Gauge Invariance in Compton Scattering

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The scattering of vector mesons off a pseudoscalar-meson target is considered and the requirements of gauge invariance in the limit of vanishing mass of the vector meson are discussed. Explicit conditions are obtained which ensure the vanishing of helicity amplitudes of longitudinal helicity components in this limit and it is shown that the transition to the limit is smooth.

The formulation and enforcement of gauge-invariance requirements is an important part of the theoretical discussion of photonic reactions. In a recent paper, Ebata and Lassila[1] investigated gauge-invariance conditions in Compton scattering[2] and showed that helicity amplitudes with longitudinal helicity components vanish with the behavior m^N as m approaches zero, where m is the mass of the vector meson, i.e., photon, and N is the number of longitudinal helicity components of the amplitude. Since their conclusions are obtained rather indirectly, and since a clear understanding of gauge invariance in the limit of vanishing photon mass is an unavoidable necessity in many problems, it is the purpose of this note to show that the gauge-invariance conditions are nontrivial and may be discussed in a simpler and more transparent way. In particular it will be shown that the conditions have the form of derivative relations which guarantee that the transition to the limit of vanishing photon mass is smooth.

We consider the u-channel reaction

$$V_\mu(k)+K(p_1)\to V_\nu(k')+K(p_2),$$

where V represents the vector meson of mass m and K a pseudoscalar meson of mass M. We use the metric $(+---)$ and notation of Ebata and Lassila. Moreover, since the point we wish to emphasize is one of principle, rather than application, we restrict our discussion throughout (without loss of generality) to the u-channel forward direction, i.e., $t=0$. The amplitude for the process, i.e., $\epsilon_{k'}^\nu T_{\nu\mu}\epsilon_k^\mu$ [ϵ_k and $\epsilon_{k'}$ are polarization vectors of $V(k)$, and $V(k')$, respectively], may be expressed in terms of four invariant amplitudes A_i. In the helicity basis the amplitudes containing longitudinal helicity components are (for $t=0$ and expanded in rising powers of m^2)

$$\begin{aligned} m^2 f_{00} &= (u-M^2)^2 A_2 \\ &\quad - m^2[A_1+2(u+M^2)A_2]+O(m^4), \\ -\left(\frac{\sqrt{2}\, m f_{0+}}{\sin\theta}\right)_{\theta=0} &= \frac{(u-M^2)}{2\sqrt{u}}[A_1-(u-M^2)(A_2-A_3)] \\ &\quad + \frac{m^2}{2\sqrt{u}}[A_1+2(u+M^2)(A_2-A_3)] \\ &\quad + O(m^4). \end{aligned} \tag{1}$$

Of course, conservation of angular momentum requires f_{0+} to vanish in the forward direction, not, however, the ratio $f_{0+}/\sin\theta$. Also, it is well known[3] that the limit of forward direction may be taken before the photon mass is allowed to vanish.

One might argue now that in order to ensure the vanishing of f_{00} as $m^2\to 0$, we may impose conditions such as, for instance,

$$\begin{aligned} (u-M^2)^2 A_2 &= O(m^4), \\ A_1+2(u+M^2)A_2 &= O(m^2). \end{aligned} \tag{2}$$

We show that in general such freely chosen conditions are incompatible with gauge invariance, although the vanishing of f_{00} is maintained. Gauge invariance implies a unique set of conditions of this type. We recall that gauge invariance means invariance of the amplitude under the replacement of ϵ_k by $\epsilon_k+\lambda k$, where λ is arbitrary. Thus we obtain the conditions

$$k'^\nu T_{\nu\mu} k^\mu = a m^2,$$

$$\lambda \epsilon_{k'}^\nu T_{\nu\mu} k^\mu + \lambda' k'^\nu T_{\nu\mu}\epsilon_k^\mu = b\, m^2,$$

where λ, λ' are arbitrary, and a, b are of order zero in m^2 or higher. Expressing these conditions in terms of invariant amplitudes, we find (for $t=0$, $P\equiv p_1+p_2$)

$$am^2=(u-M^2)^2A_2+m^2[A_1-2(u-M^2)(A_2-A_3)]+O(m^4),$$
$$bm^2=[\lambda(\epsilon'\cdot k)+\lambda'(\epsilon\cdot k)][A_1+(u-M^2)A_3-m^2(A_3-A_4)]+[\lambda(\epsilon'\cdot P)+\lambda'(\epsilon\cdot P)][(u-M^2)A_2-m^2(A_2-A_3)]. \quad (3)$$

From these relations we conclude that

$$(u-M^2)A_2-m^2(A_2-A_3)=b_1m^2$$

and hence (4)

$$A_1+(u-M^2)A_3-m^2(A_3-A_4)=b_2m^2,$$

b_1, b_2 of order zero in m^2. It is these conditions which ensure gauge invariance, not freely chosen relations such as (2).

Next we determine the behavior of f_{00}, f_{0+}. From (3) we have

$$(u-M^2)^2A_2=[a-A_1+2(u-M^2)(A_2-A_3)]m^2+O(m^4)$$
$$=[a-A_1-2(u-M^2)A_3]m^2+O(m^4). \quad (5)$$

Substituting the latter into (1) we obtain

$$f_{00}=[a-2A_1-2(u-M^2)A_3]+O(m^2). \quad (6)$$

Thus if f_{00} is to vanish as m^2 approaches zero, we must have

$$a=2A_1+2(u-M^2)A_3, \quad (7)$$

where (in principle) we could still add something of the order of m^2 on the right-hand side. The quantity a in (3) is thereby uniquely determined at least to lowest order in m^2. Hence (3) and (5) become

$$(u-M^2)^2A_2=A_1m^2+O(m^4). \quad (8)$$

This relation is compatible with the first of Eqs. (4) if

$$A_1=(u-M^2)(b_1-A_3), \quad (9)$$

which determines b_1 at least to order zero in m^2.

On the other hand we see from the second of Eqs. (4) that

$$\tfrac{1}{2}a=A_1+(u-M^2)A_3$$
$$=m^2(b_2+A_3-A_4). \quad (10)$$

Thus a is itself of order m^2, as it must be in order to ensure that f_{0+} approaches zero as m.

We see from (8) and (10) that A_1 plays a particular role in gauge-invariance conditions. This may be understood by recalling that A_1 is the coefficient of the invariant representing the so-called contact interaction, i.e., the coupling constant which is assumed to be nonzero. We emphasize that the gauge-invariance conditions (3) and (4), i.e., (8) and (10), are nontrivial. They are universal in the sense that once found from the kinematics of one channel, they ensure the vanishing of amplitudes containing longitudinal vector-meson helicity components of either of the other channels (one may verify this by application to the t-channel amplitudes given by Ebata and Lassila). Finally we remark that the gauge-invariance constraints may be interpreted as derivative conditions in m^2. The helicity amplitudes are linear combinations of the invariant amplitudes. With the help of (8) and (10) we may eliminate two of these amplitudes. We then obtain expressions for the amplitudes which have the form of a Taylor expansion, i.e.,

$$f(u,m^2)=f(u,0)+m^2g(u,0).$$

Since $g\neq 0$, the transition to the limit $m^2=0$ is continuous. Finally we mention for completeness that the gauge-invariance conditions can also be formulated entirely within the helicity formalism alone as has meanwhile been shown by the authors of Ref. 1 together with other authors.[4]

[1]T. Ebata and K. E. Lassila, Phys. Rev. 185, 1717 (1969).

[2]For a recent discussion of various related aspects of Compton scattering see G. E. Hite and H. J. W. Müller, University of Trier-Kaiserslautern report, 1972 (unpublished).

[3]See, for instance, F. Arbab and R. C. Brower, Phys. Rev. 181, 2124 (1969); N. Dombey, Nuovo Cimento 32, 1696 (1964); J. S. Ball and M. Jacob, *ibid.* 32, 1696 (1964); J. P. Ader, M. Capdeville, and P. Salin, Nucl. Phys. B17, 221 (1970).

[4]T. Akiba, M. Sakuraoka, and T. Ebata, Nucl. Phys. B31, 381 (1971); T. Ebata, T. Akiba, and K. E. Lassila, Progr. Theoret. Phys. (Kyoto) 44, 1684 (1970).