mate that $r \approx 0.5$ unless the mass of the unexcited particle is very small. Figure 5 shows the angular distribution for $r=0.5$.

On the basis of the comparison between Figs. 2 and 5 we conclude therefore that the phenomenon observed at 11.8 GeV/c should not persist at infinite energy within the framework of the diffractive excitation model conceived at present.

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Possible Existence of a New Absorption Effect in Dual Absorptive Model and the Structure of πN Helicity Amplitude

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A possible new absorption effect in Harari's dual absorptive model is suggested and calculated in order to study the amplitude structure of pion-nucleon scattering. It is predicted that the imaginary part of the $I_t=1$, s-channel helicity-nonflip amplitude does not have the second zero near $|t|=0.8$ (GeV/c)^2, although the same amplitude in the original dual absorptive model has such a zero due to the second zero of the Bessel function of the zeroth order in addition to the crossover zero at $|t| \simeq 0.15$ (GeV/c)^2. This result is consistent with those of the recent analytic amplitude analysis by Kelly and of the model-independent amplitude analysis by the Saclay group.

I. INTRODUCTION

Harari's dual absorptive model based on the two-component duality has been quite successful in describing the gross structure of the differential cross sections of many two-body processes.[1] Particularly, this model has been claimed to correctly predict the crossover phenomena and dip structures of various differential cross sections. Since the concept of duality holds for the imaginary part of the amplitude,[2] it can safely be said that the gross structure of the imaginary part of the amplitude can be predicted by the dual absorptive model.

The dual absorptive model predicts that the imaginary parts of the s-channel helicity-nonflip and -flip amplitudes in the $I_t=1$ state of pion-nucleon scattering should be proportional to the Bessel functions of the zeroth and first orders, respectively, as

$$\operatorname{Im}G_{++}^{I_t=1}(s,t) \propto J_0(r\sqrt{-t}) \tag{1}$$

and

$$\operatorname{Im}G_{+-}^{I_t=1}(s,t) \propto J_1(r\sqrt{-t}), \tag{2}$$

where r is an interaction radius. With $r=5.01$ (GeV/c)$^{-1}$ (i.e., 1 F) Harari predicted the crossover zero of $\operatorname{Im}G_{++}^{I_t=1}(s,t)$ at $|t|=0.23$ (GeV/c)^2 and a zero of $\operatorname{Im}G_{+-}^{I_t=1}(s,t)$ at $|t|=0.6$ (GeV/c)^2, which corresponds to the dip structure of the differential cross section.

However, the recent phenomenological analyses[3,4] of the πN amplitude have shown that the crossover zero of $\operatorname{Im}G_{++}^{I_t=1}(s,t)$ is located at $|t|=0.13-0.15$ (GeV/c)^2 rather than at $|t|=0.23$ (GeV/c)^2 as predicted by Harari. Therefore, if we use the interaction radius $r=6.21$ (GeV/c)$^{-1}$, which is determined in such a way as to produce a crossover zero at $|t|=0.15$ (GeV/c)^2, we obtain the second zero of $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ at $|t|=0.8$ (GeV/c)^2 due to the second zero of the Bessel functions of the zeroth order. And the dip zero of $\mathrm{Im}G_{+-}^{I_t=1}(s,t)$ is located at $|t|=0.38$ (GeV/c)^2.

On the other hand, the recent measurement of $\pi^- p$ charge-exchange polarization[5,6] and $\pi^\pm p$ spin rotation parameters[7] have made it possible to study the amplitude structure of pion-nucleon scattering in the relatively large $|t|$ regions very accurately. In particular, Kelly[8] has shown by using the method of analytic data analysis that the imaginary part of the $I_t=1$, s-channel helicity nonflip amplitude does not have the second zero near $|t| \simeq 0.8$ (GeV/c)^2. A similar result has also been obtained with the model-independent amplitude analysis by the Saclay group.[9] Although these amplitude analyses beyond $|t| \geqslant 0.5$ (GeV/c)^2 are less reliable because of lack of the experimental data on the spin rotation parameters, the results suggest the possible deviation of the amplitude structure from the prediction of Harari's model.

The direct comparison of

$$\mathrm{Im}G_{++}^{I_t=1}(s,t) \sim \frac{d\sigma}{dt}(\pi^- p) - \frac{d\sigma}{dt}(\pi^+ p)$$

with experimental data shows the possible existence of the second zero at $|t|=1.2-1.5$ (GeV/c)^2 but not at $|t|=0.8$ (GeV/c)^2.

The location of the dip zero at $|t|=0.38$ (GeV/c)^2 in $\mathrm{Im}G_{+-}^{I_t=1}(s,t)$ with $r=6.21$ (GeV/c)$^{-1}$ is, of course, inconsistent with the experimental data which show a dip at $|t|=0.5-0.6$ (GeV/c)^2.

From the above discussions we may conclude that although Eqs. (1) and (2) are conjectured to be the result of the complete absorption of ρ-Regge-pole exchange, further absorption to Eqs. (1) and (2) is necessary to produce the consistent results with the current amplitude analyses. It is, therefore, worthwhile to study the possible new absorption effect in order to resolve the discrepancy between Harari's dual absorptive model and the results of the amplitude analyses.

In this paper we show that the above inconsistency can be resolved by including the Regge-cut-like new absorption effect in the dual absorptive model. It is shown that in the dual absorptive model with the Regge-cut-like new absorption effect added the location of the second zero of $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ would be shifted to the larger $|t|$ value than that of the prediction of Eq. (1). In fact, in our numerical calculations of πN amplitude the second zero is either located at $|t|>1.5$ (GeV/c)^2 or does not exist in the region $|t|<2.0$ (GeV/c)^2.

Since our result is consistent with those of the phenomenological amplitude analyses, this may indicate the existence of the new absorption effect in Harari's dual absorptive model.

In Sec. II, the formulation for the new absorption effect proposed here is given and in Sec. III the results of the numerical fit to the data and detailed amplitude structure are presented. Some concluding remarks are also given.

II. FORMALISM

In order to resolve the difficulty of Harari's dual absorptive model, we propose here a new Regge-cut-like absorption effect which should be included in Harari's model.

First of all, it should be noted that in the t-channel description of the dual absorptive models, Eqs. (1) and (2) are the results of the sums of the conventional Regge poles and their associated Regge cuts.

In order to distinguish the dual absorptive model description from the conventional Regge-pole description, we use the notations $\bar{P}$, $\bar{P}'$, $\bar{\rho}$, and so on for the physical quantities represented by the sum of a Regge pole and its associated Regge cut in the dual absorptive model as opposed to P, P', ρ and so on in the Regge-pole model.

We employ Glauber's formalism in order to generate the new absorption.[10] The s-channel helicity-nonflip amplitude $G_{++}(s,t)$ and helicity-flip amplitude $G_{+-}(s,t)$ in pion-nucleon scattering are given by

$$G_{++}(s,t) = ikW\cos(\tfrac{1}{2}\theta) \times \int_0^\infty b\,db\,J_0(b\sqrt{-t})[1-e^{i\chi_0(s,b)}\cos\chi_f(s,b)], \tag{3}$$

and

$$G_{+-}(s,t) = kW\int_0^\infty b\,db\,J_1(b\sqrt{-t}) \times e^{i\chi_0(s,t)}\sin\chi_f(s,b), \tag{4}$$

where the eikonals $\chi_0(s,b)$ and $\chi_f(s,b)$ are assumed to be obtained by Fourier-Bessel transformations of "Born" terms $G_{++}^{(\bar{R})}(s,t)$ and $G_{+-}^{(\bar{R})}(s,t)$ in the dual absorptive model, respectively. $G_{++}^{(\bar{R})}(s,t)$ and $G_{+-}^{(\bar{R})}(s,t)$ are given by the sums of $\bar{P}$, $\bar{P}'$, and $\bar{\rho}$ exchanges as

$$G_{++}^{(\bar{R})}(s,t) = G_{++}^{(\bar{P})}(s,t) + G_{++}^{(\bar{P}')}(s,t) \pm G_{++}^{(\bar{\rho})}(s,t) \tag{5}$$

and

$$G_{+-}^{(\bar{R})}(s,t) = G_{+-}^{(\bar{P})}(s,t) + G_{+-}^{(\bar{P}')}(s,t) \pm G_{+-}^{(\bar{\rho})}(s,t), \tag{6}$$

where $\pm$ correspond to the $\pi^\mp p$ scattering processes, respectively.

The individual eikonals can be obtained in the following way:

$$\chi_0^{(\bar{i})}(s,b)=\frac{1}{kW}\int_0^\infty \sqrt{-t}\,d(\sqrt{-t})\times J_0(b\sqrt{-t})\frac{G_{++}^{(\bar{i})}(s,t)}{\cos\frac{1}{2}\theta} \tag{7}$$

and

$$\chi_f^{(\bar{i})}(s,b)=\frac{1}{kW}\int_0^\infty \sqrt{-t}\,d(\sqrt{-t})\times J_1(b\sqrt{-t})G_{+-}^{(\bar{i})}(s,t), \tag{8}$$

where i stands for $\bar{P}$, $\bar{P}'$, or $\bar{\rho}$ "Born" terms in the dual absorptive model.

Since the s-channel helicity in $I_t=0$ state is approximately conserved,[11-13] we neglect $G_{+-}^{(\bar{P})}(s,t)$ and $G_{+-}^{(\bar{P}')}(s,t)$ in Eq. (6).

In the dual absorptive model, $\bar{P}'$ and $\bar{\rho}$ terms in Eqs. (5) and (6) should be expressed according to the Harari's idea.[1] However, since we are concerned with only the structure of $I_t=1$ amplitude and since $\bar{P}$ term dominates over $\bar{P}'$ term in $I_t=0$ amplitude, we assume the conventional Regge-pole expression for $G_{++}^{(\bar{P}')}(s,t)$ as well as for $G_{++}^{(\bar{P})}(s,t)$.[14] Only $G_{++}^{(\bar{\rho})}(s,t)$ and $G_{+-}^{(\bar{\rho})}(s,t)$ are expressed according to Harari's prescription.[1]

With the above assumptions, we now write

$$G_{++}^{(\bar{i})}(s,t)=-C_0^{\bar{i}}\exp(C_1^{\bar{i}}t)\times\tau_i(t)(s/s_0)^{\alpha_i(t)}\cos(\tfrac{1}{2}\theta)\quad \text{for } i=\bar{P} \text{ and } \bar{P}', \tag{9}$$

where the signature factor $\tau_i(t)$ is given in an approximate form as

$$\tau_i(t)=\exp[-i\tfrac{1}{2}\pi\alpha_i(t)], \tag{10}$$

and we note here that we are using the conventional Regge trajectories.

Following Harari's idea,[1] the imaginary parts of the $\bar{\rho}$-exchange amplitudes are assumed to be

$$\mathrm{Im}G_{++}^{(\bar{\rho})}(s,t)\sim C_0^{\bar{\rho}}\exp(C_1^{\bar{\rho}}t)\times J_0(r\sqrt{-t})(s/s_0)^{\alpha_\rho(t)} \tag{11}$$

and

$$\mathrm{Im}G_{+-}^{(\bar{\rho})}(s,t)\sim D_0^{\bar{\rho}}\exp(D_1^{\bar{\rho}}t)\times J_1(r\sqrt{-t})(s/s_0)^{\alpha_\rho(t)}. \tag{12}$$

It is difficult to make any specific statements concerning the real parts of the $\bar{\rho}$-exchange amplitudes. The contributions of the s-channel resonances to the real part are not as localized in energy as the contribution to the imaginary part. However, the detailed structure of the real parts of the amplitudes is not vital to the present problem; we, therefore, use one of the models at the high-energy limit suggested by Harari for the finite-energy region in this paper.[1] That is, we assume that the full amplitudes are obtained by multiplying the signature factor $\tau_\rho(t)$ to Eqs. (11) and (12). In order to remove the poles at integer $\alpha_\rho(t)$ in the signature factor, we approximate $\tau_\rho(t)$ as

$$\tau_\rho(t)=-\frac{1-e^{-i\pi\alpha_\rho(t)}}{\sin\pi\alpha_\rho(t)}\simeq -ie^{-i\pi\alpha_\rho(t)/2}. \tag{13}$$

With Eq. (13), we obtain the $\bar{\rho}$-pole-exchange amplitudes in the dual absorptive model as

$$G_{++}^{(\bar{\rho})}(s,t)=-C_0^{\bar{\rho}}\,\tau_\rho(t)\exp(C_1^{\bar{\rho}}t)\times J_0(r\sqrt{-t})(s/s_0)^{\alpha_\rho(t)}\cos(\tfrac{1}{2}\theta) \tag{14}$$

and

$$G_{+-}^{(\bar{\rho})}(s,t)=-D_0^{\bar{\rho}}\,\tau_\rho(t)\exp(D_1^{\bar{\rho}}t)\times J_1(r\sqrt{-t})(s/s_0)^{\alpha_\rho(t)}. \tag{15}$$

Introducing Eq. (9) into Eq. (7), we obtain the eikonals for $i=\bar{P}$ or $\bar{P}'$ as

$$\chi_0^{(\bar{i})}(s,b)=-\frac{C_0^{(\bar{i})}e^{\alpha_i(0)\xi}}{2kW\eta_0^{(\bar{i})}}\exp\left(-\frac{b^2}{4\eta_0^{(\bar{i})}}\right), \tag{16}$$

where

$$\alpha_i(t)=\alpha_i(0)+\alpha_i'(0)t, \tag{17}$$

$$\xi=\ln(s/s_0)-i\tfrac{1}{2}\pi, \tag{18}$$

and

$$\eta_0^{(\bar{i})}=\alpha_i'(0)\xi+C_1^{\bar{i}}. \tag{19}$$

Introducing Eq. (14) into Eq. (7) and Eq. (15) into Eq. (8), respectively, we obtain the eikonals for $\bar{\rho}$ pole in the dual absorptive model as

$$\chi_0^{(\bar{\rho})}(s,b)=-i\frac{C_0^{(\bar{\rho})}e^{\alpha_\rho(0)\xi}}{2kW\eta_0^{(\bar{\rho})}}\exp\left(-\frac{r^2+b^2}{4\eta_0^{(\bar{\rho})}}\right)I_0\left(\frac{rb}{2\eta_0^{(\bar{\rho})}}\right) \tag{20}$$

and

$$\chi_f^{(\bar{\rho})}(s,b) = -i\,\frac{D_0^{\bar{\rho}} e^{\alpha_\rho(0)\xi}}{2kW\eta_f^{(\bar{\rho})}}\,\exp\left(-\frac{r^2+b^2}{4\eta_f^{(\bar{\rho})}}\right) I_1\left(\frac{rb}{2\eta_f^{(\bar{\rho})}}\right), \tag{21}$$

where

$$\eta_0^{(\bar{\rho})} = \alpha_\rho'(0)\xi + C_1^{\bar{\rho}} \tag{22}$$

and

$$\eta_f^{(\bar{\rho})} = \alpha_\rho'(0)\xi + D_1^{\bar{\rho}}\,. \tag{23}$$

$I_0(Z)$ and $I_1(Z)$ are the modified Bessel functions of the first kind.

In order to calculate $G_{++}(s,t)$ and $G_{+-}(s,t)$ we expand the right-hand sides of Eqs. (3) and (4) in terms of eikonals and retain only the terms of the first and second powers in eikonals.

Thus, we obtain

$$G_{++}(s,t) = G_{++}^{(\bar{P})}(s,t) + G_{++}^{(\bar{P}')}(s,t) \pm G_{++}^{(\bar{\rho})}(s,t) + G_{++}^{(\bar{P}\bar{P})}(s,t) + G_{++}^{(\bar{P}\bar{P}')}(s,t) \pm G_{++}^{(\bar{P}\bar{\rho})}(s,t) + G_{++}^{(\bar{P}'\bar{P}')}(s,t) \pm G_{++}^{(\bar{P}'\bar{\rho})}(s,t) \tag{24}$$

and

$$G_{+-}(s,t) = \pm G_{+-}^{(\bar{\rho})}(s,t) \pm G_{+-}^{(\bar{P}\bar{\rho})}(s,t) \pm G_{+-}^{(\bar{P}'\bar{\rho})}(s,t)\,. \tag{25}$$

Each Regge-cut-like absorption term in Eqs. (24) and (25) has the following expression after integration over the impact parameter:

$$G_{++}^{(\bar{i}\bar{i})}(s,t) = i\,\frac{(C_0^{\bar{i}})^2 e^{2\alpha_i(0)\xi}}{8kW\eta_0^{(\bar{i})}}\,\exp(\tfrac{1}{2}\eta_0^{(\bar{i})}t)\cos(\tfrac{1}{2}\theta), \tag{26a}$$

$$G_{++}^{(\bar{i}\bar{\rho})}(s,t) = -\frac{C_0^{\bar{i}}C_0^{\bar{\rho}} e^{[\alpha_i(0)+\alpha_\rho(0)]\xi}}{2kW(\eta_0^{(\bar{i})}+\eta_0^{(\bar{\rho})})}\,\exp\left(-\frac{r^2}{4\eta_0^{(\bar{i})}}\right)\exp\left\{\frac{\eta_0^{(\bar{i})}\eta_0^{(\bar{\rho})}}{\eta_0^{(\bar{i})}+\eta_0^{(\bar{\rho})}}\left[\left(\frac{r}{2\eta_0^{(\bar{\rho})}}\right)^2+t\right]\right\} J_0\left(\frac{\eta_0^{(\bar{i})}r}{\eta_0^{(\bar{i})}+\eta_0^{(\bar{\rho})}}\sqrt{-t}\right)\cos(\tfrac{1}{2}\theta), \tag{26b}$$

$$G_{++}^{(\bar{P}\bar{P}')}(s,t) = i\,\frac{C_0^{\bar{P}}C_0^{\bar{P}'} e^{[\alpha_P(0)+\alpha_{P'}(0)]\xi}}{2kW(\eta_0^{(\bar{P})}+\eta_0^{(\bar{P}')})}\,\exp\left(\frac{\eta_0^{(\bar{P})}\eta_0^{(\bar{P}')}}{\eta_0^{(\bar{P})}+\eta_0^{(\bar{P}')}}\,t\right)\cos(\tfrac{1}{2}\theta), \tag{26c}$$

and

$$G_{+-}^{(\bar{i}\bar{\rho})}(s,t) = -\frac{C_0^{\bar{i}}D_0^{\bar{\rho}} e^{[\alpha_i(0)+\alpha_\rho(0)]\xi}}{2kW(\eta_0^{(\bar{i})}+\eta_f^{(\bar{\rho})})}\,\exp\left(-\frac{r^2}{4\eta_f^{(\bar{\rho})}}\right)\exp\left\{\frac{\eta_0^{(\bar{i})}\eta_f^{(\bar{\rho})}}{\eta_0^{(\bar{i})}+\eta_f^{(\bar{\rho})}}\left[\left(\frac{r}{2\eta_f^{(\bar{\rho})}}\right)^2+t\right]\right\} J_1\left(\frac{\eta_0^{(\bar{i})}r}{\eta_0^{(\bar{i})}+\eta_f^{(\bar{\rho})}}\sqrt{-t}\right). \tag{26d}$$

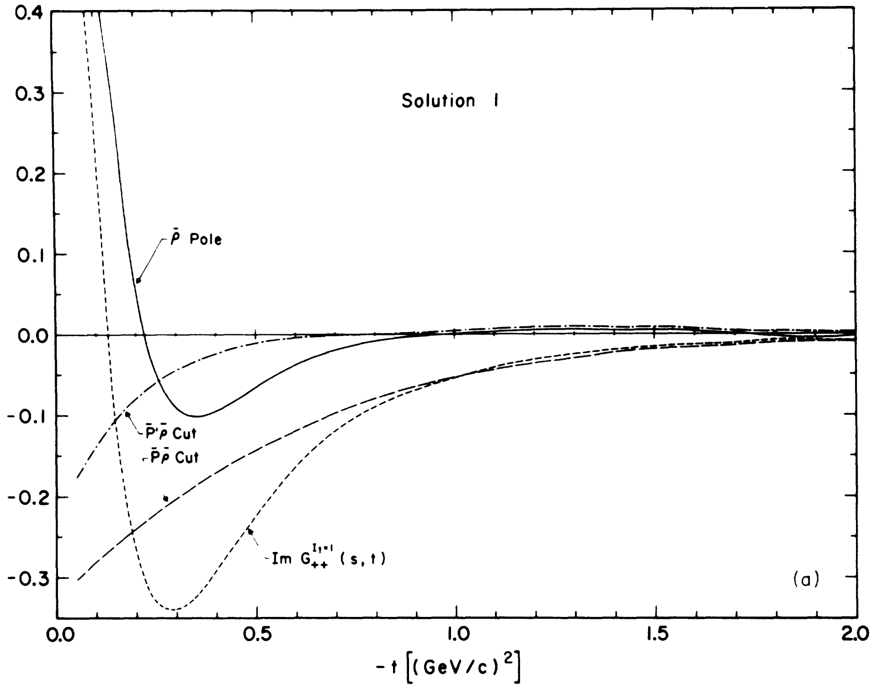
In Eqs. (26a), (26b), and (26d) $\bar{i}$ stands for $\bar{P}$ or $\bar{P}'$.

It is interesting to note that the Regge-cut-like new absorption term (26b) due to $\bar{P}$ and $\bar{\rho}$ double exchange is again proportional to the zeroth-order Bessel function but with different argument.

The real part of the coefficient of $\sqrt{-t}$ in the argument of Bessel function is generally smaller than $\frac{1}{3}$ of the interaction radius r. In fact, as is shown later, the above real part is so small that the first zero of the real part of the zeroth-order Bessel function in Eq. (26b) does not occur in the region $|t| < 1.5$ $(\mathrm{GeV}/c)^2$.

III. NUMERICAL RESULTS AND DISCUSSION

Since we are concerned with only the gross structure of $\mathrm{Im}G_{++}^{I_t=1}(s,t)$, we may safely use the fixed interaction range r and the fixed Regge trajectory parameters. We present here two sets of solutions. In solution 1, the following values for r and Regge parameters are used: $r = 5.11$ $(\mathrm{GeV}/c)^{-1}$, $\alpha_P(0) = 1.0$, $\alpha_P'(0) = 0.5$ $(\mathrm{GeV}/c)^{-2}$, $\alpha_{P'}(0) = 0.5$, $\alpha_{P'}'(0) = 1.0$ $(\mathrm{GeV}/c)^{-2}$, $\alpha_\rho(0) = 0.5$, and $\alpha_\rho'(0) = 1.0$ $(\mathrm{GeV}/c)^{-2}$. In solution 2, we use $r = 5.52$ $(\mathrm{GeV}/c)^{-1}$, $\alpha_P(0) = 1$, and $\alpha_P'(0) = 0.6$



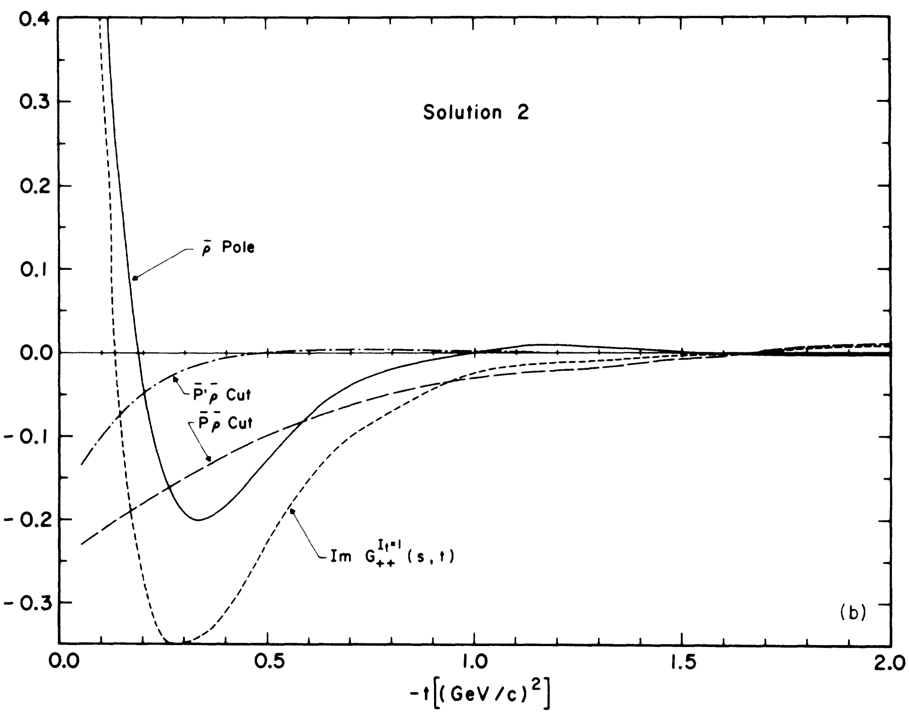


FIG. 1. Structure of $\mathrm{Im}G_{++}^{I_t=1}(s,\,t)$ of pion-nucleon scattering at 6 GeV/c. Also shown are the contributions of a $\bar{\rho}$ pole and $\bar{P}\bar{\rho}$ and $\bar{P}'\bar{\rho}$ Regge-cut-like new absorption terms in the dual absorption model. (a) Solution 1. (b) Solution 2.

$(\mathrm{GeV}/c)^{-2}$, while P' and ρ Regge trajectory parameters remain the same as in solution 1.

In order to determine eight residue parameters, $C_0^{\bar{P}}$, $C_1^{\bar{P}}$, $C_0^{\bar{P}'}$, $C_1^{\bar{P}'}$, $C_0^{\bar{\rho}}$, $C_1^{\bar{\rho}}$, $D_0^{\bar{\rho}}$, and $D_1^{\bar{\rho}}$ in each solution, we apply the least-χ^2 fit to the experimental data[15] with our model.

The determined parameter values are given in Ref. 16 and the details of result of fit are given in Ref. 17.

It has been known that with models such as the dual absorption model and the nonsense–wrong-signature–zero model for the ρ Regge pole, we cannot reproduce the large positive polarization of the $\pi^- p$ charge-exchange reaction in the region $0.2<|t|<0.5$ $(\mathrm{GeV}/c)^2$.[4,18] The problem to fit $\pi^- p$ charge-exchange polarization seems closely associated with the structure of the real parts of the helicity amplitudes.[19] Since the real part of the amplitude is not our central problem in this paper and it does not affect our conclusion, we do not attempt to improve the fit to $\pi^- p$ charge-exchange polarization.

With the residue parameters determined as above, we compute the imaginary part of the helicity-nonflip amplitude in $I_t=1$ state. The results are shown in Figs. 1(a) and 1(b) for solutions 1 and 2, respectively, along with the contributions from the $\bar{\rho}$ term, $\bar{P}\bar{\rho}$ and $\bar{P}'\bar{\rho}$ Regge-cut-like new absorption terms.

$\mathrm{Im}G_{++}^{(\bar{\rho})}(s,t)$ has, of course, two zeros in both solutions between $|t|=0.0$ and 2.0 $(\mathrm{GeV}/c)^2$ corresponding to the zeros of the Bessel function. The first zero is the well-known crossover zero. However, both the results of the phenomenological amplitude analyses by Kelly[8] and the Saclay group[9] do not show any sign of the existence of the second zero near $|t|\simeq 0.8$ $(\mathrm{GeV}/c)^2$. In our calculations, the $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ which is the sum of a $\bar{\rho}$ pole and $\bar{P}\bar{\rho}$ and $\bar{P}'\bar{\rho}$ absorption terms does not have a second zero near $|t|\simeq 0.8$ $(\mathrm{GeV}/c)^2$ in agreement with the results of the amplitude analyses. The physical reason for this is easily understood because the $\bar{P}\bar{\rho}$ absorption term Eq. (26b) is proportional to the Bessel function of zero order with a different argument from that of Eq. (14). With our parameters in solution 1, the real part of the Bessel function in Eq. (26b) does not have any zero in the region $|t|<2.0$ $(\mathrm{GeV}/c)^2$. Furthermore, the magnitude of the $\bar{P}\bar{\rho}$ absorption term in the dual absorptive model is already larger than that of the $\bar{\rho}$ pole term at $|t|=0.3$ $(\mathrm{GeV}/c)^2$, and for this reason, the monotonic behavior for $|t|>0.3$ $(\mathrm{GeV}/c)^2$ without change of sign of $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ is obtained. The crossover zero of $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ is located at $|t|=0.13$ $(\mathrm{GeV}/c)^2$ rather than at $|t|=0.22$ $(\mathrm{GeV}/c)^2$ of $\mathrm{Im}G_{++}^{(\bar{\rho})}(s,t)$ in accordance with the experimental data.

With parameters in solution 2, the real part of the coefficients of $\sqrt{-t}$ in the argument of the Bessel function in Eq. (26b) is larger than that of solution 1 and smaller than $\frac{1}{3}r$. The first zero of $\mathrm{Im}G_{++}^{(\bar{P}\bar{\rho})}(s,t)$ in this case is located at $|t|\simeq 1.7$ $(\mathrm{GeV}/c)^2$. By adding $\mathrm{Im}G_{++}^{(\bar{P}\bar{\rho})}(s,t)$ to $\mathrm{Im}G_{++}^{(\bar{\rho})}(s,t)$, we obtain the rough structure of $\mathrm{Im}G_{++}^{(I_t=1)}(s,t)$. It is seen that the first zero of $\mathrm{Im}G_{++}^{(\bar{P}\bar{\rho})}(s,t)$ approximately coincides with the second zero of $\mathrm{Im}G_{++}^{(I_t=1)}(s,t)$ since the contributions from $\bar{\rho}$ and $\bar{P}'\bar{\rho}$ terms are negligible in this $|t|$ region. The crossover zero of $\mathrm{Im}G_{++}^{(I_t=1)}(s,t)$ is again shifted to $|t|=0.13$ $(\mathrm{GeV}/c)^2$ from $|t|=0.19$ $(\mathrm{GeV}/c)^2$ of $\mathrm{Im}G_{++}^{(\bar{\rho})}(s,t)$ as in solution 1.

Although it is not certain whether the second zero of $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ exists or not at this stage of amplitude analysis, the prediction of the second zero at $|t|\simeq 0.8$ $(\mathrm{GeV}/c)^2$ in $\mathrm{Im}G_{++}^{I_t=1}(s,t)$ by Harari's model with the crossover zero at $|t|=0.15$ $(\mathrm{GeV}/c)^2$ seems inconsistent with the results of the current amplitude analyses. On the other hand, our result shows that the second zero should be located at a larger $|t|$ region or does not exist at all.

Finally it is interesting to study the similar absorption effects discussed here in other collision processes. Particularly, in pp and $\bar{p}p$ scattering, it seems that there is no second zero at all while the crossover zero is located at $|t|=0.13$ $(\mathrm{GeV}/c)^2$.[20]

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[15]The experimental data used in the analysis are as follows.

σ_t: K. J. Foley *et al.*, Phys. Rev. Letters 19, 193 (1967).

$\alpha = \mathrm{Re}G_{++}(s,t=0)/\mathrm{Im}G_{++}(s,t=0)$: K. J. Foley *et al.*, *ibid.* 14, 862 (1965).

$d\sigma/dt$: C. T. Coffin *et al.*, Phys. Rev. 159, 1169 (1967); K. J. Foley *et al.*, Phys. Rev. Letters 11, 423 (1963); M. Yvert *et al.*, 1968 (private communication).

$P(\theta)$: M. Borghini *et al.*, Phys. Letters 24B, 77 (1967); DESY-Saclay Collaboration, Ref. 6.

$R(t)$: Ref. 7.

[16]The residue parameters determined with the least-χ^2 fit to the data are as follows. The numbers without parentheses indicate the results of solution 1, and the results of solution 2 are in the parentheses.

$C_0^{\bar{P}} = 2.49$ (2.45), $C_0^{\bar{P}'} = 5.34$ (4.97), $C_0^{\bar{\rho}} = 0.81$ (0.78). $D_0^{\bar{\rho}} = -1.65$ (-1.60) [unit is $(\mathrm{GeV}/c)^{-1}$].

$C_1^{\bar{P}} = 0.76$ (0.56), $C_1^{\bar{P}'} = 3.88$ (5.28), $C_1^{\bar{\rho}} = 3.08$ (1.72), $D_1^{\bar{\rho}} = -0.04$ (0.12) [units are $(\mathrm{GeV}/c)^{-2}$].

[17]Total χ^2 are 309.6 (369.6) for solution 1 (solution 2) for 161 data points. χ^2/No. of data points for each process are as follows.

$\sigma_t^{\pi^- p}$: 33.9 (31.8)/16

$\sigma_t^{\pi^+ p}$: 4.0 (2.1)/8

$\alpha^{\pi^- p}$: 39.3 (35.4)/13

$\alpha^{\pi^+ p}$: 12.8 (11.4)/8

$\Delta = \sigma_t^{\pi^- p} - \sigma_t^{\pi^+ p}$: 0.9 (1.1)/10

$(d\sigma/dt)^{\pi^- p}$: 16.0 (23.3)/14

$(d\sigma/dt)^{\pi^+ p}$: 26.1 (36.5)/14

$(d\sigma/dt)^{\mathrm{C.E.}}$: 46.4 (63.8)/17

$P^{\pi^- p}(\theta)$: 26.3 (36.5)/16

$P^{\pi^+ p}(\theta)$: 67.2 (80.1)/13

$P^{\mathrm{C.E.}}(\theta)$: 24.6 (34.6)/16

$R^{\pi^- p}(t)$ 4.1 (3.2)/8

$R^{\pi^+ p}(t)$: 8.0 (9.9)/8.

[18]B. Schrempp-Otto and F. Schrempp, review talk at the Titisee discussion meeting on Two-Body Reactions at High Energies, 1971 (unpublished).

[19]M. F. Coirier, J. L. Guillaume, Y. Leroyer, and Ph. Salin, Université de Bordeaux I Report No. PTB-45, 1971 (unpublished).

[20]V. Barger and F. Halzen, Report No. COO-881-323, 1971 (unpublished).