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Correlations Between Neutral and Charged Pions in Multiparticle Production*

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We investigate the dynamical significance of correlations between $\langle n_0\rangle_-$ and n_-, where $\langle n_0\rangle_-$ is the mean number of neutral pions observed in conjunction with n_- negative tracks in high-energy multiparticle production processes. Several models are treated in turn. Pions are assumed to be produced either singly or in clusters, according to a Poisson or inverse-power distribution. Influences of charge, isospin, and energy-momentum conservation are studied. The linear rise of $\langle n_0\rangle_-$ vs n_- observed experimentally at very high energies is shown to rule out some models of multiperipheral type, but is in accord with fragmentation models and with a multiperipheral model in which ρ- or ω-type meson clusters are emitted.

I. INTRODUCTION

An interesting correlation measured in multiparticle production experiments is the variation of the average number of neutral pions $\langle n_0\rangle_-$ as a function of the number of charged particles.[1] Recently, new data have become available from the National Accelerator Laboratory (NAL), Serpukhov, and from the CERN Intersecting Storage Rings (ISR). These very-high-energy results[2] show a linear rise of $\langle n_0\rangle_-$ vs n_-, the number of negative tracks, significantly different from the rather flat behavior observed at 12, 19, and 25 GeV/c.[1] Data are summarized in Fig. 1.

In this paper we investigate the dynamical content of the observed correlation. We study the behavior of the function $\langle n_0\rangle_-$ vs n_- in different theoretical models in order to find out to what extent it may be used to discriminate between them. All the models have two common characteristics: First, they involve only production of pions either singly or in clusters, but other effects, such as K and $\bar{p}$ production are neglected. In addition, the models lead asymptotically to the same average number of neutral, negative, or positive pions,

$$\langle n_0\rangle = \langle n_+\rangle = \langle n_-\rangle, \tag{1}$$

although most models deviate from Eq. (1) at finite energies. The abundance of pions is a well-known property of multiparticle production, and Eq. (1) follows from a Mueller analysis[3] of the central region. It is substantiated by recent experimental data[4] from the ISR.

The averages in Eq. (1) are taken for all the data. We ask now for the behavior of $\langle n_0\rangle_-$, namely, the average of n_0 for a fixed value of n_-. A simple example of such a function is

$$\langle n_0\rangle_- = a\langle n_0\rangle + (1-a)n_-, \tag{2}$$

which obeys the constraints of Eq. (1). The case $a=1$ is the result of models in which no correlation exists between n_0 and n_-. Once a correlation is introduced many functional forms can result. We shall show that for a wide variety of models, when there is a correlation, it is approximately linear.

We start by discussing pion production in neutral pairs (σ model). We differentiate between Poisson and inverse-power distributions for the production cross sections of the pairs. Whereas the first leads to no correlations, we find that the second leads to a linear increase. Similar results are obtained for production of single pions which are constrained to have a fixed net charge (π model). Then we turn to the production of charged and neutral pairs that could result from decay of ρ resonances (ρ model) formed in the collision. Here we find correlations for both the Poisson and inverse-power distributions. We show that the constraints of charge conservation determine the main characteristics of the model. Explicit calculations of isospin and energy-momentum constraints show only minor changes of the correlations.

Our results do not depend on details of specific multiperipheral or fragmentation models which lead to the general Poisson or inverse-power distributions, respectively, which we consider. Moreover, the use of the symbols σ and ρ does not imply that such resonances are actually formed in the collision. Rather, the notation is chosen primarily for convenience in labeling properties of certain conditional probability functions.

The σ, π, and ρ models are treated in Secs. II through IV. In Sec. V, we consider modifications which arise from assuming that the production process is described by the convolution of two clusters of particles. Finally, in Sec. VI, results are summarized and compared with data.

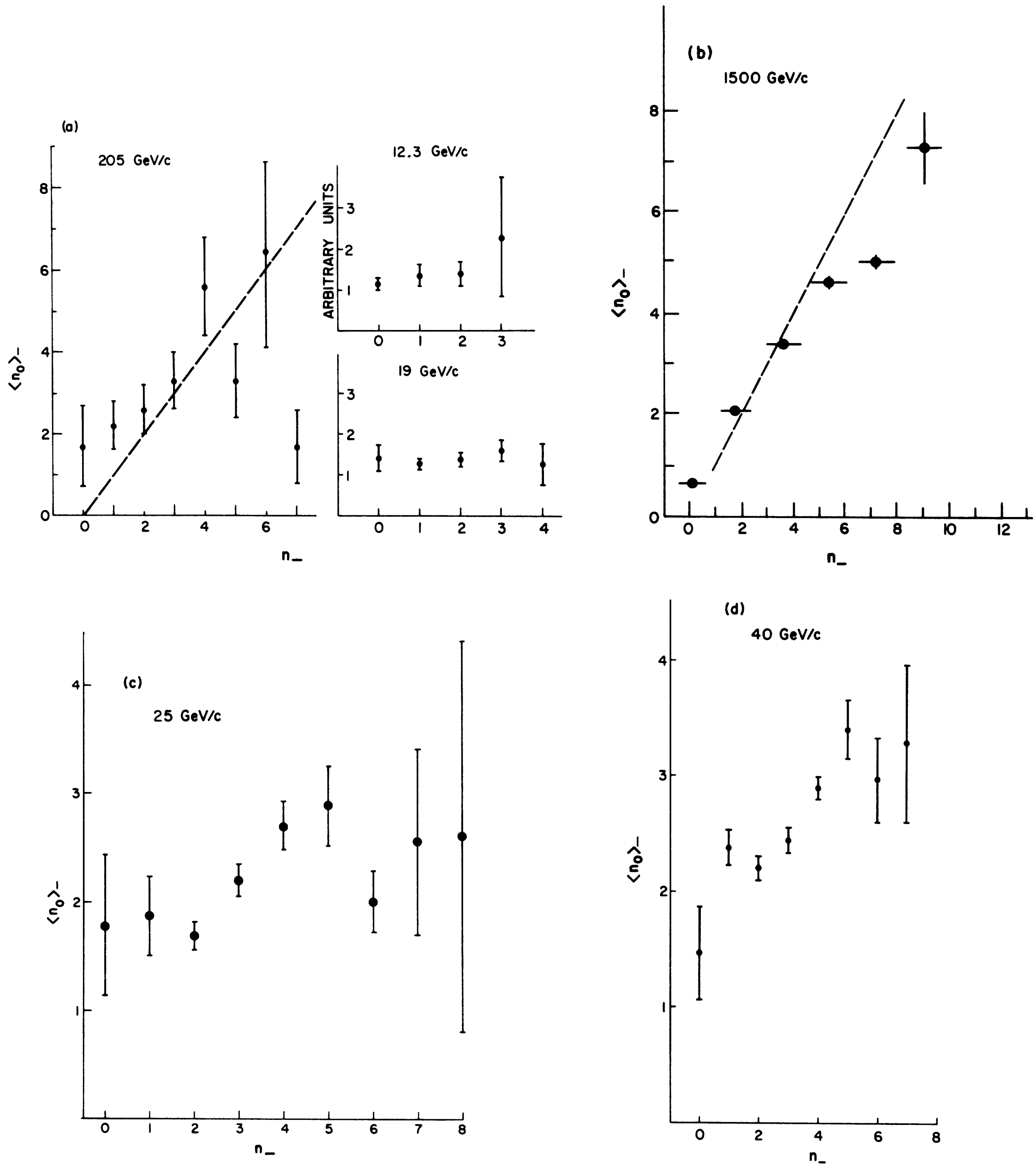

FIG. 1. (a) Data from pp interactions at beam momenta of 12.3, 19, and 205 GeV/c. This figure is taken from Charlton *et al.* (Ref. 2). Plotted is the mean number of neutral pions $\langle n_0 \rangle_-$ per inelastic pp collision as a function of the number of negative particles n_-. Note that for pp collisions the number of charged tracks $n_{\mathrm{ch}} = 2n_- + 2$. The dashed line expresses $\langle n_0 \rangle_- = n_-$. (b) Data from the CERN ISR at $\sqrt{s} = 53$ GeV (1500-GeV/c equivalent beam momentum) showing the correlation of $\langle n_0 \rangle_-$ with the recorded number of negative tracks in the same solid angle. This figure is adapted from Fig. 5(b) of Flügge *et al.* (Ref. 2). We have taken $\langle n_0 \rangle_- = 0.5$ times the measured number of γ's. As in (a), the dashed line expresses $\langle n_0 \rangle_- = n_-$. (c) Data from $\pi^- p$ interactions at a beam momentum of 25 GeV/c (Ref. 1). Plotted is the average number of neutral pions per inelastic collision as a function of n_-. (d) Data as in (c) for $\pi^- p$ interactions at a beam momentum of 40 GeV/c (Ref. 2).

II. σ MODEL

The σ model is defined as the production of neutral pion pairs. We designate by n and k the number of $\pi^+\pi^-$ and $\pi^0\pi^0$ pairs, respectively. We find then

$$n_- = n, \quad n_0 = 2k. \tag{3}$$

Given a set of $N = n + k$ pion pairs, we assume a binomial probability distribution

$$P_N(n,k) = \binom{N}{n} q^n p^{N-n}, \quad \binom{N}{n} = \frac{N!}{n!\,(N-n)!} \tag{4}$$

for producing n charged and k neutral pairs. Relation (1) follows when

$$q = 2p = \tfrac{2}{3}. \tag{5}$$

A production model has a certain probability distribution for N pairs which we designate by $D(N)$. The over-all probability is then given by

$$P(n,k) = \sum_N D(N) P_N(n,k), \tag{6}$$

where P_N is determined by (4) for the σ model. We investigate two cases:

(a) $D(N)$ is given by a Poisson distribution, as one would expect in multiperipheral models.[5]

(b) $D(N)$ is given by an inverse-power law. The interesting case is $D(N) \sim N^{-2}$, as in specific limiting-fragmentation models.[6]

The first case leads to

$$D(N) = e^{-z}\frac{z^N}{N!}, \quad P_{\sigma 1}(n,k) = e^{-z}\frac{(zq)^n}{n!}\frac{(zp)^k}{k!}. \tag{7}$$

The probability for producing n_- negative particles is given by

$$P(n_-) = \sum_k P(n,k) = e^{-z}\frac{(zq)^n}{n!}e^{zp}. \tag{8}$$

It is easy to see that the quantity $\langle n_0\rangle_-$ is defined by

$$\langle n_0\rangle_- = 2\langle k\rangle_- = \frac{1}{P(n_-)}2p\frac{\partial}{\partial p}P(n_-) = 2pz. \tag{9}$$

Hence we find that

$$\langle n_0\rangle_- = \tfrac{2}{3}z = \langle n_0\rangle. \tag{10}$$

This is to be expected because we started from a Poisson distribution and were therefore led to a form $P(n,k)$ which explicitly factors into independent q and p components.

A different situation is reached when one deals with a power law distribution. For convenience of the calculation we choose

$$D_\delta(N) = \frac{A}{N(N-1)\cdots(N-\delta+1)} = A\frac{(N-\delta)!}{N!} \to \frac{A}{N^\delta} \quad \text{as } N\to\infty. \tag{11}$$

The interesting fragmentation case corresponds to $\delta = 2$. After substitution, it follows that

$$P_{\sigma 2}(n,k) = \frac{A(n-\delta)!}{n!}\binom{n+k-\delta}{n-\delta}q^n p^k, \tag{12}$$

from which we obtain

$$P(n_-) = \sum_k P(n,k) = \frac{Aq^n(n-\delta)!}{n!}(1-p)^{\delta-1-n}, \tag{13}$$

$$\langle n_0\rangle_- = 2p\frac{\partial}{\partial p}\ln P(n_-) = \frac{2p}{1-p}(n-\delta+1). \tag{14}$$

Substituting now $p = \tfrac{1}{3}$, we find that for $n_- \geq \delta$

$$\langle n_0\rangle_- = n_- - \delta + 1, \tag{15}$$

which is a linear increase with slope of unity. The behavior of $\langle n_0\rangle_-$ for $n_- < \delta$ is discontinuous due to the discontinuous nature of $D_\delta(N)$, which may be defined as zero for $N < \delta$. We shall return to this point later when we consider numerical calculations using continuous functions $D(N)$.

Our conclusion drawn from the above calculations is that the correlation between n_- and $\langle n_0\rangle_-$ can indeed differ according to the underlying dynamics, in this case depending upon whether $D(N)$ is described by a multiperipheral-like or fragmentation-like model.

III. π MODEL

We consider a model in which single pions are emitted independently, constrained only to have an over-all charge Q. The resulting distribution is a Poisson in π^0 and a Bessel function in charged pairs,[7]

$$P_{\pi 1}(n_-, n_0) = \frac{e^{-z/3}}{I_Q(\frac{2}{3}z)}\frac{(zq)^{2n_-+Q}(zp)^{n_0}}{(n_-+Q)!\,n_-!\,n_0!}, \tag{16}$$

where we denote by zq the probability of producing a charged pion and by zp that for a neutral pion. The variables p and q are introduced to keep track of the various terms. Both p and q are set equal to $\tfrac{1}{3}$ at the end of the calculation. Equation (16) leads to

$$\langle n_+\rangle = \frac{z}{3}\frac{I_Q'}{I_Q} + \frac{Q}{2},$$

$$\langle n_- \rangle = \frac{z}{3}\frac{I_Q'}{I_Q} - \frac{Q}{2}, \tag{17}$$

$$\langle n_0 \rangle = \tfrac{1}{3} z .$$

Here I_Q' is the derivative of the Bessel function $I_Q(x)$ with respect to $x = \frac{2}{3}z$. Because of the factorization of Eq. (16) we find

$$\langle n_0 \rangle_- = \langle n_0 \rangle \tag{18}$$

as expected from the absence of correlations.

For large values of n_-, Eq. (16) may be approximated by

$$P_{\pi 1}(n_-, n_0) \approx \frac{e^{-z/3}}{I_Q(\frac{2}{3}z)} \frac{(2zq)^{2n_- + Q}}{(2n_- + Q)!} \frac{(zp)^{n_0}}{n_0!} \frac{1}{(\pi n_-)^{1/2}} . \tag{19}$$

This distribution may be written as a product

$$P_{\pi 1}(n_-, n_0) \approx \frac{e^{-z} z^N}{N!} \frac{N!}{(2n_- + Q)!\, n_0!} \times p^{n_0} (2q)^{2n_- + Q} \frac{1}{(\pi n_-)^{1/2}} \tag{20}$$

which is close to a Poisson distribution in $N = 2n_- + Q + n_0$, the total number of pions, multiplied by a binomial distribution in p and $2q$. If we now replace the Poisson distribution by an inverse-power law, we arrive at the following approximate form:

$$P_{\pi 2}(n_-, n_0) = \frac{A(N-\delta)!}{N!} \frac{N!}{(2n_- + Q)!\, n_0!} p^{n_0} (2q)^{2n_- + Q} . \tag{21}$$

This lends itself to a calculation similar to that of Sec. II. Thus,

$$P(n_-) = \frac{A (2q)^{2n_- + Q} (2n_- + Q - \delta)!}{(2n_- + Q)!} (1-p)^{-2n_- - Q + \delta - 1} \tag{22}$$

and

$$\langle n_0 \rangle_- = p \frac{\partial}{\partial p} \ln P(n_-) \approx 2n_- \frac{p}{1-p} = n_- , \tag{23}$$

where we have used $p = \frac{1}{3}$. We keep only the leading term in (23) since we start with an asymptotic form for the distribution.

The distribution of the π model takes the constraint of charge conservation into account. There are obviously other constraints in production models, namely, isospin and energy-momentum conservation. To see their effects we calculate $\langle n_0 \rangle_-$ as a function of n_- in an independent emission model[8] with isospin statistical weights.[9] The basic assumption of the model is that the square of the N-particle S matrix has the factorized form

$$(\text{Isospin weight}) \times \prod_{\text{nucleons}} e^{-R^2 p_T^2} e^{2\lambda E_p / \sqrt{s}} \times \prod_{\text{pions}} e^{-R^2 p_T^2} \exp\left[\frac{2\lambda}{\sqrt{s}} (p_T^2 + m_\pi^2)^{1/2}\right] ,$$

which reproduces the observed small transverse momentum p_T of produced particles and the relatively flat longitudinal-momentum distribution of the leading protons. The parameters R^2, λ and the relative weights of the N-particle matrix elements are adjusted to give a reasonable description of data. For details see Ref. 8. The isospin weights used are the Cerulus coefficients[9] for $pp \to NN$ + pions. The results of this numerical calculation are shown in Fig. 2, and should be compared with the constant prediction of Eq. (18), where $\langle n_0 \rangle = 2.8$. It is clear that both isospin and energy-momentum constraints lead to only small modifications.

IV. ρ MODEL

We turn to a model in which pions can be created in charged pairs. This will be the case if, for example, ρ production is a prominent effect. Since ρ mesons carry charge and isospin, we have to take into account the constraints that result from the conservation of these quantum numbers. Analogous to the π model, we find here also that charge conservation is the important constraint. We start therefore from a distribution of ρ mesons that looks like Eq. (16). Let us denote by N_+ the number of $\pi^+\pi^0$ pairs, N_- the number of $\pi^-\pi^0$ pairs, and N_0 the number of $\pi^+\pi^-$ pairs. We use n_+, n_-, and n_0 for the numbers of π^+, π^-, and π^0, respectively; ν describes the over-all charge of the ρ cloud. We find then

$$\begin{aligned} n_+ &= N_+ + N_0 , \\ n_- &= N_- + N_0 , \\ n_0 &= N_+ + N_- , \\ \nu &= N_+ - N_- \\ &= n_+ - n_- . \end{aligned} \tag{24}$$

It is useful to define a quantity k by

$$2k = n_0 - \nu \tag{25}$$

from which it follows that

$$\begin{aligned} N_+ &= k + \nu , \\ N_- &= k , \\ N_0 &= n_- - k , \\ N &= N_+ + N_- + N_0 \\ &= n_- + k + \nu . \end{aligned} \tag{26}$$

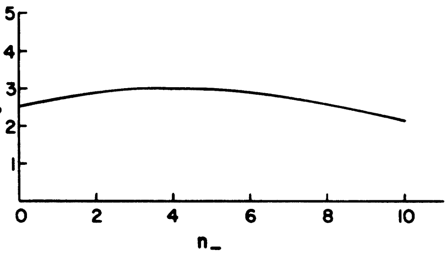

FIG. 2. An independent-emission model for pions (Ref. 8) with isospin conservation is used here to compute $\langle n_0\rangle_-$ at 200 GeV/c. This calculation is described in Sec. III.

Let us define the distribution

$$P_{\rho 1}(n_-, n_0) = A_\nu \frac{(zp)^{N_+}(zq)^{N_-}(zr)^{N_0}}{N_+!\,N_-!\,N_0!}$$

$$= A_\nu \frac{(zp)^{k+\nu}(zq)^k (zr)^{n-k}}{(k+\nu)!\,k!\,(n_- -k)!} \qquad (27)$$

in analogy with Eq. (16). As before, we introduce p, q, and r to be able to trace the various terms. They are set to $\frac{1}{3}$ at the end of the calculation. The probability of producing n_- negative pions is given by

$$P(n_-) = \sum_{n_0} P(n_-, n_0)$$

$$= A_\nu (zp)^\nu (zr)^{n_-} \sum_k \left(\frac{zpq}{r}\right)^k \frac{1}{(k+\nu)!\,k!\,(n_- -k)!}\,. \qquad (28)$$

Defining a new variable $s = pq/r$, we find from Eq. (28)

$$P(n_-) = \frac{A_\nu (zp)^\nu (zr)^{n_-}}{(\nu+n)!} L_n^{(\nu)}(-zs)\,, \qquad (29)$$

where $L_n^{(\nu)}$ denotes the Laguerre polynomial.[10] The Eq. (28) leads directly to the formula

$$\langle n_- - k\rangle_- = zr\frac{P(n_- -1)}{P(n_-)}\,, \qquad (30)$$

from which the following result can be derived:

$$\langle n_0\rangle_- = 2n_- + \nu - 2(\nu + n_-)\frac{L_{n-1}^{(\nu)}(-\frac{1}{3}z)}{L_n^{(\nu)}(-\frac{1}{3}z)}\,. \qquad (31)$$

The same result follows also from

$$\langle n_0\rangle_- = 2s\frac{\partial}{\partial s}\ln P(n_-)\,. \qquad (32)$$

Using this relation, we can also find the expression for $\langle n_0\rangle_-$ for a superposition of distributions of the form (27). An interesting case is the superposition of $\nu = 0, 1, 2$, representing the reactions $pp \to pp$ + pions, $pp \to pn$ + pions, and $pp \to nn$ + pions. Such a superposition leads to a result which has the general properties of Eq. (31). For two limiting cases, $n_- \ll \frac{1}{3}z$ and $n_- \gg \frac{1}{3}z$, these expressions are

$$\langle n_0\rangle_- \approx 2n_- + \nu_{\max}\,, \quad n_- \ll \tfrac{1}{3}z \qquad (33)$$

$$\langle n_0\rangle_- \approx 2(\tfrac{1}{3}zn)^{1/2}\,, \quad n_- \gg \tfrac{1}{3}z\,. \qquad (34)$$

The parameter z is roughly the average number of pairs. This number does not exceed 10 even at the highest energies available. Therefore, the limit (33) is not applicable for existing data.

In Fig. 3 we show explicit evaluations of Eq. (31) for $\nu = 0, 1, 2$ calculated with $z = 4.5$. This number corresponds to the result of the recent 200-GeV/c pp experiment.[2] The various curves in Fig. 3 show a linear increase at small n_- with a slope of about 0.3. On the same figure we draw also the results of a statistical isospin model,[9] which turns out to be very close to the $\nu = 0$ and 1 curves, thus showing again that charge conservation is the strong constraint. Once these results are incorporated into a longitudinal phase-space calculation (similar to the one used in the previous section), the high-n_- part of the curve is modified considerably, as may be expected. The low-n_- part however has the same characteristics of a moderate rise of $\langle n_0\rangle_-$ as a function of n_-.

Finally, we consider the results of a ρ model with over-all inverse-power behavior assumed to be the dominant production mode of particles. Using similar approximations to those of the pre-

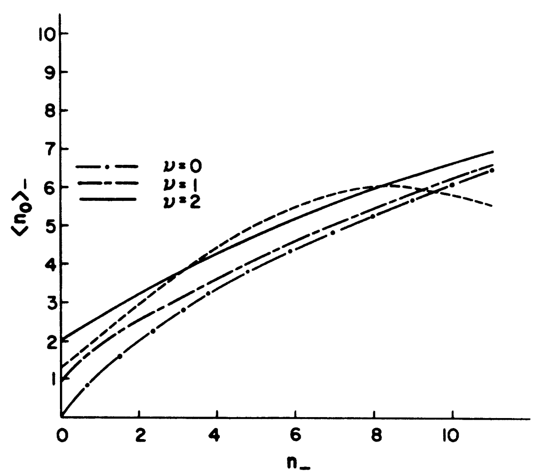

FIG. 3. Labeled curves give $\langle n_0\rangle_-$ for the ρ model [Sec. IV, Eq. (31)] with parameters $z = 4.5$ and $\nu = 0, 1, 2$. A ρ model in which isospin is conserved for the ρ's by means of a statistical isospin model (Ref. 9) gives a result identical to that of the —x— curve, for $n_- \geq 1$. Finally, the dashed curve gives $\langle n_0\rangle_-$ for a ρ model with isospin and energy-momentum conservation imposed by means of the independent-emission model described in the text (Ref. 8); the same parameters are used as for the production of pions. All curves are for 200 GeV/c.

vious section, we realize that for high n_0, Eq. (27) becomes

$$P_{\rho 1}(n_-, n_0) \approx C_\nu \frac{z^N}{N!}\left(\frac{p}{q}\right)^{\nu/2} \times \sum_k \binom{N}{2k+\nu} (2\sqrt{pq})^{2k+\nu} r^{N-2k-\nu} . \tag{35}$$

Therefore, if we want to have an asymptotic $N^{-\delta}$ behavior of $D(N)$, we can look at the distribution

$$P_{\rho 2}(n_-, n_0) = C_\nu \frac{(n+k+\nu-\delta)!}{(k+\nu)!\,k!\,(n-k)!} s^k p^\nu r^n . \tag{36}$$

It follows that

$$P(n_-) = \sum_k P_{\rho 2}(n_-, n_0) = C_\nu \frac{p^\nu r^n (n+\nu-\delta)!}{(n+\nu)!} P_n^{(\nu,-\delta)}(2s+1), \tag{37}$$

where $P_n^{(\nu,-\delta)}$ is a hypergeometric series[10] (which gives a Jacobi polynomial when the indices ν and $-\delta$ are greater than -1). Once again we apply Eq. (32) and find that

$$\langle n_0\rangle_- = \tfrac{5}{4} n_- \frac{n_- + \frac{1}{5}(\nu - 4\delta)}{n_- + \frac{1}{2}(\nu-\delta)} - \tfrac{3}{4} n_- \frac{(n_- + \nu)(n_- - \delta)}{n_- + \frac{1}{2}(\nu-\delta)} \frac{P_{n_- - 1}^{(\nu,-\delta)}(\frac{5}{3})}{P_{n_-}^{(\nu,-\delta)}(\frac{5}{3})} . \tag{38}$$

The leading terms in n_- are

$$\langle n_0\rangle_- \approx \tfrac{5}{4} n_- - \tfrac{3}{4} n_- \frac{P_{n_- - 1}^{(\nu,-\delta)}(\frac{5}{3})}{P_{n_-}^{(\nu,-\delta)}(\frac{5}{3})} . \tag{39}$$

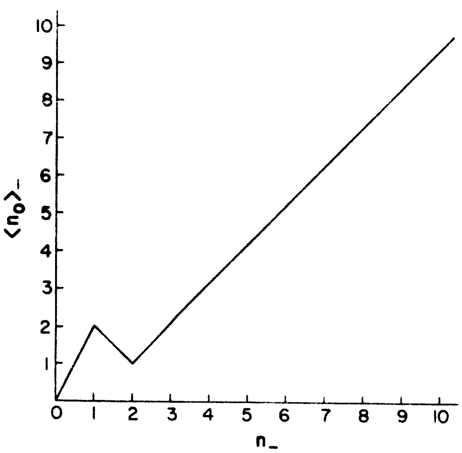

FIG. 4. The quantity $\langle n_0\rangle_-$ for a ρ model with an inverse-square law for the production distribution. [Eq. (38), $\delta = 2$ and $\nu = 0$]. The behavior of $P_{\rho 2}$ ($\equiv 0$ for $N < 2$) is discontinuous, giving rise to a discontinuous behavior of $\langle n_0\rangle_-$ for $n_- < 2$.

Numerical results are plotted in Fig. 4 for the special case $\delta = 2$ of Eq. (38).

V. TWO-CLUSTER PRODUCTION

In this section we discuss the convolution of two clusters in the σ model in order to study to what extent the conclusions of Sec. II are modified. This is of particular interest for fragmentation models which take two-cluster formation into account.

In terms of the notation of Sec. II, the over-all probability for producing n charged and k neutral pairs in two clusters is

$$P(n,k) = \sum D_1(n_1+k_1) P_{n_1+k_1}(n_1,k_1) \times D_2(n_2+k_2) P_{n_2+k_2}(n_2,k_2), \quad n = n_1+n_2, \quad k = k_1+k_2, \tag{40}$$

where $P_{n+k}(n,k)$ is given by Eq. (4). Keeping $N_1 = k_1 + n_1$, $N_2 = k_2 + n_2$, and $k = k_1 + k_2$ fixed, and summing over k_1, we get a simple relation using the binomial weights of Eq. (4):

$$\sum_{k_1} \binom{N_1}{k_1}\binom{N_2}{k-k_1} = \binom{N}{k}, \tag{41}$$

where $N = N_1 + N_2$. Thus we have

$$P(n,k) = \sum_{N_1}\Big(\sum_{N_1} D_1(N_1) D_2(N-N_1)\Big) P_N(n,k) \equiv \sum_N D^{(2)}(N) P_N(n,k), \tag{42}$$

which is identical in form to (6). In other words, for the σ model, the difference for $P(n,k)$ between single- and double-cluster formation is due entirely to the dynamics of the convolution $D^{(2)}(N)$.

As an example of this convolution, we consider first the Poisson distribution Eq. (7). In this case the result is

$$D^{(2)}(N) = e^{-2z} \sum_{N=N_1+N_2} \frac{z^{N_1+N_2}}{N_1!\,N_2!} = \frac{(2z)^N}{N!} e^{-2z} , \tag{43}$$

which is a Poisson distribution characterized by twice the mean of the single cluster. This leads to

$$\langle n_0\rangle_- = \langle n_0\rangle = \tfrac{4}{3} z , \tag{44}$$

where $\frac{2}{3}z$ is the average for the single-cluster distribution.

Numerical calculations for $D^{(2)}(N)$ arising from the convolution of two inverse-square distributions show that the resulting correlation $\langle n_0\rangle_-$ has the same asymptotic behavior as for single clusters when n_- is large. In Fig. 5, for the σ model, we contrast numerical results obtained from single-cluster formation with those from a convolution of two clusters. For $N \geq 1$, we employ the distribution

$$D(N) = N^{-2} e^{-3/N}, \quad (45)$$

$D(0) \equiv 0$. This form is chosen rather than the one described in Sec. II in order to avoid discontinuous behavior of $\langle n_0 \rangle_-$. The distribution in Eq. (45) also results directly from the choice of the excitation function $\rho(M)$ presently used in nova-model calculations,[11] and provides a good fit to the experimental distribution $\sigma_{n_{ch}}$ vs n_{ch} at 200 GeV/c. For both single- and double-cluster formation, the fall of these curves from $n_- = 0$ to 1 may be traced directly to the rapid rise of $D(N)$ from $N = 0$ to its maximum at $N = 1.5$. At small n_-, observe that $\langle n_0 \rangle_-$ is twice as large for double excitation as it is for single-cluster formation. For large n_-, our curves turn over because we arbitrarily truncate the numerical sums at $N = 15$; in a more realistic calculation this turnover would arise from energy-momentum limitations on the maximum number of produced pairs.

VI. DISCUSSION

New data from NAL, Serpukhov, and ISR (cf. Fig. 1) showing a linear rise of $\langle n_0 \rangle_-$ vs n_- clearly

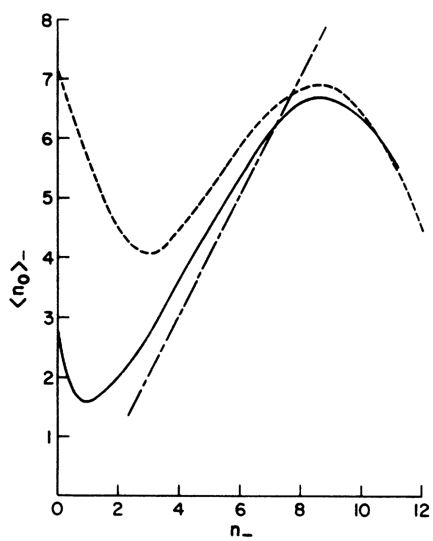

FIG. 5. The solid curve results from a numerical evaluation at 200 GeV/c of the σ model (Sec.II) with $D(N)$ given by Eq. (45). The dashed curve is obtained from the σ model when two clusters are convoluted [Eq. (42)] with $D(N)$ again given by Eq. (45). For comparison, the dot-dashed curve gives $\langle n_0 \rangle_- = n_- - 1$, Eq. (15) for $\delta = 2$.

rule out dynamical models in which single pions are emitted independently, or produced in neutral pairs according to a Poisson distribution in the pairs. However, the other four models we considered are presently consistent with the data. These four survivors are, first, emission of ρ-like objects according to a Poisson distribution in the ρ's and, second, all fragmentation-type inverse-power models (whether decay follows σ, π, or ρ statistics). All give a linear rise for $\langle n_0 \rangle_-$ with slopes varying between 0.3 and 2 in agreement with the trend of the very-high-energy data. Thus, the new data allow some discrimination between models, which was not possible with data in the range $p_{lab} \leq 30$ GeV/c. In the low-energy retion, the flat behavior (cf. Fig. 1 and also Ref. 8) of $\langle n_0 \rangle_-$ vs n_- may result from phase-space effects which cause the $\langle n_0 \rangle_-$ curve to turn over. Although existing data on $\langle n_0 \rangle_-$ allow us to eliminate some models, present statistical precision on $\langle n_0 \rangle_-$ is insufficient for further discrimination.

It is interesting that the surviving model of the multiperipheral type is one in which ρ-like objects are produced. The consequences of such a model for $\langle n_0 \rangle_-$ were discussed some time ago by Caneschi and Schwimmer.[12] If taken literally, this suggests presence of other two-particle correlation effects, for example, correlations in momentum and angle variables, besides the charge correlation we consider here. In spite of the fact that a clean ρ signal is not seen in multiparticle data, perhaps due to combinatorial background, it can be argued that ρ production may nevertheless be significant. On the one hand this is an appealing theoretical assumption,[13] and on the other hand it has been recognized for some time that multiparticle production data readily allow for the pairing of pions at low pair energies. Furthermore, measurements of $\langle n_{ch} \rangle$ at NAL and ISR require a large value of the coefficient b in fits of the form $\langle n_{ch} \rangle = a + bY$, where Y is the full rapidity interval, ($Y \simeq \ln s$).[14] Thus b^{-1}, the average spacing of particles in rapidity, is small. In turn, this determines rather small mean subenergies in the multiperipheral chain ($\bar{s}_i \simeq 0.4$ GeV^2), consistent with strong two-pion interactions and, presumably, formation of ρ mesons among pairs of pions. We should stress, however, that our results are more general than specific models in which ρ mesons are actually produced. As will be noted from the analysis in Sec. IV, we require only that *clusters* of pions be produced and that the statistics of their decay approximate that associated with isospin $I = 1$ formation of $\pi\pi$ pairs. What we required is that $(\pi^+\pi^-)$, $(\pi^+\pi^0)$, and $(\pi^-\pi^0)$ pairs be equally likely. Thus, in particular, formation of ω-like mesons would do just as well, not to mention nonresonant cluster formation.

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These ensure the early scaling of $\langle p_T\rangle$ vs s. In the notation of Ref. 8, we use the parameter values $z = 3.0\ \text{GeV}^{-2}$, $\lambda = 5$, and $R^2 = 6\ \text{GeV}^{-2}$. For a more complete discussion see G. Thomas, Phys. Rev. D (to be published).

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