such as the change in the slope of elastic scattering.

The model is a phenomenological one, and we have not attempted to justify it by any underlying dynamical scheme. We therefore feel that the main result of this work is to provide some further support to the optical approach in dealing with diffractive scattering of hadrons.

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Charge Transfer in High-Energy Fragmentation*

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Nontransference of charge is an essential aspect of the hypothesis of limiting fragmentation for infinite-energy hadron-hadron collisions. One can define experimentally a charge transfer u from one c.m. momentum-space hemisphere to another. At finite energies, u is not zero because the fragments may "spill over" to the other hemisphere. A model is discussed which yields an estimate of u. The general validity of the energy and multiplicity dependence of this estimate is then commented upon.

INTRODUCTION

In the fragmentation picture[1] of high-energy hadron-hadron collisions, the fragments of the target, taken together, have the charge of the target, and similarly for the fragments of the projectile. Define the charge transfer u, an integer, by

$$u = [\tfrac{1}{2}(\text{total charge})_R - \tfrac{1}{2}(\text{total charge})_L]_{\text{final}} - [\tfrac{1}{2}(\text{total charge})_R - \tfrac{1}{2}(\text{total charge})_L]_{\text{initial}} . \quad (1)$$

Here R and L refer to the c.m. momentum-space forward (i.e., projectile) hemisphere and backward (i.e., target) hemispheres, respectively. If all the target fragments were to go to the left hemisphere and all projectile fragments to the right hemisphere, u would be zero. That is the expected state of affairs at incoming energy $E=\infty$ in the fragmentation picture. At a finite energy, u is not zero because there are "spill-over" fragments. Such fragments are, of course, not exactly definable. However, the name is useful since it can suggest models which we hope may capture some essential features of the dependence of u on the energy and on the multiplicity.

Experimentally, in hadron-hadron collisions, such as in πp and pp collisions, the transformation from the laboratory system to the c.m. system for a final charged particle is relatively free of ambiguities if the particle is negatively charged, since it is then most likely a π^-. For a positively charged fast particle, oftentimes an ambiguity exists because it could be a π^+ or a p. We therefore choose as variables l (and l'), the number of negative pions in the left, i.e., backward (and right, i.e., forward) hemisphere. The allowed points in an (l, l', u) plot form a lattice, and one wants to study the partial cross sections at each lattice point, and various averages over them.

The Michigan State University Bubble Chamber group (Oh and Smith) has kindly shown us some rough preliminary data, which are summarized in Table I. It is seen that as one increases the incoming energy, indeed the average of u^2 for fixed l, l' decreases. We are making no efforts to fit the decrease with a formula since the data are preliminary.

We interpret this preliminary result as indicating that indeed the existence of nonzero charge transfers is due to spill-overs. To gain some feeling for the magnitude of the averages of u and u^2 we shall discuss a detailed example which is consistent with the fragmentation picture. We then analyze various quantitative features of this model and discuss their possible general validity.

TABLE I. Preliminary data of the Michigan State University Bubble Chamber Group on the averages of u (top two rows) and u^2 (bottom three rows).

l	l' \ p_0 (GeV/c)	13	18	21	24	28
0	1	−0.51	−0.44	−0.46	−0.46	−0.40
0	2	−1.10	−1.00	−0.94	−1.02	−0.88
0	1	0.73	0.66	0.67	0.67	0.62
0	2	1.82	1.64	1.46	1.73	1.25
1	1	0.66	0.57	0.70	0.60	0.69

A SPECIAL MODEL

While at this moment any specific model for high-energy collisions is not likely to be correct in the strict sense, some general features of high-energy collisions may nevertheless be revealed by specific models if the reasons for these features in the model are understood and are found to be consistent with the essential points of the physical description of high-energy collisions. In this spirit we shall investigate a specific model in which the two incoming protons in a pp collision fragment, respectively, as follows:

$$\text{target} \qquad p \to p + l(\pi^-) + l(\pi^+) + l(\pi^0)\,, \tag{2}$$

$$\text{projectile} \quad p \to p + l'(\pi^-) + l'(\pi^+) + l'(\pi^0)\,. \tag{3}$$

Consistent with the hypothesis of limiting fragmentation,[1] at c.m. incoming momentum $p_0^*=\infty$, the $(3l+1)$ fragments in (2) all move in the same hemisphere in the c.m. momentum space. So do the fragments in (3). Thus the charge transfer is $u=0$. At a finite value of p_0^*, however, there are possible "spill-overs," in which a π^+ or several of them may go into the "wrong" hemisphere. Such spill-overs lead to nonvanishing values of u. The problem we want to address ourselves to in this section is to formulate a reasonable model where the number of such spill-overs can be estimated. To this end we make the following additional assumptions:

(a) In terms of the x variable (i.e., c.m. longitudinal momentum divided by incoming c.m. longitudinal momentum) we further assume in process (1) the outgoing protons to always have $x=7/(7+3l)$, and the sum of the x's of the outgoing π's to always be

$$\sum x = \gamma = \frac{3l}{7+3l}\,. \tag{4}$$

[7:3l is the ratio of the proton mass to the static total mass of the 3l pions. Equation (4) is of course not exactly correct, but we believe some general features of the fluctuation of the quantity u, the charge transfer, are not very sensitively dependent on this point.] Thus we assume that the proton does not "spill over."

(b) We further assume that at $p_0^*=\infty$ the pion momenta are distributed so that, aside from (4), only the phase-space factor is important:

$$\prod_1^{3l} dx\, \delta\left(\sum x - \gamma\right). \tag{5}$$

The two fragmentation processes (2) and (3) are assumed to be independent.

(c) At finite values of p_0^*, (5) is not correct because the "limiting" fragmentation has not yet been reached. To estimate the "spill-overs" we assume that the ∞ energy distribution (5) for the fragments (2) still holds, but write x_∞ for x:

$$\prod_{1}^{3l} dx_\infty \delta(\sum x_\infty - \gamma)\,.$$

At a finite energy, however, $x = p_\parallel^*/p_0^*$ is not exactly x_∞. For small $|x_\infty|$, there is appreciable probability that x and x_∞ have opposite signs, i.e., there is appreciable probability for spill-overs. We assume that for a π^+ or a π^0, there is a probability $\nu(x_\infty)$ of spill-over, where $\nu(x)$ is only appreciable for

$$0 < x < \frac{m_\pi}{p_0^*}\,. \tag{6}$$

Where there is no confusion, we shall drop the subscript ∞ from x_∞. We have neglected the effect of spill-overs of π^-. In other words, we assume that the l π^-'s in one hemisphere and the l' π^-'s in the other are all non-spill-overs. Since particles with small x are more likely to spill over, this model is therefore not quite realistic if any of the negative pions have small c.m. momenta. The model is thus more relevant for smaller multiplicities.

Under these assumptions it is straightforward to evaluate various moments of the charge transfer u in terms of those of the function $\nu(x)$. To do this one notices that

$$u = u_T - u_P\,,$$

where

u_T = contribution to u from spill-over positive pions in process (2)

$$= \sum_{i=1}^{l} \nu(x_i)\,,$$

with the summation extending over the l positive pions in (2). Similarly,

$$u_P = \sum_{i=1}^{l'} \nu(x_i)\,,$$

with the summation extending over the l' positive pions in (3).

Thus for fixed l and l', the averages of u and u^2 are

$$\langle u \rangle = l\{\nu\}_l - l'\{\nu\}_{l'} \tag{7}$$

and

$$\langle u^2 \rangle = l\{\nu^2\}_l + l(l-1)\{\nu(x_1)\nu(x_2)\}_l + (\text{same with } l \to l') - 2ll'\{\nu\}_l\{\nu\}_{l'}\,. \tag{8}$$

In these formulas $\{\nu\}_l$ means the average of $\nu(x_1)$ over the space $x_1 \cdots x_l$ with the weight (5).

To obtain some explicit results let us try the case where

$$\nu(x) = 0 \quad \text{for} \quad x > \frac{a}{p_0^*} \qquad (a = \text{constant} \sim 0.2\ \text{GeV}/c)$$

and

$$\nu(x) = 1 \quad \text{for} \quad \frac{a}{p_0^*} > x > 0\,. \tag{9}$$

Then

$$\{\nu\}_l = \{\nu^2\}_l = 1 - \left(1 - \frac{a}{\gamma p_0^*}\right)^{3l-1} \approx (3l-1)\frac{a}{\gamma p_0^*}\,, \tag{10}$$

which we assume to be $\ll 1$. (We assume $l \neq 0$.) Also,

$$\{\nu(x_1)\nu(x_2)\}_l = 1 - 2\left(1 - \frac{a}{\gamma p_0^*}\right)^{3l-1} + \left(1 - \frac{2a}{\gamma p_0^*}\right)^{3l-1} \approx \left(\frac{a}{\gamma p_0^*}\right)^2 (3l-1)(3l-2)\,. \tag{11}$$

Formulas (7)–(11) give the averages $\langle u \rangle$ and $\langle u^2 \rangle$ for fixed l and l' for this specific model:

$$\langle u \rangle \approx \frac{3a}{p_0^*}(l - l')(l + l' + 2) \qquad (l, l' \neq 0)\,, \tag{12}$$

$$\langle u^2 \rangle - \langle u \rangle^2 \approx \frac{a}{3p_0^*}[(3l-1)(3l+7) + (3l'-1)(3l'+7)] \qquad (l, l' \neq 0)\,. \tag{13}$$

DISCUSSIONS

(1) The sign of $\langle u \rangle(l, l')$ as exhibited in (12) is in agreement with Table I.

(2) For fixed l and l', $\langle u \rangle \to 0$ and $\langle u^2 \rangle \to 0$ as $p_0^* \to \infty$. In fact, the cross section $\sigma \to 0$ for all events for which $u \neq 0$, while the cross section $\sigma \to$ finite limit $\neq 0$ for the case $u = 0$. This is a general feature of the hypothesis of limiting fragmentation.

It may be that the observed decrease[2] of the cross section for 4-prong events in pp collisions, roughly in proportion to $p_{\text{lab}}^{-0.4}$, from 28 to 200 GeV/c, is largely due to the decrease of cross sections for events for which there is a charge or isotopic spin transfer. It would be interesting to study whether, for 4-prong events with $u = 0$, there is no decrease, or a slower decrease than $p_{\text{lab}}^{-0.4}$.

(3) In the specific model, for fixed l and l', $\langle u \rangle$ and $\langle u^2 \rangle - \langle u \rangle^2$ decrease with increasing incoming energy like $(p_0^*)^{-1} \sim p_{\text{lab}}^{-0.5}$. This dependence stems mainly from assumption (9) about the spill-over probability $\nu(x)$. It seems to us that while assumption (9) is certainly too crude, spill-overs prob-

ably do occur only appreciably for pions with c.m. momentum which are less than some constant. Thus, with the scaling property of c.m. longitudinal momentum, it is reasonable to expect $\langle u\rangle$ and $\langle u^2\rangle - \langle u\rangle^2$ to decrease like $(p_0^*)^{-1}$.

Furthermore, since most events have small multiplicities, for almost all events the average of u^2 would approach zero. To be more precise, for any given number $f< 1$, consider that fraction f of the collision with the smallest number of charged prongs. For these collisions the average of u^2 approaches zero as $E \to \infty$, for *any* fixed $f< 1$.

(4) As far as l and l' dependence is concerned, (12) and (13) show that $\langle u\rangle$ and $\langle u^2\rangle - \langle u\rangle^2$ are quadratically dependent on l and l'. The origin of the quadratic dependence lies in the fact that large l values imply large multiplicities, and large multiplicities cause individual pions to have small x values, which leads to large probabilities of spillover.

If particles are emitted independently, then for fixed l and l' the $l+l'$ positive pions would be equally distributed in the two hemispheres on the average, and

$$\langle u\rangle = \tfrac{1}{2}(l-l')\,. \qquad (14)$$

This has a very different dependence both on the incoming energy and on l and l'.

(5) The statement that $\langle u\rangle \to 0$ as the incoming energy approaches ∞ at a fixed l and l' holds also for a Lorentz frame that maintains a finite fixed velocity relative to the c.m. system. The essential point is that at very high energies the projectile fragments all travel with very large velocities in the c.m. system, toward one side. So do the target fragments toward the other side. The probability that a fragment changes hemispheres under a Lorentz transformation with a fixed finite velocity is small, and approaches zero as the energy approaches ∞.

(6) Let us denote by a second angular bracket averages over all l and l', such as $\langle\langle u\rangle\rangle$, $\langle\langle u^2\rangle\rangle$, etc. Symmetry of the two incoming protons requires

$$\langle\langle u\rangle\rangle = 0\,. \qquad (15)$$

In the specific model above, the value of $\langle\langle u^2\rangle\rangle$ becomes large at high energies because according to (12) and (13) the term

$$\langle u\rangle^2 \approx \frac{9a^2}{p_0^{*2}}(l-l')^2(l+l'+2)^2$$

contains quartic terms in l. We believe[3] that in a power fit

$$\langle l^4\rangle \propto (p_0^*)^\alpha$$

one has

$$\alpha = 3\,.$$

Thus

$$\langle\langle u^2\rangle\rangle \propto p_0^* \qquad (16)$$

at very high energies.

Is this statement inconsistent with the second paragraph of item 3 above? The answer is no. The large contributions to $\langle\langle u^2\rangle\rangle$ in (16) all come from a small fraction of the collisions with very large multiplicities and large values of $l-l'$ (i.e., unsymmetrical fragmentations). Of course, since our model may not be realistic for large multiplicities, (16) may not be accurate. But the existence of large contributions to $\langle\langle u^2\rangle\rangle$ due to large unsymmetrical fragmentation seems a safe guess.

(7) The discussion above about the charge transfer u could be extended to the nucleon number transfer v:

$$v = [\tfrac{1}{2}(\text{nucleon number})_R - \tfrac{1}{2}(\text{nucleon number})_L]_{\text{final}} - [\tfrac{1}{2}(\text{nucleon number})_R - \tfrac{1}{2}(\text{nucleon number})_L]_{\text{initial}}\,.$$

They can also be extended to the transfer of other quantum numbers, such as the strangeness.

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