Unified Gauge Theories of Hadrons and Leptons*

I. Bars,† M. B. Halpern, and M. Yoshimura‡
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 6 November 1972)

If we insist on SU(3)⊗SU(3) classification for hadrons, in the presence of the known low-lying multiplets, we are led to models of the following nature: Before spontaneous breakdown, we have two commuting gauge groups, hadronic and leptonic. This divides such models into three sectors: hadrons, leptons, and a third unconventional set of (presumably high-mass) scalar mesons which serve to connect the two "known" worlds. Spontaneous breakdown induces appropriate masses and all usual strong, weak, and electromagnetic couplings. Intimate connections are seen between these three fundamental forces.

I. INTRODUCTION

Gauge principles have been a guiding light in elementary particle theory for a very long time. However, by itself, gauge invariance is useful only for theories involving certain massless particles (electrodynamics and gravitation). Taken together with spontaneous breakdown of gauge symmetries, and the Higgs-Kibble phenomenon,[1] the possibility of an elegant gauge structure for all physical forces is emerging in the context of renormalizable field theories.

Weinberg[2] and Salam[3] were first to move in this direction, by constructing a unified gauge model of leptonic weak interactions and electromagnetism. At that time they also conjectured what is probably the most fascinating bonus of this gauge approach, namely that such models may be renormalizable.

The subject lay dormant until 't Hooft[4] and Lee[4] showed that, modulo anomalies,[5] this conjecture was correct. Various mechanisms for canceling (known) anomalies have since been discussed, so that with some confidence the community has begun a search through various (presumably) renormalizable gauge models for the one "chosen" by nature.

Most effort has gone into constructing alternative models for weak interactions and electromagnetism, and a number[2,6] have recently appeared in the literature. In spite of the lack of theoretical "uniqueness" of these models, they all share in an elegance and force that has, we believe, opened a new era in weak-interaction physics.

Recently, also, Bardakci and Halpern[7] constructed a similar renormalizable gauge model of the hadronic vector mesons. This model realizes then the Yang-Mills ideas about strong vector mesons, and thus moves further toward a unified gauge theory of particle forces. Such a unified model of hadrons and leptons has now been briefly presented in the literature.[8] It is the purpose of this paper to discuss the model in some depth.

For strong interactions a Lagrangian is not very useful from the practical point of view: Such a Lagrangian can at best be used to describe low-energy hadronic data, but we feel it will be extremely illuminating as a guide to a better understanding of hadron dynamics, and of the interplay of strong, weak, and electromagnetic interactions. For example, strictly from the hadronic viewpoint, such a model suggests that it will be useful to consider a hadron dynamics in which the strong vector mesons, at some intermediate stage in the calculation, have zero mass. (We remind the reader that this is indeed exactly what is happening in dual models at the moment.) It has been shown that there is an intimate connection between dual models and gauge theories.[9] Now the search is beginning for a dual Higgs-Kibble mechanism to raise the ρ mass. We believe there is an intimate connection between our hadron model and the future spontaneously broken dual model with internal symmetries, and hope our efforts may serve as a guide in the duality situation.

Further, as we shall see below, the presence of the leptons does dictate in a certain way the structure of hadronic symmetry breaking. Thus, a full understanding of strong interactions seems to require the simultaneous understanding of weak and electromagnetic forces. Intimate connections between strong, weak, and electromagnetic forces, such as shown in our model, will, we believe, be of much more than passing interest in future theory and experiment.

Our goal in this paper is then a unified renormalizable gauge theory of strong, weak, and electromagnetic forces. Our approach is based on the following reasoning: It is the hadrons whose symmetries are "known" – not the leptons. This is reflected by the plethora of lepton models, but only one $SU(3)\otimes SU(3)$ hadron model.[7] For this reason we *start* firmly with the $SU(3)\otimes SU(3)$ hadrons, and search through the various lepton universes for one which "fits."

In this, we are extremely encouraged by the structure of the hadron world. As shown in Ref. 7, the symmetric hadron theory *necessarily* begins (before spontaneous symmetry breakdown) with a local $SU(3)\otimes SU(3)$ *and* an extra "global" group at least as large. The final symmetry is the product group. With Bardakci and Halpern, we thus interpret the hadrons as "welcoming" a lepton theory as a local subgroup of its "extra" group. In this paper, the extra group will be called "primed" or "leptonic."

The program is orderly. We study embedding the leptons in progressively larger "primed" groups of the hadron model. Taking $SU(3)'\otimes SU(3)'$ for the "primed" group leads to trouble with strangeness-changing processes, but when we go to $SU(4)'\otimes SU(4)'$ everything falls in place beautifully.

In our search through lepton universes, we first set ourselves the following additional boundary conditions:

(1) We require the leptons to allow a $(3,\bar{3})+(\bar{3},3)$ symmetry-breaking mechanism for hadrons.

(2) In keeping with having only an $SU(3)\otimes SU(3)$ hadron world (3 quarks), we want the lepton model to contain only the *known leptons*. Requirement (2) limits us to Weinberg's theory, and is relaxed later. It is worth remarking here that although some other lepton models can "fit" our hadrons, none is as natural as Weinberg's.

In any case, of course, some extra (heavy) quarks and leptons are required to cancel anomalies. Our models lead uniquely to an anomaly-removal scheme which, for hadrons, is very much in the spirit of dual models: In particular, we find that removal of anomalies and a proper rate for $\pi^0\to 2\gamma$ imply the existence of a heavy pion.

The plan of this paper is as follows. Section II contains a general formulation of gauge theories. In Sec. III we reformulate Weinberg's theory in a suggestive notation, involving a new classification of the leptons. Section IV is a review of the $U(3)\otimes U(3)$ hadron theory; we include here a discussion of the hadronic currents. In Sec. V, we discuss a physical induction of the lepton world from this hadron world. Section VI contains the model itself, details, and possible alternatives. There are two appendixes. Appendix A discusses the spontaneous breakdown in the somewhat involved system of scalar mesons. Appendix B mentions the alternative but unsuccessful attempt to embed the leptons in $SU(3)'\otimes SU(3)'$.

II. GENERAL GAUGE FORMALISM

In order to present our analysis in an organized way, we outline here an operational approach, independent of representations, for writing a gauge-invariant Lagrangian. We will always follow just three steps in each model we consider in this paper:

Step 1. Classify the particles in the theory with an appropriate group.

Step 2. Write a gauge-invariant Lagrangian with dimension ≤ 4.

Step 3. Break the symmetry spontaneously, guided by physical arguments.

We emphasize here that the requirements of gauge invariance and dimension $d\leq 4$ are so restrictive that the physical content of the theory is essentially determined by the classification of the particles. Therefore, step 1 contains the most important ingredients in building a model.

Step 1. We assume we have chosen a group whose generators are denoted by F_α, $\alpha=1\cdots n$. The operator which generates local transformations is $\mathfrak{U}(x)=\exp i[F_\alpha\omega_\alpha(x)]$. The transformation properties of the particles are determined by the linear representation to which they have been assigned (nonlinear representations are excluded from our analysis, because of the criterion $d\leq 4$ for renormalizability). Thus, denoting the particles as $\phi_i(x)$, we have

$$\phi_i(x)\to\mathfrak{U}\phi_i(x)\mathfrak{U}^{-1}=S_{ij}(x)\phi_j(x)\,,\qquad(2.1)$$

where $S_{ij}(x)$ defines the representation. For example:

(a) If the group is SU(2) with generators $\vec{T}$, and

$$\phi_i=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

is a doublet, then

$$\phi\to\mathfrak{U}\phi\mathfrak{U}^{-1}=e^{+i\vec{\tau}\cdot\vec{\omega}(x)/2}\phi,\quad \vec{\tau}=\text{Pauli matrices}.\qquad(2.2)$$

(b) If the group is $SU(3)_L\otimes SU(3)_R$ with generators F_L^α, F_R^α, and $\phi_{ij}(x)$ is a 3×3 matrix in the $(3,\bar{3})$ representation, then

$$\phi\to\mathfrak{U}\phi\mathfrak{U}^{-1}=e^{i\lambda\cdot\omega_L(x)/2}\phi e^{-i\lambda\cdot\omega_R(x)/2}\,,\qquad(2.3)$$

where λ_α are the usual 3×3 SU(3) matrices. The infinitesimal form of the transformation equation *defines* the commutators of the generators with the fields. In examples (a) and (b) above we get, respectively,

$$\text{(a)}\quad [T_i,\phi]=\tfrac{1}{2}\tau_i\phi\,,\qquad(2.4)$$

$$\text{(b)}\quad [F_L^\alpha,\phi]=\tfrac{1}{2}\lambda^\alpha\phi,\quad [F_R^\alpha,\phi]=-\phi\tfrac{1}{2}\lambda^\alpha\,.\qquad(2.5)$$

Step 2. The derivative $\partial_\mu \phi = -i[P_\mu, \phi]$ (P_μ = momentum operator) does not transform covariantly when ϕ is replaced by $\mathfrak{U}(x)\phi\mathfrak{U}^{-1}(x)$. To define a covariant derivative independent of representation, we first define a covariant momentum operator $\mathcal{P}_\mu$ by introducing as many vector gauge bosons $V_\mu^\alpha(x)$ as there are generators,

$$\mathcal{P}_\mu = P_\mu + gV_\mu \cdot F\,. \tag{2.6}$$

(V_μ^α are considered as c numbers with respect to F^α in the following formal manipulations.) We demand that under $\mathfrak{U}(x)$, $\mathcal{P}_\mu$ transform covariantly:

$$\mathcal{P}_\mu \to \mathfrak{U}\mathcal{P}_\mu \mathfrak{U}^{-1} = P_\mu + \mathfrak{U}(gV_\mu \cdot F + i\partial_\mu)\mathfrak{U}^{-1}\,, \tag{2.7}$$

where the right-hand side is found by calculating $\mathfrak{U}P_\mu\mathfrak{U}^{-1} = P_\mu + \mathfrak{U}[P_\mu, \mathfrak{U}^{-1}] = P_\mu + i\mathfrak{U}\partial_\mu\mathfrak{U}^{-1}$. Here we have assumed that $[P_\mu, F^\alpha] = 0$, which means that F^α are internal symmetry generators. If we allow the more general case of $[P_\mu, F^\alpha] \neq 0$, for example F^α being generators of Lorentz transformations, or dilatations, etc., then we have to consider general relativity in curved space. This displays the known close relationship that exists between general relativity and the Yang-Mills approach.[10] Equation (2.7) induces a transformation on $V_\mu^\alpha(x)$:

$$V_\mu \cdot F \to V_\mu' \cdot F = \mathfrak{U}\left(V_\mu \cdot F + \frac{i}{g}\,\partial_\mu\right)\mathfrak{U}^{-1}\,. \tag{2.8}$$

With these properties we can see that the covariant derivative is

$$\nabla_\mu \phi = -i\,[\mathcal{P}_\mu, \phi] = \partial_\mu \phi - igV_\mu^\alpha[F^\alpha, \phi]\,, \tag{2.9}$$

where the commutator $[F^\alpha, \phi]$ is specified in step 1 and depends on representation. Indeed under a simultaneous transformation of ϕ and V_μ^α we get

$$\nabla_\mu \phi \to -i[\mathfrak{U}\mathcal{P}_\mu\mathfrak{U}^{-1}, \mathfrak{U}\phi\mathfrak{U}^{-1}] = \mathfrak{U}(\nabla_\mu \phi)\mathfrak{U}^{-1}\,. \tag{2.10}$$

Thus $\nabla_\mu \phi$ transforms covariantly (like ϕ). The covariant derivatives for V_μ^α can be found from the commutator

$$[\mathcal{P}_\mu, \mathcal{P}_\nu] \equiv igF_{\mu\nu}^\alpha F^\alpha\,. \tag{2.11}$$

Since the left-hand side is covariant, so is the right-hand side. We get

$$F_{\mu\nu}^\alpha = \partial_\mu V_\nu^\alpha - \partial_\nu V_\mu^\alpha + gf^{\alpha\beta\gamma}V_\mu^\beta V_\nu^\gamma\,, \tag{2.12}$$

where $f^{\alpha\beta\gamma}$ are the structure constants of the group under consideration.

Using only covariant derivatives we can now write an invariant Lagrangian as if we had global invariance, as usual.[11] Mass terms for V_μ^α should be omitted since thay are not invariant. For renormalizability we should also require that any term in the Lagrangian have dimension $d \leq 4$.

Step 3. The gauge particles acquire mass through the Higgs-Kibble mechanism.[1] The local gauge symmetry is broken spontaneously by introducing a set of scalar mesons which acquire nonvanishing vacuum expectation value. A counting argument due to Kibble[1] shows that, when one considers this scalar meson system *alone*, the number of massless Goldstone bosons generated by the spontaneous breakdown is equal to the difference of the number of *global* symmetries existing within the scalar system before and after spontaneous breakdown. In simple[12] physically reasonable models, these Goldstone bosons are completely eliminated from the Lagrangian by a gauge transformation, and they become the longitudinal components of the vector gauge bosons which acquire mass. Thus, the scalar mesons must be assigned to a representation such as to generate, through spontaneous breakdown, the *same number* of Goldstone bosons as the number of gauge particles that are desired to be raised in mass.

The restrictive power of the procedure is self-evident. The model is essentially completed in the first step, simply by the classification of the particles. The form of the vacuum expectation value is further restricted by physical requirements such as the existence of a massless and universal photon, masses of fermions, masses of gauge mesons (if known), physical values of coupling constants, etc. This procedure also produces many relations between (bare) masses and coupling constants, which are adjusted to best fit experimental data (say to zeroth order). As a result, few possible classifications of particles are capable of yielding a viable theory. If in addition we restrict ourselves to as small a group as possible which can describe all possible interactions, and fit the data as close as possible to first order, then the form of the theory that one can write is extremely limited. This will be illustrated in the following sections.

III. WEINBERG'S THEORY EMBEDDED IN $SU(2)' \otimes SU(2)'$

In this section we present Weinberg's theory as an example of the procedure outlined in the preceding section. We classify the leptons with $SU(2)_L' \otimes SU(2)_R'$ with generators $\vec{F}_L', \vec{F}_R'$ among which only $\vec{F}_L', F_{3R}'$ [corresponding to a subgroup $SU(2) \otimes U(1)$] generate local transformations; the other generators correspond only to global transformations. We are denoting our generators with a prime for notational convenience which will become clear later. This formalism, as shown below, suggests that the electronic and muonic systems form a (badly) broken $SU(2)_L' \otimes SU(2)_R'$ multiplet. The classification is such that it generates exactly Weinberg's theory for leptons. Thus, it appears that as long as we consider *only* the leptonic system without any reference to the hadrons this classification is equivalent to Weinberg's. However, it will be shown that it suggests a *natural extension to the hadrons* (not implied by Weinberg) which will be crucial in the building of a unified theory of strong, weak, and electromagnetic interactions,[8] such that the group associated with strong interactions is $SU(3)\otimes SU(3)$.

The known leptons are embedded within $(2,\bar{2})$ and $(1,3)$ representations of $SU(2)'_L\otimes SU(2)'_R$ ($\nu_R=\nu_\mu^C$, D = doublet, S = singlet):

$$\psi_D=\begin{pmatrix}\nu_L & \mu_R^+\\ e_L^- & -\nu_R\end{pmatrix},\quad \psi_S=\begin{pmatrix}0 & \mu_L^+\\ e_R^- & 0\end{pmatrix}, \tag{3.1}$$

with the following transformation properties under $\mathfrak{U}(x)$, where $\mathfrak{U}(x)=\exp i[\vec{F}'_L\cdot\vec{\alpha}_L(x)+F'_{3R}\alpha_{3R}(x)]$:

$$\mathfrak{U}\psi_D\mathfrak{U}^{-1}=S'_L\psi_D S_R'^{-1},$$

$$\mathfrak{U}\psi_S\mathfrak{U}^{-1}=S'_R\psi_S S_R'^{-1},$$

where

$$S'_L(x)=e^{i\vec{\tau}\cdot\vec{\alpha}_L(x)/2},$$

$$S'_R(x)=e^{i\tau_3\cdot\alpha_{3R}(x)/2}.$$

The commutators of the generators with the fields are formally defined by the infinitesimal form of the transformation equations. Thus $[\vec{F}_L,\psi_D]=\frac{1}{2}\vec{\tau}\psi_D$, $[F_{3R},\psi_D]=-\psi_D\frac{1}{2}\tau_3$, $[F_{3R},\psi_S]=\frac{1}{2}[\tau_3,\psi_S]$. The electric charge is given as $Q=F'_{3L}+F'_{3R}$, which identifies the weak hypercharge $Y_W=F'_{3R}$. It can be checked that this charge assigns the correct charges to each field by commuting it with ψ_D and ψ_S. Electronic- and muonic-type leptons are not mixed by any local transformation (only F'_{3R} is included in our local group, while F'_{1R} and F'_{2R} are excluded). It is due to this choice of a local group that $SU(2)'_R$ is broken, and thus, electron- and muon-type leptons are distinguished by their $SU(2)'_R$ quantum numbers. We remark that the doublet

$$\hat{\psi}_\mu=\begin{pmatrix}\mu_R^+\\ -\nu_R\end{pmatrix}$$

is the "G-parity" conjugate of

$$\psi_\mu=\begin{pmatrix}\nu_{\mu L}\\ \mu_L^-\end{pmatrix},$$

i.e., $\hat{\psi}_\mu=i\tau_2\psi_\mu^C$. As is well known, under SU(2) both ψ_μ and $\hat{\psi}_\mu$ transform in exactly the same way; that is, if $\psi_\mu\to S'_L\psi_\mu$ (like electronic doublet) then also $\hat{\psi}_\mu\to S'_L\hat{\psi}_\mu$. Therefore, our local group is equivalent to Weinberg's when we consider *only the leptons*.

To make our classification more transparent, we remark that this is not the only possible 2×2 matrix classification of leptons. Another one given by Gürsey and Feinberg[13] (which can also be fitted to give Weinberg's theory of leptons) is very close to Weinberg's formulation

$$\psi_L=\begin{pmatrix}\nu_{eL} & \nu_{\mu L}\\ e_L^- & \mu_L^-\end{pmatrix},\quad \psi_R=\begin{pmatrix}0 & 0\\ e_R^- & \mu_R^-\end{pmatrix}. \tag{3.2}$$

In this case, we must take $Y_W=-(F'_{0L}+2F'_{0R})$ instead of F'_{3R}, so that $Q=F'_{3L}-F'_{0L}-2F'_{0R}$. Here ψ_L transforms only with $F'_{\alpha L}$ from the left, and ψ_R only with F'_{0R} from the left. It turns out that only the previous classification can be joined to an $SU(3)\otimes SU(3)$ classification of hadrons. These considerations hinge on the respective charge operators, and will become clear in Sec. V. From here on we concentrate on the classification of Eq. (3.1).

The $SU(2)'_L$ triplet of weak gauge bosons, $\vec{W}_\mu$, are assigned to a $(3,1)$ representation, the singlet B_μ is part of $(1,3)$, and Weinberg's scalars ϕ belong to a $(2,\bar{2})$ representation. We define

$$W_\mu=\sum_{i=1}^{3}W_\mu^i\tfrac{1}{2}\tau_i,$$

$$\phi=\phi_0\tfrac{1}{2}\tau_0+i\vec{\tau}\cdot\vec{\phi} \tag{3.4}$$

$$=\begin{pmatrix}\phi_0^+ & \phi_+\\ -\phi_- & \phi_0\end{pmatrix},$$

where $\mathfrak{U}\phi\mathfrak{U}^{-1}=S'_L\phi S_R'^{-1}$, etc. The covariant momentum is

$$\mathcal{P}_\mu=P_\mu+g\vec{W}_\mu\cdot\vec{F}'_L+g'B_\mu F'_{3R}. \tag{3.5}$$

By commuting $\mathcal{P}_\mu$ with each field, we obtain the covariant derivatives

$$\nabla_\mu\psi_D=\partial_\mu\psi_D-igW_\mu\psi_D+ig'\psi_D\tfrac{1}{2}\tau_3B_\mu,$$

$$\nabla_\mu\psi_S=\partial_\mu\psi_S-ig'B_\mu[\tfrac{1}{2}\tau_3,\psi_S], \tag{3.6}$$

$$\nabla_\mu\phi=\partial_\mu\phi-igW_\mu\phi+ig'\phi\tfrac{1}{2}\tau_3B_\mu.$$

The $F_{\mu\nu}^W$ and $F_{\mu\nu}^B$ are obtained from Eq. (2.12). We are now ready to write the most general (electron- and muon-number-conserving) gauge-invariant Lagrangian with dimension $d\leq 4$,

$$\mathcal{L} = -\tfrac{1}{4}F^{W}_{\mu\nu}F^{\mu\nu}_{W} - \tfrac{1}{4}F^{B}_{\mu\nu}F^{\mu\nu}_{B} - i\,\mathrm{Tr}\bar{\psi}_D \nabla\!\!\!/\, \psi_D - i\,\mathrm{Tr}\bar{\psi}_S \nabla\!\!\!/\, \psi_S + \mathrm{Tr}(G\bar{\psi}_D \phi\psi_S + \mathrm{H.c.})$$
$$+ m_0{}^2\,\mathrm{Tr}\phi^\dagger\phi - h(\mathrm{Tr}\phi^\dagger\phi)^2 - \tfrac{1}{4}\mathrm{Tr}[(\nabla^\mu\phi)^\dagger\nabla_\mu\phi]\,. \tag{3.7}$$

In order not to violate the local gauge invariance, as required by renormalizability, the numerical matrix G should satisfy $G = S_R' G S_R'^{-1}$ or $[\tau_3, G]=0$. Therefore, G is any diagonal matrix,

$$G = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \end{pmatrix}.$$

The photon can immediately be found by rewriting the covariant momentum in terms of a canonical redefinition of fields such that the charge operator $Q = F_{3L}' + F_{3R}'$ appears:

$$\mathcal{P}_\mu = P_\mu + gF_{1L}'W_{1\mu} + gF_{2L}'W_{2\mu} + g\sin\phi(F_{3L}' + F_{3R}')(\sin\phi W_{3\mu} + \cos\phi B_\mu)$$
$$+ \frac{g}{\cos\phi}(\cos^2\phi F_{3L}' - \sin^2\phi F_{3R}')(\cos\phi W_{3\mu} - \sin\phi B_\mu)\,, \tag{3.8}$$

where $\tan\phi = g'/g$. Thus, we can read off Weinberg's photon and electric charge as the coefficient of the charge operator;

$$A_\mu = \sin\phi W_{3\mu} + \cos\phi B_\mu\,,$$
$$e = g\sin\phi = \frac{gg'}{(g^2 + g'^2)^{1/2}}\,. \tag{3.9}$$

We emphasize that we have found the photon *before* spontaneous breakdown. This is because we knew *a priori* the form of the charge operator by making sure it assigned the correct physical charges to the particles in the theory. In fact, of course, it is quite general that specification of Q defines the photon and charges independent of spontaneous breakdown. In our later analysis we found it very convenient to follow this procedure, because it could show *a priori* whether a certain classification of particles involving both leptons and hadrons could give a massless, universal photon or not.

The spontaneous breakdown should be arranged such that the photon remains massless, while W_1, W_2, and $Z = \cos\phi W_3 - \sin\phi B$ become massive. For a massless photon we must demand

$$[Q, \phi]_{\phi = \langle\phi\rangle} = 0\,. \tag{3.10}$$

Thus, by taking $\langle\phi\rangle = \lambda\frac{1}{2}\tau_0$, i.e., $\langle\phi_0\rangle = \lambda$, we can give masses to the gauge bosons as well as the electron and muon. It is more convenient to use the covariant derivatives obtained with $\mathcal{P}_\mu$ of Eq. (3.8). With Weinberg we obtain

$$m_W = \tfrac{1}{2}g\lambda\,,$$
$$m_Z = \tfrac{1}{2}(g^2 + g'^2)^{1/2}\lambda\,,$$
$$G = \frac{2}{\lambda}\begin{pmatrix} m_e & 0 \\ 0 & m_\mu \end{pmatrix},$$
$$\lambda^2 = \frac{1}{\sqrt{2}\,G_W}\,. \tag{3.11}$$

With the single proviso below, the structure of our classification is just that of Weinberg.

The form of G emphasizes the point suggested earlier, that the source for the difference between the electron and the muon might be the breaking of $SU(2)_R'$. Furthermore, there is one amusing philosophical consequence of our representation. In Weinberg's original classification, the universality of electromagnetic charge is fixed by hand. However, by imagining that the leptons belong to a badly broken $SU(2)_L' \otimes SU(2)_R'$, universality of the electric charge is automatically obtained due to the construction of Q as a generator belonging to a non-Abelian group.

The present $SU(2)_L' \otimes SU(2)_R'$ classification of the leptons has been our starting point for a search through schemes to unify strong, weak, and electromagnetic interactions [such that strongly interacting particles are classified with $SU(3)_L \otimes SU(3)_R$]. We shall see that the introduction of the Cabibbo angle, resulting in unwanted $\Delta S = 1$ neutral currents, will suggest embedding the above classification in progressively larger matrices, finally resulting in the scheme of Sec. VI.

IV. MASSIVE GAUGE THEORY OF STRONG INTERACTIONS

Some time ago, Bardakci and Halpern[7] considered the problem of giving mass to strongly interacting vector and axial-vector gauge systems. Here we will, for completeness, give only the model with a final $U(3)_L \otimes U(3)_R$ symmetry – using the notation of Sec. II. We also indicate the direction we shall follow in unifying this model with a model of leptons like that of Weinberg, or with other such models.

The generators of the local group $U(3)_L \otimes U(3)_R$ are indicated as $F_{\alpha L}$ and $F_{\alpha R}$, $\alpha = 0, \ldots, 8$, with the representation $\frac{1}{2}\lambda_\alpha$ (left or right), where λ_α are the usual 3×3 SU(3) matrices.

We introduce the vector (V_μ) and axial-vector (A_μ) gauge fields in the matrix form

$$V_\mu^L = \sum_0^8 \tfrac{1}{2}\lambda_\alpha (V_\mu^\alpha - A_\mu^\alpha),$$
$$V_\mu^R = \sum_0^8 \tfrac{1}{2}\lambda_\alpha (V_\mu^\alpha + A_\mu^\alpha). \tag{4.1}$$

These transform as (8, 1) and (1, 8) representations, respectively, under the local gauge-transformation operator $\mathfrak{U}(x) = \exp i[\alpha_L \cdot F_L + \alpha_R \cdot F_R]$:

$$V_\mu^L \to S_L\left(V_\mu^L + \frac{i}{f}\partial_\mu\right)S_L^{-1},$$
$$V_\mu^R \to S_R\left(V_\mu^R + \frac{i}{f}\partial_\mu\right)S_R^{-1},$$
$$S_L(x) = \exp i[\tfrac{1}{2}\lambda \cdot \alpha_L(x)],$$
$$S_R(x) = \exp i[\tfrac{1}{2}\lambda \cdot \alpha_R(x)]. \tag{4.2}$$

The Higgs-Kibble mechanism which will give mass to *all* gauge particles (no photon) is generated by introducing two 3×3 complex matrices, M_L and M_R, which transform under $\mathfrak{U}(x)$ like sets of $SU(3)_{L,R}$ triplets (3, 1) and (1, 3);

$$\mathfrak{U} M_L \mathfrak{U}^{-1} = S_L(x) M_L,$$
$$\mathfrak{U} M_R \mathfrak{U}^{-1} = S_R(x) M_R. \tag{4.3}$$

We remark here that the 3×3 matrices M_L and M_R transform only from *one side* with the *local group* generated by F_L^α and F_R^α. There is also the freedom of applying more transformations from the *second side* with a "primed" group which will be generated by some other operators $F_L'^\alpha$, $F_R'^\alpha$,

$$M_L \to M_L S_L'^{-1}, \quad M_R \to M_R S_R'^{-1}. \tag{4.4}$$

The "primed" group here is the "global" group of Ref. 7. Then, under the group generated by $(F_L^\alpha, F_L'^\alpha)$, M_L would be a set of fields in the $(3, \overline{3})_L$ representation, and similarly for M_R. The "primed" transformations are *not local* transformations in this discussion. However, in the coming sections where we introduce the leptons and weak gauge bosons as well, they will be classified with a *local* $SU(2)_L' \otimes U(1)'$ subgroup embedded in the "primed" group.

The covariant derivatives are easily obtained from the covariant momentum operator

$$\mathcal{P}_\mu = P_\mu + f V_\mu^{L\alpha} F_{L\alpha} + f V_\mu^{R\alpha} F_{R\alpha}, \tag{4.5}$$

with the same coupling f for both left and right gauge bosons to preserve parity. We get

$$\nabla_\mu M_L = \partial_\mu M_L - if V_\mu^L M_L,$$
$$\nabla_\mu M_R = \partial_\mu M_R - if V_\mu^R M_R, \tag{4.6}$$

and $F_{\mu\nu}^{L\alpha}$ and $F_{\mu\nu}^{R\alpha}$ obtained from Eq. (2.12) in terms of $V_\mu^{\alpha L}$ and $V_\mu^{\alpha R}$. The gauge-invariant Lagrangian with dimension $d \le 4$ is

$$\mathcal{L} = -\tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu}^L F_L^{\mu\nu} + F_{\mu\nu}^R F_R^{\mu\nu}) - \tfrac{1}{2}\mathrm{Tr}[(\nabla_\mu M_L)^\dagger \nabla^\mu M_L + (\nabla_\mu M_R)^\dagger \nabla^\mu M_R] + m_0^2\,\mathrm{Tr}(M_L^\dagger M_L + M_R^\dagger M_R)$$
$$+ h_1\{[\mathrm{Tr}(M_L^\dagger M_L)]^2 + [\mathrm{Tr}(M_R^\dagger M_R)]^2\} + h_2\,\mathrm{Tr}[(M_L^\dagger M_L)^2 + (M_R^\dagger M_R)^2] \tag{4.7}$$

where we have written $F_{\mu\nu}^L = \sum_0^8 \frac{1}{2}\lambda^\alpha F_{\mu\nu}^{\alpha L}$, etc.

The gauge symmetry is broken spontaneously by taking

$$\langle M_L \rangle = \langle M_R \rangle = \kappa 1. \tag{4.8}$$

There are 18 massless Goldstone bosons, which are identified as $M_L^\dagger - M_L$ and $M_R^\dagger - M_R$, and which are eliminated by using the 18 degrees of freedom generated by F_L^α and F_R^α. The Goldstone bosons become the longitudinal degrees of freedom of the massive vector and axial vector mesons with masses

$$m_V^2 = m_A^2 = f^2\kappa^2. \tag{4.9}$$

The remaining scalar particles are the Hermitian part of M_L and M_R, and have arbitrary $SU(3) \otimes SU(3)$-invariant masses. The final Lagrangian then is obtained from (4.7) by replacing $M_L \to M_L + \kappa 1$ and $M_R \to M_R + \kappa 1$, where now M_L and M_R are *Hermitian* matrices. As observed by Bardakci and Halpern, this final Lagrangian is *invariant* under a *global final group* $U(3)_L \otimes U(3)_R$ generated by $(F_L + F_L')_\alpha$ and $(F_R + F_R')_\alpha$.

Hadron currents. Here we also discuss the structure of the currents associated with the *final (product)* group. As in massive Yang-Mills theories (in general), we distinguish two kinds of currents, both conserved but with equal charges. The first is the usual Noether current(s) J_N^μ generated by the transformation

$$V_\mu \to S V_\mu S^{-1},$$
$$M + \kappa \to S(M + \kappa)S^{-1}, \tag{4.10}$$

(left and right); the second kind (J^μ) is associated with the transformation

$$V_\mu \to S\left(V_\mu + \frac{i}{f}\partial_\mu\right)S^{-1},$$
$$(M + \kappa) \to S(M + \kappa)S^{-1} \tag{4.11}$$

(left and right). In an ordinary massive Yang-Mills model, J^μ is proportional to the vector-meson field. In our case, because masses arise spontaneously, we will obtain a modified field-current identity – however, as is generally true, we maintain

$$(+f^{-1})\partial_\mu F^{\mu\nu}+J^\nu=J_N^\nu\,. \tag{4.12}$$

As seen below, weak interactions and electromagnetism do in fact couple to J^ν rather than J_N^ν, so we give next some more of its structure. The variation (4.11) gives $[S\approx 1+i\alpha(x)]$

$$\delta\mathfrak{L}=\mathrm{Tr}[J^\mu(x)\partial_\mu\,\alpha(x)]\,,$$
$$J_\mu=+\tfrac{1}{2}i[M,\partial_\mu M]+f(M+\kappa)V_\mu(M+\kappa)$$

in the unitary gauge. We notice that, in the absence of the M's, we have the usual field-current identity.

The algebra of the J_μ is found in the usual manner, using (4.12). The results are almost those of field algebra, with the exception that the usual c-number Schwinger term C in the space-time algebra becomes an operator. Where algebra of fields has $\delta_{\alpha\beta}C$, $C=m_0^{\,2}/f^2$, we obtain

$$c_{\alpha\beta}(\text{operator})=(M+\kappa)_{\alpha\beta}{}^2 = \frac{m_0^{\,2}}{f^2}\,\delta_{\alpha\beta}+(\text{operator terms})_{\alpha\beta}\,. \tag{4.13}$$

The algebra including time derivatives of currents is more complicated and will be discussed elsewhere.[14]

In the presence of additional hadrons, such as quarks, pions, etc., J^μ does not change, while J_N^μ acquires extra terms involving the additional fields.

V. UNIFICATION OF HADRONIC AND LEPTONIC GAUGE THEORIES

A. Extension of Hadron Theory and General Considerations

As already suggested, the path we will explore for the unification of hadronic and leptonic models is the freedom of making *local* a certain subgroup of the "primed" group, and classifying the leptons with it. The following picture emerges: Hadrons are classified and transform only with the unprimed $\mathrm{U}(3)_L\otimes\mathrm{U}(3)_R$ chiral local group, while leptons are classified and transform with only the primed group. Gauge invariance does not allow any direct lepton-hadron interactions before spontaneous breakdown. The only fields that transform with both groups are M_L and M_R, and they couple to both strong and weak gauge bosons. Thus, M_L and M_R play the role of a "bridge" between hadrons and leptons. Before spontaneous breakdown all semileptonic and nonleptonic weak interactions occur through intermediate M_L and M_R loops. After spontaneous breakdown, however, we generate direct mixings between strong and weak gauge bosons, so that at low energies semileptonic and nonleptonic weak interactions occur through vector-meson dominance.

For hadrons, we consider $\mathrm{U}(3)_L\otimes\mathrm{U}(3)_R$ chiral theory,[15] which includes the usual quarks, and $(3,\overline{3})$ scalar and pseudoscalar mesons $(\Sigma=\sigma+i\pi)$, as well as vector and axial-vector mesons. These fields transform *only* with the unprimed generators F_L^α and F_R^α of Sec. IV:

$$\begin{aligned}&q_L\to S_Lq_L,\quad q_R\to S_Rq_R,\quad \Sigma\to S_L\Sigma S_R{}^{-1}\\ &V_L^\mu\to S_L[V_L^\mu+(i/f)\partial^\mu]S_L{}^{-1},\\ &V_R^\mu\to S_R[V_R^\mu+(i/f)\partial^\mu]S_R{}^{-1}.\end{aligned} \tag{5.1}$$

In this model, gauge invariance does not allow mass terms for V_L, V_R, and q. Masses for these fields can only be generated through spontaneous breakdown. For quarks we need a term in the Lagrangian of the type $\alpha\overline{q}_L\Sigma q_R+\mathrm{H.c.}$ (this is another reason for introducing Σ), and to generate masses for <u>all</u> vector and axial-vector mesons we have to introduce the Bardakci-Halpern scalars[7] M_L and M_R of Sec. IV. Notice that we cannot break $\mathrm{SU}(3)_L\otimes\mathrm{SU}(3)_R$ in the usual way, by adding a linear term in Σ, like $\mathrm{Tr}(f\Sigma)$ (f is a numerical matrix). This would spoil the gauge invariance (and hence renormalizability). Such a linear term must be induced only through spontaneous breakdown from a gauge-invariant term. If only a hadron theory is desired, such a term is easy to find: $\mathrm{Tr}(GM_L^\dagger\Sigma M_R)+\mathrm{H.c.}$, where G is a numerical "insertion" of form

$$\begin{pmatrix} a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & b\end{pmatrix}.$$

In the presence of leptons the term is a bit harder, but we shall find later just such a term which, in the limit of no weak or electromagnetic interactions, reduces to just the above hadronic term.

Again, if only a hadron theory is desired, M_L and M_R may be taken 3×3. In the presence of leptons, however, we will need to take them as 3×4 matrices (to eliminate neutral strangeness-changing currents). In general of course, we can enlarge to $3\times n$, $n\geqslant 3$, thus enlarging the "primed" group to $\mathrm{U}(n)'\otimes\mathrm{U}(n)'$. No Goldstone bosons will couple as long as the "new" columns do not develop any vacuum expectation values (the extra global symmetries associated with the extra columns should be broken by hand). Thus

$$\langle M_L\rangle=\langle M_R\rangle=\kappa=\begin{pmatrix}\kappa_1 & 0 & 0 & 0\cdots 0\\ 0 & \kappa_1 & 0 & 0\cdots 0\\ 0 & 0 & \kappa_2 & 0\cdots 0\end{pmatrix}. \tag{5.2}$$

We find that the smallest n we need is $n=4$. This, we find below, eliminates all neutral strangeness-changing processes to first order.

B. "Induction" of Leptonic Structure from Hadrons

Among the constraints on the unification, two are particularly worth focusing on. These are (a) proper introduction of Cabibbo angle and (b) having a universal, massless photon. These play particularly crucial roles in the choice of a local group to classify the leptons, as a subgroup of the "primed" group (as well as its representations).

We first consider the photon, whose structure is closely related to the construction of the charge operator Q. In the type of theory we want to propose Q will be constructed from the unprimed as well as the primed generators. The unprimed part, which will assign the correct charges to quarks, Σ, V_μ, and A_μ, is the usual $SU(3)_L \otimes SU(3)_R$ charge operator, namely

$$(F_{3L}+F_{3R})+\frac{1}{\sqrt{3}}\,(F_{8L}+F_{8R}).$$

The primed part of the charge operator should first of all give the correct charges for the leptons. We assume that we have chosen $SU(2)'_L \otimes Y'$ as a subgroup of a $U(n)'_L \otimes U(n')_R$ group. We choose $SU(2)'_L \otimes Y'$ both because it is the natural group of the known leptons, and because, as it will turn out, our hadrons will not connect to any smaller leptonic group. Of course we will search for the smallest n compatible with data.

Thus, the total charge operator is

$$Q=F_{3L}+F_{3R}+\frac{1}{\sqrt{3}}\,(F_{8L}+F_{8R})+F'_{3L}+Y'. \tag{5.3}$$

The "primed" part of the charge operator $F'_{3L}+Y'$ determines the weak hypercharge Y'. At this point, the crucial ingredient in our induction is the known charges of the Bardakci-Halpern scalars $M_{L,R}$; e.g., their diagonal entries, which acquire a vacuum expectation value, must be neutral.

More precisely, the charges of $M_{L,R}$ are determined by the *representations* of the unprimed and primed parts of Q, which we denote by $Q_s^{L,R}$ and $Q_w^{L,R}$, respectively (s=strong, w=weak). We have already determined $Q_s^{L,R}$ as the usual SU(3) charge matrix,

$$Q_s^{L,R}=\begin{pmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\frac{1}{3} & 0\\ 0 & 0 & -\frac{1}{3}\end{pmatrix}.$$

The charges of each entry of the matrices M_L and M_R are found by computing

$$[Q,M_L]=Q_s^L M_L - M_L Q_w^L, \tag{5.4}$$

etc. The above commutator is determined as explained in Sec. II, with the transformation properties of M_L and M_R as in Eqs. (4.3) and (4.4).

To assure a massless photon we have to satisfy

$$[Q,M_L]_{M_L=\langle M_L\rangle}=[Q,M_R]_{M_R=\langle M_R\rangle}=0. \tag{5.5}$$

Since $\langle M_L\rangle=\langle M_R\rangle=\kappa$ as in Eq. (5.2), Q_w must then satisfy

$$Q_s^{L,R}\kappa-\kappa Q_w^{L,R}=0. \tag{5.6}$$

Therefore, $Q_w^{L,R}$ must have the form

$$Q_w^{L,R}=\begin{bmatrix}\frac{2}{3} & 0 & 0 & & & \\ 0 & -\frac{1}{3} & 0 & & 0 & \\ 0 & 0 & -\frac{1}{3} & & & \\ & & & ? & & \\ & 0 & & & ? & \\ & & & & & \ddots \\ & & & & & & ?\end{bmatrix}. \tag{5.7}$$

That is, the 3×3 submatrix has the *same* form as $Q_s^{L,R}$. Now, if we embed $SU(2)'_L \otimes Y'$ in $U(3)'_L \otimes U(3)'_R$ and take $SU(2)'_L$ as the isospin subgroup of $U(3)'_L$, then the primed part of the charge operator is uniquely determined as $(F'_{3L}+F'_{3R})+(1/\sqrt{3})(F'_{8L}+F'_{8R})$; therefore

$$Y'=F'_{3R}+\frac{1}{\sqrt{3}}\,(F'_{8L}+F'_{8R}). \tag{5.8}$$

This suggests that Weinberg's leptons should be embedded in a 3×3 matrix with the notation of Sec. III:

$$\psi_D=\begin{pmatrix}\nu_L & \mu_R^+ & 0\\ e_L^- & -\nu_R & \\ 0 & 0 & \end{pmatrix},\qquad \psi_S=\begin{pmatrix}0 & \mu_L^+ & 0\\ e_R^- & 0 & \\ 0 & 0 & \end{pmatrix}. \tag{5.9}$$

Notice that the operator $(F'_{8L}+F'_{8R})$ commutes with *both* ψ_D and ψ_S, if these fields transform as specified in Sec. III [taking the isospin subgroup of SU(3) with λ_α matrices instead of the 2×2 Pauli matrices in S'_L and S'_R]. Therefore, the hypercharge Y' is assigned to ψ_D and ψ_S only by F'_{3R}, just as in Sec. III. Thus the charge operator that we have just chosen also assigns the correct charges to the leptons. This is why we think Weinberg's leptons are much more suggestive if classified as in Eq. (3.1) [rather than as in Eq. (3.2)]. Thus (as mentioned above) the presence of the hadrons in a sense distinguishes the electron- and muon-type leptons by their $SU(2)'_R$ quantum numbers, and gives a zero-charge neutrino.

As already mentioned and as shown in Appendix A, $U(3)'_L \otimes U(3)'_R$ will turn out to be unsatisfactory. The next most economic scheme is an embedding of $SU(2)'_L \otimes Y'$ in $U(4)'_L \otimes U(4)'_R$. Out of the generators $F'^{L,R}_\alpha$, $\alpha = 0, \ldots, 15$, we choose $F'^{L}_{1,2,3}$ to form an $SU(2)'_L$ plus a $U(1)'$ operator Y'. For reasons that will become clear shortly, the representation we want to use for these generators has the form (left or right)

$$t_\alpha = \begin{pmatrix} \frac{1}{2}\tau_\alpha & 0 \\ 0 & \frac{1}{2}\hat{\tau}_\alpha \end{pmatrix},$$

where (5.10)

$$\tau_\alpha = \text{Pauli matrices} = (\tau_0, \tau_1, \tau_2, \tau_3),$$

$$\hat{\tau}_\alpha = \tau_2 \tau_\alpha \tau_2 = (\tau_0, -\tau_1, \tau_2, -\tau_3).$$

Clearly $t_{0,1,2,3}$ form a U(2) algebra. Equation (5.7) is satisfied if we take $Q_w^{L,R} = t_3 + \frac{1}{3} t_0$. This suggests that the primed charge operator has the form $(F'_{3L} + F'_{3R}) + \frac{1}{3}(F'_{0L} + F'_{0R})$, determining

$$Y' = F'_{3R} + \tfrac{1}{3}(F'_{0L} + F'_{0R}). \tag{5.11}$$

Now, if we embed the leptons of Sec. III in a 4×4 matrix,

$$\psi_D = \begin{pmatrix} \nu_L & \mu_R^+ & & \\ e_L^- & -\nu_R & & \\ & & 0 & \\ & & & ? \end{pmatrix},$$

$$\psi_S = \begin{pmatrix} 0 & \mu_L^+ & & \\ e_R^- & 0 & & \\ & & 0 & \\ & & & ? \end{pmatrix}, \tag{5.12}$$

and let them transform as before (but with t_α replacing $\frac{1}{2}\tau_\alpha$ in S'_L and S'_R), we see that $F'_{0L} + F'_{0R}$ commutes with both ψ_D and ψ_S. Again only F'_{3L} assigns the value of the hypercharge Y' to the leptons as in Sec. III. The question marks (?) in (5.12) are suggestive of the presence of new heavy leptons. In fact, in order to cancel anomalies we will need new leptons which will fill the spaces marked by (?); these will also fix the fourth entry of $Q_w^{L,R}$ in Eq. (5.7) exactly as given by (5.11). Discussion of heavy leptons will be continued in Sec. VI.

So far we have seen how the determination of the charge operator has greatly restricted our choice of the $SU(2)'_L \otimes Y'$ subgroup. However, we still need to introduce the Cabibbo angle. This and the requirement of no $\Delta S = 1$ neutral currents will finally determine the representation of the $SU(2)'_L \otimes Y'$ algebra which we need.

According to Cabibbo's theory, the weak currents which are part of an SU(2) multiplet "see" a rotated picture of the hadrons. Therefore, the weak $SU(2)'_L$ group must be rotated with respect to the strong $SU(3)_L$ group in such a way that the weak gauge bosons "see" a left-handed strong isospin current slightly rotated in the λ_7 direction. Weak gauge boson couplings occur in our model through the scalars M_L and M_R, which after spontaneous breakdown generate terms of the form (see later sections)

$$\operatorname{Tr}(V_L^\mu \kappa \tilde{W}_\mu \kappa^\dagger), \tag{5.13}$$

where $\tilde{W}_\mu$ is a rotated matrix $\tilde{W}_\mu = R W_\mu R^{-1}$, $W_\mu = \sum_{\alpha=1}^{3} W_\mu^\alpha t_\alpha$, $R =$ Cabibbo rotation. With the form of κ given in Eq. (5.2) we see that R must be chosen as

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \cdots 0 \\ 0 & \cos\theta & \sin\theta & 0 \cdots 0 \\ 0 & -\sin\theta & \cos\theta & 0 \cdots 0 \\ 0 & 0 & 0 & 1 \cdots \vdots \\ \vdots & \vdots & \vdots & \ddots\ 0 \\ 0 & 0 & 0 & 0 \cdots 1 \end{bmatrix}, \tag{5.14}$$

where θ is the Cabibbo angle.

Thus our group is now generated by the operators $\tilde{F}'^{L}_{1,2,3}$, $\tilde{Y}'$ (rotated with respect to $F'^{L}_{1,2,3}$ and Y'). In the simplest case of $SU(3)'_L \otimes SU(3)'_R$ these are represented by $\tilde{\lambda}_\alpha = R\lambda_{1,2,3,8}R^{-1}$ for the left-handed generators, and unrotated $\lambda_{3,8}$ for the right-handed generators. For the case of $SU(4)'_L \otimes SU(4)'_R$, we represent the left- or right-handed generators by $\tilde{t}_\alpha = R t_\alpha R^{-1}$, where t_α are as given in Eq. (5.10). The representation of Q is invariant under R in both cases. However, it is only in the latter case that the neutral operators do not (Cabibbo) rotate $\tilde{t}_3 = t_3$, $\tilde{t}_0 = t_0$. As will be seen, this is why the neutral strangeness-changing currents are eliminated. For this reason, we relegate further discussion of the $SU(3)'_L \otimes SU(3)'_R$ to Appendix B, and continue here with the preferred $SU(4)'_L \otimes SU(4)'_R$ scheme.

Finally, we remark that to maintain the couplings of the unrotated leptonic world unchanged we need of course rotate the entire leptonic representation (so that they would not be aware of the rotated generators). Thus, we will formally introduce a fully rotated notation:

$$\tilde{\psi}_D \equiv R\psi_D R^{-1}, \quad \tilde{\psi}_S = R\psi_S R^{-1},$$
$$\tilde{S}'_L = R S'_L R^{-1}, \quad \tilde{S}'_R = R S'_R R^{-1} \tag{5.15}$$

which transform as above, namely, under the "primed" group

$$\begin{aligned}&\tilde{\psi}_D \to \tilde{S}'_L \tilde{\psi}_D \tilde{S}'^{-1}_R ,\\ &M_L \to M_L \tilde{S}'^{-1}_L ,\\ &M_R \to M_R \tilde{S}'^{-1}_R ,\end{aligned} \tag{5.16}$$

etc. Actually, this is equivalent to saying that weak gauge bosons couple to hadrons ($M_{L,R}$) with rotated $\tilde{t}_\alpha$, and to leptons with unrotated t_α. In particular, $\tilde{S}'_R = S_R$, so some of this formalism is for notational convenience.

We have given reasons for our choice of the $SU(2)'_L \otimes Y'$ group, and its representations. Thus much has followed from no $\Delta S=1$ neutral currents and known hadron charges. More problems remain to be solved, such as medium-strong $SU(3) \otimes SU(3)$ breaking in the presence of leptons (with no physical Goldstone bosons), cancellation of anomalies, etc. These will be discussed after we construct explicitly our model in the coming sections.

VI. UNIFIED MODEL

A. Construction of Lagrangian

As we saw in the last section and Appendix B, the use of $SU(3)'_L \otimes SU(3)'_R$ as the primed group for embedding the leptons led to trouble in general with strangeness-changing processes. Speaking generally, then, we have at this juncture two possible directions: One choice, followed by most authors,[6] is to try enlarging the hadronic group (more quarks, etc.); as explained above, we consider this unesthetic at least, and in fact such attempts do not solve the "strangeness" problems in our case anyway. Thus our choice[8] will be, as anticipated above, to enlarge the primed group to $U(4)'_L \otimes U(4)'_R$.

For the sake of elegance, we will present the model in a unified (strong, weak, and electromagnetic) supermatrix notation. For example, the general local operator transformation is represented by the 14×14 supermatrix

$$\mathfrak{U} = \exp i(\alpha_L \cdot F_L + \alpha_R \cdot F_R + \beta \cdot \tilde{F}'_L + \gamma Y) ,$$

$$S = \begin{pmatrix} S_L & 0 & 0 & 0 \\ 0 & S_R & 0 & 0 \\ 0 & 0 & \tilde{S}'_R & 0 \\ 0 & 0 & 0 & \tilde{S}'_L \end{pmatrix} , \tag{6.1}$$

where, as detailed in Sec. V,

$$\begin{aligned}&\tilde{S}'_L = \exp i(\tilde{t} \cdot \beta + \tfrac{1}{3} t_0 \gamma),\\ &\tilde{S}'_R = \exp i(t_3 + \tfrac{1}{3} t_0)\gamma ,\end{aligned} \tag{6.2}$$

and so on. Recall that the operation denoted by a tilde is the Cabibbo rotation.

In the same notation we represent all vector mesons (strong, weak, and electromagnetic) as a similar 14×14 matrix with diagonal entries,

$$V^\mu : \; [f V^\mu_L , f V^\mu_R , g'(t_3 + \tfrac{1}{3} t_0) B^\mu , g \tilde{W}^\mu + \tfrac{1}{3} g' t_0 B^\mu] . \tag{6.3}$$

Then the unified gauge transformation on V^μ is just

$$\mathfrak{U} V_\mu \mathfrak{U}^{-1} = S(V_\mu + i\partial_\mu) S^{-1} . \tag{6.4}$$

Similarly, for Weinberg's leptons, we introduce the SU(2) doublets ψ_D and the SU(2) singlets ψ_S as in Eq. (5.12). To fit them into a supermatrix notation l, we define rotated quantities

$$\tilde{\psi}_D = R \psi_D R^{-1} , \quad \tilde{\psi}_S = R \psi_S R^{-1} , \tag{6.5}$$

$$l = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\psi}_S & \frac{1}{\sqrt{2}} \tilde{\psi}^C_D \\ 0 & 0 & \frac{1}{\sqrt{2}} \tilde{\psi}_D & 0 \end{bmatrix} \tag{6.6}$$

(ψ^C_D: charge conjugated in Dirac space and transposed in matrix space). We then specify the transformation $\mathfrak{U} l \mathfrak{U}^{-1} = S l S^{-1}$. This means that the unrotated ψ_D and ψ_S transform with the *unrotated* representations S'_L and S'_R:

$$\psi_D \to S'_L \psi_D S'^{-1}_R , \quad \psi_S \to S'_R \psi_S S'^{-1}_R . \tag{6.7}$$

Thus, they belong respectively to $(4, \bar{4})$ and $(1, 15)$ representations of $U(4)'_L \otimes U(4)'_R$. For the scalar mesons, the supermatrix notation is most symmetric:

$$M \equiv \begin{pmatrix} 0 & \Sigma & 0 & M_L/\sqrt{2} \\ \Sigma^\dagger & 0 & M_R/\sqrt{2} & 0 \\ 0 & M^\dagger_R/\sqrt{2} & 0 & \tilde{\phi}^\dagger \\ M^\dagger_L/\sqrt{2} & 0 & \tilde{\phi} & 0 \end{pmatrix} ,$$
$$\mathfrak{U} M \mathfrak{U}^{-1} = S M S^{-1} . \tag{6.8}$$

Here $\Sigma \equiv \sigma + i\pi$ is the usual $(3, \bar{3})$ multiplet of scalars and pseudoscalars; $M_{L,R}$ are now *three-by-four* complex matrices (one extra column to support the enlarged primed group), and $\tilde{\phi}$ is the rotated Weinberg scalar $\tilde{\phi} = \phi_0 t_0 + i \phi \cdot \tilde{t}$. The notation emphasizes that (a) $M_{L,R}$ are the *only* fields in the model which transform under both the hadronic and leptonic groups, and (b) Weinberg's ϕ is to the leptonic system precisely what Σ is to the hadronic. It is this symmetry which will allow us to construct a $(3, \bar{3}) + (\bar{3}, 3)$ symmetry-breaking term in the model. We take leptons and quarks, for the moment at least, as discussed in the previous sections.

Covariant derivatives are formed in the usual fashion, leading to our unified gauge-invariant Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}\mathrm{Tr}(F_L^{\mu\nu}F_{\mu\nu}^L + F_R^{\mu\nu}F_{\mu\nu}^R) - \tfrac{1}{4}F_{\mu\nu}^B F_B^{\mu\nu} - \tfrac{1}{4}\mathrm{Tr}(F_{\mu\nu}^W F_W^{\mu\nu}) - i\bar{q}\nabla^\mu\gamma_\mu q - i\,\mathrm{Tr}(\bar{l}\nabla^\mu\gamma_\mu l) - \tfrac{1}{8}\mathrm{Tr}[(\nabla^\mu M)^\dagger\nabla_\mu M]$$
$$+\beta(\bar{q}_L\Sigma q_R + \mathrm{H.c}) + \mathrm{Tr}(\bar{\psi}_D\phi\psi_S G + \mathrm{H.c}) + V(M_L) + V(M_R) + V(\Sigma) + V(\phi)$$
$$+\mathrm{Tr}(G_2\tilde{\phi}^\dagger M_L^\dagger M_L\tilde{\phi} + G_2 M_R^\dagger M_R\tilde{\phi}^\dagger\tilde{\phi}) + \mathrm{Tr}(G_1\tilde{\phi}^\dagger M_L^\dagger\Sigma M_R + \mathrm{H.c}). \tag{6.9}$$

Here the $V(\cdots)$'s are the usual quartic and quadratic terms,[15] with certain G "insertions",[7] which are 4×4 numerical matrices which, because they break only F_R'-type symmetries, do not spoil the unified gauge invariance. In particular, we write their diagonal entries as

$$G:\ \frac{2}{\lambda}(m_e, m_\mu, ?, ?),$$
$$G_1:\ \frac{\sqrt{2}}{2\lambda}\left(\frac{f_\pi m_\pi^2}{\kappa_1^2}, \frac{f_\pi m_\pi^2}{\kappa_1^2}, \frac{2f_K m_K^2 - f_\pi m_\pi^2}{\kappa_2^2}, d\right),$$
$$G_2:\ (a, a, b, c)\,. \tag{6.10}$$

The interpretation of these parameters will be clarified in the following paragraph.

We come now to the spontaneous breakdown. First, we use 21 degrees of gauge freedom (all but Q) to eliminate the 3×3 submatrices of $M_L - M_L^\dagger$ and $M_R - M_R^\dagger$, and all the components of ϕ except ϕ_0. With an eye to the charge operator (5.3), we next assign the charge-conserving vacuum expectation values

$$\langle\phi\rangle = \lambda t_0, \quad \langle M_L\rangle = \langle M_R\rangle = 2\kappa. \tag{6.11}$$

We shall return in a moment to the specific allowed form of κ, but first notice that (6.11) generates, through the last term in $\mathcal{L}$, a linear term in the Σ field. Thus Σ itself acquires a vacuum expectation value $\langle\Sigma\rangle \equiv v$, which is the *usual* $(3,\bar{3}) + (\bar{3},3)$ hadronic symmetry breaking in the spirit of Gell-Mann, Oakes, and Renner.[16]

The allowed forms for κ and v require a detailed discussion of this complicated scalar system. Such is tedious and not terribly illuminating, but can be found in Appendix A. Here we only state that to lowest order, neglecting weak effects, we can take the following isospin- and strangeness-conserving vacuum expectation values[17]: λ arbitrary and

$$\kappa = \begin{pmatrix}\kappa_1 & 0 & 0 & 0\\ 0 & \kappa_1 & 0 & 0\\ 0 & 0 & \kappa_2 & 0\end{pmatrix},$$
$$v = \begin{pmatrix} v_1 & 0 & 0\\ 0 & v_1 & 0\\ 0 & 0 & v_2\end{pmatrix}, \tag{6.12}$$

with no massless Goldstone mesons. Except for d, the interpretation of the parameters in G_1 and v is standard,[15] while G_2, d, and $V(\cdots)$ can be adjusted to give arbitrarily large masses to ϕ_0 and the remaining scalars in M_L and M_R. The vacuum expectation values v_i are directly related to the pseudoscalar decay constants f_π and f_K.[15]

To illustrate the meaning of the κ_i, we also list (ignoring electromagnetic mixing of ρ, ϕ, ω, etc. for the moment) some (bare) vector-meson masses after spontaneous breakdown[18]:

$$m_\rho^2 = m_\omega^2 = f^2\kappa_1^2,$$
$$m_\phi^2 = f^2\kappa_2^2,$$
$$m_{K^*}^2 = \tfrac{1}{2}f^2[\kappa_1^2 + \kappa_2^2 + (v_1 - v_2)^2],$$
$$m_{A_1}^2 = m_{\omega_A}^2 = f^2(\kappa_1^2 + 2v_1^2),$$
$$m_{\phi_A}^2 = f^2(\kappa_2^2 + 2v_2^2),$$
$$m_{K_A}^2 = \tfrac{1}{2}f^2[\kappa_1^2 + \kappa_2^2 + (v_1 + v_2)^2].$$

With proper choices of κ_1, κ_2, v_1, and v_2, these formulas for the vector-meson masses are well satisfied by experiment. Further, $W^\pm$ and Z also get a small extra contribution to their masses, due to κ. Such relations should be taken together with a number of remarks: (1) Electromagnetic mixing, to be discussed below, gives order-e^2 corrections to $\rho\omega\phi$ masses. (2) Ignoring (1) and the presence of the remaining $M_{L,R}$ terms – which influence the known hadrons only through loops – the hadron theory is just a familiar mass-mixing Yang-Mills σ model. Of course, with $\kappa_1 \neq \kappa_2$, we lose the second Weinberg sum rule, so in general we prefer $\kappa_1 = \kappa_2$, leaving ω-ϕ splitting to higher order. (3) Frankly, we do not know whether our Lagrangian will be more useful as an effective Lagrangian or as a guide to nonperturbative structure and the currents of the strong interactions. In general, we will discuss whichever view (or both) when they appear interesting.

B. Photon System and Vector-Meson Diagonalization

The structure of our theory with respect to these topics is somewhat unusual. As discussed above, the (massless, universal) photon is found as the coefficient of Q in the covariant momentum $(F_\alpha = F_{\alpha L} + F_{\alpha R})$:

$$f(F_3V_3+F_8V_8)+gF'_{3L}W_3+g'Y'B=eQ\cdot A+\frac{f}{\cos\eta}\,[(\tfrac{1}{2}\sqrt{3}\,F_3+\tfrac{1}{2}F_8)-\tfrac{1}{2}\sqrt{3}\sin^2\eta\,Q]\,V_{38}$$
$$+f(-\tfrac{1}{2}F_3+\tfrac{1}{2}\sqrt{3}\,F_8)V'_{38}+\frac{g}{\cos\phi}(\cos^2\phi\,F'_{3L}-\sin^2\phi\,Y')Z\,, \tag{6.14}$$

where

$$e=g\sin\phi\cos\eta,$$
$$\tan\phi=\frac{g'}{g},\ \tan\eta=\frac{2g\sin\phi}{\sqrt{3}f}\,,$$
$$A^\mu=\cos\eta(\sin\phi\,W_3^\mu+\cos\phi\,B^\mu)+\sin\eta(\tfrac{1}{2}\sqrt{3}\,V_3^\mu+\tfrac{1}{2}V_8^\mu)\,, \tag{6.15}$$
$$V_{38}\equiv\cos\eta(\tfrac{1}{2}\sqrt{3}\,V_3+\tfrac{1}{2}V_8)-\sin\eta(\sin\phi\,W_3+\cos\phi\,B)\,,$$
$$V'_{38}\equiv-\tfrac{1}{2}V_3+\tfrac{1}{2}\sqrt{3}\,V_8\,.$$

With $f^2/4\pi\sim 2$ and g, g' small, we obtain approximately Weinberg's $e\sim gg'/(g^2+g'^2)^{1/2}$.

Because of the diagonalization, our picture of electromagnetic effects is unusual. As a first indication of this, we notice that we are forced to electromagnetically mix the strong vector mesons. The $\rho\phi\omega$ mass matrix becomes

$$\mathrm{Tr}[(\sqrt{3}\,Qf_1V_{38}+fU_3V'_{38}+f'\lambda_0V_0)^2\kappa^2]\,. \tag{6.16}$$

Here we have allowed a different universal coupling constant $f'\neq f$ for the ninth vector and axial-vector mesons V_0 and A_0; κ stands for the 3×3 submatrix of κ [Eq. (6.12)] with $\kappa_1\neq\kappa_2$. The other symbols are given as

$$f_1=f^2(f^2-\tfrac{4}{3}e^2)^{-1/2}\,,$$
$$Q=\begin{pmatrix}\frac{2}{3}&0&0\\0&-\frac{1}{3}&0\\0&0&-\frac{1}{3}\end{pmatrix},$$
$$U_3=\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}, \tag{6.17}$$
$$\lambda_0=(\tfrac{2}{3})^{1/2}1\,.$$

It turns out that f' must be close to f to obtain the usual ϕ-ω (canonical nonet) mixing angle. We also find that, aside from small electromagnetic mass corrections, the ρ-ω mixing angle can be fitted to data, and is very sensitive to variations in f'-f of order e^2.

Further, of course, the eigenvectors of the mass matrix, which are the physical ρ, ω, and ϕ, have *direct* order-e^2/f electromagnetic couplings to the leptons. This can be easily seen from Eq. (6.14): V_{38} is also associated with the *total* charge Q, like the photon. Thus electromagnetic effects will not be describable purely in terms of a $J^\mu_{\rm em}\cdot A_\mu$ coupling. In explicit calculation, say in electron quark (electromagnetic) scattering, we find that the hadronic vector couplings always add to the photon in just such a way as to simulate vector-meson-dominated electromagnetic form factors in lowest order: e.g.,

$$\frac{e^2}{q^2}-f\left(\frac{e^2}{f}\right)\frac{1}{q^2-m_\rho{}^2}=\frac{e^2}{q^2}\left(\frac{-m_\rho{}^2}{q^2-m_\rho{}^2}\right). \tag{6.18}$$

Further, these couplings give a hadronic correction to the muonic $\frac{1}{2}(g-2)$ of order 5×10^{-8}, agreeing with previous estimates.[19]

We will discuss the effect of the diagonalization on currents after specifying our prescription for the other weak vector-meson couplings. These we choose *not* to diagonalize, leaving charged $W^\pm$ and neutral Z terms of the form

$$\mathcal{L}'=gf\,\mathrm{Tr}[V_L(M_L+\kappa)\tilde{W}(M_L^\dagger+\kappa^\dagger)]+(Z\text{ terms}) \tag{6.19}$$

as they are. Thus the charged lowest-order currents proceed via vector exchange at low energies. Actually, of course, one can diagonalize, but this is quite lengthy, and the theory is easily interpreted without doing so.

C. Currents and Universality

Hadronic currents in our model are, of course, determined by the Lagrangian $\mathcal{L}$. The physical weak currents $J^\mu_\pm$ can immediately be read off as the hadronic coefficient of $W^\mu_\pm$ in $\mathcal{L}$ (M's considered hadrons). In the limit $g, g'\to0$ (and mass of the fourth column of M's large), these currents are just those discussed in Sec. IV. The electromagnetic current is also the hadronic coefficient of A^μ, but as stressed above, this current is not useful in the usual manner, due to the "other" electromagnetic effects from neutral strong vector mesons.

Although, as in Sec. IV, these currents can be found through hadronic considerations, it is perhaps more illuminating to consider their structure from the point of view of the "primed" transformations and the W equations of motion. As an example, we discuss the Noether derivation of $J^\mu_\pm$ from this viewpoint. For simplicity, we consider only transformations *within* the *unitary* gauge $[(M-M^\dagger)_{3\times3}=0,\ \phi-\phi^\dagger=0]$ – in this case, those generated by $\tilde{F}'_{1,2}$. Thus ϕ, M, and the other hadrons do not transform (otherwise we cannot maintain the gauge), while the leptons transform as usual. With respect to W, we follow Sec. IV to consider two classes of transformations (and hence two groups of currents). As in Sec. IV, the two W transformations are those with or without an extra $\tilde{S}'^{-1}\partial_\mu \tilde{S}'$. The transformation *with* the extra derivative term leads to the physical currents $J_\pm^\mu$ defined just above; these can be written as

$$J_\pm = \frac{\delta}{\delta W_\pm}(\mathcal{L}_M + \mathcal{L}_\phi),$$

where $\mathcal{L}_{M,\phi}$ are the covariant kinetic energy terms for M and ϕ in the unitary gauge. The transformations *without* the derivative lead to

$$J_{\pm N} = \frac{\delta}{\delta W_\pm}(\mathcal{L} - \mathcal{L}_M - \mathcal{L}_\phi),$$

which are the ordinary Noether currents of the leptons and W fields, and not the hadron currents. As always in these algebra-of-fields-type situations (see Sec. IV), the two sets of currents are related by a total divergence, which this time is $\partial_\mu F^{\mu\nu}_{W_\pm}$. Thus $J_\pm^\mu$ is related to the weak Noether current $J_{\pm N}^\mu$ through the W equations of motion; simultaneously, forms essentially $J_\pm^\mu$ are related, as in Sec. IV, to the hadronic Noether current through the strong vector-meson equations of motion.

It is clear from the above discussion that the hadronic charge algebra is that of the leptonic charges; hence universality is guaranteed.[20]

A further remark about the electromagnetic current: It would be useful to have an "effective" electromagnetic current that takes into account the hadronic corrections mentioned above. We would conjecture that such an object is the current coupling to Weinberg's photon (i.e., do not diagonalize; Weinberg's photon is the real photon taken at $\eta = 0$).

Neutral strangeness-changing currents and a correspondence principle. In lowest order, we get no $\Delta S = 1$, $\Delta Q = 0$ currents, because our Cabibbo rotation (5.14) does not rotate neutral weak vector mesons $[\tilde{t}_3 = t_3,\ \tilde{t}_0 = t_0]$; we have accomplished this only by increasing the size of $M_{L,R}$, without extra quarks.

On the other hand, it is clear that the four columns of $M_{L,R}$ are acting like the $(\mathcal{P}, \mathcal{N}, \lambda, \mathcal{P}')$ quarks of other models.[6] In fact, we see a type of "correspondence principle" at work here in the sense that, from the structure of some n-quark "direct coupling theory" (e.g., $\bar{q}Wq$), we can read an n-column "M theory" (our models here) – or vice versa. This principle will be useful below when we consider inclusion of other lepton models.

Preliminary calculations indicate that *higher-order* induced strange currents are suppressed by factors of hadron masses and mass splittings divided by $M_W{}^2$: Before vacuum expectation values, such processes are zero to order g^4, due to a cancellation between *internal* $\mathcal{P}$- and $\mathcal{P}'$-type column exchanges [as in the Glashow-Iliopoulos-Maiani SU(4) model]. Therefore, after spontaneous breakdown, these amplitudes are suppressed by hadron masses and mass differences divided by $M_W{}^2$. These conclusions are being checked in detail and will be presented elsewhere.[21]

D. Fermions and Anomalies

As thus far presented, our model has anomalies. Further, in the presence of both strong and weak vector mesons, it does not appear possible to cancel quark-vs-lepton anomalies. Hence we will discuss a simple doubling scheme which, at least for the hadrons, is very much in the spirit of dual models. In particular, our approach will lead us directly to the existence of a heavy pion. The scheme is as follows.

We introduce (heavy) q', $\psi'_{S,D}$ that couple to gauge bosons just as q, $\psi_{S,D}$ but with the opposite sign of γ_5. The new leptons go where we had question marks in the 4×4 lepton matrices:

$$\psi_D = \begin{bmatrix} \nu_{eL} & \mu_R^+ & 0 & 0 \\ e_L^- & -(\nu_\mu^C)_R & 0 & 0 \\ 0 & 0 & -\nu'_{L\mu} & e_R'^- \\ 0 & 0 & \mu_L'^+ & (\nu_e'^C)_R \end{bmatrix},$$
$$\psi_S = \begin{bmatrix} 0 & \mu_L^+ & 0 & 0 \\ e_R^- & 0 & 0 & 0 \\ 0 & 0 & \beta'\nu'_{\mu R} & e_R'^- \\ 0 & 0 & \mu_R'^+ & \alpha'\nu'_{eR} \end{bmatrix}. \qquad (6.20)$$

In the leptonic system, anomalies are canceled without complication. However, now q and q' loops with an odd number of γ_5 couplings tend to cancel (because both type of quarks are picking up masses and interactions from the same type of terms $\bar{q}_L\Sigma q_R$, $\bar{q}'_R\Sigma q'_L$) – suppressing $\pi^0 \to 2\gamma$. This we cannot allow. The only solution to this dilemma appears to require the introduction of *a heavy pion'*-Σ' field $\Sigma' = \sigma' + i\pi'$. For simplicity, we *choose* to couple q *only* to Σ, and q' *only* to Σ'. (Most general couplings do not affect the conclusion.) Now it is easy to arrange that the masses of q' and Σ' are high while keeping $\langle\Sigma'\rangle/\langle\Sigma\rangle = v'/v \ll 1$ so that the new Σ' has negligible effect on all *low-lying* hadrons, including V and A. Now, of course, $\pi^0 \to 2\gamma$ proceeds only through q. To get an extra factor of 3 in the $\pi^0 \to 2\gamma$ amplitude there are a number of choices. We can go to sets of integrally charged quarks, with Y providing a "charm," or we can, most perversely, e.g., introduce two more such "pairs" of canceling quarks with large mass.[22] Such schemes appear quite flexible with regard to quark classification, the only common denominator being the apparent necessity for a heavy pion. Implications of such ideas in the operatorial formulation of PCAC (partially conserved axial-vector current) will be explored elsewhere.

E. Other Lepton Models

Among the other lepton models in the literature, none fits our hadrons as well as Weinberg's. However, with more scalar mesons (etc.), some other models can be incorporated, and we will make some brief remarks on this subject.

The second model of Prentki and Zumino looks good at first sight.[8] Indeed, their leptons fit naturally into our $\psi_{D,S}$. However, due to the $(2)^{-1/2}$ in their neutrino classification, we would violate hadronic universality (by this factor) if we coupled directly to our hadrons above.[23] Consulting the correspondence principle again, we find that universality is restored with the addition of one *more* set of 3×4 matrices $M'_{L,R}$. At this stage, however, we consider this unattractive.

A more economical generalization follows lines between those of Weinberg and of Georgi and Glashow. As discussed in Sec. V, the presence of the M's requires four weak gauge bosons. Keeping the *same* weak gauge bosons, we now classify the leptons under SU(2) with Georgi and Glashow. The leptons are singlets under Y. The Georgi-Glashow scalar fields $\phi_G = \vec{t}\cdot\vec{\phi}_G$ are needed to retain their lepton mass pattern; Weinberg's scalars ϕ_W are also needed to construct our $(3,\bar{3})$ symmetry-breaking term (see above). Further, $\langle\phi_W\rangle = \lambda$ can be taken to provide the bulk of weak gauge boson masses. To avoid Goldstone bosons in lowest order, terms like $\mathrm{Tr}[\phi_W^\dagger\phi_G\phi_W]$ must be included in the potential. In this model, then, with only three extra scalars (ϕ_G), we suppress neutrino processes in the manner of Georgi and Glashow. Without the extra U(1), the original O(3) model of Georgi and Glashow does not seem possible to incorporate.

We have not found a way of incorporating (without Goldstone bosons in lowest order) the model of Lee, Prentki, and Zumino.

F. Final Remarks and Directions

We would like to discuss briefly perturbation expansion around the "hadron" theory. We choose to hold fixed the masses of W, Z, and ϕ. This leaves one parameter, say e (electric charge), to expand all weak and electromagnetic effects. As $e\to 0$, we reach the pure hadronic system, which is of interest in itself. The hadron Lagrangian is in most respects the model of Bardakci and Halpern. A notable exception, of course, is the $(3,\bar{3})$ symmetry-breaking term. Since $\lambda^{-1} = gM_W^{-1}\sim O(e)$, then G_1 [Eq. (6.9)] is also $O(e)$. Thus in the term $\mathrm{Tr}[G_1(\tilde{\phi}+\lambda)M_L^\dagger\Sigma M_R]$, the term $\mathrm{Tr}(\lambda G_1 M_L^\dagger\Sigma M_R)$ survives as (another e-independent) part of the hadron world. Thus *all hadron symmetry breaking occurs in terms of dimension* $d\le 3$. Since we have required parity-, isospin-, and hypercharge-conserving strong interactions, this conclusion about symmetry-breaking dimensionality follows directly from the structure of the leptons.

The question of deep-inelastic scaling for our model remains to be investigated. Although the current algebra generally resembles algebra of fields, still there are a number of special features here that interest us in a reexamination of the possible scaling. (1) The theory is renormalizable, i.e., longitudinally damped in some sense. Can this connect with the physical fact $\sigma_L/\sigma_T\to 0$? (2) Whereas in algebra of fields one has current dimension one, here we naturally obtain asymptotic dimension three. (3) Possibly relevant to this question is the further fact that the unified theory can be taken scale-invariant before spontaneous breakdown, all except for the ϕ mass term. Hadronic scalar masses are generated along with other masses, as long as we include also potential terms like $\mathrm{Tr}(M_L M_L^\dagger\Sigma\Sigma^\dagger)$, etc. The theory cannot be taken completely scale-invariant (or a Goldstone dilaton appears).[24]

Finally, we want to make a few brief remarks concerning the introduction of baryons in the model. In lieu of a Bethe-Salpeter bound-state calculation, we have the option of introducing elementary baryons, but the resulting picture is not very attractive. To give (renormalizable) mass to the usual $(8,1)=B_L$, $(1,8)=B_R$ baryons, one is *forced* to introduce an $(8,8)$ scalar field χ which couples to baryons as $\bar{B}_L\chi B_R$ and to Σ as $\chi^\dagger_{\alpha\beta}\mathrm{Tr}(\lambda_\alpha\Sigma\lambda_\beta\Sigma^\dagger)$. The last term is needed to avoid new Goldstone bosons in lowest order. χ of course, involves 128 new scalars, whose masses can be taken large. An alternate possibility is the old $(3,\bar{3})+(\bar{3},3)$ baryons,[25] whose masses can be generated by Σ alone (no new scalars); such classification of course leads to bad D/F ratios to lowest order.

APPENDIX A: SPONTANEOUS BREAKDOWN AND THE SCALAR SYSTEM

The part of the Lagrangian we wish to study here is

$$\mathcal{L}' = V_1(M) + V_2(\Sigma) + V_3(\phi) + \mathrm{Tr}(G_1\tilde{\phi}^\dagger M_L^\dagger \Sigma M_R + \mathrm{H.c.}) + \mathrm{Tr}(G_2\tilde{\phi}^\dagger M_L^\dagger M_L\tilde{\phi} + G_2 M_R^\dagger M_R \tilde{\phi}^\dagger\tilde{\phi}) . \tag{A1}$$

We will take the potential terms $V(\cdots)$, as follows:

$$V_1(M) = \alpha\,\mathrm{Tr}(M_L^\dagger M_L) + \beta(\mathrm{Tr}M_L^\dagger M_L)^2 + \gamma\,\mathrm{Tr}[(M_L^\dagger M_L)^2] + (L \leftrightarrow R) , \tag{A2}$$

$$V_2(\Sigma) = \alpha'\,\mathrm{Tr}(\Sigma^\dagger\Sigma) + \beta'(\mathrm{Tr}\Sigma^\dagger\Sigma)^2 + \gamma'\,\mathrm{Tr}[(\Sigma^\dagger\Sigma)^2] ,$$

$$V_3(\phi) = \delta\,\mathrm{Tr}(\phi^\dagger\phi) + \epsilon(\mathrm{Tr}\phi^\dagger\phi)^2 .$$

G_1 and G_2 are diagonal "insertion" matrices, as detailed in the text.

We remark that the $\mathrm{SU}(2)_L'\times\mathrm{U}(1)'$ gauge invariance allows more general insertions, and in more places, than indicated. The only further restriction is that, to preserve *CP* invariance both before and after spontaneous breakdown, all insertions must be real matrices ($G_i^* = G_i$), which commute with the $\mathrm{SU}(2)_L'\times\mathrm{U}(1)'$ group. However, in lowest order in the weak and electromagnetic couplings, we would like to have isospin-, hypercharge-, and parity-invariant strong interactions. For this reason, we only allow the insertions indicated. We wait until higher-order divergent weak-interaction loops demand a certain insertion as a counterterm, and do not introduce it otherwise. This is a device to make their effect show only in higher orders, and thus be physically negligible.

For the moment, we will not introduce the term $\det\Sigma + \det\Sigma^\dagger$, but will return later to remark on the circumstances of its inclusion. Now, we assign vacuum expectation values $\langle\phi\rangle = \lambda t_0$, $\langle M_L\rangle = \langle M_R\rangle = \kappa$, $\langle\Sigma\rangle = v$, where

$$\kappa = \begin{bmatrix} \kappa_1 & 0 & 0 & 0 \\ 0 & \kappa_2 & 0 & 0 \\ 0 & 0 & \kappa_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} , \qquad v = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{pmatrix} , \tag{A3}$$

and κ_i, λ, and v_i are real numbers.

Let us first deal with the particles of the fourth column of $M_{L,R}$. Writing

$$M_{L,R} = \begin{pmatrix} \xi_{L,R} & \chi_{L,R} \\ 000 & 0 \end{pmatrix} , \tag{A4}$$

where $\xi_{L,R}$ are 3×3 matrix fields and $\chi_{L,R}$ is the fourth column, we notice that we are inducing *no* linear terms in χ, $\chi^\dagger$. This is consistent with (A3). In fact, the set of quadratic terms involving $\chi_{L,R}$ is just the usual terms from $V(M)$, plus

$$\frac{1}{2\sqrt{2}}\, d\chi_L^\dagger v\chi_R + \tfrac{1}{4}\lambda^2 c\chi_L^\dagger\chi_L + (L\leftrightarrow R) , \tag{A5}$$

where c, d are the numbers in the 4, 4 position of $G_{1,2}$. The parameters c and d can be adjusted to give arbitrary masses to $\chi_L \pm \chi_R$. Thus, though $\chi_{L,R}$ are extremely important in the structural connection between strong and nonstrong interactions, they play no important role in the analysis of the scalar system. In what follows, we regard $M_{L,R} \sim \xi_{L,R}$ as just 3×3 matrices.

Proceeding, we list the (matrix) relations obtained on requiring the absence of linear terms in ϕ, Σ, and M:

$$\tfrac{1}{2}\lambda v G_1' + \tfrac{1}{4}\lambda^2 G_2' + \alpha + 2\beta\,\mathrm{Tr}(\kappa^2) + 2\gamma\kappa^2 = 0 ,$$

$$\tfrac{1}{2}\lambda\kappa^2 G_1' + [\alpha' + 2\beta'\,\mathrm{Tr}(v^2) + 2\gamma' v^2]v = 0 , \tag{A6}$$

$$\mathrm{Tr}(v\kappa^2 G_1') + [\mathrm{Tr}(\kappa^2 G_2') + 2\delta + 16\epsilon\lambda^2]\lambda = 0 ,$$

where $\kappa G_i'$ are the 3×3 parts of κ, G_i. Because of the "insertions" $G_{1,2}'$, the equations are well underdetermined even allowing arbitrary diagonal κ, v, λ.

In preparation for writing down the quadratic terms, we use our 21 degrees of gauge freedom (all but Q) to eliminate the 21 scalar degrees of freedom $(\xi^\dagger - \xi)_{L,R}$ and $(\phi - \phi^\dagger)$. Then, using (A6) to simplify, we have

$$\begin{aligned} &\mathrm{Tr}[(2\gamma\kappa^2 - \tfrac{1}{2}\lambda v G_1')\xi_L{}^2 + 2\gamma\kappa\xi_L\kappa\xi_L + \tfrac{1}{2}\lambda\, G_1'\xi_L v\xi_R] + 4\beta(\mathrm{Tr}\kappa\xi_L)^2 + \xi_L \leftrightarrow \xi_R \\ &\quad + \mathrm{Tr}(-\tfrac{1}{2}\lambda\kappa^2 v^{-1} G_1'\Sigma^\dagger\Sigma + 2\gamma' v^2\Sigma\Sigma^\dagger + \gamma'\Sigma v\Sigma v + \gamma'\Sigma^\dagger v\Sigma^\dagger v) \\ &\quad + \beta'[\mathrm{Tr}\,v(\Sigma+\Sigma^\dagger)]^2 + \tfrac{1}{4}[\epsilon\lambda^2 + (2/\lambda)\mathrm{Tr}(v\kappa^2 G_1')]\,\phi_0{}^2 + \mathrm{Tr}[\lambda\kappa G_1'(\xi_L\Sigma + \Sigma\xi_R + \Sigma^\dagger\xi_L + \xi_R\Sigma^\dagger)] \\ &\quad + \phi_0\,\mathrm{Tr}[(\xi_L + \xi_R)(v\kappa G_1' + \lambda\kappa G_2') + (\Sigma+\Sigma^\dagger)\kappa^2 G_1'] , \end{aligned} \tag{A7}$$

where we have written $\xi^\dagger = \xi$.

It is relatively easy to see that this system contains in general *no* Goldstone bosons. Further, if we choose to fix ϕ_0 and $\xi_{L,R}$ to have large masses (e.g., by large λ, γ), while $\sigma + i\pi$ stay at lower masses (see interpretation of entries of v in text), then the mixings of physical particles (say just the pseudoscalars) with ϕ_0 and $\xi_{L,R}$ are very small and there is no practical need to diagonalize further.

Thus far, we have analyzed the system without a $\det\Sigma + \det\Sigma^\dagger$ term, keeping the ninth axial-vector meson. This leaves us with a problem as far as considering our Lagrangian as an *effective* Lagrangian. To zeroth order, then, we have a π-η degeneracy. The ninth axial-vector current is *not* conserved in the model, however, so this degeneracy is not expected to persist to all orders.

We would however feel more comfortable with the conventional "effective" π-η dynamics. This can be achieved by omitting the ninth axial-vector meson entirely. Thus, with no need for ninth axial gauge freedom, we can add the $\det\Sigma$ term. Now our 20 degrees of gauge freedom just suffices to remove the resulting 20 Goldstone scalars (the scalar system of course starts with one less symmetry). There is a problem, however. A *low-mass* (about 1 GeV) η'', with the quantum numbers of η', remains; it is the pseudoscalar that would have been absorbed into A_9. It would be of interest to carry out some detailed calculations on its mixing with η, η', and its decays, etc., with the possibility of its identification with $M(953)$.[26]

APPENDIX B: EMBEDDING OF THE WEAK GROUP IN $SU(3)'_L \otimes SU(3)'_R$

We present here a scheme for embedding the leptons in $SU(3)'_L \otimes SU(3)'_R$. This would seem to be the most natural extension of the hadronic theory to include also weak interactions. Unfortunately, the simplest scheme leads to conflict with experimental evidence on strangeness-changing neutral currents. This effect is well known and will not be discussed further. (It is however, the $g'_2 = 0$ limit of the following scheme.)

Therefore, we extend the leptonic gauge group by one more $U(1)'_2$ operator in addition to $SU(2)'_L \otimes U(1)'_1$. Then, as seen below, we succeed in suppressing greatly the $\Delta S = 1$ neutral currents in semileptonic decays. However, $\Delta S = 2$ nonleptonic interactions are found not small enough to lowest order. We consider this result as a failure of this scheme. This is why we are finally led to embed the leptons in $U(4)'_L \otimes U(4)'_R$ as shown in Sec. VI.

Groups and Representations

The hadronic group is as chosen in the main text. The local leptonic group is $SU(2)'_L \otimes U(1)'_1 \otimes U(1)'_2$ embedded in the primed $SU(3)'_L \otimes SU(3)'_R$. We call the latter's generators $\tilde{F}'_{\alpha L}, F'_{\alpha R}$ $(\alpha = 1, \ldots, 8)$. These are represented by $\tilde{\lambda}_\alpha$ and λ_α, respectively, where for the left-handed group we have applied a Cabibbo rotation ($\tilde{\lambda}_\alpha = e^{i\theta\lambda_7}\lambda_\alpha e^{-i\theta\lambda_7}$). Only five of these generators are realized locally; these are $\tilde{F}'_{\alpha L}$, $(\alpha = 1, 2, 3)$, $\tilde{Y}_1 = F'_{3R} + (1/\sqrt{3})(\tilde{F}'_{8L} + F'_{8R})$, and $\tilde{Y}_2 = (1/\sqrt{3})(\tilde{F}'_{8L} + F'_{8R})$. The charge operator is

$$Q = F_{3L} + F_{3R} + \frac{1}{\sqrt{3}}(F_{8L} + F_{8R}) + F'_{3L} + Y_1 .$$

We remark that since the charge combination $F'_{3R} + (1/\sqrt{3})F'_{8R}$ is invariant under a right-handed Cabibbo rotation, we could write

$$\tilde{Y}_1 = \tilde{F}'_{3R} + \frac{1}{\sqrt{3}}(\tilde{F}'_{8L} + \tilde{F}'_{8R}) .$$

Local Transformations

The general local operator

$$\mathfrak{U} = \exp i(\alpha_L \cdot F_L + \alpha_R \cdot F_R + \beta \cdot \tilde{F}'_L + \gamma_1 \tilde{Y}_1 + \gamma_2 \tilde{Y}_2) \tag{B1}$$

is represented in a unified supermatrix notation:

$$S = \begin{bmatrix} S_L & 0 & 0 & 0 \\ 0 & S_R & 0 & 0 \\ 0 & 0 & S'_R & 0 \\ 0 & 0 & 0 & \tilde{S}'_L \end{bmatrix} , \tag{B2}$$

where

$$\begin{aligned} \tilde{S}'_L(x) &= \exp \tfrac{1}{2} i \left[\vec{\tilde{\lambda}} \cdot \vec{\beta} + \frac{1}{\sqrt{3}} \tilde{\lambda}_8 (\gamma_1 + \gamma_2) \right] , \\ S'_R(x) &= \exp \tfrac{1}{2} i \left[\left(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right) \gamma_1 + \frac{1}{\sqrt{3}} \lambda_8 \gamma_2 \right] , \text{ etc.} \end{aligned} \tag{B3}$$

Fields and Classification

The hadronic part includes quarks, vector and axial-vector mesons, $\Sigma = \sigma + i\pi$ multiplet, etc. The only change from the model in the text is that M_L and M_R are now 3×3 matrices, transforming as

$$\begin{aligned} \mathfrak{U}\, M_L \mathfrak{U}^{-1} &= S_L M_L \tilde{S}_L'^{\,-1} , \\ \mathfrak{U}\, M_R \mathfrak{U}^{-1} &= S_R M_R S_R'^{\,-1} . \end{aligned} \tag{B4}$$

The weak gauge bosons are $\vec{W}_\mu, B_{1\mu}, B_{2\mu}$, associated with the generators $\tilde{F}'_L, \tilde{Y}_1, \tilde{Y}_2$ with couplings g,

g_1', g_2', respectively.

The $SU(2)_L'$ doublet leptons ψ_D are assigned to part of the $(3,\bar{3})'$ representation while the singlet leptons ψ_S belong to $(1,8)'$:

$$\psi_D=\begin{pmatrix}\nu_L & \mu_R^+ & 0\\ e_L^- & -\nu_R & 0\\ 0&0&0\end{pmatrix},\qquad \psi_S=\begin{pmatrix}0&\mu_L^+&0\\ e_R^-&0&0\\0&0&0\end{pmatrix}. \tag{B5}$$

Defining the rotated representation for ψ_D as $\tilde{\psi}_D = e^{i\lambda_7\theta}\psi_D$, we specify the transformations

$$\mathfrak{U}\,\tilde{\psi}_D\,\mathfrak{U}^{-1}=\tilde{S}_L'\tilde{\psi}_D S_R'^{-1} \text{ and } \mathfrak{U}\,\psi_S\,\mathfrak{U}^{-1}=S_R'\psi_S S_R'^{-1}. \tag{B6}$$

(This means that the unrotated ψ_D transforms with the unrotated S_L' as follows: $\psi_D \to S_L'\psi_D S_R'^{-1}$.) Finally we introduce a $(3,\bar{3})'$ complex 3×3 scalar ϕ ($\tilde{\phi}=e^{i\theta\lambda_7}\phi$) transforming just like ψ_D, and satisfying the invariant *linear* constraint

$$\mathrm{Tr}[(\tilde{\phi}-\tilde{\phi}^\dagger)P]=0. \tag{B7}$$

Here

$$P=\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$$

commutes with the local $SU(2)_L'\otimes U(1)_1'\otimes U(1)_2'$ group. This last constraint is necessary to avoid a Goldstone boson in lowest order in the spontaneous breakdown scheme we wish to consider below.

Lagrangian and Spontaneous Breakdown

Covariant derivatives are written with the help of the covariant momentum operators

$$\mathcal{P}_\mu=P_\mu+f(V_{L\mu}\cdot F_L+V_{R\mu}\cdot F_R)+g\,\vec{\tilde{F}}_L'\cdot\vec{W}_\mu + g_1'Y_1B_{1\mu}+g_2'Y_2B_{2\mu}. \tag{B8}$$

The Lagrangian is constructed analogously to the $U(4)_L'\otimes U(4)_R'$ model in Sec. VI, with the G insertions now being 3×3 matrices and in particular the G_1 insertion parametrized as

$$G_1=\lambda^{-1}\kappa^{-2}\begin{bmatrix} f_\pi m_\pi^2 & 0 & 0\\ 0 & f_\pi m_\pi^2 & 0\\ 0&0&2f_K m_K^2-f_\pi m_\pi^2\end{bmatrix} \tag{B9}$$

where λ and κ are respectively the vacuum expectation values

$$\langle\tilde{\phi}\rangle=\lambda=\begin{pmatrix}\lambda&0&0\\0&\lambda'&0\\0&0&\lambda''\end{pmatrix},$$

$$\langle M_L\rangle=\langle M_R\rangle=\kappa=\begin{pmatrix}\kappa_1&0&0\\0&\kappa_1&0\\0&0&\kappa_2\end{pmatrix}.$$

(We will specialize to $\lambda''=\lambda$ below for simplicity. We do not find any interesting results with $\lambda''\neq\lambda$.) The potential term $V(\phi)$ also must contain the necessary gauge-invariant insertions that break extra symmetries by hand, thus avoiding massless Goldstone bosons in lowest order.

Further, notice that we can write a lepton mass generating term

$$G\,\mathrm{Tr}(\bar{\psi}_D\phi\psi_S)+\mathrm{H.c.} \tag{B10}$$

which gives the mass ratio $m_e/m_\mu=(\lambda'/\lambda)\cos\theta$. (This relation can actually be broken by allowing G to be a matrix inside the trace, without breaking the gauge invariance.)

Photon Diagonalization and Heavy Neutral Weak Gauge Bosons

If the photon is found as in Sec. VI B, we remain with *two* weak neutral gauge bosons Z and B_2 associated with the following generators and couplings:

$$(g^2+g_1'^2)^{1/2}Z_\mu(\cos^2\phi\,\tilde{F}_{3L}'-\sin^2\phi\,\tilde{Y}_1)+g_2'\frac{1}{\sqrt{3}}(\tilde{F}_{L8}'+F_{R8}')B_{2\mu}, \tag{B11}$$

where

$$Z=\cos\phi\,W_3-\sin\phi\,B_1,\qquad \tan\phi=g_1'/g. \tag{B12}$$

In this notation we write the mass matrix for the massive weak gauge bosons:

$$g^2[\lambda^2+\lambda'^2\cos^2\theta+\lambda''^2\sin^2\theta]\,(W_1{}^2+W_2{}^2)+(g^2+g'^2)[\lambda^2+\lambda'^2\cos^2\theta+\lambda''^2\sin^2\theta]Z^2 + g_2'^2(\lambda'^2+\lambda''^2)\sin^2\theta\, B_2{}^2 - 2g_2'(g^2+g'^2)^{1/2}\lambda''^2\sin^2\theta\, ZB_2\,. \quad \text{(B13)}$$

Defining the neutral eigenstates Z_1, Z_2 as

$$Z=\cos\alpha Z_1+\sin\alpha Z_2\,,$$
$$B_2=-\sin\alpha Z_1+\cos\alpha Z_2\,. \quad \text{(B14)}$$

We find in the approximation $\lambda'\ll\lambda\cong\lambda''$ (remember $m_e/m_\mu\cong\lambda'/\lambda$)

$$\tan 2\alpha\cong\frac{2x-\sin^2\theta}{x^2-\sin^2\theta}\,,\quad x=\frac{g^2+g'^2}{g_2'^2}\,. \quad \text{(B15)}$$

Strangeness-Changing Neutral Currents

The couplings of the heavy neutral weak gauge bosons to hadronic and leptonic currents are obtained as

$$\mathcal{L}\sim\sin\theta\cos\theta\,[J^h_{L6}+J^h_{\Delta S=\Delta Q=0}]_\mu\,[(g^2+g_1'^2)^{1/2}Z_\mu - g_2' B_{2\mu}]+(g^2+g_1'^2)^{1/2}Z_\mu\,(\cos^2\phi\, j^l_{3L}-\sin^2\phi\, j^l_{3R})_\mu\,, \quad \text{(B16)}$$

where J^h_{L6} is the left-handed hadronic current associated with λ_6, j^l_{3L} is the left-handed leptonic current associated with λ_3, etc.

Rewriting the above in terms of $Z_{1,2}$, and using the mass matrix, we calculate the effective semileptonic, nonleptonic, and leptonic Lagrangians. We find that

(1) the purely leptonic processes are almost as in Weinberg's Lagrangian, with a small change of the order of $\lambda'^2/\lambda^2\approx(m_e/m_\mu)^2$,

(2) the neutral $\Delta S=1$ currents in semileptonic processes are suppressed in the *decay rates* by a factor of $(\lambda'/\lambda)^4\approx(m_e/m_\mu)^4$, and

(3) the nonleptonic effective weak Lagrangian contains $\Delta S=2$ pieces, which, compared to the largest $\Delta S=0$ pieces, are smaller only by a factor of $\approx(\sin\theta)^2$ [there are also terms which may give an approximate $\Delta I=\frac{1}{2}$ rule ($\lambda''\neq\lambda$ may be better)].

Conclusion

In spite of a lot of effort we could not improve on item (3) above within many variations of the $\mathrm{SU}(3)'_L\otimes\mathrm{SU}(3)'_R$ scheme. To avoid this large contribution of $\Delta S=2$ processes in lowest order, we were finally led to consider enlarging the primed group to $\mathrm{U}(4)'\times\mathrm{U}(4)'$ as discussed in Sec. VI.

---

*Supported in part by the U. S. Atomic Energy Commission.

†Participating guest, Lawrence Berkeley Laboratory.

‡Present address: Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104.

[1]P. W. Higgs, Phys. Letters 12, 132 (1964); Phys. Rev. Letters 13, 508 (1964); Phys. Rev. 145, 1156 (1966); T. W. B. Kibble, *ibid.* 155, 1554 (1967).

[2]S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).

[3]A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Wiley, N.Y., 1969).

[4]G. 't Hooft, Phys. Letters 39B, 195 (1971); Nucl. Phys. B35, 167 (1971); B. W. Lee, Phys. Rev. D 5, 823 (1972); B. W. Lee and J. Zinn-Justin, *ibid.* 5, 3121 (1972); 5, 3137 (1972); 5, 3155 (1972).

[5]M. Veltman, report (unpublished); D. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972); C. Bouchiat, J. Iliopoulos, and Ph. Meyer, Phys. Letters 42B, 91 (1972); H. Georgi and S. Glashow, Phys. Rev. D 6, 429 (1972).

[6]S. Weinberg, Phys. Rev. D 5, 1962 (1972); 5, 1412 (1972); H. Georgi and S. Glashow, Phys. Rev. Letters 28, 1494 (1972); B. W. Lee, Phys. Rev. D 6, 1188 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972).

[7]K. Bardakci and M. B. Halpern, Phys. Rev. D 6, 696 (1972).

[8]I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. Letters 29, 969 (1972).

[9]A. Neveu and J. Scherk, Nucl. Phys. B36, 155 (1972); B36, 317 (1972); J.-L. Gervais and A. Neveu, *ibid.* B46, 381 (1972).

[10]R. Utiyama, Phys. Rev. 101, 1597 (1956); T. W. B. Kibble, J. Math. Phys. 2, 212 (1961).

[11]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[12]See, however, S. Coleman and E. Weinberg, report (unpublished).

[13]F. Gürsey and G. Feinberg, Phys. Rev. 128, 378 (1962); see Appendix II.

[14]Following the limit procedure of K. Bardakci,

Y. Frishman, and M. B. Halpern [Phys. Rev. 170, 1353 (1968)], we can map the gauge theory of hadrons onto a Sugawara-like theory. The limit is $f \to 0$, $m^2 \to 0$ holding $\kappa^2 = m^2/f^2$ constant. The theory seems to resemble a recent model of R. Perrin, Phys. Rev. D 6, 2089 (1972).

[15]See, e.g., S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969), and references therein.

[16]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[17]Actually, there is freedom to induce $\langle \sigma_3 \rangle$ as well, if desired, for a lowest-order model of $\eta \to 3\pi$. See also Appendix A.

[18]As mentioned in Ref. 7 and footnote 3 of Ref. 8, it is elementary to generalize to a ninth coupling f'/f. See also in this connection H. Georgi, this issue, Phys. Rev. D 7, 1258 (1973).

[19]M. Gourdin and E. de Rafael, Nucl. Phys. B10, 667 (1969). Our calculation parallels the vector contribution of the Z-exchange diagram in the weak contribution to g-2 (with Z parameters replaced by ρ parameters). See I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972); R. Jackiw and S. Weinberg, *ibid.* 5, 2396 (1972).

[20]M. Gell-Mann, Physics 1, 63 (1964).

[21]H. Aytac, I. Bars, and M. Yoshimura (unpublished).

[22]This mechanism is called "red, white, and blue quarks" by Gell-Mann and collaborators (unpublished). The mechanism was proposed independently, and at about the same time, by us in Ref. 8.

[23]We thank B. W. Lee for making this point to us.

[24]We thank S. Coleman for making this point to us.

[25]S. Coleman and S. Glashow, Ann. Phys. (N.Y.) 17, 41 (1962).

[26]Particle Data Group, Phys. Letters 39B, 1 (1972).