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Characteristics of Multipion Production at 3.9 and 11.9 GeV/c*

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A quasi-two-body model based on one-particle exchange and diffraction dissociation has been fitted to data from $\pi^- p$ interactions at 3.9 and 11.9 GeV/c in which a nucleon and 3–6 pions are present in the final state. It is used to estimate partial cross sections for the contributing interaction mechanisms and the dominant resonances which are produced at these energies. The energy dependence of the cross sections is examined and found to be consistent with expected behavior, and reactions are compared and found to agree with simple factorization.

I. INTRODUCTION

In an earlier paper[1] it was shown that the main kinematic features of the reaction $\pi^- p \rightarrow \pi^-\pi^-\pi^+ p$ at 3.9 GeV/c could be described by a simple model based on pion exchange, ρ exchange, and diffraction dissociation. It was demonstrated that the kinematic reflections of the various competing processes produced phenomena such as anisotropic Treiman-Yang and scattering angular distributions which might otherwise be interpreted as deeper dynamical effects. It was also shown that diffraction dissociation, without multiperipheralism, was adequate to describe the data in the $A_1(1070)$ and $N(1470)$ regions without the need to introduce these as resonant states.

This paper represents a considerable extension of this work to include 5-, 6-, and 7-body final states and a higher momentun, 11.9 GeV/c.

A model with a small number of adjustable parameters was developed to describe the following reactions at both momenta:

$$\pi^- p \rightarrow \pi^-\pi^-\pi^+ p \qquad [4],$$

$$\rightarrow \pi^-\pi^-\pi^+\pi^0 p \qquad [5p],$$

$$\rightarrow \pi^-\pi^-\pi^+\pi^+ n \qquad [5n],$$

$$\rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p \qquad [6],$$

$$\rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+\pi^0 p \qquad [7p],$$

$$\rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+\pi^+ n \qquad [7n].$$

The model is based on the dynamical ideas of single-particle exchange and diffraction dissociation. It incorporates only the main features expected from these mechanisms and is basically a quasi-two-body rather than multiperipheral model.

The experimental data used in this study are, for the most part, being published for the first time. The data for reactions [4] and [6] at 3.9 GeV/c have already appeared[1,2]; the remainder are new results. The 3.9-GeV/c data presented represent our complete exposure at that momentum of the 72-in. hydrogen bubble chamber at the (then) Lawrence Radiation Laboratory, Berkeley. The 11.9-GeV/c data are from the same chamber (enlarged to 82 in.) exposed at SLAC, and form only a small subsample from our total of 650 000 pictures.[3]

Events were measured using scanning and measuring projectors (SMP's) and data reduced either by PACKAGE or by TVGP and SQUAW. The number of events for each reaction at each energy and cross sections is given in Table I.

The six reactions were fitted independently at each energy. The results of the fits were used to estimate partial cross sections for the contributing interaction mechanisms and the dominant res-

TABLE I. Summary of cross sections determined for each production mechanism.

Reaction	Events	Total cross section (mb)	Cross section for production mechanism (mb) OPE	RHE	OPD	EPD	Background cross section (mb)
			(a) 3.9 GeV/c				
[4]	7980	1.92 ± 0.1^{a}	0.48 ± 0.05	0.18 ± 0.06	0.65 ± 0.06	<0.02	0.62 ± 0.04
[5p]	6870	2.08 ± 0.1^{a}	1.19 ± 0.17	<0.04	0.21 ± 0.06	<0.04	0.69 ± 0.08
[5n]	3560	1.08 ± 0.1^{a}	0.67 ± 0.10	<0.02	<0.04	<0.02	0.40 ± 0.05
[6]	456	0.12 ± 0.02^{b}	0.05 ± 0.01	<0.01	0.02 ± 0.01	<0.01	0.05 ± 0.01
[7p]	603	0.13 ± 0.03^{c}	0.04 ± 0.01	<0.01	0.05 ± 0.03	<0.01	0.04 ± 0.03
[7n]	292	0.06 ± 0.02^{c}	0.02 ± 0.01	<0.01	0.01 ± 0.01	<0.01	0.03 ± 0.01
			(b) 11.9 GeV/c				
[4]	931	0.84 ± 0.05^{c}	0.06 ± 0.02	<0.02	0.75 ± 0.09	<0.02	<0.04
[5p]	1519	1.36 ± 0.04^{c}	0.45 ± 0.08	<0.03	0.22 ± 0.08	0.52 ± 0.07	0.17 ± 0.09
[5n]	1076	0.97 ± 0.06^{c}	0.17 ± 0.09	<0.05	0.44 ± 0.15	0.30 ± 0.12	<0.10
[6]	1350	0.19 ± 0.02^{c}	0.03 ± 0.01	<0.01	0.15 ± 0.02	<0.01	0.01 ± 0.01
[7p]	2000	0.81 ± 0.08^{c}	0.05 ± 0.05	<0.03	0.24 ± 0.09	0.19 ± 0.07	0.25 ± 0.06
[7n]	1560	0.27 ± 0.03^{c}	0.02 ± 0.02	<0.02	0.04 ± 0.04	0.17 ± 0.06	0.05 ± 0.05

[a]Ref. 12. [b]Ref. 2. [c]Determined in this experiment.

onances which are produced at these energies, as discussed in Sec. II.

Since pion exchange and diffraction dissociation each have a characteristic energy dependence, we have used this to test the consistency of our partial cross sections for the two beam momenta. These results are presented in Sec. III.

Further consistency checks on our diffraction-dissociation particle cross sections are summarized in Sec. IV. These were based on the idea of factorization and allow us to compare several different final states at each energy.

II. DESCRIPTION OF THE EXPERIMENT

This section includes a discussion of the fits made separately to the six reactions at each of the two beam momenta.

A. The Model

We do not attempt to describe the data in minute detail. We concern ourselves with the primary features of each reaction. In particular we neglect all resonance production but the most prominent, Δ, ρ, ω, and A_2. These we represent by relativistic fixed-width Breit-Wigner amplitudes with masses and widths taken from the data tables and not allowed to vary.

The strong peripheralism of the data, as manifested in sharply peaked momentum transfer or longitudinal-momentum distributions, is a primary feature which one cannot neglect and still claim to describe the data. Two possible mechanisms are known to be able to account for this: pion exchange[4] and diffraction dissociation.[5]

For pion exchange, the peaking in four-momentum transfer squared, t, results from the pole in t at the pion mass. Although the many theories and phenomenologies for pion exchange require various t-dependent vertex factors, the experimental facts are that the t distributions can be

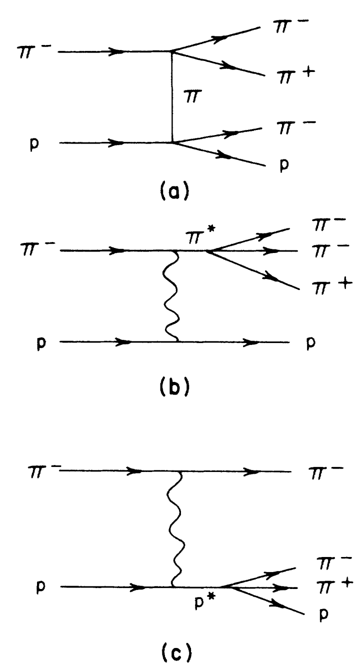

FIG. 1. Diagrams illustrating (a) pion exchange, (b) diffraction dissociation of pion, (c) diffraction dissociation of proton.

adequately fitted by the pole alone in many cases.[1]

In the case of diffraction dissociation, the production process is regarded as quasielastic and an e^{bt} dependence, analogous to $\pi^- p$ elastic scattering with $b \approx 8$ $(\text{GeV}/c)^{-2}$, is expected and agrees well with the data.[1]

As an example of how these two basic mechanisms have been incorporated, let us consider reaction [4]. In Fig. 1(a) we show the exchanged pion undergoing a scattering on the incident pion at the upper vertex which will be dominated by the ρ^0 resonance, and a scattering on the incident proton at the lower vertex which will have a strong contribution from the Δ^0. The main features of this process are described by the amplitude representing pion exchange:

$$T(s,t) = \frac{B(\rho)B(\Delta)}{t - m_\pi^{\,2}} \left(\frac{s}{s_0}\right)^{\alpha_\pi(0)}, \tag{1}$$

where $B(R)$ is a relativistic fixed-width Breit-Wigner amplitude for the resonance R. In Fig. 2(b) we illustrate the diffraction dissociation of the incident pion. If M is the 3π mass, then

$$T(s,t) = e^{bt}\left(\frac{s}{M^2}\right)^{\alpha_P(0)} \tag{2}$$

In both (1) and (2) the t-dependence of $T(s,t)$ is taken simply to be that given by the pion pole or diffractive exponential, respectively. The s-dependence is taken from Regge theory where $\alpha_\pi(0) \approx 0$ and $\alpha_P(0) \approx 0.9$ are the pion and Pomeranchukon intercepts. Since we fit our reactions independently at each energy, the s dependence is irrele-

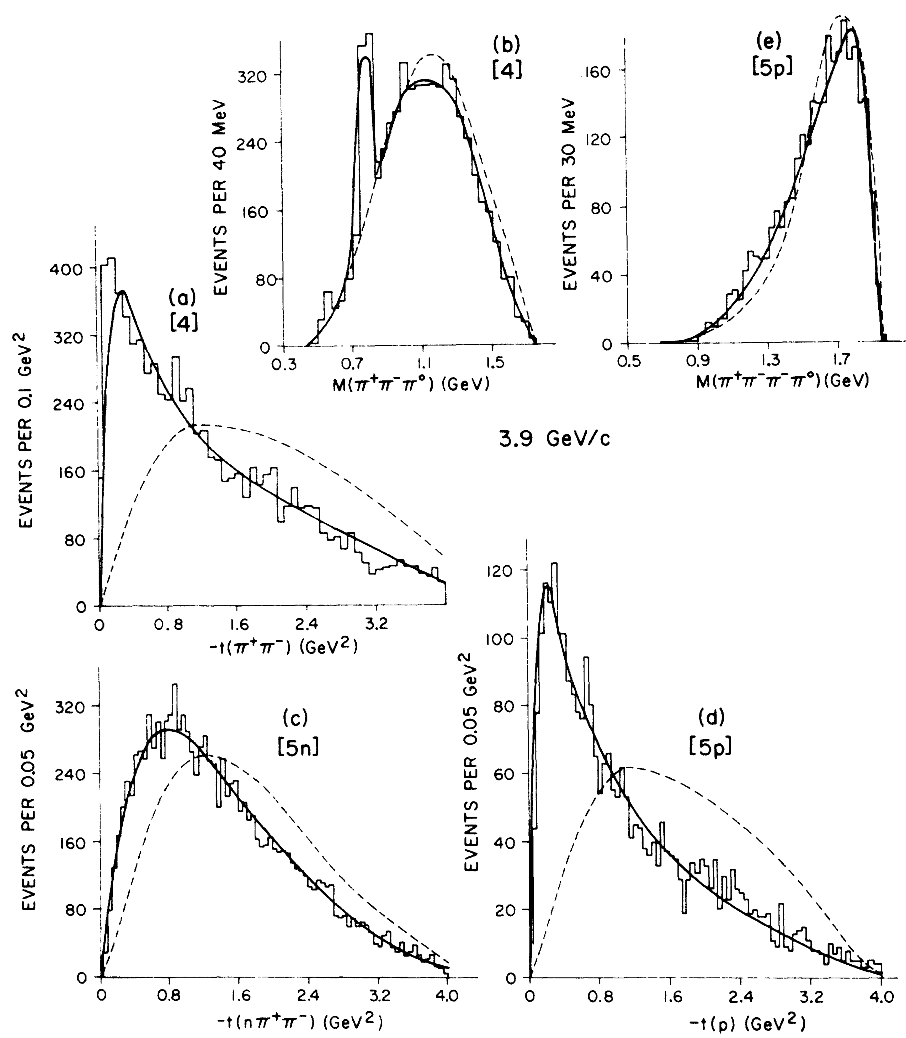

FIG. 2. Selected plots showing fits to 4- and 5-body reactions at 3.9 GeV/c. Solid curves indicate the distributions of Monte Carlo events generated according to the model. The dashed curves are predictions of uniform phase space normalized to the total number of events.

vant to the fitting process and enters only in the energy consistency tests discussed in Sec. III. Any t-dependence implied by Regge theory through $\alpha(t)$ is absorbed in the pole or expenential factors.

With the value $b \approx 8\,(\mathrm{GeV}/c)^{-2}$ as suggested by elastic scattering, it has been shown that the amplitude (2) multiplied by the appropriate phase-space factor for reaction [4] and integrated over t gives a broad enhancement in M peaking at 1.07 GeV, independent of beam momentum above a few GeV/c and nicely fitting the A_1 enhancement seen in a wide range of experiments.[6] The studies on angular distributions which show that the A_1 region is dominantly $J^P = 1^+$ with some 0^- contribution agree with the diffraction interpretation.[7]

In the model discussed here a slight variation on form (2) was used with M^2 replaced by $M^2 - M_0{}^2$, where M is the mass of the dissociating particle (pion or proton). Such a form might be expected, for example, in the Pomeranchukon-exchange picture where the scattering of the Pomeranchukon and incident particle have an "s-channel pole" at the mass of the incident particle. In any case, this was found to give somewhat better results in the case of proton dissociation and makes no difference in the case of pion dissociation, since $M_\pi{}^2$ is very small.

In general, the slope parameter b is expected to vary with s, M^2, and particle multiplicity. Having no *a priori* knowledge of the dependence, and for reasons of simplicity, we have taken it to be constant. The magnitude of b, however, is expected to be comparable to that for $\pi^- p$ elastic scattering, viz., $\approx 8\,(\mathrm{GeV}/c)^{-2}$. We have found this to be a good choice for the processes where the pion dissociates, as in Fig. 1(b), but a smaller value of 5 $(\mathrm{GeV}/c)^{-2}$ was used for proton dissociation as in Fig. 2(c). In these choices we are consistent with other authors.[8,9]

Our model utilizes these two basic processes expressed in this simplified way. Breit-Wigner functions are used for vertex amplitudes whenever a known prominent resonance (ρ, Δ, ω, A_2) is assumed to be produced; otherwise s-wave scattering of the exchanged and incident particles is assumed. These vertex amplitudes are calculated for the exchanged particle on the mass shell. No effects of spins of resonances, e.g., decay correlations, are incorporated explicitly. This does not mean that the usual decay correlation angular distributions are isotropic except for the particular process in which the resonance is produced. No attempt is made to include multiperipheral effects, so this work represents to some extent a test of how well one can do without invoking multiparticle exchange.

The procedure utilizes a maximum-likelihood method in which several processes are assumed to contribute incoherently to a given reaction at a given energy. Coherent effects are important in cases where two or more processes contribute strongly to the same region of phase space. While there is some overlapping for the mechanisms we consider, the main contribution from each process is in a unique region. The parameters which are unknown *a priori* are the relative amounts of each process which are determined by a fit to the density in the four-vector phase space. The fit is made independently for each reaction and energy. In this way the fractional contribution of each process is obtained. The number of processes needed to give reasonable fits varies from 3 to 8 and thus, with over-all normalization, no more than 7 parameters are ever required for a given reaction.

We have classified the basic production mechanisms considered as follows:

(1) one-pion exchange (OPE),

(2) ρ exchange $- A_2$ production only (RHE),

(3) diffraction dissociation with an odd number of pions at the pion vertex (OPD),

(4) diffraction dissociation with an even number of pions at the pion vertex (EPD).

The distinction between the latter two was made since in the Pomeranchukon-exchange view of diffraction dissociation, G-parity conservation allows only an odd number of pions to dissociate from the incident pion, while if one looks at the diffraction dissociation mechanism as involving the πN system as a whole, G parity need not be conserved since the πN system is not in an eigen-

TABLE II. Results of fit to reaction [4] at each energy. The background is taken as uniform phase space. Parentheses indicate which particles or resonances emerge from each vertex.

Process $\pi^- p \to$	Mechanism	Fraction fitted	Cross section (mb)
	(a) 3.9 GeV/c		
$(\pi^-\pi^-)(\Delta^{++})$	OPE	0.13	0.25 ± 0.03
$(\rho^0)(\Delta^0)$	OPE	0.12	0.23 ± 0.04
$(\rho^0\pi^-)(p)$	OPD	0.13	0.26 ± 0.04
$(\pi^-)(\pi^-\Delta^{++})$	OPD	0.20	0.39 ± 0.04
$(A_2^-)(p)$	RHE	0.09	0.18 ± 0.04
Background	···	0.32	0.62 ± 0.04
	(b) 11.9 GeV/c		
$(\rho^0)(\Delta^0)$	OPE	0.07	0.06 ± 0.02
$(\pi^-\pi^-\pi^+)(p)$	OPD	0.20	0.17 ± 0.07
$(\rho^0\pi^-)(p)$	OPD	0.34	0.28 ± 0.04
$(\pi^-)(\pi^-\Delta^{++})$	OPD	0.26	0.22 ± 0.03
$(\pi^-)(\pi^-\pi^+p)$	OPD	0.09	0.08 ± 0.03
Background	···	0.04	<0.04

TABLE III. Results of fit to reaction [5p] at each energy. The background is taken as uniform phase space. Parentheses indicate which particles or resonances emerge from each vertex.

Process $\pi^- p \to$	Mechanism	Fraction fitted	Cross section (mb)
	(a) 3.9 GeV/c		
$(\pi^-)(\pi^-\pi^0\Delta^{++})$	OPD	0.10	0.21± 0.06
$(\pi^-\pi^0)(\pi^-\Delta^{++})$	OPE	0.10	0.21± 0.07
$(\pi^-\pi^0)(\pi^+\Delta^0)$	OPE	0.07	0.14± 0.07
$(\pi^-\pi^-\pi^0\pi^+)(p)$	OPE	0.07	0.15± 0.06
$(\rho^0\pi^0\pi^-)(p)$	OPE	0.08	0.16± 0.08
$(\rho^-\pi^-\pi^+)(p)$	OPE	0.10	0.21± 0.08
$(\omega\pi^-)(p)$	OPE	0.15	0.32± 0.07
Background	···	0.33	0.69± 0.08
	(b) 11.9 GeV/c		
$(\pi^-)(\pi^-\pi^0\Delta^{++})$	OPD	0.08	0.11± 0.05
$(\pi^-)(\pi^-\pi^0\pi^+p)$	OPD	0.08	0.11± 0.06
$(\rho^0\pi^-\pi^0)(p)$	EPD	0.38	0.52± 0.07
$(\rho^-)(\pi^-\pi^+p)$	OPE	0.22	0.30± 0.06
$(\rho^0)(\pi^-\pi^0p)$	OPE	0.11	0.15± 0.06
Background	···	0.13	0.17± 0.10

state of G (see note added in proof, Ref. 5). The reason for the inclusion of this debatable process in our model was the apparent requirement of the 5- and 7-body data at 11.9 GeV/c for a process yielding low four-momentum transfer to the nucleon, yet not as low as produced by our particular form of pion exchange nor as high as one expects from heavy-meson exchange. In general, our 5- and 7-body fits are not as good as the 4- and 6-body fits so we cannot make a strong statement on this, and for our present purposes we consider the EPD process as a undetermined but probably diffractive background.

B. Summary of Results

The fitting procedure allows us to estimate the fractional contributions of the competing processes for a given reaction and energy. Knowing the cross section for the reaction at that energy we can then determine the partial cross section for each process. These results are presented in Table I. The errors are statistical with an estimated systematic error for the fitting process folded in. This is generally of the order of 10–20 μb and is determined by our experience of the level at which our fitting programs seemed to be able to detect the presence of the various processes.

A more detailed breakdown of the fits is given in Tables II and III for reactions [4] and [5p]. These show how a given mechanism may account for several resonances or how a given resonance may be produced by several mechanisms. For example, the total OPE cross section presented in Table I(a) for reaction [5p] at 3.9 GeV/c is the sum of the cross sections for Δ^{++}, ρ^0, ρ^-, ω and nonresonant production by OPE listed in Table III(a).

Similarly we may sum the cross sections for a given resonance produced by each mechanism to get the total cross section for that particular resonance. These results are given in Table IV. We wish to point out, however, that these represent *lower limits* on the total resonance production since not all possible mechanisms have been considered. Or, put another way, they represent our estimate of the resonance production due to

TABLE IV. Summary of partial cross sections for resonance production by mechanisms considered in this work.

Reaction	Cross section for resonance (mb) Δ^{++}	Δ^+	Δ^0	Δ^-	ρ^0	ρ^-	ω	A_1^- [a]	A_2^-
				(a) 3.9 GeV/c					
[4]	0.64± 0.06	···	0.23± 0.04	···	0.41± 0.06	···	···	<0.01	0.18± 0.04
[5p]	0.42± 0.08	<0.04	0.14± 0.07	···	0.16± 0.08	0.21± 0.08	0.32± 0.07	<0.04	<0.04
[5n]	···	<0.02	···	0.28± 0.06	0.27± 0.07	···	···	<0.02	<0.02
[6]	0.05± 0.01	···	<0.01	···	0.02± 0.01	···	···	<0.01	<0.01
[7p]	0.05± 0.03	<0.02	<0.02	···	<0.02	<0.02	0.04± 0.01	<0.02	<0.02
[7n]	···	<0.02	···	<0.03	···	···	···	<0.02	<0.02
				(b) 11.9 GeV/c					
[4]	0.22± 0.03	···	0.06± 0.02	···	0.34± 0.05	···	···	<0.02	<0.02
[5p]	0.11± 0.05	<0.03	<0.03	···	0.15± 0.05	0.30± 0.06	<0.03	<0.03	<0.03
[5n]	···	0.11± 0.08	···	0.26± 0.11	0.09± 0.09	···	···	<0.05	0.09± 0.09
[6]	0.03± 0.01	···	0.07± 0.01	···	0.12± 0.01	···	···	0.03± 0.01	0.03± 0.01
[7p]	0.24± 0.09	<0.03	<0.03	···	0.13± 0.06	<0.1	<0.03	<0.03	<0.03
[7n]	···	<0.09	···	<0.09	0.12± 0.05	···	···	<0.03	<0.03

[a] Nondiffractive.

the mechanisms considered.

The column marked A_1 in Table IV refers to the nondiffractive (ρ-exchange) production of an A_1 Breit-Wigner resonance. In all cases the cross sections are consistent with zero and the hypothesis that the entire A_1 enhancement is a diffractive, nonresonant, kinematic effect.

The columns labeled "background" in the tables refer to our estimate of the cross section for all of the nonperipheral processes which are present. These are represented in the model by the statistical average behavior of uniform phase space.

Monte Carlo events generated according to the model and compared with the real data proved to be a valuable tool in evaluating the fits. It is unrealistic to expect to describe in detail complicated reactions with such a simple model, so the usual statistical tests are not useful. The Monte Carlo distributions permit one to see if the model describes the broad trends of the data and are especially valuable in comparing the contributions for various processes. Of course, many model variations were tried before the final results presented here were obtained. Many processes not included in the final model were discarded because the fitting program found little contribution for these processes when used in competition with the processes we have preserved.

Space does not permit a comprehensive view of our description of the 6 reactions at the two energies.[3] Here we show in Figs. 2–5 only a few selected histograms to give the reader a feeling for the quality of the fit to the data and how he should interpret our results.

In these figures the real data are shown superimposed with fake events weighted according to the

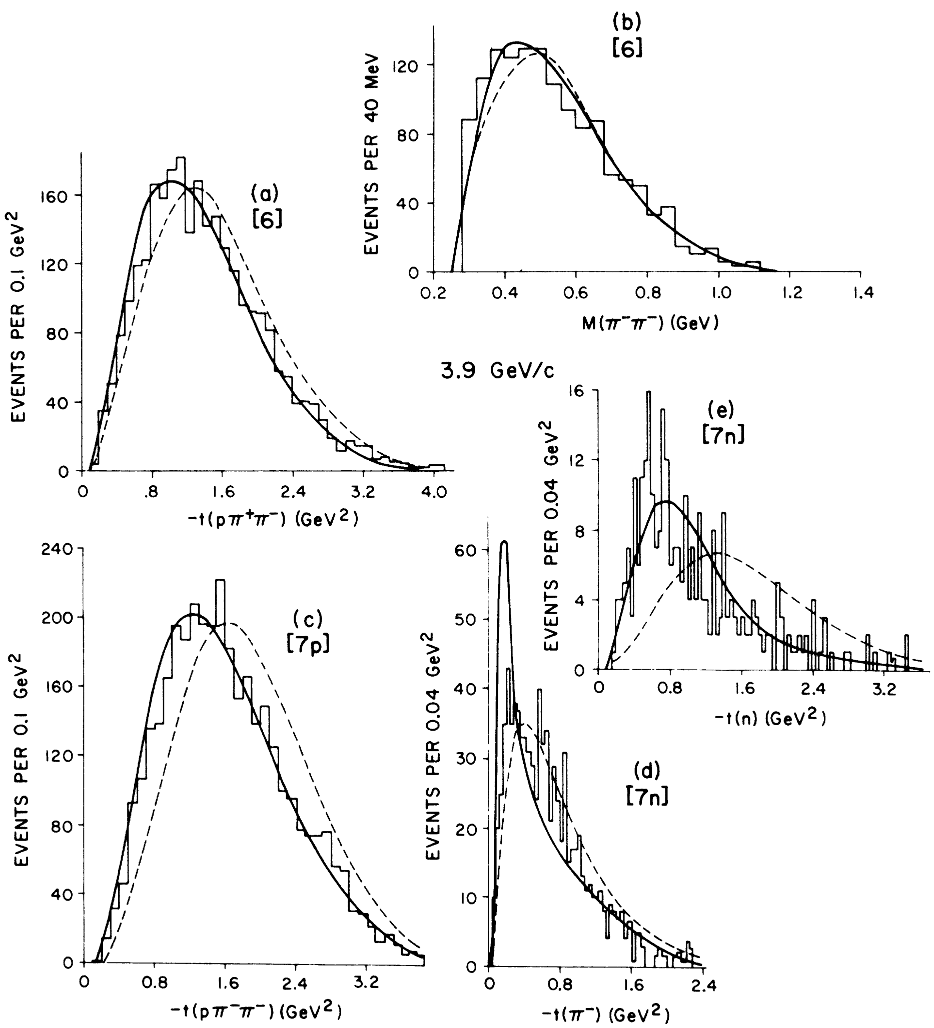

FIG. 3. Selected plots showing fits to 6- and 7-body reactions at 3.9 GeV/c.

model shown as solid curves and the distributions predicted by uniform phase space indicated by the dashed curves.

We have chosen to present only invariant mass and momentum-transfer distributions since these manifest best the dynamical features of the model and are most sensitive to the model parameters. Monte Carlo distributions of other variables such as Jackson angles, Treiman-Yang angles, and transverse and longitudinal momenta have been studied and, in general, agree well with the data. The well-known leading particle and limiting transverse-momentum distributions are adequately simulated by the model.[3]

The histograms presented were *not* selected to put our results in the best possible light. Virtually all of the important discrepancies between our fits and the data are shown and many of the good fits necessarily left out.

One can see from this limited sample of figures that the model does describe the basic trends of the data in most cases. As with our previous work, we find that we can describe the basic shapes of the mass distributions which show deviations from what is expected from phase space plus resonances. The kinematic reflections of resonances in other channels and the peripheral production mechanism are illustrated in Figs. 2(b), 2(e), 3(b), 4(b) and 4(e). The model tends to underestimate resonance production somewhat, probably because of the limited number of production mechanisms considered.

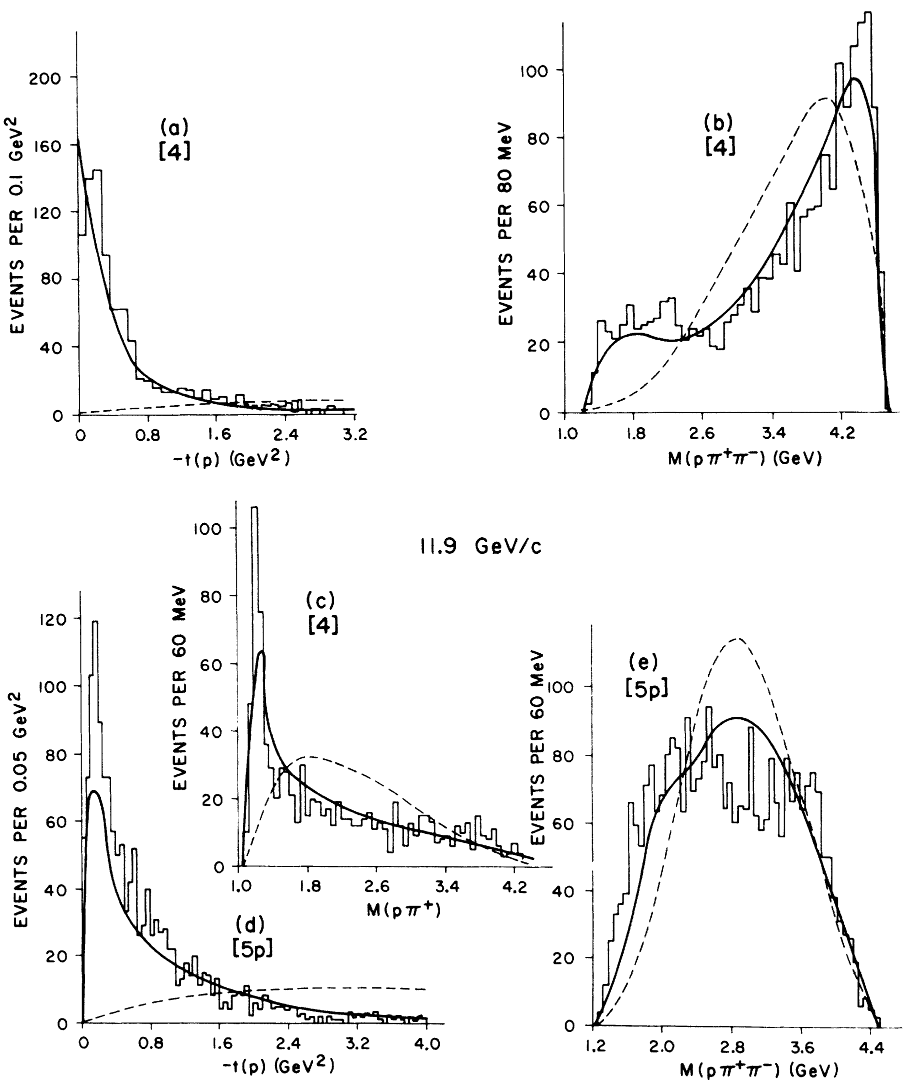

FIG. 4. Selected plots showing fits to 4- and 5-body reactions at 11.9 GeV/c.

The t distributions show varying amounts of peaking depending on reaction, multiplicity, and particle combination. We wish to point out that the basic agreement of our model with the data is not simply the result of parameter freedom. Discrepancies do occur. For example, in the n-6π final state at 3.9 GeV/c we find that $t(n)$, the momentum transfer to the neutron, is more peaked than phase space and is well fitted by our model as seen in Fig. 3(e), while $t(\pi^-)$ follows phase space, the model giving too much peaking as observed in Fig. 3(d). With the latter as the only significant exception, the t distributions of all six reactions at both energies are well described.

The point to be emphasized is that we do not fit the individual histograms shown but rather the multidimensional phase space density for the reaction as a whole. Correlations are considered which cannot be simply illustrated but which do provide important constraints on the result of a fit. Thus a particular mechanism, say Δ^{++} production by diffraction dissociation of the nucleon in reaction (1), will produce a correlation between $m(\pi^+p)$ and $t(\pi^-)$ which will be different from that for production by pion exchange or some other mechanism. The program is not free to get the best possible fit to the $m(\pi^+p)$ distribution, or any other particular histogram, but must fit the reaction as a whole, with all correlations. This, together with the relatively small number of adjustable parameters, leads us to have some confidence that the basic processes considered are present in the data in about the proportions listed in Tables I–IV.

III. COMPARISON OF ENERGIES

We summarize in Table I the partial cross sections for the primary production mechanisms

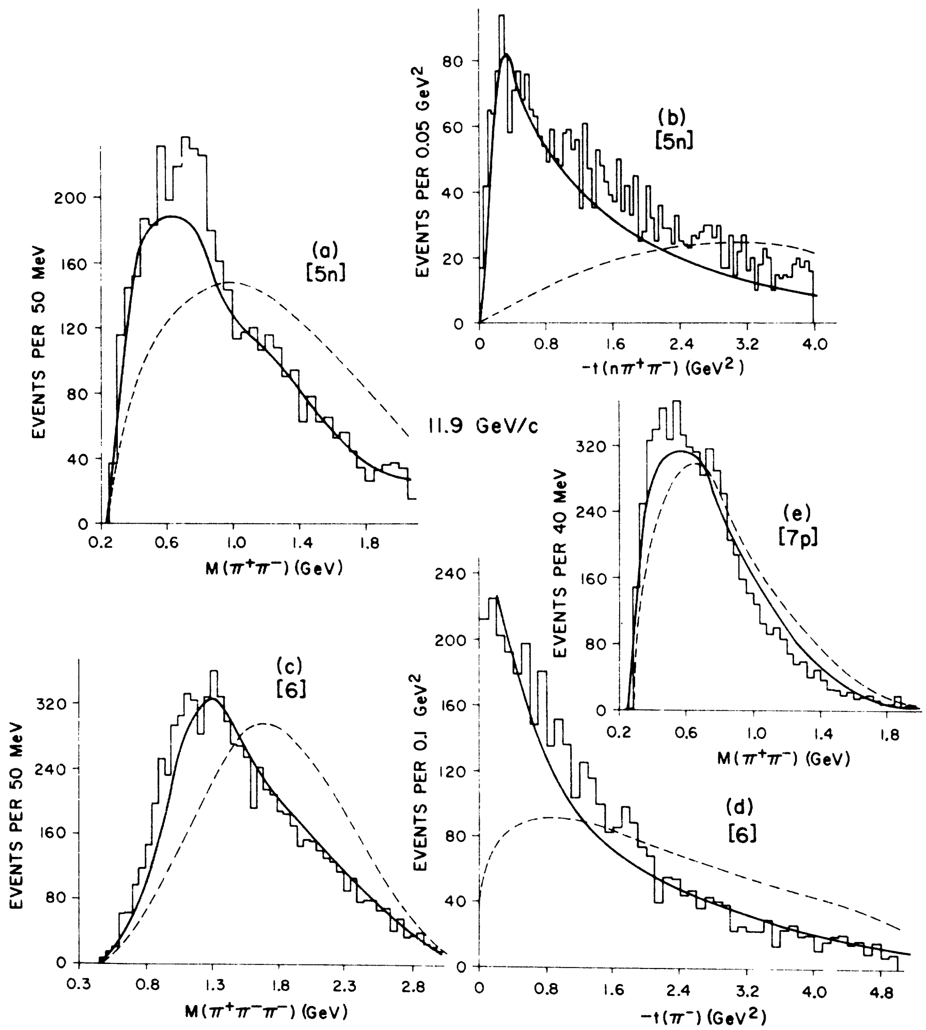

FIG. 5. Selected plots showing fits to 5-, 6-, and 7-body reactions at 11.9 GeV/c.

which we have determined for the two energies. As discussed in Sec. II, Pomeranchukon exchange is taken to be synonymous with OPD–diffraction dissociation with an odd number of pions at the pion vertex. EPD, even-pion diffraction, we take to be part of the background. Now we would like to compare the two energies to see if there is basic agreement.

In general, it is expected that two-body pion exchange cross sections will have an energy dependence like p_{lab}^{-2} while diffraction dissociation will be essentially constant. It has been shown that for a wide range of multibody reactions good agreement is obtained if one takes

$$\sigma \propto (\text{phase space volume}) p_{\text{lab}}^{2\alpha - n} , \qquad (3)$$

where $\alpha = 0$ for pion exchange and 0.9 for Pomeranchukon exchange (the Regge intercepts), and n is the number of particles in the final state.[10]

Taking our experimentally determined partial cross sections at 11.9 GeV/c we have used (3) to predict the corresponding partial cross sections at 3.9 GeV/c. The predictions are given in Fig. 6, where they are compared with the experimental values. We see that our results are basically consistent with the proposed energy dependence. The only significant exception is the 5-body-neutron final state, where essentially no diffraction dissociation was found at 3.9 GeV/c, while an appreciable amount appears at 11.9 GeV/c. There is some tendency of the other points to cluster above the line which could indicate an overall systematic error of about 20 μb in the total cross section at either momentum.

We feel that our results, then, are essentially consistent with what is expected for pion exchange and diffraction dissociation.

IV. COMPARISON OF REACTIONS

We have made an attempt to test further whether the partial cross sections we have obtained make sense by comparing reactions of different multiplicities at a given energy. To do this we utilize the concept of factorization[11] which might be expected to hold, at least crudely, for quasi-two-body reactions dominated by Pomeranchukon exchange.

We consider three classes of reactions as illustrated in Fig. 7. In each case we see that the numerator and denominator diagrams have a common vertex process whose amplitude cancels out if one assumes a simplified factorization. This predicts a correspondence between two sets of cross sections, $\{\sigma_N\}$ and $\{\sigma_D\}$, for the reactions in numerator and denominator which will hold independent of the nature of the common vertex process.

We have examined our results which allow us to

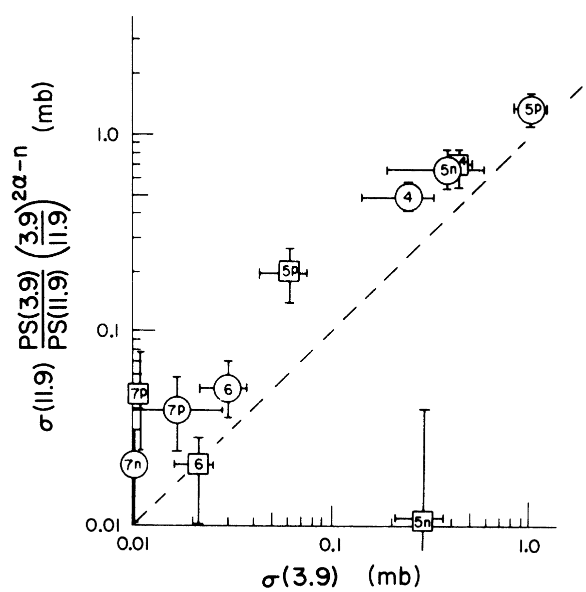

FIG. 6. Plot illustrating the energy correlation of the mechanisms studied. The measured partial cross sections at 3.9 GeV/c are compared with the "predicted" cross sections at that energy using the measured cross sections at 11.9 GeV/c.

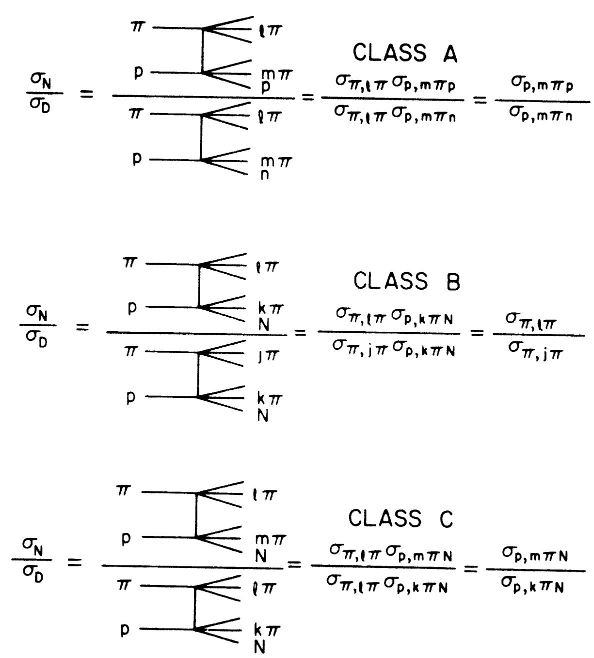

FIG. 7. Predictions of factorization for the relationships between partial cross sections for three classes of reactions.

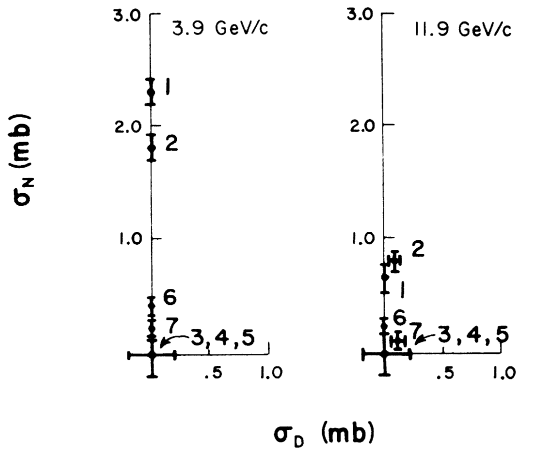

FIG. 8. Pomeranchukon-exchange partial cross sections σ_N plotted vs σ_D for reactions of class B with $l=1$, $j=3$. The specific reactions are of the type $\pi^- p \to \pi^- B$ compared to $\pi^- p \to (3\pi)^- B$, where B is the baryon system resulting from the dissociated proton. The numbers near the data points specify B as follows: (1) $\pi^0 p$, (2) $\pi^+ n$, (3) $\pi^+\pi^- p$, (4) $\pi^-\pi^-\pi^+ n$, (5) $\pi^+\pi^-\pi^0 p$, (6) $\pi^-\Delta^{++}$, (7) $\pi^-\pi^0\Delta^{++}$.

test these correspondences in a large number of cases. We find that there are no clear violations of the predictions of factorization. Perhaps the best illustration of this is provided in the case of class B reactions when, in the notation of Fig. 7, $l=1$ and $j=3$. Factorization predicts

$$\frac{\sigma_N}{\sigma_D}=\frac{\sigma_{\pi,l\pi}}{\sigma_{\pi,j\pi}}=\frac{\sigma_{\pi,\pi}}{\sigma_{\pi,3\pi}}\,. \tag{4}$$

In Fig. 8 we have plotted σ_N vs σ_D for 7 reactions of this class at each momentum. The particular reactions are given in the figure caption. We see that, at both 3.9 and 11.9 GeV/c, the points lie along a vertical line. That is, within our sensitivity, $\sigma_D=0$ is consistent with the coupling $\sigma_{\pi,\pi} \gg \sigma_{\pi,3\pi}$.

Unfortunately, many of the reactions which one would like to test have cross sections below 20 μb, which is about the sensitivity of our method with the current data. The general observation is that "double dissociation," that is diffraction dissociation of both incident particles, is considerably weaker than the dissociation of the pion or proton singly. That we find no exception to this rule is at least consistent with factorization.

One interesting and possibly significant observation made in this experiment and others is that the cross section for $\pi^- p \to (l\pi)^-(\pi^0 p)$ is greater than that for $\pi^- p \to (l\pi)^-(\pi^+ n)$ for all $l \geq 1$. If the interaction is dominated by $I=0$ exchange, one would expect the second to be twice the first since the $(\pi^0 p)$ and $(\pi^+ n)$ systems would be pure $I=\frac{1}{2}$. The observation then implies that Pomeranchukon exchange with the proton dissociating into πN is not the dominant process even at 11.9 GeV/c, where pion exchange is weak. The fact that this happens for $l=1$ and the higher multiplicities studied here is consistent again with factorization since, if it is suppressed for one, it should be suppressed for all.

V. SUMMARY AND CONCLUSIONS

We have found that the main kinematic features for multipion reactions at 3.9 and 11.9 GeV/c can be described by a model which incorporates the basic structure of pion exchange and diffraction dissociation as quasi-two-body processes. The fits to the model have been used as a means for estimating the partial cross sections for the interaction mechanisms and the dominant resonances which are present at these energies. The cross sections which are obtained appear to be consistent with the energy dependence expected for the two mechanisms and with a simplified factorization.

Pion exchange, as manifested by extremely sharp t distributions, is found to be important only at the lower energy. Diffraction dissociation, as manifested by moderately sharp t distributions analogous to that for elastic scattering and a peaking at low invariant mass of the dissociated fragments, appears to dominate most of the reactions studied at the higher energy.

Pomeranchukon exchange does not seem to be the whole story at high energy, however. Some process is evident in the data which produces moderately peaked t distributions in a diffraction-like manner, yet cannot be described by quasi-two-body Pomeranchukon exchange diagrams. We have not explored all possible explanations, such as multiperipheralism, but have simply included in our model an additional non-Pomeranchukon-exchange type of diffraction dissociation.

Undoubtedly many other processes, expecially heavy-meson exchanges, are present in the reactions studied. Since these generally do not produce the t distributions much different from phase space, we have taken these to be undetectable, except for some A_1 and A_2 production. Rather than attempt to fit a given reaction at a given energy in detail, we have looked for common features among reactions at two widely spaced energies and studied only the broadest characteristics. We have seen that these are consistent, for the most part, with what is expected for pion exchange and diffraction dissociation. The cross sections obtained for these mechanisms and the main resonances they produce in multipion events at these energies are presumably, then, not unrealistic. At the same time, we have shown that the main features of the reactions studied can be described by extremely simple ideas, and only with experiments with considerably greater statistics could one hope to disentangle the details of the dynamics involved.

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Reaction $pn \to \Delta^{++}(1238)\Delta^{-}(1238)$ at 5.9 GeV/c*

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We present a study of the reaction $pn \to p\pi^+\pi^-n$. The cross section is found to be 4.75 ± 0.25 mb at 5.9 GeV/c. Decay spherical-harmonic moments are presented for peripheral $|T_z| = \frac{3}{2}\,\pi N$ systems as a function of πN-invariant mass. The cross section for the quasi-two-body process $pn \to \Delta^{++}\Delta^-$ is found to be 1.90 ± 0.14 mb; the differential cross sections and isobar decays are also studied. The data are not consistent with the predictions of simple one-pion exchange.

I. INTRODUCTION

In this work we present a study of the reaction

$$pn \to \Delta^{++}(1238)\Delta^{-}(1238) \qquad (1.1)$$

at an incident proton momentum of 5.9 GeV/c. Previous analyses of reaction (1.1) have been presented[1,2] at 3.7 and 7.0 GeV/c. Double-Δ(1238) production data have been compared with the one-pion-exchange model and the Białas-Zalewski[3] quark-model predictions in studies of reaction (1.1) as well as in other reactions. In this analysis we simply present the data and compare with previous experimental results.

The reaction (1.1) data are studied in detail in the peripheral region. The possible exotic exchange process with the Δ^-(1238) emitted in the forward c.m. hemisphere is not considered here.[4] We present total and differential cross sections for reaction (1.1); in addition, we calculate the joint $\Delta\Delta$ decay density-matrix elements, in order to help expedite any future theoretical studies of decay correlations, etc.

In Sec. II we discuss the experimental separation of the reaction (1.1) data from accepted kinematic fits of the reaction