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ρ^0-ω Interference in Antiproton Annihilations at 0.65–1.1 GeV/c*

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Evidence for ρ^0-ω interference observed in the $\pi^+\pi^-$ mass spectrum from the annihilation $\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-$ at incident momenta between 0.65 and 1.1 GeV/c is presented. The best-fit values for the branching ratio $R(\omega \rightarrow \pi^+\pi^-/\omega \rightarrow \pi^+\pi^-\pi^0)$, phase φ, and coherence factor α are 1.3%, 100°, and 0.95.

INTRODUCTION

Recently the G-parity-violating 2π decay mode of the ω meson has been observed in studie of the $\pi^+\pi^-$ mass spectrum in the ρ region where interference effects between the ρ and ω amplitudes may occur. The experiments have utilized a variety of production techniques.[1] The experiment reported here is complementary to the $\bar{p}p$ experiments of Allison *et al.*[2] and Chapman *et al.*[3] (hereafter referred to as ARG and UM, respectively), which used incident beam momenta in the regions 1.26–1.65 GeV/c and 1.63–2.20 GeV/c with 1448 and 3135 events, respectively, in the channel

$$\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^- . \tag{1}$$

Our experiment covers the beam momentum region between 0.65 and 1.1 GeV/c and has 6093 events in the $\pi^+\pi^+\pi^-\pi^-$ final state. For purposes of comparison our analysis of the ρ-ω interference effect closely follows that of the above authors.

EXPERIMENTAL PROCEDURE

The exposure of 220 000 pictures was taken in the BNL 30-in. hydrogen bubble chamber at six evenly spaced energies spanning the above momentum region. The events were measured on image-plane digitizers and analyzed with the TVGP-SQUAW reconstruction and fitting programs. This work uses the 4C (4-constraint) kinematical fits to the hypothesis of reaction (1) to investigate the ρ-ω interference in the $\pi^+\pi^-$ mass spectrum, and uses the 1C kinematical fits to the hypothesis

$$\bar{p}p \rightarrow \pi^+\pi^+\pi^-\pi^-\pi^0 \tag{2}$$

to determine the number of $\omega \rightarrow \pi^+\pi^-\pi^0$ decays, $N_{\omega \rightarrow 3\pi}$.

Due to the narrow width of the ω, the experimental mass resolution is an important consideration in the study of the $\pi^+\pi^-$ mass histogram for reaction (1). We studied the $K^0 \rightarrow \pi^+\pi^-$ decays and established that the $\pi^+\pi^-$ mass resolution at the kaon mass has a full width at half-maximum of approximately 8 MeV, establishing that 10-MeV binning of the $\pi^+\pi^-$ mass histogram is reasonable. A resolution function was Gaussian-folded into our subsequent analysis. To determine the accuracy of our invariant-mass measurements, we measured the kaon mass, as well as the mass of the ω meson determined from ω's produced in reaction (2), for each of the six energies separately, to be within one MeV of the table values.[4]

The 4π final state was cut so that the SQUAW-determined χ^2 was <8, while the 5π final state was cut at $\chi^2 < 1.5$. The 4C χ^2 was scaled by a factor of about 1.3 in order to agree with the expected χ^2 distribution. The 1C χ^2 required no scaling.

Figure 1 shows the $\pi^+\pi^-$ mass histogram in the ρ-meson region for all of our data. Each event may contribute four mass combinations to the histogram.

FOUR-PION FINAL STATE EXCLUDING INTERFERENCE

As reported previously,[5] a detailed fit has been made to the four-pion final state of reaction (1), without including the ρ-ω interference effect, by using a properly symmetrized sum of incoherent amplitudes for resonance production (parametrized by Breit-Wigner amplitudes) and phase space. The average resonance production distribution in our energy region is approximately 10% $\rho\pi\pi$, 10% $f\pi\pi$, 10% $\rho\rho$, 36% ρf, and 8% $A_2\pi$. These percentages were determined by using both the maximum-likelihood method and χ^2 fits of Monte Carlo-generated events to various mass histograms (50-MeV bins) of the data. These fits required $m_\rho \approx 745$ MeV. A new χ^2 fit [fit (a) of Table I to be discussed in detail below] to the combined $\pi^+\pi^-$ mass histogram in 10-MeV bins yields a χ^2 of 77

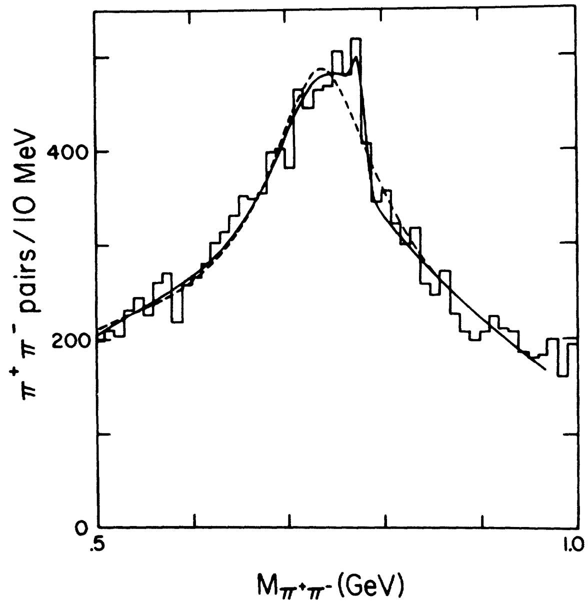

FIG. 1. The $\pi^+\pi^-$ mass spectrum for all of our $\pi^+\pi^+\pi^-\pi^-$ events. The dashed curve is the no-interference fit to Eq. (4) ($a_\omega=0$) and the solid curve corresponds to Fit (b) discussed in the text.

for 47 bins and $m_\rho = 749$ MeV. The ρ width was fixed at $\Gamma_\rho = 145$ MeV.

FIVE-PION FINAL STATE

The $\bar{p}p \to \pi^+\pi^+\pi^-\pi^-\pi^0$ final state also contains abundant resonance production. A detailed fitting procedure similar to that used for the 4π events is in progress. For the present discussion we need only determine $N_{\omega\to 3\pi}$. Then, given the number of $\omega \to \pi^+\pi^-$ decays, $N_{\omega\to 2\pi}$, as determined from the interference effect, the decay branching ratio

$$R = N_{\omega\to 2\pi}/N_{\omega\to 3\pi} \tag{3}$$

may be calculated. We have determined the number of $\bar{p}p \to \omega + (\text{anything})$ in reaction (2) by a χ^2 fit to the data near m_ω using a polynomial background with a Breit-Wigner ω amplitude. A Gaussian resolution function with the width left as a free parameter was folded into the expected distributions. The data at each energy were fitted separately and, after correcting for the χ^2 cut on the fitted events, we found a total of 5532 ω events. These include the $\omega\pi\pi$, $\omega\rho$, and ωf channels. Using our earlier results,[5] we find that the ratio of ρ to ω production with two charged pions is 0.8.

FOUR-PION FINAL STATE WITH INTERFERENCE EFFECTS

A number of different methods have been used to parametrize the ρ-ω mixing.[1] The fits to the 4π final state show that $\rho\pi\pi$, $\rho\rho$, and ρf channels are available to interfere with the $\omega\pi\pi$, $\omega\rho$, and ωf channels, respectively, of the 5π final state. One could parametrize the $\pi^+\pi^-$ mass spectrum of (1) by assuming that the individual channels are incoherent and allow for interference between properly symmetrized amplitudes for the ρ and ω in each channel. However, since the interference manifests itself only over a very narrow region (i.e., just a few bins) there is insufficient information to determine amplitudes and phases for the individual channels. Further, the data for each energy are statistically poor but are qualitatively very similar. Meaningful fits are obtained only by using the total data sample, represented by combining the six histograms, as shown in Fig. 1.

Both ARG and UM have fitted their $\pi^+\pi^-$ mass distribution to the form

$$\frac{dN}{dm} = P[a_{PS}^2 + a_\rho^2|B_\rho|^2 + a_\omega^2|B_\omega|^2 + 2\alpha a_\omega a_\rho \mathrm{Re}(e^{i\varphi}B_\omega B_\rho^*)] \,, \tag{4}$$

where

TABLE I. Parameters from production fits.

Fit	Description	χ^2	R	φ
(a)	Equation (4), no interference	77.2	0	...
(b)	Equation (4)	52.8	1.3%	1.71
(a′)	Equation (7), no interference	65.8	0	...
(c)	Equation (7) with ρ-background interference, no ω production	62.2	0	...
(d)	Equation (7) with ρ-ω interference	51.4	>1.15%	...
(e)	Equation (7) with ρ-ω interference and ρ-background interference	51.3	>0.99%	...

$$B_\rho = \frac{\Gamma(m)^{1/2}}{(m_\rho{}^2 - m^2) - i m_\rho \Gamma(m)} ,$$

$$\Gamma(m) = \Gamma_\rho \frac{m_\rho}{m}\left(\frac{q}{q_\rho}\right)^3 ,$$

$$B_\omega = \frac{\Gamma_\omega{}^{1/2}}{(m_\omega{}^2 - m^2) - i m_\omega \Gamma_\omega} .$$

In this expression P is the two-out-of-four pion background which we have approximated in this region by a polynomial

$$P = A_1 + A_2(m - 0.75) + A_3(m - 0.75)^2 . \qquad (5)$$

The momentum of a pion from the ρ decay in the rest frame of the ρ is denoted by q. The coherence factor α $(0 \leqslant \alpha \leqslant 1)$ arises because there may be several channels contributing to the effect and the corresponding amplitudes may not be coherent. The parameters a_ρ, a_ω, and φ are real, and can be related to the production amplitudes for ρ and ω for all channels. More general expressions are given by UM.

In terms of the basic parameters of ρ-ω mixing phenomenology,[1] we have $\varphi = \overline{\varphi}_P + \varphi_\epsilon$, where $\overline{\varphi}_P$ is the averaged relative production phase and φ_ϵ is the phase of

$$\epsilon = \frac{\delta}{(m_\rho - m_\omega) - \frac{1}{2} i(\Gamma_\rho - \Gamma_\omega)} .$$

Here $-\delta$ is the off-diagonal element of the ρ-ω mass matrix. If δ is real and positive then $\varphi_\epsilon \approx 100°$ (using the table values[4] for ρ and ω parameters).

Even with the combined sample of events the resolution and statistics only permit a poor independent determination of α, a_ω, and Φ. The condition $\alpha = 1.0$ leads to a lower limit on a_ω.

For this parametrization the number of 2π decays of the ω was calculated from

$$N_{\omega\to 2\pi} = \int P a_\omega^2 |B_\omega|^2 dm . \qquad (6)$$

We found, as was the case with ARG and UM, that fits to Eq. (4) with m_ρ and Γ_ρ as free parameters gave ρ widths on the order of 180 MeV. The interference parameters are not very sensitive to the value of the width and therefore we set $\Gamma_\rho = 145$, which is one standard deviation from the

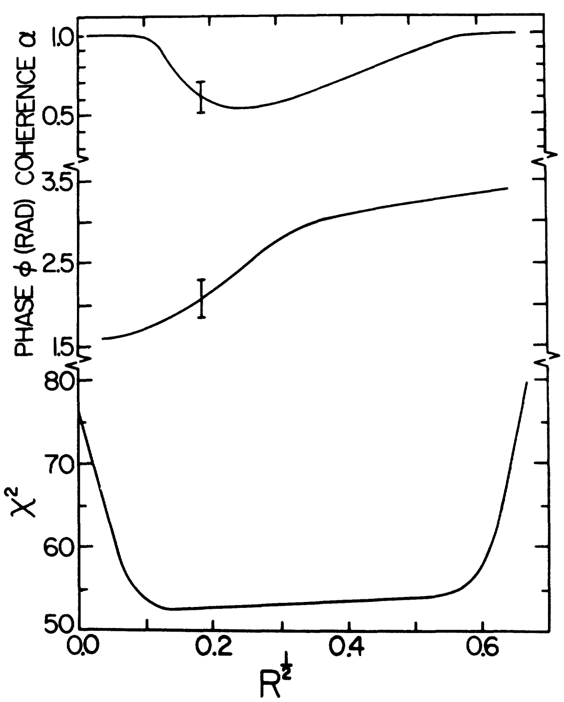

FIG. 2. The values of χ^2, phase φ, and coherence α as a function of $R^{1/2}(\omega\to\pi^+\pi^-/\omega\to\pi^+\pi^-\pi^0)$ for fits using Eq. (4). The error bars indicate typical uncertainties for φ and α.

table value.[4] The fit results [fit (b) of Table I] gave $m_\rho = 749$ MeV, $\varphi = 1.74$ rad ($\varphi = 100^0$), $\alpha = 0.95$. For this fit, $\chi^2 = 53$ for 47 bins and $N_{\omega\to 2\pi} = 72.1$ (corrected for χ^2 cuts) which gives $R = 1.3\%$. The fit results for no interference, $a_\omega = 0$ [fit (a) of Table I], had a $\chi^2 = 77.2$. These results are displayed with the data in Fig. 1. In Fig. 2, χ^2, α, and φ are shown as a function of $R^{1/2}$. Here the background parameters and resonance parameters were held fixed at the values determined for fit (b).

It is clear from Fig. 2 that α, φ, and R are strongly correlated in the fit. Because of this we feel it is misleading to quote errors in the standard way. In Table II, we give the error estimates which result from two choices for the limits of acceptable deviation from the best fit. We display the corresponding results for ARG and UM together with results from our own experiment.

TABLE II. Best fits of three experiments.

Cut	Irvine	UM	ARG
$\Delta\chi^2 < 2.7$ $1.3\% \leq R \leq 6\%$	$\alpha > 0.55$ $0.57\% \leq R \leq 35\%$ $95° \leq \varphi \leq 125°$	$\alpha > 0.4$ $0.75\% \leq R \leq 40\%$ $95° \leq \varphi \leq 125°$	$\alpha > 0.4$ $1.4\% \leq R \leq 42\%$ $60° \leq \varphi \leq 90°$

We do not present results for combined data because the combined result depends sensitively on instrumental effects (such as slightly differing mass scales among laboratories) which we have not been able to evaluate adequately.

Following Flatté,[6] Abramovich *et al.*,[7] and UM we have tried another form which allows for coherence between the ρ and ω amplitude and also a P-wave component of the nonresonant background. The expression fitted was

$$\frac{dN}{dm} = P(a_1 + a_2 |B_\rho|^2 + a_3 |B_\omega|^2 + a_4 \,\mathrm{Re} B_\rho + a_5 \,\mathrm{Re} B_\omega) \ , \qquad (7)$$

where the approximations have been made that B_ρ is constant near m_ω over a few ω widths, and the imaginary part of B is a constant times $|B|^2$. The numerators of the Breit-Wigner formulas have been replaced by $m_x \Gamma_x$. With $a_4 = 0$, Eq. (4) and Eq. (7) are equivalent within these approximations. Fit (a′) with $a_3 = a_4 = a_5 = 0$ is equivalent to Eq. (4) with $a_\omega = 0$ except for the modifications to the Breit-Wigner form. Three additional fits have been tried using Eq. (7). Fit (c) used $a_3 = a_5 = 0$, which allows for ρ with background interference but no ω production. Fit (d) had $a_4 = 0$ which contains ω interference with the background (which contains the ρ) but no ρ-background interference, while fit (e) has all a_i nonzero. In these fits we set $m_\rho = 749$ and $\Gamma_\rho = 145$ MeV. If m_ρ is allowed to vary it becomes unreasonably large (~790 MeV), as also noted by UM.

The branching ratio for these fits is determined by calculating

$$N_{\omega\to 2\pi} \geq A \int |B_\omega|^2 dm \ ,$$

where

$$A = b - (b^2 - a_3^{\,2} - a_5^{\,2})^{1/2} \ ,$$

$$b = 2a_1 + 2a_2 |B_\rho|^2_{m=m_\omega} + a_3 + 2a_4 (\mathrm{Re} B_\rho)_{m=m_\omega} \ .$$

Referring to Table I, we note that the fits show a large decrease in χ^2 with the inclusion of ρ-ω interference: between fits (a) (no interference) and (b) $\Delta\chi^2 = 24.4$, while between fits (a′) and (e) $\Delta\chi^2 = 14.5$. The inclusion of ρ interfering with background gave insignificant changes in χ^2 from the corresponding fits without such interference [i.e., the changes in χ^2 comparing (c) with (a′), and (e) with (d) are small]. UM found that the inclusion of ρ-background interference weakened the evidence for $\rho\omega$ interference in both their own data and in the data of ARG. This is decidedly not true for our data.

Changes of the resolution function by ±2 MeV did not alter the fit results. The results are also insensitive to variations of the ρ parameters. Momentum-transfer cuts or channel cuts tend only to reduce the sensitivity of the data. In particular, selection of f production channels, ρf in the 4π and ωf in the 5π events, as done by UM, reduce our sample significantly, since we are near threshold for ρ-f and ω-f production. If this is the predominant channel, as suggested by UM, then the $1000\omega f$ events in our sample lead to $R \simeq 7\%$. The comparable number is not given by UM.

CONCLUSION

This experiment strengthens the evidence for ρ-ω interference in $\bar{p}p \to \pi^+\pi^+\pi^-\pi^-$ annihilations; the results are quite consistent among the several $\bar{p}p$ experiments spanning the range from 0.65 to 2.2 GeV/c. The high degree of ρ-ω production coherence observed may be a useful clue in constructing models for $\bar{p}p$ annihilation. Further, since $\alpha \simeq 1$, φ is related directly to the relative production phase φ_P of ρ and ω, i.e., $\varphi = \varphi_P + \varphi_\epsilon$. In the model of Goldhaber, Fox, and Quigg[8] where the $\omega \to 2\pi$ decay proceeds through a virtual ρ with a real off diagonal ρ-ω mass-matrix element, $\delta = -\langle \rho^0 | M | \omega \rangle$, the decay phase is known and is given by the ρ amplitude at the ω mass, $\varphi_\epsilon \approx 106°$. This gives a production phase between the ρ and ω amplitude of approximately zero. Furthermore an SU(3) estimate[9] of δ is of the order of 2.5 MeV. The first-order relation between R and δ is[1]

$$R \simeq \frac{4|\delta|^2}{\Gamma_\rho \Gamma_\omega} \ ,$$

so that our best value of $R = 1.3\%$ yields $\delta \approx 2.4$ MeV.

Since our data and the published $\bar{p}p$ data of ARG and UM are quite similar, we have taken the liberty of combining all data and find best values of $R = 2\%$, $\varphi = 1.8$ rad, $m_\rho = 750$ MeV, and $\alpha = 1.0$ with Γ_ρ fixed at 145 MeV. The χ^2 for this fit is 49.4 for 37 bins which is an improvement of $\Delta\chi^2 = 59$ over the no interference fit for the same combined data. The large decrease in χ^2 indicates the necessity of including the interference term; however, the actual values of the parameters determined in this fit must be taken with caution, because of the pitfalls inherent in combining data from different experiments.

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Characteristics of Multipion Production at 3.9 and 11.9 GeV/c*

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A quasi-two-body model based on one-particle exchange and diffraction dissociation has been fitted to data from $\pi^- p$ interactions at 3.9 and 11.9 GeV/c in which a nucleon and 3–6 pions are present in the final state. It is used to estimate partial cross sections for the contributing interaction mechanisms and the dominant resonances which are produced at these energies. The energy dependence of the cross sections is examined and found to be consistent with expected behavior, and reactions are compared and found to agree with simple factorization.

I. INTRODUCTION

In an earlier paper[1] it was shown that the main kinematic features of the reaction $\pi^- p \to \pi^-\pi^-\pi^+ p$ at 3.9 GeV/c could be described by a simple model based on pion exchange, ρ exchange, and diffraction dissociation. It was demonstrated that the kinematic reflections of the various competing processes produced phenomena such as anisotropic Treiman-Yang and scattering angular distributions which might otherwise be interpreted as deeper dynamical effects. It was also shown that diffraction dissociation, without multiperipheralism, was adequate to describe the data in the $A_1(1070)$ and $N(1470)$ regions without the need to introduce these as resonant states.

This paper represents a considerable extension of this work to include 5-, 6-, and 7-body final states and a higher momentun, 11.9 GeV/c.

A model with a small number of adjustable parameters was developed to describe the following reactions at both momenta:

$$\pi^- p \to \pi^-\pi^-\pi^+ p \qquad [4],$$
$$\to \pi^-\pi^-\pi^+\pi^0 p \qquad [5p],$$
$$\to \pi^-\pi^-\pi^+\pi^+ n \qquad [5n],$$
$$\to \pi^-\pi^-\pi^-\pi^+\pi^+ p \qquad [6],$$
$$\to \pi^-\pi^-\pi^-\pi^+\pi^+\pi^0 p \qquad [7p],$$
$$\to \pi^-\pi^-\pi^-\pi^+\pi^+\pi^+ n \qquad [7n].$$

The model is based on the dynamical ideas of single-particle exchange and diffraction dissociation. It incorporates only the main features expected from these mechanisms and is basically a quasi-two-body rather than multiperipheral model.

The experimental data used in this study are, for the most part, being published for the first time. The data for reactions [4] and [6] at 3.9 GeV/c have already appeared[1,2]; the remainder are new results. The 3.9-GeV/c data presented represent our complete exposure at that momentum of the 72-in. hydrogen bubble chamber at the (then) Lawrence Radiation Laboratory, Berkeley. The 11.9-GeV/c data are from the same chamber (enlarged to 82 in.) exposed at SLAC, and form only a small subsample from our total of 650 000 pictures.[3]

Events were measured using scanning and measuring projectors (SMP's) and data reduced either by PACKAGE or by TVGP and SQUAW. The number of events for each reaction at each energy and cross sections is given in Table I.

The six reactions were fitted independently at each energy. The results of the fits were used to estimate partial cross sections for the contributing interaction mechanisms and the dominant res-