PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOL. 7, NO. 5 | 1 March 1973

Nonvanishing Cross Section for the Reaction $\pi+N \rightarrow (\pi+\pi)+N$ at $t=0$†

S. Hagopian,* V. Hagopian,* J. D. Kimel, and N. D. Pewitt
Department of Physics, Florida State University, Tallahassee, Florida 32306

and

W. Selove‡
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 31 October 1972)

It is shown from experimental data that the extrapolated cross section for the reaction $\pi^- p \rightarrow \pi^-\pi^+ n$ at 2.3 GeV/c is not only nonzero at $t=0$, but negative. This is in contrast with higher-energy results where the cross section at $t=0$ is positive. One-pion-exchange calculations modified by absorptive corrections give results in general agreement with these experiments.

The determination of the elastic $\pi\pi$ cross section is of current interest and several groups[1-4] have attempted to calculate this quantity using the Chew-Low[5] extrapolation method of determining the $\pi\pi$ cross section by extrapolating the differential cross section to the pion pole, i.e., $t=\mu^2$ (t is the square of the four-momentum transfer from initial to final nucleon; μ is the pion mass). The problem is always that, with the limited statistics available, the results of a proper extrapolation always have large statistical errors and are essentially meaningless. The basic relation used for determining $\sigma(\pi\pi)$ at the pion pole (i.e., $\lim t \rightarrow \mu^2$) in the reaction $\pi^- p \rightarrow \pi^-\pi^+ n$ is

$$\frac{d^2\sigma}{dt d\omega^2} = \frac{-f^2\omega(\frac{1}{4}\omega^2-\mu^2)^{1/2}t}{2\pi p^2(\mu^2-t)^2\mu^2}\sigma(\pi\pi), \tag{1}$$

where $\omega = m(\pi\pi)$, f is the π-nucleon coupling constant, and p is the momentum of the incident π in the laboratory system.[5] The usual method of determining $\sigma(\pi\pi)$ is to extrapolate smoothly either

$$\sigma(\pi\pi) = \lim_{t\rightarrow\mu^2}[-F(s,t,\omega^2)] \tag{2}$$

or

$$\sigma(\pi\pi) = \lim_{t\rightarrow\mu^2}\frac{-F(s,t,\omega^2)}{t/\mu^2}, \tag{3}$$

where

$$F(s,t,\omega^2) = N(s,\omega^2)(\mu^2-t)^2\frac{d\sigma}{dt d\omega^2}(s,t,\omega^2),$$
$$N(s,\omega^2) = \frac{2\pi p^2}{f^2\omega(\frac{1}{4}\omega^2-\mu^2)^{1/2}}, \tag{4}$$

and s is the c.m. energy squared.

The expression (3) assumes the evasive hypothesis $d^2\sigma/dt d\omega^2=0$ at $t=0$, while (2) does not. Unfortunately, even though expression (2) is more likely to be correct, the statistical errors obtained by its use are very large.[2] Several experimenters have observed that F is close to zero at $t=0$ and so have assumed it to be zero for all $\pi\pi$ masses and even for extrapolation of moments determined from angular distributions.[1,2,6] This assumption was not seen as very unreasonable, as extrapolations using $F(s,0,\omega^2)=0$ gave π-π cross sections at the ρ mass consistent with the

p-wave unitarity limit. This was not true at all incident momenta. At about 2 GeV/c or so the extrapolated $\sigma(\pi^+\pi^-)$ at the ρ mass was about 20% lower than the unitarity limit.[2]

Of more direct bearing on the question of whether F vanishes at $t=0$ are the SLAC data at 15 GeV/c,[7] which suggest that $d\sigma/dtd\omega^2$ (or F) is *positive* at $t=0$ for ω in the ρ region and that a low-order polynomial extrapolation of Eq. (3) is incorrect for $p=15$ GeV/c.

Our data ($p=2.29$ GeV/c) also show that the evasive hypothesis is incorrect, but in contrast to Ref. 7 our experimental results indicate that $F(s, 0, \omega^2)$ is substantially *negative* for ω in the ρ region. (Note: $t=0$ is the unphysical region.) We find, however, that this s dependence of $F(s, 0, \omega^2)$ is expected on the basis of a reasonable absorption-model calculation. Further, we shall show that the absorption model can explain a number of the difficulties met experimentally in evasive Chew-Low extrapolations.

Our experimental analysis was performed on a sample of 8291 events of the type

$$\pi^- + p \to \pi^- + \pi^+ + n. \qquad (5)$$

Several other aspects of these data have already been published.[8] These data were obtained from an exposure of 2.29-GeV/c π^- mesons in the 15-in. hydrogen-filled bubble chamber at the Princeton-Pennsylvania Accelerator. About 50 000 2-prong events were measured using the University of Pennsylvania Flying Spot Digitizer (Hough-Powell Device or HPD) and processed through the University of Pennsylvania's Automatic Track-Following (ATF) and minimum-guidance event-recognition computer programs. About 8% of the events failed the semiautomatic system and were remeasured using conventional measuring machines. Because of the very accurate nature of the measuring technique, the fraction of misidentified events was estimated to be less than 1% for reaction (5).

The present analysis was done with 2900 events with $-t \leq 10\mu^2$. The cross section at $t=0$ was determined by extrapolating F in Eq. (4) using two separate methods. In the first method F was expanded in a Taylor series of the form

$$F(s, t, \omega^2) = A + Bt + Ct^2, \qquad (6)$$

where A, B, and C are functions of ω and were fitted to the data in 20–40-MeV-wide $\pi\pi$ mass intervals. The second method used the Padé approximant form

$$F(s, t, \omega^2) = a + \frac{bt}{1+ct}, \qquad (7)$$

where a, b, and c are functions of ω. This parametrization is equivalent in form[9] to that employed recently[6] by Baton *et al.*, and the physically reasonable value for c is $-0.11/\mu^2$, which comes from the cut[6] due to the $N\bar{N} \to 3\pi$ channel at $t=9\mu^2$. As proposed by Froggatt and Morgan,[10] the parameter a takes into account possible nonzero $F(s, t, \omega^2)$ at $t=0$.

Figure 1(a) shows fits to Eqs. (6) and (7) to the $F(s, t, \omega^2)$ vs t for one sample of data, namely for the $\pi\pi$ mass interval between 750 and 780 MeV. The two fits are quite similar and we have noticed no statistically significant deviations between the two forms of fits for all the mass ranges that we considered. Figure 1(b) shows the value of $F(s,0,\omega^2)$ as obtained by using the Padé approximant form [i.e., Eq. (7)]. The results of fitting Eqs. (6) and (7) for other $\pi\pi$ mass bins are very similar in nature to those shown and are not presented in this paper.[11] We would like to point out that $F(s, 0, \omega^2)$ is significantly below zero, especially for masses above 750 MeV. These data alone show that $F(s, 0, \omega^2)$ cannot be assumed to be zero for all values of ω and incident momentum p.

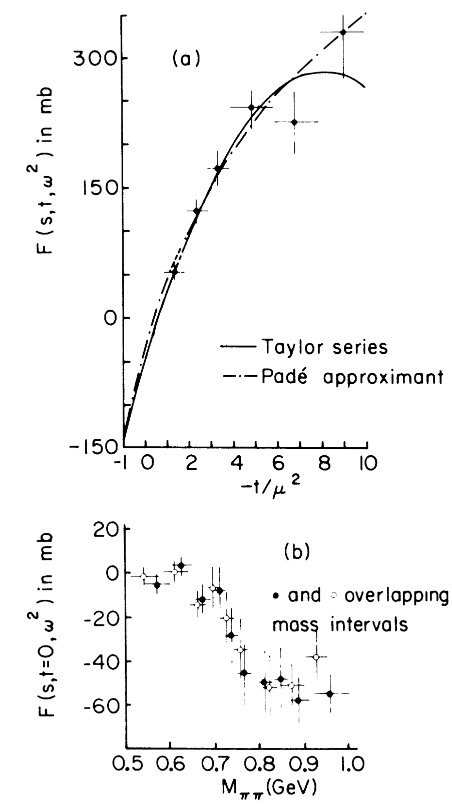

FIG. 1. (a) Fits of the extrapolation function $F(s, t, \omega^2)$, for the dipion mass interval 750–780 MeV, for the two forms of F. (b) Extrapolation function $F(s, t, \omega^2)$ at $t=0$ versus invariant dipion mass ($M_{\pi\pi}$), using the Padé approximant form of F.

These results are in disagreement with the evasive one-pion-exchange (OPE) model, but it has long been recognized that absorptive corrections[12,13] constitute a substantial background to the OPE contribution in $\pi^- p \to \pi^-\pi^+ n$. The data of Ref. 7 at 15 GeV/c show qualitative agreement with absorption-model predictions[14] even in the very near-forward direction where $0 < -t \leq \mu^2$, and thus substantiate the presence of a sizeable absorption background near $t=0$. Here we investigate the effects of absorption corrections at $t=0$ at lower beam momenta as well.

In the Gottfried-Jackson frame, one can obtain information on the individual $\pi\pi$ phase shifts by measuring the $\cos\theta$ (θ is the $\pi\pi$ scattering angle) distribution of $F(s, t, \omega^2)$,

$$\frac{dF}{d\cos\theta}(s, t, \omega^2, \theta) = N(s, \omega^2)(\mu^2 - t)^2 \frac{d\sigma(s, t, \omega^2, \theta)}{dt d\omega^2 d\cos\theta}, \tag{8}$$

which, if only s- and p-wave $\pi\pi$ phase shifts contribute, can be written

$$\frac{dF}{d\cos\theta}(s, t, \omega^2, \theta) = A_0(s, t, \omega^2) + A_1(s, t, \omega^2)\cos\theta + A_2(s, t, \omega^2)\cos^2\theta \tag{9}$$

and

$$F(s, t, \omega^2) = 2A_0(s, t, \omega^2) + \tfrac{2}{3}A_2(s, t, \omega^2). \tag{10}$$

Detailed absorption-modified OPE models for the three-body final state $\pi^- p \to \pi^-\pi^+ n$ have recently been formulated.[4,15,16] In these models the density matrix elements $\rho^{ll'}_{mm'}$ arising from $\pi\pi$ partial waves l and l' are proportional to $b_l b^*_{l'}$, where in terms of $\pi\pi$ phase shifts δ^I_l

$$b_l = \sum_I C_I \exp(i\delta^I_l)\sin\delta^I_l,$$

and $C_0 = \frac{2}{3}$, $C_1 = 1$, and $C_2 = \frac{1}{3}$.

Thus the cosine moments of (9) can be written in the evocative way,

$$A_0(s, t, \omega^2) = \left(\frac{2\pi}{k^2}\right)|b_0|^2 \times\left[F^{ss}_0(s, t, \omega^2) + \left|\frac{b_1}{b_0}\right|^2 F^{pp}_0(s, t, \omega^2)\right],$$

$$A_1(s, t, \omega^2) = \left(\frac{2\pi}{k^2}\right) 6\,\mathrm{Re}(b_0 b_1^*) F^{sp}_1(s, t, \omega^2), \tag{11}$$

$$A_2(s, t, \omega) = \left(\frac{2\pi}{k^2}\right) 9|b_1|^2 F^{pp}_2(s, t, \omega^2),$$

where k is the momentum of each of the final pions in the dipion rest frame. The dimensionless functions $F^{ll'}_m(s, t, \omega^2)$ are independent of phase shifts and have the convenient normalization

$$F^{ss}_0(s, \mu^2, \omega^2) = F^{sp}_1(s, \mu^2, \omega^2) = F^{pp}_2(s, \mu^2, \omega^2) = -1, \tag{12}$$

$$F^{pp}_0(s, \mu^2, \omega^2) = 0.$$

If absorptive corrections were neglected, the OPE contribution alone would predict $F^{ll'}_m(s, 0, \omega^2) = 0$. We are interested here in the deviation from this result arising from the absorption corrections, which are calculated using the absorption model of Ref. 16.

The OPE helicity amplitudes for the reaction $\pi^- p \to (\pi^-\pi^+)_l n$ are of the form (suppressing helicity labels)[16]

$$B^l(t) = \frac{(-\tau)^{n/2}}{\mu^2 - t}\left[P^l(\tau)|_{t=\mu^2}\right], \tag{13}$$

where $\tau = t - t_0$ (t_0 is the kinematical limit of t) and n is the net helicity flip. We have followed Durand and Chiu[13] in eliminating the often unitarity-violating "extraordinary" terms of (13) by evaluating the polynomial $P^l(\tau)$ at the pion pole. The absorption-modified helicity amplitudes are then calculated in the usual way according to Ref. 16 and depend on the absorption parameters C and A. We take $A = \frac{4}{3}A_{el} \simeq 12$ (GeV/c)$^{-2}$ and $C = 1.4$, which corresponds to $\Lambda = 2.6$ ($\Lambda = 4\pi AC/\sigma$; $\sigma = 32$ mb) and agrees with the Λ value of "2 to 3" obtained by Ross, Henyey, and Kane in their absorption-model analyses of reactions dominated by π exchange.[17]

Figures 2(a)–2(d) show the absorption-model prediction for $F^{ll'}_m$ at $t=0$ as a function of beam momentum and dipion effective mass. Like all absorption-model calculations, these are somewhat dependent in magnitude on the absorption strength C, but for C in the "canonical" strong absorption

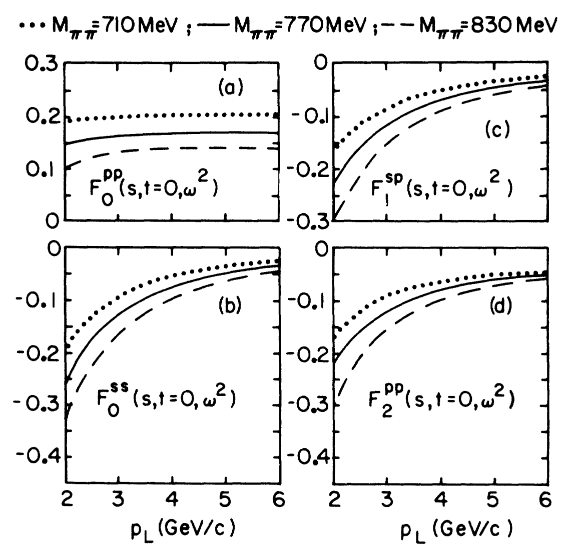

FIG. 2. Absorption-model predictions of $F^{ll'}_m(s, t, \omega^2)$ at $t=0$.

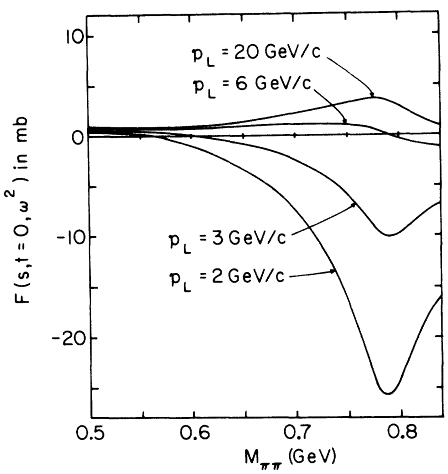

FIG. 3. Absorption-model prediction of the extrapolating function $F(s, t, \omega^2)$ at $t=0$.

range 1–1.4 the following qualitative features hold:

(a) $F_0^{ss}(s, 0, \omega^2)$, $F_1^{sp}(s, 0, \omega^2)$, and $F_2^{pp}(s, 0, \omega^2)$ are negative and deviate increasingly from the evasive hypothesis $F_n^{ll'}(s, 0, \omega^2)=0$ with decreasing beam momenta and increasing $\pi\pi$ effective mass. Equations (9) and (11) show that an evasive extrapolation of A_1 and A_2 would tend to underestimate $\mathrm{Re}(b_0 b_1^*)$ and $|b_1|^2$. Experimental evidence at low beam momenta supports the latter prediction.[2]

(b) The p-wave "leakage" contribution, $F_0^{pp}(s,0,\omega^2)$, is positive. Since from (11) one sees that in the effective-mass region of the ρ $A_0(s, 0, \omega^2)$ is dominated by the leakage term $F_0^{pp}(s, 0, \omega^2)$, evasive extrapolation of A_0 is expected to overestimate $|b_0|^2$. This prediction also describes the experimental situation.[18]

(c) Asymptotically in s, $F_0^{ss}(s, 0, \omega^2) \to 0$, $F_1^{sp}(s, 0, \omega^2) \to 0$, and $F_0^{pp}(s, 0, \omega^2) = -9F_2^{pp}(s, 0, \omega^2)$, all in agreement with the nonevasive high-energy extrapolation models of Refs. 15 and 19.

To calculate $F(s, 0, \omega^2) = 2A_0(s, 0, \omega^2) + \frac{2}{3}A_2(s, 0, \omega^2)$ requires from Eq. (11) a knowledge of the $\pi\pi$ phase shifts, and we show in Fig. 3 the absorption-model results using the $\pi\pi$ phase shifts listed in Ref. 18, corresponding to the "down-down" solution for δ_0^0.[20] This figure shows dramatically the nonevasive character of the absorption-model prediction. For C in the "canonical" range 1–1.4, the absorption model predicts that in the dipion mass region of the ρ, $F(s, 0, \omega^2)$ moves from a significantly negative value at $p=2.0$ GeV/c to a positive value at higher energies.

A comparison of Figs. 1(b) and 3 shows a qualitative agreement between our experimental determination of $F(s, 0, \omega^2)$ and the absorption-model prediction, particularly in the ω^2 dependence and in the rather surprising negative sign of $F(s, 0, \omega^2)$ obtained experimentally in the ρ region. The s dependence of $F(s, 0, \omega^2)$ shown in Fig. 3 is also consistent with the data of Ref. 7 at 15 GeV/c, which indicate that $F(s, 0, \omega^2)$ is positive at 15 GeV/c. There is a difference in magnitude of about a factor of 2 between the model prediction and our experimental determination of $F(s, 0, \omega^2)$ at 2.3 GeV/c, but since the absorption model neglects s-channel effects, as well as $\pi^- p \to \pi^- \Delta^+$ effects (although the latter are almost nonexistent for events with $-t \leq 10\mu^2$), we do not expect exact agreement at the relatively low energy of this experiment and have not attempted to adjust the model for a more quantitative fit. The nearby s channel $N(2190)$, which is known to decay into the $N\pi\pi$ channel, could influence the value of $F(s, 0, \omega^2)$ at our energy. However, the energy trend predicted by the absorption model in Fig. 3 does suggest that the seemingly contradictory experimental results that $F(s, 0, \omega^2)$ is negative for $p=2.3$ GeV/c, zero (within large error bars) for $p=2.77$ GeV/c,[6] and positive at high (15-GeV/c) energies[7] may be qualitatively understood on the basis of absorptive pion exchange.

Another recent experiment by Grayer *et al.* at high energy, $\pi^- p \to \pi^- \pi^+ n$ at 17.1 GeV/c,[21] finds their data not compatible with zero cross section at $t=0$.

Thus we conclude from our experimental results that $F(s, 0, \omega^2)$ is significantly *negative* for ω in the ρ region and that an evasive extrapolation of $F(s, t, \omega^2)$ is incorrect for $p=2.29$ GeV/c. It can be inferred that $F(s, 0, \omega^2)$ is *positive* for ω in the ρ region from the data of Ref. 7 at 15 GeV/c. We find that a reasonable absorption model qualitatively interpolates between these results. On the basis of the absorption-model calculations, we conclude in addition that this model predicts that the evasive hypothesis is incorrect in extrapolations of F, A_0, and A_2 to determine $\pi\pi$ phase shifts. The same holds true for A_1 for beam momenta below approximately 6 GeV/c.

We wish to thank Professor J. Lannutti and especially Professor P. K. Williams for their helpful discussions. We are grateful for the support of the staffs of the Princeton-Pennsylvania Accelerator and the Bubble Chamber. Also our thanks to the staff of the University of Pennsylvania Hough-Powell Device. We would like to thank Dr. E. Bogart, Dr. R. O'Donnell, and Dr. Y. L. Pan for their help in the data reduction.

†Research supported in part by the U. S. Atomic Energy Commission.

*Before 1970 at the University of Pennsylvania, where the data reduction was performed.

‡John S. Guggenheim Fellow, 1972.

[1]J. P. Baton *et al.*, Phys. Letters 25B, 419 (1967).

[2]S. Marateck *et al.*, Phys. Rev. Letters 21, 1613 (1968).

[3]L. Chan *et al.*, Phys. Rev. D 2, 583 (1970).

[4]B. Y. Oh *et al.*, Phys. Rev. D 1, 2494 (1970).

[5]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[6]J. P. Baton *et al.*, Phys. Letters 33B, 525 (1970).

[7]F. Bulos *et al.*, Phys. Rev. Letters 26, 1453 (1971); P. Baillon *et al.*, Phys. Letters 38B, 555 (1972).

[8]S. Hagopian *et al.*, Phys. Rev. Letters 24, 1445 (1970); 25, 1050 (1970); Phys. Rev. D 5, 2684 (1972).

[9]J. Zinn-Justin, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1970), Vol. 55, p. 43.

[10]C. D. Froggatt and D. Morgan, Phys. Rev. 187, 2044 (1969).

[11]N. D. Pewitt, Master's thesis, Florida State University, 1972 (unpublished).

[12]K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).

[13]L. Durand III and Y. T. Chiu, Phys. Rev. 137, B1530 (1965).

[14]P. Baillon *et al.*, Phys. Letters 35B, 453 (1971).

[15]P. K. Williams, Phys. Rev. D 1, 1312 (1970).

[16]J. D. Kimel and L. M. Nath, Nucl. Phys. B29, 616 (1971).

[17]M. Ross *et al.*, Nucl. Phys. B23, 269 (1970).

[18]V. Hagopian *et al.*, in *Proceedings of the Conference on ππ and Kπ Interactions, Argonne National Laboratory, 1969*, edited by F. Loeffler and E. D. Malamud (Argonne National Laboratory, Argonne, Ill., 1969), p. 149.

[19]G. L. Kane and M. Ross, Phys. Rev. 177, 2353 (1969).

[20]The calculation of $F(s,0,\omega^2)$ is dominated in the mass range considered by the p-wave phase shifts and is very insensitive to the s-wave phase shifts used. Changing the s-wave phase shifts from "down-down" to "down-up" alters $F(s,0,\omega^2)$ by less than 5% over the mass range of the ρ.

[21]G. Grayer *et al.*, in *Experimental Meson Spectroscopy-1972*, proceedings of the Third International Conference, Philadelphia, 1972, edited by Kwan-Wu Lai and A. H. Rosenfeld (A.I.P., New York, 1972), p. 5.

PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

Search for Q Production in Charge-Exchange Reactions*

B. Werner,† P. Slattery, and T. Ferbel‡
University of Rochester, Rochester, New York 14627
(Received 24 August 1972)

The low-mass $K\pi\pi$ enhancement (the Q effect) has been observed in $K^{\pm}N$ interactions only in the same charge state as the incident beam particle. The lack of observation of the Q in reactions involving nucleon charge exchange has been cited as evidence for the diffractive nature of Q production. We have searched for Q^0 and Q^{++} production in $K^{\pm}$ induced reactions by combining relevant world data. We have also carried out a double-Regge exchange calculation in order to estimate the magnitude of the expected signal for Q production in charge-exchange processes.

The low-mass $K\pi\pi$ (Q) enhancement[1] has been studied extensively in $K^{\pm}N$ interactions by means of bubble chambers exposed to beams of high-momentum kaons. The puzzling situation now exists in which the spin-parity of the entire Q region is measured to be predominantly 1^+, but there remains considerable controversy as to whether this enhancement is due to one, two, or more resonances, or whether the entire phenomenon can be understood as arising from threshold kinematic effects.

The Q enhancement has thus far only been observed to occur in $K\pi\pi$ systems which are produced in association with the target nucleon. No Q enhancement has ever been observed in a $K\pi\pi N$ final state in which the nucleon undergoes charge exchange. This, along with the observation of a slow variation of the production cross section as a function of incident beam momentum, a steep momentum-transfer dependence, and an alignment of the polarization vector for the $K\pi\pi$ system perpendicular to the incident beam direction, is consistent with a diffractive production mechanism.

Production mechanisms other than the exchange of vacuum quantum numbers appear to contribute to Q production.[2] Consequently, the Q should also be observed, although at a reduced level, in reactions involving charge exchange to the nucleon (hereafter referred to as Q_{CE} production). An individual experiment (such as our 10-event/μb 12.7-GeV/c K^+p bubble-chamber exposure) would very likely be insensitive to the expected level of