) \geq [\lambda^2\beta^2\Gamma(K_L \to 2\gamma) - R_L]^{1/2} - \Gamma^{1/2}(K_L \to 2\mu), \tag{21a}$$

$$\mathrm{Re}\epsilon_L \Gamma^{1/2}(K_S \to 2\mu) \leq [\lambda^2\Gamma(K_L \to 2\gamma) - R_L]^{1/2} + \Gamma^{1/2}(K_L \to 2\mu). \tag{21b}$$

When $R_L = 0$ and $\epsilon_L = \epsilon$, one obtains the inequalities for the CPT-invariant case, i.e., Eqs. (7), but then $(K_S \to 2\mu)$ must be big. If we do not allow a big enhancement of $(K_S \to 2\mu)$, we obtain

$$\lambda^2\beta^2\Gamma(K_L \to 2\gamma) - 1.9\times 10^{-9} \lesssim R_L \lesssim \lambda^2\beta^2\Gamma(K_L \to 2\gamma),$$

i.e.,

$$3.5\times 10^{-9} \lesssim R_L \lesssim 5.4\times 10^{-9}. \tag{22}$$

There are two possible sources of contribution to R_L: (A) $\tilde{T}_{\pm L}$ and (B) $\tilde{\Sigma}_{\pm L}$.

Class A. If CPT is violated solely in a direct coupling of K_L to $\mu^+\mu^-$ (References 18 and 19 give explicit Hamiltonian models which belong to this class.), $\tilde{\Sigma}_{\pm L} = 0$ and

$$R_L(\tilde{\Sigma}_{\pm L} = 0) = -[\tfrac{1}{4}|\tilde{T}_{+L}|^2 + \tfrac{1}{4}|\tilde{T}_{-L}|^2 + \lambda\beta\Gamma^{1/2}(K_L \to 2\gamma_+)\,\mathrm{Im}(\tilde{T}_{+L}) + \lambda\Gamma^{1/2}(K_L \to 2\gamma_-)\,\mathrm{Im}(\tilde{T}_{-L})]. \tag{23}$$

Taking into account the limits on R_L, Eq. (22), we expect

$$2\times 10^{-9} \lesssim |\tilde{T}_{+L}|^2 + |\tilde{T}_{-L}|^2 \lesssim 8\times 10^{-8}. \tag{24}$$

The absolute magnitude of such violation is so small that one cannot expect it to be observed elsewhere in the near future (if the corresponding CPT-nonconserving interaction does not manifest itself explicitly in other places).

Class B. A nonzero value for $\tilde{\Sigma}_{\pm L}$ means a breaking of CPT invariance in one of the intermediate states contributing to the absorptive part of the $K_L \to 2\mu$ amplitude.

This case is more promising than the previous one, as long as possible detection of the CPT-nonconservation effect is concerned. Also, $\tilde{\Sigma}_{\pm L}$ will induce an effective CPT violation in the $K_L \to 2\mu$ amplitude itself. This will show CPT violation in the absorptive part only, while CPT violation due to the "direct" interaction is limited to the dispersive part. An explicit calculation of the induced CPT-violating part of $K_L \to 2\mu$ gives

$$\begin{aligned} \text{“}\tilde{T}_{\pm L}\text{”} &= \text{“}\tilde{T}^*_{L\pm}\text{”} \\ &= i(\mathrm{Abs}T_{\bar{L}\mp} - \mathrm{Abs}T_{\pm L}) \\ &= i\tilde{\Sigma}_{\pm L}. \end{aligned} \tag{25}$$

Hence,

$$\begin{aligned} R_{\pm L} &= \tilde{\Sigma}_{\pm L} - \tfrac{1}{2}i\,\text{“}\tilde{T}^*_{\pm L}\text{”} \\ &= \tfrac{3}{2}\tilde{\Sigma}_{\pm L}. \end{aligned} \tag{26}$$

Therefore, using (22) we estimate

$$2\times 10^{-10} \lesssim |\tilde{\Sigma}_{+L}|^2 + |\tilde{\Sigma}_{-L}|^2 \lesssim 1\times 10^{-8}. \tag{27}$$

In principle, the induced CPT violation may originate in any intermediate state contributing to the unitarity sum. However, in view of assumption (iii) (2γ dominance), it is most natural to generate $\tilde{\Sigma}_{\pm L}$ by a breaking of CPT invariance in $K_L \to 2\gamma$. In this case

$$\begin{aligned} \tilde{\Sigma}_{+L} &\approx \lambda\beta\tilde{T}_{2\gamma_+L}, \\ \tilde{\Sigma}_{-L} &\approx \lambda\tilde{T}_{2\gamma_-L}, \end{aligned} \tag{28}$$

and using (26),

$$2\times 10^{-5} \lesssim 0.8|\tilde{T}_{2\gamma_+L}|^2 + |\tilde{T}_{2\gamma_-L}|^2 \lesssim 1\times 10^{-3}. \tag{29}$$

The first thing to observe here is that one does not expect the CPT-violating amplitude to contribute to the $K^0, \bar{K}^0$ mass difference, more than the upper bound set by the mass difference of K_L, K_S. This is due to the fact that we deal here with a second-order weak-electromagnetic ($\Delta S = 1$) effect $[(GM^2\sin\theta\,\alpha/\pi)^2 \approx 10^{-18}]$.

Is it possible to detect such a CPT violation elsewhere? In the $K^0, \bar{K}^0$ complex, CPT violation requires in general unequal ϵ_L and ϵ_S. A rough estimate gives[17,20]

$$\begin{aligned} |\delta| &= \tfrac{1}{2}|\epsilon_S - \epsilon_L| \\ &\approx \frac{\Gamma_L}{\Gamma_S}|\tilde{T}_{2\gamma L}|^2 \approx 10^{-5} - 10^{-7}, \end{aligned} \tag{30}$$

while experimentally[17,21]

$$|\epsilon_L| \approx 10^{-3},$$

$$|\mathrm{Re}\delta| \lesssim 10^{-2},$$

$$|\mathrm{Im}\delta| \lesssim 10^{-3}.$$

The same order of magnitude is expected for the contributions of $\tilde{T}_{2\gamma L}$ to the Bell-Steinberger unitarity sum and this may affect the value of ϕ_{+-}, the phase of η_{+-}. If we assume that the main decay modes of K_L do not violate CP in their amplitudes (i.e., obey the superweak theory), then

$$\tan\phi_{+-} = \frac{2(M_{K_L} - M_{K_S})}{\Gamma_S - \Gamma_L} \quad .$$

Our $\tilde{T}_{2\gamma L}$ may violate this equality by an amount of $10^{-2}-10^{-4}$.

Within several years we expect experiments to reach the accuracy level needed to look for quantities such as

$$|\delta| \approx 10^{-5}$$

or

$$\left|\tan\phi_{+-} - \frac{2}{\Gamma_S - \Gamma_L}(M_{K_L} - M_{K_S})\right| \approx 10^{-2} .$$

The decay $K^\pm \to \pi^\pm\gamma\gamma$ resembles the $K_L \to \gamma\gamma$ from many points of view. If we assume that, in this case also, CPT is violated by the same amount as in $K_L \to \gamma\gamma$, a difference may be induced between the total decay rates of K^+ and K^-.[22] This difference is expected to be at the same level as the partial rates of $K^\pm \to \pi^\pm\gamma\gamma$, i.e.,

$$\frac{\tau^+ - \tau^-}{\overline{\tau}} \lesssim 10^{-5} . \tag{31}$$

Experiments are now on the level of 10^{-3}.[3]

To conclude, we studied phenomenologically, in a systematic way, possible effects of CPT violations in the $K_L \to \mu^+\mu^-$ decay. It is found that all these effects are concentrated in one term. This term may be blamed on the suppression of the unitarity bound on the rate of $K_L \to 2\mu$, irrespective of the rate of $K_S \to 2\mu$. In the CPT-invariant CP-violating case an unexpected big rate for $K_S \to 2\mu$ is needed, and this may contradict experiment. The needed amount of counter contribution leads to bounds on the CPT-noninvariant part of the amplitudes which may generate such a contribution. We showed that in the class of possibilities where the violation of CPT is due to a direct coupling of K_L to $\mu^+\mu^-$, it is not possible to observe CPT nonconservation elsewhere, while a little more promising from this point of view is the case where the breaking of CPT invariance is in the absorptive part of $K_L \to 2\mu$ (e.g., in $K_L \to 2\gamma$).

In any case, to resolve the $K_L \to 2\mu$ puzzle, CPT violation must show up in one amplitude at least. Pure superweak theory does not allow such a possibility and hence it is inconsistent with experiment, under assumptions (i), (iii), and (iv), even in its CPT-violating version. The discussed CPT violation may however play, in the case of $K_L \to 2\mu$, a role similar to that played by the superweak theory for $K_L \to 2\pi$, i.e., it may resolve the corresponding puzzle without leaving a trace elsewhere.

What is the mechanism which might give rise to CPT violation? One can show that the $K_{L,S}$ decays are especially suitable for the construction of a CPT-violating nonlocal theory.[23] One possibility would be to couple K_L to neutral leptonic currents in a nonlocal way. (This may "explain" the suppression of the effective coupling constant as well as its imaginary part.)

The author would like to thank Professor J. S. Bell, Professor M. K. Gaillard, Professor J. Prentki, and Professor B. Stech, as well as his colleagues at the Institut für Hochenergiephysik, for discussions and comments.

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Some Absolute Bounds for K_{l3} Decay Parameters

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Assuming that the scalar [or vector] K_{l3} form factor $D(t)$ [or $f_+(t)$] is either univalent or a function satisfying at most a once-subtracted dispersion relation with a definite sign for its imaginary part, we can derive several exact bounds for these form factors without introducing any arbitrary parameters.

There are many theoretical calculations on K_{l3} decay parameters. However, most of these calculations are based upon varieties of approximations whose validity is not obvious at all. Recently, exact inequalities which are relatively free of theoretical uncertainties were derived by several authors.[1-7] In particular, the exact bound for the scalar slope parameter Λ_0 defined by

$$\Lambda_0 = \lambda_+ + \frac{m_\pi^2}{m_K^2 - m_\pi^2}\xi \tag{1}$$

was found[3,5] to be very small and positive, with

$$0.008 \le \Lambda_0 \le 0.019 . \tag{2}$$

This must be compared with the world-averaged experimental value[8] of

$$\Lambda_0 = -0.11 \pm 0.03 . \tag{3}$$

However, in view of mutually contradicting experimental data,[8,9] this discrepancy should not be regarded as final. The main assumptions needed in deriving the bound in Eq. (2) are the following: (i) the chiral SW(3) Hamiltonian[10] of Gell-Mann, Oakes, and Renner and of Glashow and Weinberg, (ii) the K_{l3} soft-pion theorem,[11] (iii) a weak form[12] of the Ademollo-Gatto theorem, i.e., $f_+(0) \lesssim 1$, and finally (iv) some technical assumptions[2,3] concerning two-point Green's functions for divergences of weak currents.

In this note, we shall present an entirely different approach with fewer assumptions and show that we can derive similar exact bounds for K_{l3} decay parameters. With this objective, we shall consider two different classes of analytic functions. Hereafter, $f(t)$ will always represent a real analytic function of t [i.e., $f^*(t^*) = f(t)$] with a right-hand cut on the real axis at $t_0 \le t < \infty$. Without loss of generality, we can assume that the threshold t_0 is positive. First, a univalent function is a function $f(t)$ which never assumes the same value twice, i.e., it satisfies $f(t_1) \ne f(t_2)$ in the entire cut plane whenever $t_1 \ne t_2$. Second, let us introduce another class of analytic functions, which we shall call semimonotonic for lack of a better terminology. This is a real analytic function $f(t)$ which satisfies (i) a standard dispersion relation with a finite number of subtractions, and (ii) a condition that $\mathrm{Im} f(t+i\epsilon)$ does not change its sign on the entire cut at $\infty > t \ge t_0$. Then, the main conclusions of this paper are as follows: We assume either that $f(t)$ is a univalent function of t in the cut plane or that $f(t)$ is semimonotonic and satisfies an at most once-subtracted dispersion relation. Under either of these conditions, we can prove the