PHYSICAL REVIEW D VOLUME 7, NUMBER 1 1 JANUARY 1973

Dual Model for the Diffractive Photoproduction of "ρ" Mesons*

J. L. Uretsky†
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
(Received 28 August 1972)

Diffractive vector-meson photoproduction is studied in a dual model due to Neveu and Scherk that incorporates a gauge-invariant vector current. The structure of the amplitude turns out to be much more complicated than the vector dominance plus "Drell-Söding" structure often assumed for analyzing experimental data, although it is shown how even that structure involves some degree of "double counting." Experience with this model shows the need for a model with a realistic Pomeranchukon in order to understand experimental photoproduction data.

I. INTRODUCTION

In principle, the idea of vector dominance[1] provides a simple and beautiful description of the diffraction production of vector mesons by photons. In the real world where experiments are done, however, some of the beauty and simplicity is lost because there are ambiguities in the interpretation of the experimental data. Mostly, the difficulties are concerned with distinguishing between the vector meson and the accompanying "background." This would not be a problem if vector mesons were stable objects. However, because they undergo strong decays the vector mesons have appreciable widths and their presence can only be inferred by measuring the mass spectra and angular distributions of decay products. Since the same decay products can be produced through other mechanisms, the problem arises of identifying the vector-meson contribution to the measurements. The problem is more difficult if the width of the decaying particle is very large (some appreciable fraction of the resonance mass, say) as in the case of the ρ meson. In that case the shape of the resonance "bump" is especially sensitive to distortion by the background thereby confusing its identification.

The point of all this is that the definition of "background," crucial as it is, is model-dependent. To the extent that one wants a reliable determination of the vector-meson production parameters (cross section and density-matrix elements), then to that extent one also needs a reliable model for the background and the way in which it interferes with the "off-mass-shell" production of the resonance. Here "off-mass-shell" means far (in some sense) from the S-matrix pole of the resonance – which is always the case for broad resonances.

There is also another aspect of vector-meson production for which it would be nice to have a predictive model. This has to do with the fact that in photoproduction (or electroproduction) there is a mass change involved in the diffractive process. The effect of this mass change is not known, but one might suspect that at least it would affect the value of the vector-meson–photon coupling constant as compared to values found when there is no mass shift (as in e^+e^- annihilation). In fact, recent electroproduction experiments suggest that the ρ-photon effective coupling decreases appreciably with increasingly negative values of the square of the photon mass. The interpretation of the apparent decrease would be simpler if a good model of the production process were available.

To date the most popular model for interpreting diffractive ρ-meson photoproduction has been the "Drell-Söding" interference picture.[2] In this model the production amplitude is composed of two terms, a "vector-dominance" term and a background term describing the diffraction photoproduction of two free pions (see Fig. 1). The interference between the two can be used to duplicate the experimentally observed shift of the ρ-meson mass peak and distortion of its shape from what is observed in e^+e^- annihilation.

The only attempt of which I am aware to justify such a model is contained in the work of Kramer and Quinn.[3] These authors start with the background term just described as the "driving" mechanism. Then a model of the final-state interaction between the two outgoing pions gives a Breit-Wigner "vector-dominance" term plus a Drell-Söding term. The latter is multiplied by a factor that vanishes at the peak of the ρ resonance in the two-pion mass spectrum. This formulation provides a realization of an earlier suggestion by Pumplin[4] for avoiding "double counting" in adding the two amplitudes. Gauge invariance was a problem for Kramer and Quinn and had to be imposed "by hand."

With the advent of dual models of electromagnetic currents interacting with hadrons it becomes

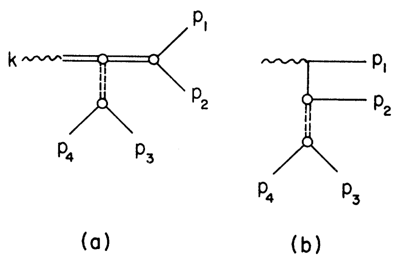

FIG. 1. (a) The vector-dominance picture. The photon (wavy line) turns into a vector meson (double line) which scatters from the target by Pomeranchukon exchange (double dashed line) and decays into two "pions." (b) One Drell-Söding term. The photon produces a pion pair and one pion scatters from a Pomeranchukon.

tempting to see how vector-meson photoproduction looks in a more comprehensive theory. In this paper I will utilize a narrow-resonance model[5] that has the desirable features of gauge invariance, duality, Regge behavior, factorization, and a reasonable mass spectrum. Here "reasonable" means that the hadronic mass spectrum is not altered by the introduction of interaction with the electromagnetic current. It would also be nice if the model were unitary and contained a reasonable Pomeranchukon, but that seems to be asking too much at this stage of development. Thus, there will be important questions having to do with final-state interactions and triangle singularities[6] that the model will not answer. It should, however, give more detailed insight into the nature of the "background" amplitude and the extent to which it can be modeled by a Drell-Söding term, the effects of varying the photon mass, and the way in which gauge invariance comes about.

In the next section I introduce and briefly discuss a model of a vector current interacting with dual hadrons that is due to Neveu and Scherk.[5] The model, for the case of n hadrons interacting with a single current, has all the desirable features described in the last paragraph. The extension to multicurrent amplitudes was an unsolved problem in the Neveu-Scherk work that is irrelevant to the purposes of the present discussion. Notational conventions and kinematical quantities are also described in the next section, and the independent invariant amplitudes are exhibited.

If the square of the four-momentum transfer between the incident photon and the produced vector meson vanishes, then there is a single invariant amplitude contributing to photoproduction. In Sec. III this amplitude is expanded in a way that exhibits its vector dominance content in the high-energy diffraction limit. It is shown in the model how the process is considerably more complicated than the simple picture of the "ρ contents" of the photon rescattering from the target. In fact if one imagines that the ρ in the model has finite width, then there are corrections to vector dominance that persist even at the resonance peak, contrary to the spirit of Pumplin's conjecture. The corrections are seen to be related to the virtual production of higher mass mesons in the "initial" and "final" states. All of this structure is dependent upon the nature of the "exchange" trajectory and its couplings, so that it is *possible* that in the physical production process ("Pomeranchukon" exchange) many of the apparent complications are absent.

In Sec. IV the leading term of an alternative expansion of the amplitude, "dual" to the expansion just described, is obtained and found to be the Drell-Söding amplitude. Finally, in the concluding Sec. V some experimental implications are touched upon.

II. THE MODEL

In the formalism of Neveu and Scherk a "tree diagram" with one external electromagnetic current (of four-momentum k^μ) and n scalar-meson legs (of momenta p_i^μ, $i=1,\ldots n$, $p_i^2=m^2$) is represented by the expression

$$A_n^\mu = -\tfrac{1}{4} g^{n-1}\int_0^{\pi/2} d\theta_1 \int_0^{\theta_1} d\theta_2 \cdots \int_0^{\theta_{n-2}} d\theta_{n-1}\left(\prod_{i=1}^{n}\prod_{j>i}[\sin(\theta_i-\theta_j)]^{-2p_i\cdot p_j}\right)\left(\sum_{i=1}^{n-1} p_i^\mu \sin 2\theta_i\right)\Bigg|_{\theta_n=0} - \text{"twist"}, \tag{1}$$

where the "twisted" expression contains the anticyclical permutation of all the momenta (i.e., the sequence $1,2,\ldots,n$ is replaced by the sequence $n,\ n-1,\ldots,1$), the integration variables remaining unchanged. In this form the Regge intercepts are all taken to be unity, and the mass scale is chosen so that the Regge slopes are unity. We shall be interested in the case $n=4$. All the scalar-meson momenta are taken to be outgoing and the photon momentum is

$$k^\mu = \sum_{i=1}^{n} p_i^\mu \ . \tag{2}$$

The configuration is shown in Fig. 2. We take M^2 the mass of the final "diffractive" two-meson state, and set

$$M^2=(p_1+p_2)^2,$$
$$s=(p_1+p_2+p_3)^2=(k-p_4)^2,$$
$$t=(p_3+p_4)^2=(k-p_1-p_2)^2, \tag{3}$$
$$s_{234}=(p_2+p_3+p_4)^2=(k-p_1)^2,$$
$$s_{23}=(p_2+p_3)^2.$$

The trajectory functions are written

$$\alpha(x)=\alpha_0+x, \tag{4}$$

where x is any of the variables in Eq. (3).

Following Neveu and Scherk,[5] in part, I make the substitutions

$$y=\cos\theta_1, \tag{5a}$$

$$\frac{1-z}{z}=\frac{\sin\theta_2}{\sin(\theta_1-\theta_2)}, \tag{5b}$$

$$x\left(\frac{1-z}{z}\right)=\frac{\sin\theta_3}{\sin(\theta_1-\theta_3)}, \tag{5c}$$

and A_4^μ takes on the Veneziano-like form,

$$A_4^\mu=\tfrac{1}{2}g^3\int_0^1 ydy(1-y^2)^{-[1+\alpha(k^2)]/2}\int_0^1 dz\, z^{-1-\alpha(s_{234})}(1-z)^{-1-\alpha(s)}[1-2z(1-z)(1-y)]^{p_3\cdot k}$$
$$\times\int_0^1 dx\, x^{-1-\alpha(M^2)}(1-x)^{-1-\alpha(s_{23})}[z^2+2xyz(1-z)+x^2(1-z)^2]^{p_2\cdot k}$$
$$\times\left[(k-2p_1)^\mu-2p_2^\mu z\left(\frac{z+xy(1-z)}{z^2+2xyz(1-z)+x^2(1-z)^2}\right)-2p_3^\mu z\frac{z+y(1-z)}{1-2z(1-z)(1-y)}\right]$$
$$\equiv(k-2p_1)^\mu F_1-2p_2^\mu F_2-2p_3^\mu F_3. \tag{6a}$$

It is straightforward to verify that A_4^μ is gauge-invariant since $k_\mu A_4^\mu$ turns out to be the total differential of the z integrand. From this it follows that the F_i are not all independent but that

$$F_1=\frac{2}{(k-p_1)^2-m^2}(p_2\cdot kF_2+p_3\cdot kF_3). \tag{7}$$

At this point it is convenient to relax the condition of unit intercept on the Regge trajectories and let α_0 be arbitrary. Note that

$$\alpha(s_{234})=(k-p_1)^2-m^2 \tag{8}$$

and that the factor of z multiplying the p_2^μ and p_3^μ in Eq. (6a) guarantees that there are no poles in F_2 and F_3 at $\alpha_{234}=0$. But a pole at $\alpha_{234}=0$ is just the one-pion pole that gives the "Drell-Söding" terms, so this term can only come from F_1.

Equation (6a) may be rewritten in the form

$$A_4^\mu=(k-2p_1-2p_2)^\mu F_1+2p_2^\mu(F_1-F_2)-2p_3^\mu F_3 \tag{6b}$$

so that the first term will not contribute in the case of real photons (choose the Coulomb gauge in the rest frame of p_1+p_2). The photoproduction amplitude is just $\epsilon^{(r)}\cdot A_4$, where $\epsilon_\mu^{(r)}$ is the photon polarization 4-vector, and it is easily seen that $\epsilon\cdot p_3$ is proportional to $(-t)^{1/2}$. For diffraction production the interesting kinematical region is the limit $s\to\infty$, $-t\to 0$, and in this limit only the amplitude F_1-F_2 contributes.

III. VECTOR DOMINANCE

It is a simple matter to expand F_1-F_2 in the poles in the variable M^2 by expanding[7] the factor

$$[z^2+2xyz(1-z)+x^2(1-z)^2]^{2p_2\cdot k-1}(1-z)[yz+x(1-z)]$$

in powers of x. Then taking the $\alpha(s)\to-\infty$ limit in the usual way [by letting $z=-u/\alpha(s)$ and keeping only lowest powers of $\alpha(s)^{-1}$], we obtain for the contribution of the first three poles

$$F_{12} \equiv F_1 - F_2$$
$$= -\tfrac{1}{4} g^3 (-s)^{\alpha(t)+1-\alpha(M^2)} \Gamma(\alpha(M^2) - \alpha(t) - 3) H(k^2, t, M^2, p_2 \cdot k, p_2 \cdot p_3, s)\,, \tag{7}$$

where H may be written

$$H = B_2(t, M^2, k^2)[\alpha(M^2) - \alpha(t) - 1]\{[\alpha(M^2) - \alpha(t) - 2] B(1 - \alpha(M^2), -\alpha(s_{23})) - \alpha(s) B(2 - \alpha(M^2), -\alpha(s_{23}))\}$$
$$+ B_3(t, M^2, k^2)(p_2 \cdot k - 1)\{3\alpha(s)^2 B(3 - \alpha(M^2), -\alpha(s_{23})) - 2\alpha(s) B(2 - \alpha(M^2), -\alpha(s_{23}))[\alpha(M^2) - \alpha(t) - 2]\}$$
$$+ 2B_4(t, M^2, k^2)(p_2 \cdot k - 1)^2 B(3 - \alpha(M^2), -\alpha(s_{23})) + \cdots \tag{8a}$$

and

$$B_n(t, M^2, k^2) = B(\tfrac{1}{2}[\alpha(t) - \alpha(M^2)] + n,\ \tfrac{1}{2}[1 - \alpha(k^2)])\,. \tag{8b}$$

Here $B(x, y)$ is the usual beta function, and $\Gamma(x)$ the gamma function. It is easily verified that at the pole where $\alpha(M^2) = l$, H is proportional to s^{l-1} so that to leading order in s the amplitude $F_{12} \sim s^{\alpha(t)} \Gamma(-\alpha(t))$.

The physical content of Eqs. (7) and (8) is quite explicit. The photon couples to the ρ trajectory which then scatters and produces "diffractively" all the particles that lie on that trajectory. The result comes about, in this model, because the exchange trajectory [with trajectory function $\alpha(t)$] couples the incident ρ [characterized by the pole at $\alpha(k^2) = 1$] to all of the two-pion resonances in the model.

Experimentally, it is found the ρ photoproduction is dominated at high energy by Pomeranchukon exchange. Since the Pomeranchukon does not carry g parity, it does not couple the ρ (or photon) to the even-spin resonances on the ρ trajectory. Whether it couples the ρ to the higher odd-spin resonances is still uncertain although coherent photoproduction of the g meson ($J = 3$) at about 1650 MeV seems to be consistent with existing experimental evidence.[8] It may be concluded, then, that if g-parity considerations are ignored, the prediction that high-spin resonances may be photoproduced is a physically reasonable feature of this model.

The t-channel exchange in this model does not, of course, model the t dependence of Pomeranchukon exchange. Thus, the factors in Eqs. (7) and (8) that depend upon t probably do not represent real physics even if the t-channel trajectory is given unit intercept and arbitrary slope (recall that the interest here is in $t \approx 0$). In order for F_{12} to represent real physics, the factors like $\Gamma(-\alpha(t))$ and $[\alpha(M^2) - \alpha(t) - m]$, m an integer, should probably be replaced by arbitrary form factors to be determined from experiment. Nevertheless, the message contained in Eqs. (7) and (8) seems clear. Even in the vicinity of the vector-meson poles in k^2 and M^2 the photoproduction amplitude contains contributions in these variables (from all the higher resonances) that do not occur in elastic π-π scattering. Stated in the language of Veneziano models: If elastic π-π scattering is represented by a Veneziano amplitude without satellites, then the π-π production amplitude in diffractive photoproduction must contain an infinite set of satellite terms as in Eqs. (7) and (8). It is these satellite terms that constitute the "background" in the ρ-photoproduction amplitude while the $B(1 - \alpha(M^2), -\alpha(s_{23}))$ represents the elastic π-π scattering contribution.

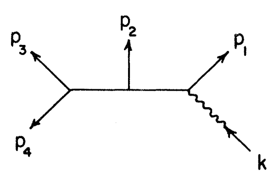

FIG. 2. The "tree" diagram described by Eq. (1) of the text.

Now the meaning of vector dominance, as it is usually applied, is that the photon interacts with hadrons (in the context of the present model) as though it were a ρ meson, at least in the case of photons with k^2 zero or moderately negative. That is to say, the k^2 dependence in Eq. (8) is given by the contribution of the pole at $\alpha(k^2) = 1$. The contribution of this pole to β_n in Eq. (8a) is just given by

$$B_n(t, M^2, k^2) = \frac{2}{1 - \alpha(k^2)} + \cdots\,, \tag{9}$$

where the dots represent the contribution of the higher-mass poles in the electromagnetic current. It follows, then, that near the ρ pole

$$H \approx \frac{2}{1 - \alpha(k^2)} \sum_{n=1}^{\infty} A_n(M^2, t) B(n - \alpha(M^2), -\alpha(s_{23})) + \cdots\,, \tag{10}$$

where the first few A_n may be read off from Eq. (8a).

An interpretation of Eq. (10) can be made by noting that in the present model the pion form factor may be interpreted as being just

$$F_\pi = \tfrac{1}{2}g\int_0^1 y\,dy(1-y^2)^{-[1+\alpha(k^2)]/2} = \frac{\frac{1}{2}g}{1-\alpha(k^2)}\,. \quad (11)$$

Although a one-pole form factor is much too simple to be realistic, the qualitative inference that can be made from the model is suggestive. That is, the k^2 dependence in photoproduction processes may be quite different from that given by simple vector-dominance considerations. More specifically, the effective "ρ-photon coupling" in diffractive photoproduction need not be simply related to the coupling constant determined from $e^+e^- \to \rho$. The difference comes about because the higher-mass resonances "in the photon" are behaving differently in the two cases. In fact, Eq. (8) suggests that the ρ-photon coupling might be represented by the vector-dominance part, exhibited in Eq. (10) plus an additive complex slowly varying (with k^2) part.

Finally, to conclude this section, I record the "pure vector-dominance" content of Eqs. (7) and (8) by evaluating the ρ-pole contribution at $\alpha(M^2) = 1$. Under the assumption that $\alpha(t) = 1$ at $t = 0$, but that $\Gamma(-\alpha(t))$ is replaced by a finite constant f in this limit, the result is

$$F_{12}^\rho = \frac{g^3 f}{-[1-\alpha(k^2)][3-\alpha(k^2)]}\, s\, \frac{1}{1-\alpha(M^2)}\,, \quad (12)$$

which, for photoproduction, is to be evaluated at $k^2 = 0$.

IV. THE DRELL-SÖDING AMPLITUDE

An alternative representation of F_{12}, dual to that given in Eqs. (7) and (8), may be obtained by expanding in the poles of $\alpha(s_{234})$. This is obtained by expanding the integrand in Eq. (6) in powers of z. The leading term turns out to be just

$$F_{12}^\pi = -\frac{\frac{1}{2}g^3}{[1-\alpha(k^2)][(k-p_1)^2-m^2]} \times B(\alpha(s_{234})-\alpha(t), -\alpha(s_{234}))\,,$$

$$\approx -\frac{\frac{1}{2}g^3}{[1-\alpha(k^2)][(k-p_1)^2-m^2]} \times (-s_{23})^{\alpha(t)}\Gamma(-\alpha(t)) \quad \text{for} \quad \alpha(s_{234}) \approx 0, \quad (13)$$

where $(s_{23})^{1/2}$ is just the total energy of pion-2 scattering from the target (3) in the 23 barycentric system. The term exhibited in Eq. (13) is, of course, just the expression for the Drell-Söding process. I have thereby exhibited the expected result that in a dual, factorizable model the simple addition of a vector-dominance amplitude plus a Drell-Söding amplitude may involve double counting since both Eq. (12) and (13) are approximate representations of the same amplitude.[9] That the addition of the two amplitudes would involve double counting was conjectured by Pumplin.[4] Pumplin suggested that the double counting be compensated by multiplying the Drell-Söding term by $\cos\delta_{\pi\pi}$ in order to make it vanish at the resonance position where $\delta_{\pi\pi} = \frac{1}{2}\pi$. This prescription is in accord with the usual application of final-state-interaction theory.[6]

In fact, the present model shows that the $\cos\delta_{\pi\pi}$ prescription is too simple. In the present model elastic π-π scattering is described by (I give only the contribution of the s and t channels)

$$A_{\pi\pi} = B(-\alpha(s_{12}), -\alpha(t_{\pi\pi}))\,, \quad (14)$$

with the ρ at $\alpha(s_{12}) = 1$. The p-wave contributions from the resonances at $\alpha(s_{12}) = n$, $n \neq 1$ presumably describe the background even at the position of the ρ (I say "presumably" out of deference to the uncertainty involved in giving a physical interpretation to a nonunitary theory). Equation (8) shows that the background is considerably modified in the photoproduction amplitude and there is nothing to suggest that the *modification* might vanish at $\alpha(s_{12}) = 1$. The Drell-Söding term, Eq. (13), gives a representation of the ρ pole plus modified background (exact at the one-pion-exchange pole) and some part of it should presumably persist even at the resonance position.

V. CONCLUSION

The underlying motivation of this work has been to obtain a qualitative feeling for the structure of a vector-meson photoproduction amplitude. I infer from the model that, in the vicinity of the resonance the amplitude at high energy can be divided into three parts, assuming Pomeranchukon dominance of the production process. One part may be taken to be a Breit-Wigner resonance term with a mass-dependent width taken, for example, from hadronic production of the vector meson.[10] A second part would be a Drell-Söding amplitude multiplied by a function of the π-π invariant mass (in the case of ρ production) to be determined from experiment. The remaining part would hopefully be a slowly varying amplitude (as a function of π-π mass) that represents the remainder of the "background."[11]

The importance of the Drell-Söding portion of the amplitude is that the one-pion pole is very close to the physical region of the π-π angular distribution. If β is the velocity (in units of the speed of light) of the decay pions in the rest frame of the two-pion system, then

$$\beta^2 = 1 - 4m_\pi^{\,2}/M_{\pi\pi}^{\,2} \tag{15}$$

and the poles in the $\pi\pi$ angular distribution are predicted to be at

$$\cos\theta_{\pi\pi} = \pm 1/\beta\,, \tag{16}$$

which is about ± 1.06 for $M_{\pi\pi}$ near the ρ mass. Thus, a Chew-Low type of extrapolation in the π-π angular distribution should be feasible in a high-statistics experiment as a means of isolating the Drell-Söding background contribution (not at $t=0$, however, where the pole residue vanishes).

Another interesting facet of the model may be seen in the different photon form factors that occur in Eqs. (7) and (13). The lesson that may be drawn is that different parts of the amplitude may be expected to have different dependences upon the photon mass so that the spectrum of pion pairs in electroproduction may look quite different from the spectrum obtained with real photons. Indications of such differences are already present in existing data. It should also be apparent that the effective ρ-photon coupling constant, as determined from simple vector-dominance considerations, might have an appreciable dependence upon the photon mass so that the coupling constants obtained from photoproduction and e^+e^- annihilation may differ appreciably, again in accord with existing data.[12]

There are three major weaknesses of the model that require comment. As has already been seen, the fact that the model is not unitary tends to obscure the meaning of "background," especially "at" the resonance position.[13] As a consequence of the zero widths, resonances do not get shifted and resonance shapes are barely discussable.

The second weakness lies in the absence of a realistic Pomeranchukon to mediate the production process. The structure of Eq. (8) in particular arises because the t-channel trajectory is basically the same one (even if its intercept is shifted) that determines the spectrum of π-π masses. The coupling of this trajectory to the higher-spin resonances in the π-π channel determines the background. A real Pomeranchukon might conceivably give rise to a much simpler amplitude.[14]

Finally, there does not appear to be any sensible way to discuss "final-state interactions" in a tree-diagram model such as the present model. The notion of final-state interactions in hadronic processes, and the triangle singularities associated with them, seems to be inseparable from calculations involving closed loops. Such calculations are beyond the scope of the present work.

In summary, it appears that the weaknesses listed above, which seem to be inherent in current dual models, make such models unsuitable as detailed guides for use in analyzing vector-meson photoproduction experiments. The model dealt with in the present investigation does illustrate some of the pitfalls that could waylay the "simple" vector-dominance model. Attempts presently in progress to analyse existing photoproduction data may be encountering difficulties with some of these pitfalls.[15] It seems to me that at the present stage of theoretical development a detailed description of vector-meson photoproduction must depend upon the efforts of the experimentalist. What is needed are high-statistics experiments and detailed "amplitude analysis."

ACKNOWLEDGMENTS

I am indebted to Stanley Fenster and Chia Tze for many discussions concerning dual models of currents and to Gustave Kramer and Donald Yennie for many discussions of vector dominance.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Address after 1 October 1972: University of Chicago, The Law School, 1111 E. 60th St., Chicago, Ill. 60637.

[1]Reviews may be found in: G. Kramer, in *Proceedings of the Daresbury Study Weekend, 1970,* edited by E. A. Donnachie and E. Gabathuler (Science Research Council, Daresbury Nuclear Physics Laboratory, Lancashire, England, 1970); Z. Physik 250, 413 (1972); D. W. G. S. Leith, in *Hadronic Interactions of Electrons and Photons,* edited by J. Cumming and H. Osborn (Academic, New York, 1971); G. Wolf, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N.Y., 1972). There is also a review of hadronic interactions of photons in preparation by K. Gottfried, F. Pipkin, R. Spital, and D. Yennie (private communication from D. Yennie).

[2]P. Söding, Phys. Letters 19, 702 (1966).

[3]G. Kramer and H. R. Quinn, Nucl. Phys. B27, 77 (1971). See also G. Kramer and J. L. Uretsky, Phys. Rev. 181, 1918 (1969).

[4]Jon Pumplin, Phys. Rev. D 2, 1859 (1970).

[5]A. Neveu and J. Scherk, Nucl. Phys B41, 365 (1972).

[6]Some of the uncertainties in present day theory may be gleaned from a reading of I. J. R. Aitchison and C. Kacser, Phys. Rev. 173, 1700 (1968).

[7]A *convergent* expansion can be made in powers of $(1-x)$ and the pole (in M^2) contributions contained in Eq. (8a) then calculated. The result is the same.

[8]H. Alvensleben *et al.*, Phys. Rev. Letters 26, 273 (1971); F. Bulos *et al.*, *ibid.* 26, 149 (1971); G. Barbarino *et al.*, Lett. Nuovo Cimento 3, 689 (1972).

[9]Nevertheless, the actual amount of double counting involved in adding the two amplitudes may be zero in this model. I have not, however, been able to obtain an

estimate of the overlap.

[10]The model suggests that the contribution of the ρ to a hadronic production amplitude differs from its contribution to the pion form factor (as determined from e^+e^- annihilation, for example). It should be noticed, by the way, that I have consistently ignored the nontrivial problem of ρ-ω interference.

[11]A simpler proposal for parametrizing the ρ photoproduction amplitude has been advanced by D. Yennie (private communication).

[12]K. Berkelman, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971* (Ref. 1).

[13]See Wolf in Ref. 1.

[14]As in the model of H. Satz and K. Schilling, Nuovo Cimento 67A, 511 (1970); Lett. Nuovo Cimento 3, 723 (1970). In their model the Pomeranchukon was forced to mediate a simple, vector-dominant amplitude.

[15]Private communication from D. Yennie.

PHYSICAL REVIEW D VOLUME 7, NUMBER 1 1 JANUARY 1973

Isobar Models, Analysis and Symmetrization Effects in Meson+Baryon → Meson+Meson+Baryon†

C. David Capps, P. B. Madden,* and P. K. Williams
Department of Physics, Florida State University, Tallahassee, Florida 32306

(Received 17 August 1972)

We have made a study of helicity formalisms for two-body → three-body reactions. We present what we feel is an improvement in the formalism of Namyslowski, Razmi, and Roberts, both in accuracy and in manageability. Also, we present the results of an application of the formalism to the reaction $K^-n \to \Sigma^+\pi^-\pi^-$ in an isobar model with sequential decays through intermediate Λ and Σ resonances. We find some appreciable effects due to boson symmetrization.

I. INTRODUCTION

Detailed energy-dependent phase-shift analyses of 2-body → 3-body hadron reactions are *a priori* difficult. Yet much experimental information is accumulating on hadronic interactions in resonance-formation energy regions which is of the 2-body → 3-body type. Early attempts at analysis consisted of very crude isobar models in which all resonances were assumed to decay isotropically and/or interferences arising from dynamics or symmetries were neglected.[1] More recent formulations have more properly included resonance spins, parities, and centrifugal-barrier effects into general, relativistic, and unitary schemes.[2,3] However, these formulations are not very practical in actual use for one or more of the following reasons: (a) They are difficult to understand at first sight. (b) They require major programming efforts. (c) They require large amounts of computer memory and time. (d) They may yield marginal results. (e) Parameter errors are often difficult to ascertain.

As a result, most data are not analyzed using the most sophisticated isobar models. This state of affairs is unfortunate as much can be learned in principle from the application of such models to data, e.g., resonance parameters and branching ratios. In this paper, we hope to facilitate the use of isobar models in determining sequential decay schemes through examining, correcting, and simplifying the work of Namyslowski, Razmi, and Roberts (NRR).[2] In part we follow the development of these authors. However, our examination of their work leads to correcting an apparently rather significant error in their formalism, and results in simplified expressions which are neither less general, nor as formally imposing, nor anywhere nearly as difficult or time consuming to apply. Further, we need not make any of the rather severe truncations advocated in NRR to facilitate explicit calculations. Rather, we find these truncations to be essentially incorrect and totally unnecessary. In its final form, our result is similar to that of Deler and Valadas (DV) for the special case which they consider ($\pi N \to \pi\pi N$).[3]

Finally, we present explicit calculations on the reaction

$$K^- + n \to \pi^- + \pi^- + \Sigma^+ \tag{1}$$

at center-of-mass energy $W = 1690$ MeV. We examine mass distributions and angular distributions for effects of masses, widths, spins, and parities of various isobars and decay schemes, and we look for the effects of symmetrization. We find that the angular distributions can be sensitive to