†Research supported in part by the U. S. Atomic Energy Commission.

*Before 1970 at the University of Pennsylvania, where the data reduction was performed.

‡John S. Guggenheim Fellow, 1972.

[1] J. P. Baton *et al.*, Phys. Letters 25B, 419 (1967).

[2] S. Marateck *et al.*, Phys. Rev. Letters 21, 1613 (1968).

[3] L. Chan *et al.*, Phys. Rev. D 2, 583 (1970).

[4] B. Y. Oh *et al.*, Phys. Rev. D 1, 2494 (1970).

[5] G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[6] J. P. Baton *et al.*, Phys. Letters 33B, 525 (1970).

[7] F. Bulos *et al.*, Phys. Rev. Letters 26, 1453 (1971); P. Baillon *et al.*, Phys. Letters 38B, 555 (1972).

[8] S. Hagopian *et al.*, Phys. Rev. Letters 24, 1445 (1970); 25, 1050 (1970); Phys. Rev. D 5, 2684 (1972).

[9] J. Zinn-Justin, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1970), Vol. 55, p. 43.

[10] C. D. Froggatt and D. Morgan, Phys. Rev. 187, 2044 (1969).

[11] N. D. Pewitt, Master's thesis, Florida State University, 1972 (unpublished).

[12] K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).

[13] L. Durand III and Y. T. Chiu, Phys. Rev. 137, B1530 (1965).

[14] P. Baillon *et al.*, Phys. Letters 35B, 453 (1971).

[15] P. K. Williams, Phys. Rev. D 1, 1312 (1970).

[16] J. D. Kimel and L. M. Nath, Nucl. Phys. B29, 616 (1971).

[17] M. Ross *et al.*, Nucl. Phys. B23, 269 (1970).

[18] V. Hagopian *et al.*, in *Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969*, edited by F. Loeffler and E. D. Malamud (Argonne National Laboratory, Argonne, Ill., 1969), p. 149.

[19] G. L. Kane and M. Ross, Phys. Rev. 177, 2353 (1969).

[20] The calculation of $F(s,0,\omega^2)$ is dominated in the mass range considered by the p-wave phase shifts and is very insensitive to the s-wave phase shifts used. Changing the s-wave phase shifts from "down-down" to "down-up" alters $F(s,0,\omega^2)$ by less than 5% over the mass range of the ρ.

[21] G. Grayer *et al.*, in *Experimental Meson Spectroscopy-1972*, proceedings of the Third International Conference, Philadelphia, 1972, edited by Kwan-Wu Lai and A. H. Rosenfeld (A.I.P., New York, 1972), p. 5.

PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

Search for Q Production in Charge-Exchange Reactions*

B. Werner,† P. Slattery, and T. Ferbel‡
University of Rochester, Rochester, New York 14627

(Received 24 August 1972)

The low-mass $K\pi\pi$ enhancement (the Q effect) has been observed in $K^{\pm}N$ interactions only in the same charge state as the incident beam particle. The lack of observation of the Q in reactions involving nucleon charge exchange has been cited as evidence for the diffractive nature of Q production. We have searched for Q^0 and Q^{++} production in $K^{\pm}$ induced reactions by combining relevant world data. We have also carried out a double-Regge exchange calculation in order to estimate the magnitude of the expected signal for Q production in charge-exchange processes.

The low-mass $K\pi\pi$ (Q) enhancement[1] has been studied extensively in $K^{\pm}N$ interactions by means of bubble chambers exposed to beams of high-momentum kaons. The puzzling situation now exists in which the spin-parity of the entire Q region is measured to be predominantly 1^+, but there remains considerable controversy as to whether this enhancement is due to one, two, or more resonances, or whether the entire phenomenon can be understood as arising from threshold kinematic effects.

The Q enhancement has thus far only been observed to occur in $K\pi\pi$ systems which are produced in association with the target nucleon. No Q enhancement has ever been observed in a $K\pi\pi N$ final state in which the nucleon undergoes charge exchange. This, along with the observation of a slow variation of the production cross section as a function of incident beam momentum, a steep momentum-transfer dependence, and an alignment of the polarization vector for the $K\pi\pi$ system perpendicular to the incident beam direction, is consistent with a diffractive production mechanism.

Production mechanisms other than the exchange of vacuum quantum numbers appear to contribute to Q production.[2] Consequently, the Q should also be observed, although at a reduced level, in reactions involving charge exchange to the nucleon (hereafter referred to as $Q_{\rm CE}$ production). An individual experiment (such as our 10-event/μb 12.7-GeV/c K^+p bubble-chamber exposure) would very likely be insensitive to the expected level of

TABLE I. $K\pi\pi$ charge-exchange compilation.

Contributor	Reaction	Momentum (GeV/c)	Number of events	Source
UC-LBL (Group B)	$K^+p \to K^0\pi^+\pi^+n$	4.6	170	World K^+ data tape
CERN Brussels	$K^+p \to K^0\pi^+\pi^+n$	5.0	662	World K^+ data tape
Johns Hopkins	$K^+p \to K^0\pi^+\pi^+n$	5.5	206	World K^+ data tape
UCLA	$K^+p \to K^0\pi^+\pi^+n$	7.3	198	Phys. Letters 28B, 143 (1968)
CERN Brussels	$K^+p \to K^0\pi^+\pi^+n$	8.25	589	World K^+ data tape
UC-LBL (Group B)	$K^+p \to K^0\pi^+\pi^+n$	9.0	445	World K^+ data tape
Birmingham-Glasgow-Oxford	$K^+p \to K^0\pi^+\pi^+n$	10.0	976	Private communication
UC-LBL (Group A)	$K^+p \to K^0\pi^+\pi^+n$	12.0	1235	Private communication
Rochester	$K^+p \to K^0\pi^+\pi^+n$	12.7	504	This experiment
Purdue	$K^-d \to \bar{K}^0\pi^-\pi^-pp$	4.5	642	Private communication
Vanderbilt	$K^-d \to \bar{K}^0\pi^-\pi^-pp$	5.0	375	Private communication
BNL	$K^-p \to K^0\pi^-\pi^+n$	3.9	2039	Private communication
		4.6	2341	
ANL-NU	$K^-p \to K^0\pi^-\pi^+n$	5.5	2055	Phys. Rev. 166, 1317 (1968)
Oxford-Birmingham-Glasgow-London (I.C.)-Munich-Rutherford	$K^-p \to K^0\pi^-\pi^+n$	6.0	688	Private communication
CERN Aachen-Berlin-London (I.C.)-Vienna	$K^-p \to K^0\pi^-\pi^+n$	10.0	492	Private communication
Yale	$K^-p \to K^0\pi^-\pi^+n$	12.6	249	Private communication
Purdue	$K^+d \to K^0\pi^+\pi^-pp$	9	159	Private communication
UC-LBL (Group B)	$K^+d \to K^0\pi^+\pi^-pp$	12	894	Private communication
			14 919 total events	

$Q_{\rm CE}$ production (see below). We have therefore obtained a >100-event/μb data sample by combining all currently available world data from appropriate reactions. The sources for the compiled data sample are listed in Table I. The $K^*(890)$-π mass spectrum for this combined data sample is shown in Fig. 1. No $Q_{\rm CE}$ signal is apparent. The solid curves represent the results of a maximum likelihood fit of the data to a third-order polynomial describing the background plus the $K^*(1420)$ at a mass of 1420 MeV and with a width of 95 MeV; this fit describes the data quite well. A fit to the data using background, $K^*(1420)$, and Q (approximated by a Breit-Wigner term centered at 1280 MeV with a width of 220 MeV) requires ~140 "Q" events, or less than $\frac{1}{3}$ of the required $K^*(1420)$ signal (~460 events). The fitted Q events are indicated by a dashed line in Fig. 1. The confidence level for the fit including the Q was not significantly higher than that for the fit using only background and $K^*(1420)$.

The data shown in Fig. 1 are from a wide range of incident beam momenta and from different $K\pi\pi$ charge states; the mass spectrum presented, therefore, consists of a sum of mass spectra which differ considerably in shape. In Fig. 2 we present separately the mass spectra for the neutral and for the doubly charged $[K^*(890)$-$\pi]$ samples; the $[K^*(890)$-$\pi]$ mass spectra are also further subdivided according to incident beam momentum (above and below 8 GeV/c). The solid curves in Figs. 2(b), 2(c), 2(e), and 2(f) repre-

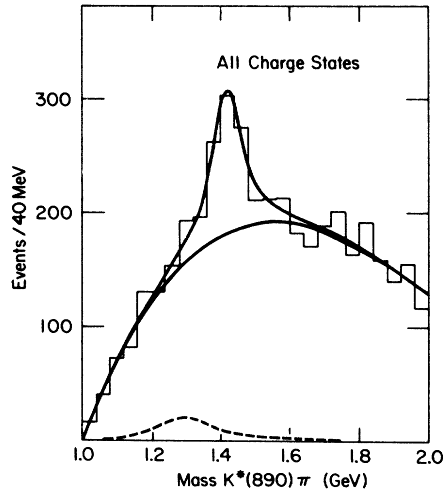

FIG. 1. The $[K^*(890)$-$\pi]$ mass spectrum for all the data from the sources listed in Table I. The solid curves represent the results of a fit to a third-order background polynomial plus $K^*(1420)$.

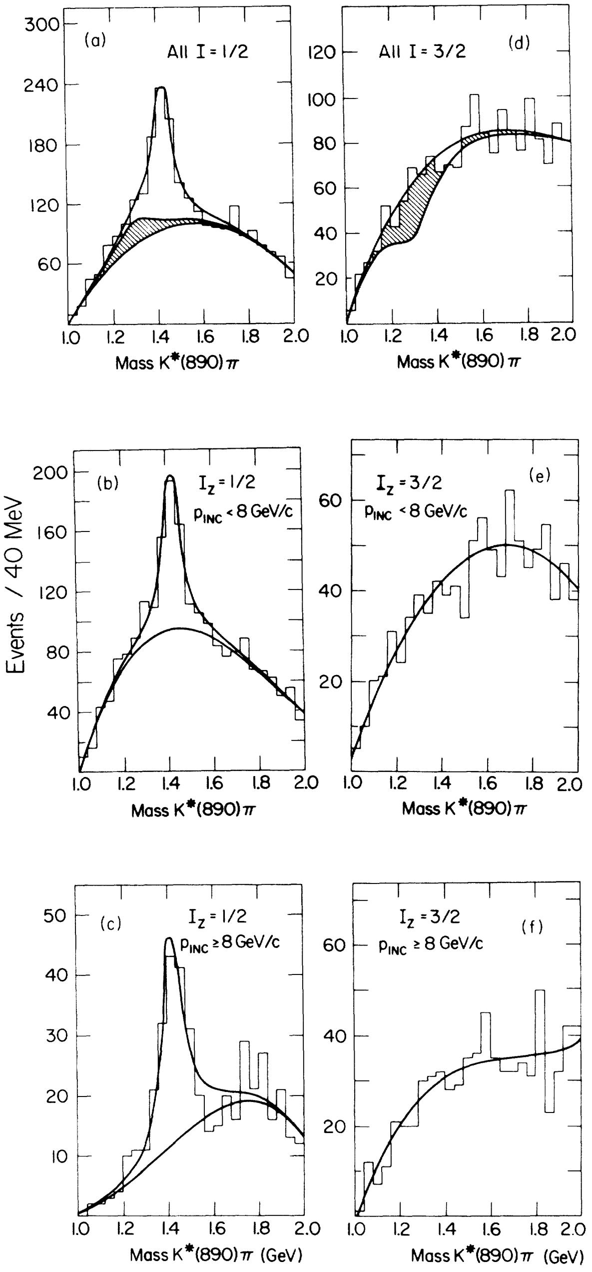

FIG. 2. (a) The [$K^*(890)$-π] mass spectrum for all neutrally charged $K^*\pi$ data. (b) The [$K^*(890$-π] mass spectrum for the neutrally charged $K^*\pi$ data from experiments with an incident beam momentum <8 GeV/c. (c) The [$K^*(890)$-π] mass spectrum for the neutrally charged $K^*\pi$ data from experiments with an incident beam momentum ≥ 8 GeV/c. (d) The [$K^*(890)$-π] mass spectrum for all doubly charged $K^*\pi$ data. (e) The [$K^*(890)$-π] mass spectrum for the doubly charged $K^*\pi$ data from experiments with an incident beam momentum < 8 GeV/c. (f) The [$K^*(890)$-π] mass spectrum for the doubly charged $K^*\pi$ data from experiments with an incident beam momentum ≥ 8 GeV/c. See text for explanation of curves.

sent fits to a third-order polynomial background for the doubly charged sample, and to a background plus a $K^*(1420)$ Breit-Wigner term for the neutral $K\pi\pi$ sample. The best fit parameters for the $K^*(1420)$ are found to be $M = 1420 \pm 5$ MeV and $\Gamma = 95 \pm 15$ MeV. These fits describe the data quite adequately (χ^2/degrees of freedom $\lesssim 1$). Fits to the $I = \frac{1}{2}$ samples in which Q production is included in the parametrization indicate that these data are consistent with the presence of some Q signal. However, fits to the $I = \frac{3}{2}$ samples do not predict a positive Q signal[3]; we estimate on this basis that there are, for example, at most 50 Q-type of events in the high-energy $I = \frac{3}{2}$ $K\pi\pi$ mass spectrum [Fig. 2(f)].[4] In Fig. 2(a) we show the result of a fit to the $I = \frac{1}{2}$ sample which allows for Q production. In Fig. 2(d) we indicate the shape which the background in the $I = \frac{3}{2}$ $K\pi\pi$ mass spectrum would have if the data contained the expected amount of Q^{++} production (see later).[4]

The diffraction dissociation[5] process has been proposed as an explanation for the Q and for other similar threshold enhancements. Reggeized versions of diffraction dissociation models[6] have been fairly successful in describing the shape of the [$K^*(890)$-π] mass enhancement for kaon beam momenta below 8 GeV/c,[7] although they have been less successful when applied to higher-momentum data.[8]

Assuming the approximate validity of these multiperipheral models, we can determine the ratio of Q_{CE} production to that of the diffractively produced Q (henceforth called Q_D); i.e., we can investigate reactions such as

$$K^{\pm}p \to K^0\pi^{\pm}\pi^{+}n\,. \tag{1}$$

In what follows we examine the nature of the expected threshold effect in reaction (1), basing our conclusions on a double-Regge model calculation analogous to the one used by Berger.[6]

The diagrams of interest are shown in Fig. 3. The left diagram, the Reggeized one-pion-exchange diagram, is used to calculate the diffraction dissociation process. The right diagram, which is used for the charge-exchange reaction,

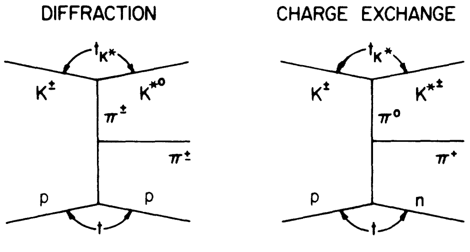

FIG. 3. Diagrams calculated to obtain the predictions of the double-Regge model.

is similar to the left diagram except that πN charge-exchange scattering replaces elastic scattering at the lower vertex.[9]

The square of the matrix element is given by

$$\sum |M|^2 = q^2 \frac{B(S_{K\pi})}{1-\cos\pi\alpha_\pi}\left(\frac{S_2}{S_0}\right)^{2\alpha_\pi} S_1 q_1^2 \frac{d\sigma}{dt}\bigg|_{\exp}, \qquad (2)$$

where q is the momentum of the K in the K^* rest frame, α_π is the pion trajectory $(t_{K^*} - m_\pi^2)$, S_0 is set to 1 GeV^2, S_1 is the square of the invariant mass of the πN system, q_1 is the momentum of N in the $N\pi$ rest frame, $S_2 = S_{K^*\pi} - t - m_K^2$, $B(S_{K\pi})$ is a Lorentzian shape describing the $K^*(890)$ line shape, and $d\sigma/dt|_{\exp}$ is the experimentally measured cross section, as a function of S_1 and t, for elastic or charge-exchange scattering.[10]

The only difference between the two diagrams in Fig. 3 is in the parametrization of the πN scattering. Since πN elastic and charge-exchange scattering are well known, a direct comparison of the strengths and shapes of the predicted threshold signals for the two processes in Fig. 3 should be reliable.

The calculation was carried out using a Monte Carlo technique in which $K^*(890)\pi N$ events were generated and weighted using phase space modified by the double-Regge matrix element. We considered only events which were contained in a kinematic region where our parametrization of the double-Regge model is expected to apply. We define this region by $|t_{K\to K^*}| < 0.5$ GeV^2, $|t_{K\to K^*\pi}| < 0.5$ GeV^2, and $M_{\pi N} > 1.5$ GeV. For an incident beam momentum of 12.7 GeV/c the $K^*\pi$ mass spectra predicted for these two diagrams are shown in Fig. 4. Both diagrams predict wide low-mass $K^*\pi$ enhancements peaking at ~1200 MeV. At 12.7 GeV/c the magnitude of the diffractive cross section for the production of Q_{CE} is 1/30th of that for the production of Q_D.[2] The ratio of the cross section for Q_{CE} to that for Q_D production decreases with increasing momentum.

From the amount of Q_D which is observed in the 10-events/μb Rochester 12.7-GeV/c K^+p bubble-chamber data, we can estimate the expected size of the Q_{CE} signal. The total cross section for Q_D production is about 450 μb. The ratio of the size of the expected Q_{CE} signal to that of the Q_D signal is given by the product of the ratios of the square of the matrix elements modified by the appropriate couplings for the production and decay of the $[K^{*+}(890)\text{-}\pi]$ system, and the probability of observing a K^0 in the bubble chamber. We conclude that the magnitude of the Q_{CE} signal in our data should be ~6 events, and would not be observed.

The magnitude of the Q_{CE} signal predicted by the calculation for the entire $I=\frac{1}{2}$ $K\pi\pi$ sample of data is ~150 events. A signal of this size cannot be excluded by the data, and, in fact, a fit to the mass spectrum [Fig. 2(a)], again using a Breit-Wigner term ($M=1280$ MeV, $\Gamma=220$ MeV) to approximate the Q,[11] yields a production rate of this order of magnitude [cross-hatched area in Fig. 2(a)]. A charge-exchange cross section of approximately this size is also expected on the basis of recent theoretical speculations.[12]

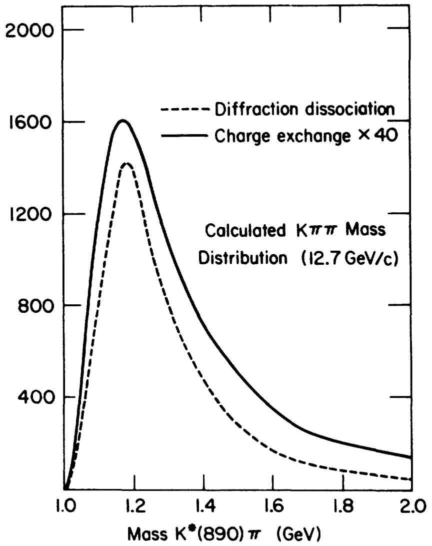

FIG. 4. The results of weighting phase space by the square of the matrix elements for the diagrams shown in Fig. 3.

The data for the $I=\frac{3}{2}$ $K\pi\pi$ sample, however, are consistent with having no Q signal present, whereas 166 Q events are expected on the basis of the calculation [cross-hatched area in Fig. 2(d)]. In Fig. 2(d) we display the shape which the background would assume if the rate for Q_{CE} production were consistent with the model; it is unlikely that even half as many events can be accommodated by the data. We conclude that unlike the situation which prevails in a previous application of the Reggeized Deck calculation to the $\rho^-\pi^-$ final state,[3] the $I=\frac{3}{2}$ data do not allow for consistency with the double-Regge model. This discrepancy is not surprising in light of recent theoretical[13] and experimental[2,3] criticisms of the Reggeized Deck model.

We thank Dr. R. Thews and Dr. H. Yuta for helpful discussions. We also wish to express our appreciation to Dr. D. Miller, Dr. T. Ludlam, Dr. P. Watkins and Dr. D. Colley, Dr. J. R. Campbell, and Dr. D. R. O. Morrison, Dr. A. Weinberg, Dr. R. Eisner, Dr. A. Firestone, Dr. M. Alston-Garnjost, and Dr. D. Carmony and their collaborators for the communication of their unpublished data.

[*] Research supported by the U. S. Atomic Energy Commission. The computer analysis has been made possible by a grant from the University of Rochester. This paper is an updated and final version of previous University of Rochester Reports No. 312 and No. 354.

[†] Presently at Monroe Community College, Rochester, New York.

[‡] A. P. Sloan Research Fellow.

[1] For a review of the status of the Q enhancement see: A. Firestone, in *Experimental Meson Spectroscopy – 1970,* edited by C. Baltay and A. H. Rosenfeld (Columbia Univ. Press, New York, 1970), p. 229; and also P. Slattery, University of Rochester Report No. UR-875-332, 1971 (unpublished).

[2] See the recent analysis of Q production in the reactions $K_L^0 p \to Q^0 p$ and $K_L^0 p \to \bar{Q}^0 p$ of G. Brandenburg *et al.* Nucl. Phys. B45, 397 (1972). The authors find that the cross section for nondiffractive Q production is approximately 30 μb near 7 GeV/c. This value is quite compatible with what we expect on the basis of the present calculation (see text).

[3] Application of the same procedure to the analogous data for the reaction: $\pi^- n \to \pi^- \rho^- p$ at 7 GeV/c, shown in Fig. 2 of D. Cohen *et al.*, Phys. Rev. Letters 28, 1601 (1972), yields $\gtrsim 100$ "A_1" events. This signal is consistent in size with the signal observed when the appropriate double-Regge cuts are applied to those data.

[4] The cross-hatched area in Fig. 2(d) corresponds to a cross section similar to that obtained for Q^0 in the fit shown in Fig. 2(a). In Fig. 2(f) we expect about 100 Q^{++} events on the basis of the model which we are about to discuss.

[5] M. Good and W. Walker, Phys. Rev. 120, 1857 (1960); R. Deck, Phys. Rev. Letters 13, 169 (1964); M. Ross and Y. Yam, *ibid.* 19, 546 (1967).

[6] E. Berger, Phys. Rev. 166, 1525 (1968).

[7] B. Werner *et al.*, Phys. Rev. 188, 2023 (1969); C. Y. Chien *et al.*, Phys. Letters 29B, 433 (1969); S. U. Chung *et al.*, Phys. Rev. 182, 1443 (1969).

[8] M. Farber *et al.*, Phys. Rev. Letters 22, 1394 (1969); J. Andrews *et al.*, *ibid.* 22, 738 (1969); H. Yuta, University of Rochester Report No. UR-875-271, 1969 (unpublished).

[9] A similar calculation performed by F. Bomse and R. Moses, Phys. Rev. 176, 2163 (1968), also predicts a low-mass $K^*\pi$ enhancement. Also see the analogous work pertaining to the A_1 in F. Winkelmann, R. Mickens, and H. J. Lubatti, Nucl. Phys. B30, 535 (1971), and in Ref. 3.

[10] The quantities $d\sigma/dt\,|_{\exp}$ were calculated in the manner described in Ref. 3.

[11] These values of mass and width are similar to those which were found to describe the Q^+ signal produced in 12.7-GeV/c K^+p interactions, see M. Farber *et al.*, Phys. Rev. D 1, 78 (1970). We have widened the Γ_{CE} to account for the different widths observed in Fig. 4. When fitting mass spectra we use a distorted Breit-Wigner: The simple Breit-Wigner form is multiplied by the polynomial employed to describe the background.

[12] G. Fox, in *Experimental Meson Spectroscopy – 1972,* proceedings of the Third International Conference, Philadelphia, 1972, edited by Kwan-Wu Li and A. H. Rosenfeld (A.I.P., New York, 1972).

[13] E. Berger, in *Phenomenology in Particle Physics, 1971,* proceedings of the conference held at Caltech, 1971, edited by C. B. Chiu, G. C. Fox, and A. J. G. Hey (Caltech, Pasadena, 1971); P. H. Frampton and N. A. Törnqvist, CERN Report No. CERN-TH 1457; H. I. Miettinen, Proceedings of the Zakopane Colloquium, 1972 (unpublished).

PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c[†]

S. D. Protopopescu,[*] M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,[‡]
J. H. Friedman,[§] T. A. Lasinski, G. R. Lynch, M. S. Rabin,[‖] and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 25 September 1972)

We present results of an energy-dependent phase-shift analysis for $\pi\pi$ energies between 550 and 1150 MeV from reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c. The $I=0$ s wave is parametrized in terms of a 2×2 M-matrix coupling $\pi\pi$ and $K\bar{K}$ channels. All the obtained solutions rule out the possibility of a narrow ϵ resonance in the ρ region and are characterized by a very rapid variation of the $I=0$ s-wave amplitude near $K\bar{K}$ threshold. We show that this rapid variation can be explained by a pole in the second Riemann sheet close to the $K\bar{K}$ threshold.

I. INTRODUCTION

Because of its theoretical simplicity, the s-wave $\pi\pi$ scattering amplitude has been the subject of substantial experimental and theoretical work over many years.[1] The experimental work has depended on the use of reactions dominated by pion exchange; the analysis of these reactions has yielded some information in the region of the ρ meson, but a persistent ambiguity in the s-wave amplitude between 750 and 900 MeV has made any conclusions drawn from the data very uncertain.

Recently, in the reaction $\pi^+p \to \pi^+\pi^-\Delta^{++}$, we have observed a strong anomaly in the $\pi\pi$ system near $K\bar{K}$ threshold[2]; this anomaly consists of a shoulder followed by a rapid drop in the cross section be-