Light-Cone Algebra, Field-Current Identity, and Deep-Inelastic Scattering*

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We reproduce the full content of the Fritzsch–Gell-Mann light-cone algebra (originally derived using the quark model) for deep-inelastic lepton-nucleon scattering using currents given by the field-current identity. These currents are modified by dilaton clothing from the usual low-energy vector-dominance form by the principles of scale invariance to possess a scale dimension three (which is the dimension of the currents in the quark model). The calculation is performed by the technique of an effective Lagrangian to be used to first nonvanishing order, and the forms of the interactions needed to maintain the light-cone algebra are determined. The usual quark-parton results are explicitly obtained, e.g., the vanishing of $q^2\sigma_L$ in the deep-inelastic limit, the relations between electron and neutrino structure functions, etc. The Callan-Gross sum rule is examined and shown to be consistent with the above results.

I. INTRODUCTION

Interest in the light-cone commutators arises mainly due to the observation that the deep-inelastic scattering experiments sample the commutator of two currents at lightlike separations. This feature draws attention to the dominant singularity structure of these commutators near the light cone. The particular relationship between the light-cone singularities and the deep-inelastic scattering is most clearly embedded in the work of Fritzsch and Gell-Mann[1,2] but is present in almost all models of deep-inelastic scattering. The work of Fritzsch and Gell-Mann is an outgrowth of the work of Wilson[3] on operator-product expansions extended by Frishman[4] and by Altarelli, Brandt, and Preparata[4] to the light cone. It starts with the chiral currents formed from the free-quark model

$$V_a^\mu(r;x) = i\bar\psi(x)(\tfrac{1}{2}\lambda_a)\gamma^\mu(1+r\gamma_5)\psi\,, \tag{1.1}$$

where $r=\pm1$ and λ_a are the SU(3) matrices. Using the free-quark anticommutation relations, current commutators of two currents $V_a^\mu(r;x)$ and $V_b^\nu(r;y)$ are then worked out, and in the limit when $z^2\to0$, where $z=x-y$, Fritzsch and Gell-Mann obtain the result

$$[V_a^\mu(r,x),\ V_b^\nu(r,y)] \to \{if_{abc}[S^{\mu\nu\alpha\beta}V_{\beta c}(r,S;x,y) + ir\epsilon^{\mu\nu\alpha\beta}V_{\beta c}(r,A;x,y)] + d_{abc}[S^{\mu\nu\alpha\beta}V_{\beta c}(r,A;x,y) - ir\epsilon^{\mu\nu\alpha\beta}V_{\beta c}(r,S;x,y)]\}\,\partial_\alpha D(z)\,. \tag{1.2}$$

Here $S^{\mu\nu\alpha\beta}=\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\nu\alpha}\eta^{\mu\beta}-\eta^{\mu\nu}\eta^{\alpha\beta}$ and $D(z)$ is the function which contains the light-cone singularity $D(z)=-(2\pi)^{-1}\epsilon(z^0)\delta(z^2)$. The right-hand side of Eq. (1.2) contains the bilocal operators defined by

$$V_a^\mu(r,S(A);x,y) = i\bar\psi(x)(\tfrac{1}{2}\lambda_a\gamma^\mu)(1+r\gamma_5)\psi(y) \pm (x \leftrightarrow y)\,. \tag{1.3}$$

The matrix elements of these bilocal operators between nucleon states are assumed to be smooth and the scaling of the structure functions in the deep-inelastic region is governed by the singular nature of the function $D(z)$ near the light cone. Modifications of these results when interactions are switched on has been examined by Gross and Treiman.[5] These authors consider the specific theory where strong interactions arise from isosinglet vector or scalar gluons. Their conclusion based on this analysis implies that so far as the formal structure of Eq. (1.2) is concerned, it remains intact when the interactions are included (though the definition of the bilocal current becomes modified).

The Fritzsch–Gell-Mann picture is a particularly simple and elegant description of light-cone phenomena. Thus Eq. (1.2) automatically predicts the observed transverse nature of deep-inelastic electron scattering ($q^2\sigma_L\to0$) as well as the main parton predictions for neutrino scattering. While the latter have yet to be tested, it is a reasonable working hypothesis to postulate the general validity of Eq. (1.2). On the other hand physical quarks have yet to be discovered, leaving the quark model origin of Eq. (1.2) somewhat in doubt. The question naturally arises if the results of Eq. (1.2) can be achieved without assuming the existence of physical quark fields. The situation is somewhat similar to the analysis in the low- and intermediate-energy region of the equal-time SU(3)×SU(3) current algebra. This algebra was again first obtained by Gell-Mann[6] using the quark model. However, in dealing with the physical meson spectrum, it was seen in the hard-meson phenomenology[7] that it was possible to realize the quark equal-time algebra without quark currents but rather using vector-meson currents. The role of the current algebra was then to constrain and determine the meson coupling structures. It is the purpose of the present work to show that a similar result holds for the light-cone algebra. Thus we will see that it is possible to realize the results of the Fritzsch–Gell-Mann commutation relations in deep-inelastic scattering making use only of observed particles and without postulating the existence of quarks. The light-cone algebra then plays a role analogous to the equal-time algebra in that it determines the form of the allowed interaction structures between the physical particles.

In order to implement the above program it is necessary to have an appropriately defined current (as in the quark model). The vector-dominance current of low-energy current algebra is unsatisfactory as it has scale dimension one. As is well known, if the dynamics is asymptotically scale-invariant the electroproduction structure functions will exhibit the Bjorken scaling[8] required by experiment[9] provided the currents possess scale dimension three. This is indeed the case for the quark currents of Eq. (1.1). In the intermediate-energy domain, when the principles of current algebra are combined with those of broken scale invariance, one finds that the vector-dominance current must in fact be modified and there does result a current of scale dimension[10] three. This current has the form

$$V_a^\mu(x) = g_\rho \tfrac{1}{2}\{\lambda(x), \rho_a^\mu(x)\}. \qquad (1.4)$$

Here ρ_a^μ are the vector-meson fields, $\lambda(x) = [1+\varphi(x)/F_\sigma]^2$ where $\varphi(x)$ is a dilaton of scale dimension one. The parameter $b \equiv 1/F_\sigma$ represents the coupling strength of the scalar dilaton to other hadrons. The dilaton nature and dimension of φ imply that $\lambda(x)$ has dimension two and hence the total dimension of $V_a^\mu(x)$ is three. While the derivation of Eq. (1.4) is based on principles appropriate for the intermediate-energy region,[10] it is natural to speculate that a current of the form of Eq. (1.4) is valid at higher energies, for it is a simple generalization of the low-energy vector-dominance current and possesses the scale dimension apparently required by nature at high energies.

We will now state the basic assumptions to be used in the light-cone commutator calculations below.

(1) Light-cone commutators: We assume the validity of the formal structure of the Fritzsch–Gell-Mann commutators (but do not assume the underlying quark definition of the currents).

(2) Asymptotic chiral symmetry: We assume that there is asymptotic chiral symmetry which implies the conservation of the entire 18-plet of vector and axial-vector currents, i.e.,

$$\partial_\mu V_a^\mu = 0. \qquad (1.5)$$

(3) Field-current identity: The 18-plet of currents has the field-current identity form modified to obey the principle of scale invariance given by Eq. (1.4), where $\rho_a^\mu(x)$ are the fields representing the 18-plet of vector mesons and $\varphi(x)$ is an SU(3)×SU(3) singlet scalar dilaton field of scale dimension one.

By condition (1) we mean that the matrix elements of the light-cone commutators calculated using Eq. (1.4) should produce the same results as taking matrix elements of Eq. (1.2). Condition (2) is most easily achieved by introducing interaction structures that depend only on the curl structure $\rho_{\mu\nu a} = \partial_\mu \rho_{\nu a} - \partial_\nu \rho_{\mu a}$. In carrying out our detailed calculations we will make two additional dynamical assumptions which also have analogs in the equal-time current-algebra analyses: (a) In writing down interaction terms we will adopt a "smoothness" assumption and restrict ourselves to structures containing the minimum number of gradients needed to satisfy the light-cone algebra. As will be seen this turns out to imply that interactions possess one more derivative than is normally allowed in the low- and intermediate-energy equal-time current-algebra analyses. The higher-derivative structure appears to represent one of the distinctions between the low-energy and high-energy dynamics. (b) As in the equal-time current algebra, we will treat our interaction Lagrangian as an "effective Lagrangian" to be used only to first nonvanishing order in ρ_a^μ couplings. However, the coupling of the dilaton to hadrons can be included to all orders, for this coupling, as will be seen below, governs in part the detailed properties of the structure functions.

Section II reviews the scale-breaking principles that lead to currents of the form of Eq. (1.4). Section III discusses the form of the effective Lagrangian needed to reproduce the light-cone algebra. In Sec. IV the light-cone commutators are explicitly calculated for deep-inelastic scattering. Section V discusses the Callan-Gross relation for this model and conclusions are presented in Sec. VI.

II. SCALE INVARIANCE AND FORM OF CURRENTS

In this section we give a brief review of the arguments that lead to currents of the type given in Eq. (1.4) possessing scale dimension three. A more detailed discussion can be found in Ref. 10.

The generator of scale transformations is the dilatation charge

$$D(t) = -\int d^3x\, x_\nu \Theta^{0\nu}, \tag{2.1}$$

where $\Theta^{0\nu}$ are the components of the total (improved) stress tensor $\Theta^{\mu\nu}$. The dilatation charge transforms fields $\chi(x)$ of fixed scale dimension d_χ according to

$$i[D(t), \chi(x)] = (d_\chi + x^\mu \partial_\mu)\chi(x). \tag{2.2}$$

A dilaton field $\tau(x)$ of scale dimension d_τ is one which transforms according to the rule

$$i[D(t), \tau(x)] = \frac{1}{b} + (d_\tau + x^\mu\partial_\mu)\tau(x). \tag{2.3}$$

The breakdown of scale invariance is governed by the trace of the stress tensor $\Theta^\mu{}_\mu$. For the case of a single dilaton field $\sigma(x)$ (which we will consider here), the simplest assumption is that $\Theta^\mu{}_\mu$ is subject to pole dominance[11] by σ, i.e., $\Theta^\mu{}_\mu = F_\sigma m_\sigma{}^2\sigma(x)$. This relation is the scale-breaking analog of partial conservation of axial-vector current (PCAC) (pion pole dominance of $\partial_\mu A_a^\mu$). In Ref. 10, it is shown that one can rephrase this condition as a constraint on the source $J_\sigma \equiv (-\Box^2 + m_\sigma{}^2)\sigma$ of the σ field[12]

$$\begin{aligned} F_\sigma J_\sigma &= \Theta_M \\ &= 2(g_{\mu\nu}\partial\mathcal{L}_M/\partial g_{\mu\nu})_{g_{\mu\nu}=\eta_{\mu\nu}}. \end{aligned} \tag{2.4}$$

In Eq. (2.4) Θ_M is the trace of the Belinfante stress tensor of all the hadron fields [and may be constructed if one likes by the usual device of replacing the Lorentz metric $\eta_{\mu\nu}$ by an arbitrary metric $g_{\mu\nu}$ and rewriting the hadron Lagrangian $\mathcal{L}_M$ as a scalar density as indicated in Eq. (2.4)].

We now apply Eq. (2.4) to the coupling of the σ to the ρ mesons. We choose for $\mathcal{L}_M$ the structure[13]

$$\begin{aligned} \mathcal{L}_M = &-\tfrac{1}{4}\lambda_1(\sigma)\rho_a^{\mu\nu}\rho_{\mu\nu a} - \tfrac{1}{2}m_\rho{}^2\lambda_2(\sigma)\rho_a^\mu\rho_{\mu a} \\ &- \tfrac{1}{2}\lambda_3(\sigma)\partial^\mu\sigma\partial_\mu\sigma - \tfrac{1}{2}m_\sigma{}^2\lambda_4(\sigma), \end{aligned} \tag{2.5}$$

where $\rho_{\mu\nu a} \equiv \partial_\mu\rho_{\nu a} - \partial_\nu\rho_{\mu a}$, $\rho_{\mu a}$ = 18-plet of vector-meson fields, and $\lambda_i(\sigma)$ are *a priori* arbitrary functions of σ. For the vector currents we assume the general form $V_a^\mu = g_\rho\lambda(\sigma)\rho_a^\mu(x)$. Constructing J_σ and Θ_M by standard means, Eq. (2.4) combined with conservation of vector current (CVC) implies

$$\begin{aligned} \lambda_1 &= 1 \\ \lambda &= \lambda_2 \\ &= \lambda_3{}^{-1} \\ &= 1 + 2b\sigma, \\ \lambda_4 &= \sigma^2, \end{aligned} \tag{2.6}$$

where $b \equiv 1/F_\sigma$. One may simplify the kinetic energy of the σ field by introducing the variable φ defined by

$$\sigma = \varphi + \tfrac{1}{2}b\varphi^2. \tag{2.7}$$

Then

$$\begin{aligned} \lambda &= \lambda_2 \\ &= (1 + b\varphi)^2, \end{aligned} \tag{2.8}$$

and the dilaton kinetic-energy term in Eq. (2.5) has the standard form $-\tfrac{1}{2}\partial^\mu\varphi\partial_\mu\varphi$.

Equation (2.8) thus implies that the currents indeed have the form of Eq. (1.4) (when appropriately Hermitized). Using Eq. (2.1) and the fact that $\Theta^{\mu\nu}$ possesses a σ-meson Huggins term,[10] $\Theta^{\mu\nu} = \Theta_M^{\mu\nu} + \tfrac{1}{3}F_\sigma(\eta^{\mu\nu}\Box^2 - \partial^\mu\partial^\nu)\sigma$, one finds

$$i[D(t), \varphi(x)] = F_\sigma + (1 + x^\mu\partial_\mu)\varphi(x), \tag{2.9}$$

and $\varphi(x)$ is indeed a dilaton field of scale dimension one. This then implies that $\lambda(x)$ is a field of scale dimension two and hence the total current has correctly dimension three.

Equation (1.4) implies that the vector current acquires a "clothing" of the φ field. The remarkable feature of Eq. (1.4) is that the scaling condition Eq. (2.4) requires that the currents have a nonpole φ-ρ couplings. Expanding $\lambda(x)$ we find that the vector current has three parts to it,

$$V_a^\mu = g_\rho\rho_a^\mu + (2bg_\rho)\varphi\rho_a^\mu + (g_\rho b^2)\varphi^2\rho_a^\mu. \tag{2.10}$$

The most natural assumption is that φ is the phenomenological field representing the $\epsilon(700)$ meson. Then the terms of Eq. (2.10) have the following interesting interpretation: The first term on the right-hand side of Eq. (2.10) is the usual vector-dominance term and controls the threshold $q^2 \approx 0$ phenomena. The second term has scale dimension two and governs the intermediate q^2 domain. This term strongly enhances the e^+-e^- annihilation into four charged pions (via $e^+ + e^- \to \rho^0 + \epsilon \to 2\pi^+ + 2\pi^-$) at Frascati beam energies and can account for the sudden rise in that cross section at about 1.4 GeV.[14] In the deep-inelastic region, the first two terms are negligible, and the last term which has scale dimension three dominates and produces the scaling of the structure functions.

III. EFFECTIVE LAGRANGIAN FOR DEEP-INELASTIC SCATTERING

The quantity of primary interest for the study of the inclusive inelastic lepton (l)-nucleon (N) scattering

$$l_1+N\to l_2+X \tag{3.1}$$

is $W^{\mu\nu}_{ab}$, defined by

$$W^{\mu\nu}_{ab}(q,p;r)=(2\pi)^{-1}\int d^4z\, e^{-iqz}N_p^{-2}\langle N,p|C^{\mu\nu}_{ab}(x,y;r)|N,p\rangle\,, \tag{3.2}$$

where $z\equiv x-y$ and $C^{\mu\nu}_{ab}$ is the current commutator

$$C^{\mu\nu}_{ab}(x,y;r)=[V^{\mu}_{a}(x;r),\ V^{\nu}_{b}(y;r)]. \tag{3.3}$$

In Eq. (3.2), the index $r=+1$ (-1) labels the chiral even (odd) currents. The normalization factor N_p is given by $N_p=[m/(2\pi)^3E]^{1/2}$, where m is the nucleon mass and E the nucleon energy. Thus the covariantly normalized states of momentum p^μ, $N_p^{-1}|N,p\rangle$, have scale dimension -1. An average over nucleon spin states is understood. For physical lepton scattering, the photon momentum q^μ is spacelike, i.e., $q^2>0$, and also $q^0>0$. $W^{\mu\nu}_{ab}$ may be expanded in terms of the structure functions F_{1ab}, F_{2ab}, and F_{3ab} according to

$$mW^{\mu\nu}_{ab}=F_{1ab}(q^2,q\cdot p;r)\eta^{\mu\nu}-F_{2ab}(q^2,q\cdot p;r)p^\mu p^\nu q\cdot p - irF_{3ab}(q^2,q\cdot p;r)\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta/q\cdot p+\cdots\,, \tag{3.4}$$

where the F_{iab} are dimensionless functions. As is well known,[1] only the value of the current commutator in the vicinity of the light cone contributes to the integral of Eq. (3.2) in the deep-inelastic region, where $q^2\to\infty$, $p\cdot q\to\infty$ but $\omega\equiv-2p\cdot q/q^2\to$ finite. Bjorken scaling[8] implies then that the F_{iab} are functions of ω only, $F_{iab}=F_{iab}(\omega;r)$.

In order to calculate the light-cone commutators using the currents of Eq. (1.4), it is necessary to assume a specific set of interactions. As discussed in Sec. I, the interactions will be chosen to maintain the content of the Fritzsch-Gell-Mann light-cone algebra, when nucleon matrix elements are taken of Eq. (1.2). In this section we discuss the form of the interaction, and, as will be seen, the imposition of Eq. (1.2) greatly limits these forms. The actual computation of the light-cone commutators using the allowed interactions is carried out in Sec. IV.

Since the vector current is proportional to the ρ^μ_a field, it is convenient to divide the Lagrangian into a part governing the interactions of the ρ with other hadrons, $\mathcal{L}_{\rho H}$, a part governing the ρ-dilaton interaction $\mathcal{L}_{\rho\varphi}$, and a remainder $\mathcal{L}_H$ describing the dilaton and hadron interactions:

$$\mathcal{L}_I=\mathcal{L}_{\rho\varphi}+\mathcal{L}_{\rho H}+\mathcal{L}_H\,. \tag{3.5}$$

The ρ-φ interaction is the ρ-meson mass term determined in Eqs. (2.5) and (2.8) from the scale-breaking condition

$$\mathcal{L}_{\rho\varphi}=-\tfrac{1}{2}m_\rho^{2}\lambda(\varphi)\rho^\mu_a\rho_{\mu a}\,, \tag{3.6}$$

where $\lambda(\varphi)=(1+b\varphi)^2$, $b=1/F_\sigma$. This term accounts for the dilaton "clothing" in the current of Eq. (1.4) since the arguments of Sec. II show that, in effect, $V^\mu_a=-(g_\rho/m_\rho^2)\partial\mathcal{L}_{\rho\varphi}/\partial\rho_{\mu a}$.

To construct $\mathcal{L}_{\rho H}$ we first note that since we are interested in calculating a nucleon matrix element of a single commutator in Eq. (3.2) we are interested in the part of the effective Lagrangian $\mathcal{L}_{\rho H}$ quadratic in the ρ fields. For then the current will be linear in the ρ field and the commutator Eq. (3.3) will contain only other hadron variables whose nucleon matrix elements determine the structure functions of Eq. (3.4).[15] The CVC condition is most directly satisfied by requiring that the ρ fields enter $\mathcal{L}_{\rho H}$ only in the curl combination $\rho_{\mu\nu a}=\partial_\mu\rho_{\nu a}-\partial_\nu\rho_{\mu a}$. Finally we note from Eq. (1.2) that the light-cone commutators have four types of structures: two with f_{abc} and two with d_{abc}, two with an $\epsilon^{\mu\nu\alpha\beta}$ factor and two without this factor. Thus the general form of the action needed to achieve Eq. (1.2) is

$$A_{\rho H}=\sum_r\int d^4x d^4y\,[-\tfrac{1}{2}\xi_1 d_{abc}\rho_{\alpha\mu a}(x;r)\rho^{\alpha}{}_{\nu b}(y;r)\Theta^{\mu\nu}_c(x,y;r)-\xi_2 f_{abc}\epsilon^{\mu\alpha\beta}{}_{\gamma}\rho_{\alpha\mu a}(x;r)\rho_{\beta\delta b}(y;r)\Theta^{\gamma\delta}_c(x,y;r) -\tfrac{1}{2}\xi_3 f_{abc}\rho_{\alpha\mu a}(x;r)\rho^{\alpha}{}_{\nu b}(y;r)\Phi^{\mu\nu}_c(x,y;r)-\xi_4 d_{abc}\epsilon^{\mu\alpha\beta}{}_{\gamma}\rho_{\alpha\mu a}(x;r)\rho_{\beta\delta b}(y;r)\Phi^{\gamma\delta}_c(x,y;r)]\,, \tag{3.7}$$

where $\Theta^{\mu\nu}$ and $\Phi^{\mu\nu}$ depend on other hadrons but are independent of the ρ field. The index $r=+1$ (-1) labels the chiral even (odd) combinations of the vector-meson fields:

$$\rho_{\mu a}(x;r)=[v_{\mu a}(x)+ra_{\mu a}(x)]/\sqrt{2}\,, \tag{3.8}$$

where $v_{\mu a}$ $(a_{\mu a})$ are the vector (axial-vector) meson fields. The function $\Theta^{\mu\nu}$ and $\Phi^{\mu\nu}$ must have scale dimension four to maintain scale invariance.

While we wish to consider *only local interactions* here, we have written Eq. (3.7) in the general form to exhibit the symmetries that must be imposed on $\Theta^{\mu\nu}$ and $\Phi^{\mu\nu}$. Thus one must have

$$\Theta_a^{\mu\nu}(x, y; r) = \Theta_a^{\nu\mu}(y, x; r),$$
$$\Phi_a^{\mu\nu}(x, y; r) = -\Phi_a^{\nu\mu}(y, x; r). \tag{3.9}$$

The simplest possibility for $\Theta_a^{\mu\nu}$ and $\Phi_a^{\mu\nu}$ is that they depend only on a scalar function f_c. If we now impose the condition that our interaction be local, there are still several different forms available for $\Theta_a^{\mu\nu}$ and $\Phi_a^{\mu\nu}$. However, the *requirement that the light-cone algebra Eq. (1.2) be maintained, and the smoothness condition that at most two derivatives appear, uniquely determines* $\Theta_a^{\mu\nu}$ *and* $\Phi_a^{\mu\nu}$:

$$\Theta_a^{\mu\nu}(x, y; r) = \delta^4(x-y)\partial^\mu\partial^\nu f_a(x; r), \tag{3.10a}$$

$$\Phi_a^{\mu\nu}(x, y; r) = \tfrac{1}{2}[\partial_x^{(\mu}\delta^4(x-y)\partial^{\nu)}\bar{f}_a(y; r) - \partial_x^{(\mu}\delta^4(x-y)\partial^{\nu)}\bar{f}_a(x; r)]. \tag{3.10b}$$

The operators f_a and $\bar{f}_a$ are functions of the hadron fields. They have scale dimension -2 to maintain the scale invariance of the interaction. As will be seen in Sec. IV, the matrix elements of f_a and $\bar{f}_a$ are directly related to the structure functions and so the detailed nature of the $f_a, \bar{f}_a$ operators (which we have not specified here) are related in part to the detailed properties of the structure functions. Inserting Eqs. (3.10) into Eq. (3.7) yields now the local Lagrangian $\mathcal{L}_{\rho H}$,

$$\mathcal{L}_{\rho H} = \sum_r [-\tfrac{1}{2}\xi_1 d_{abc}\rho_{\alpha\mu a}(x; r)\rho^\alpha{}_{\nu b}(x; r)\partial^\mu\partial^\nu f_c(x; r) - \xi_2 f_{abc}\epsilon^{\mu\alpha\beta}{}_\gamma \rho_{\alpha\mu a}(x; r)\rho_{\beta\delta b}(x; r)\partial^\gamma\partial^\delta f_c(x; r)$$
$$- \tfrac{1}{4}\xi_3 f_{abc}\{\rho^\alpha{}_{\mu a}(x; r)\overleftrightarrow{\partial}^{(\mu}\rho_{\alpha\nu b}(x; r)\}\partial^{\nu)}\bar{f}_c(x; r) - \tfrac{1}{2}\xi_4 d_{abc}\epsilon^{\mu\alpha\beta}{}_\gamma\{\rho_{\alpha\mu a}(x; r)\overleftrightarrow{\partial}^{(\gamma}\rho_{\beta\delta b}(x; r)\}\partial^{\delta)}\bar{f}_c(x; r)], \tag{3.11}$$

where $A\overleftrightarrow{\partial}_\mu B \equiv A\partial_\mu B - (\partial_\mu A)B$.

The final part of the Lagrangian $\mathcal{L}_H$ governs the coupling of the f_a operators to the other hadrons, e.g., the nucleons and the dilaton. As will be seen, the detailed nature of this coupling determines the detailed nature of the structure functions. However, the light-cone condition Eq. (1.2) strongly limits the general form of even these couplings. Thus if we assume that f_a and $\bar{f}_a$ can couple directly to the nucleons, e.g., via a ξ_5 f_a-$\bar{N}$-N and ξ_6 $\bar{f}_a$-$\bar{N}$-N type structure, then the dilaton cannot couple directly to the nucleons and the dilaton-f couplings must be via an *even* number of f fields, e.g., g_1 φ-f-f and g_2 φ^2-f-f, etc. In the following sections we will assume coupling structures of this type (without specifying their detailed nature) and show how they realize the light-cone algebra of Eq. (1.2).

The coupling structures described above give a simple physical picture of the deep-inelastic scattering process. The current of Eq. (1.4) implies that the incident photon converts to a vector meson "clothed" by the dilaton ϵ-meson factor $\lambda(\varphi)$. Equation (3.11) implies that the vector meson then interacts with the f_a and $\bar{f}_a$ fields which have been emitted by the nucleon. (The f fields absorb and emit dilatons by the dilaton-f couplings.) The scattering of the photon by the nucleon is thus done via the intermediary of the f fields. Thus one may think of the f_a and $\bar{f}_a$ as representing the "partons" in the scattering. This interpretation is strengthened by the fact that the structure functions are determined by the detailed nature of these fields. However, note that the fields are scalar (not spin $\frac{1}{2}$) and it is still possible, as will be seen in Sec. IV and Sec. V, to achieve the results of the quark-model relations Eq. (1.2).

IV. LIGHT-CONE COMMUTATORS AND DEEP-INELASTIC SCATTERING

In Sec. III, the general form of the interactions that are needed to calculate the light-cone commutators for inelastic lepton scattering were given. In this section we will see that these interactions when combined with the currents of Eq. (1.4) are indeed equivalent to the Fritzsch–Gell-Mann light-cone commutators for this inclusive process.

The Lagrangian of Eq. (3.5) consists of the ρ-φ coupling of Eq. (3.6), the ρ-ρ-f coupling of Eq. (3.11) containing coupling constants $\xi_1, \ldots, \xi_4$, the f-$\bar{N}$-N couplings with constants ξ_5 and ξ_6, and the φ-f-f couplings with constants g_1, g_2, etc. Since this Lagrangian is an effective Lagrangian, it is to be used to lowest nonvanishing order in calculating the structure functions. To see explicitly what this implies we first note that if one writes

$$\lambda(\varphi) \equiv 1 + \mu(\varphi), \tag{4.1}$$

then the strength of the φ-ρ coupling is governed by $\mu(\varphi)$. The ρ-φ coupling produces the φ clothing in the vector current of Eq. (1.4) so that V_a^μ correctly has scale dimension three. Since it is $\mu(\varphi)$ which carries the dimension two, it is necessary to calculate each V_a^μ in the commutator $C_{ab}^{\mu\nu}$ of Eq. (3.3) to first order in $\mu(\varphi)$ so that each current will correctly have dimension three. Turning next to the various ξ_i couplings, one obtains the first nonzero contribution to $W_{ab}^{\mu\nu}$ of Eq. (3.2) if these are kept to second order. More precisely, the *commutator* $C^{\mu\nu}_{ab}$ must be computed to *first* order in the $\xi_1, \ldots, \xi_4$ couplings, since one additional factor of ξ_5 or ξ_6 is needed for the f field to couple to the nucleons in the matrix element of Eq. (3.2). Finally, as we will see below, it is not necessary to specify the order to be retained for the g_1, g_2 coupling constants to obtain the general form of the functions $W^{\mu\nu}_{ab}$. We will therefore leave open the question of whether one wishes to treat these couplings also in an effective Lagrangian formalism, or in a nonperturbation fashion.

A convenient method of expressing the currents in a perturbation series is by means of the "in-field" expansion of the ρ field equations. From Eqs. (3.5), (3.6), and (3.11) one has (suppressing the chiral index r)

$$\{[-\Box^2+m^2\lambda(\varphi)]\delta^\mu_\alpha-\partial^\mu\partial_\alpha\}\rho^\alpha_a(x)=J^\mu_a(x), \qquad (4.2)$$

where $J^\mu_a(x)$ is the source obtained from Eq. (3.11). We note that J^μ_a is linear in $\xi_1, \ldots, \xi_4$ and has the general form

$$J^\mu_a=\partial_\alpha D^{\mu\alpha}, \quad D^{\mu\alpha}=-D^{\alpha\mu}, \qquad (4.3)$$

since the ρ fields enter into Eq. (3.11) only in the curl combination. From Eqs. (4.2) and (4.3) then one sees $\partial_\mu(\lambda\rho^\mu_a)=0$ so that the CVC condition is satisfied. One may solve Eq. (4.2) to first order in $\xi_1, \ldots, \xi_4$ by writing

$$\rho^\mu_a(x)=\tilde{\rho}^\mu_a(x)+\int d^4x'\,\tilde{\Delta}^{\mu\alpha}_{\rm ret}(x,x')J_{\alpha a}(x'), \qquad (4.4)$$

where

$$\{[-\Box^2+m^2\lambda(\varphi)]\delta^\mu_\alpha-\partial^\mu\partial_\alpha\}\tilde{\rho}^\alpha_a(x)=0, \qquad (4.5a)$$

and $\tilde{\Delta}^{\mu\alpha}_{\rm ret}$ is a Green's function with retarded boundary conditions obeying

$$\{[-\Box^2+m^2\lambda(\varphi)]\delta^\mu_\alpha-\partial^\mu\partial_\alpha\}\tilde{\Delta}^{\alpha\nu}_{\rm ret}(x,x')=\eta^{\mu\nu}\delta^4(x-x'). \qquad (4.5b)$$

From Eq. (3.11) one sees that the source $J_{\alpha a}$ is linear in the ρ field and also linear in the f_c and $\bar{f}_c$ fields. Since we only need the solution of Eq. (4.4) to first order in $\xi_1, \ldots, \xi_4$, one may replace $\rho_{\mu a}$ by $\tilde{\rho}_{\mu a}$ in the $J_{\alpha a}$ of Eq. (4.4). The current then has the form

$$V^\mu_a(x)=V^\mu_{a(0)}(x)+V^\mu_{a(1)}(x), \qquad (4.6)$$

where

$$V^\mu_{a(0)}=\tfrac{1}{2}\{\lambda(x),\tilde{\rho}^\mu_a(x)\}, \qquad (4.7a)$$

$$V^\mu_{a(1)}=\int d^4x'\,\tfrac{1}{2}\{\lambda(x),\tilde{\Delta}^{\mu\sigma}_{\rm ret}J_{\sigma a}\}. \qquad (4.7b)$$

As discussed above, one need only calculate the current commutator to first order in $\xi_1, \ldots, \xi_4$. Thus one can write

$$C^{\mu\nu}_{ab}=[V^\mu_{a(0)}(x),V^\nu_{b(0)}(y)]+[V^\mu_{a(1)}(x),V^\nu_{b(0)}(y)]+[V^\mu_{a(0)}(x),V^\nu_{b(1)}(y)]. \qquad (4.8)$$

Expression (4.8) may be inserted into Eq. (3.2) to calculate $W^{\mu\nu}_{ab}$. In evaluating $W^{\mu\nu}_{ab}$, we need keep only terms linear in $\mu(\varphi)$ for each current. This allows one to simplify the analysis, for it is shown in the Appendix that in the deep-inelastic limit $W^{\mu\nu}_{ab}$ becomes

$$W^{\mu\nu}_{ab}(q,p)=P^\mu{}_\alpha\overline{W}^{\alpha\beta}_{ab}P_\beta{}^\nu, \qquad (4.9)$$

where $P^{\mu\alpha}\equiv\eta^{\mu\alpha}-q^\mu q^\alpha/q^2$ and $\overline{W}^{\alpha\beta}_{ab}$ is just the structure function calculated with $\tilde{\rho}^\mu_a$ replaced by the in-field $\rho^\mu_{a\,\rm in}$ and $\tilde{\Delta}^{\mu\alpha}_{\rm ret}$ replaced by the usual retarded function $\Delta^{\mu\alpha}_{\rm ret}(x-x')$:

$$[(-\Box^2+m^2)\delta^\mu_\alpha-\partial^\mu\partial_\alpha]\Delta^{\alpha\nu}_{\rm ret}=\eta^{\mu\nu}\delta^4(x-x'). \qquad (4.10)$$

The projection operators P^μ_α and $P_\beta{}^\nu$ therefore just guarantee that the CVC conditions are maintained.

The $C^{\mu\nu}_{ab}$ of Eq. (4.8) contains three terms. However, it is easy to see that the first term does not actually contribute to $W^{\mu\nu}_{ab}$, for this structure involves terms in $\overline{W}^{\mu\nu}_{ab}$ of the type

$$\langle p|\lambda(x)\rho^\mu_{a\,\rm in}(x)\lambda(y)\rho^\nu_{b\,\rm in}(y)|p\rangle=\langle p|\lambda(x)\lambda(y)|p\rangle\langle 0|\rho^\mu_{a\,\rm in}(x)\rho^\nu_{b\,\rm in}(y)|0\rangle. \qquad (4.11)$$

Now for the interaction structures of Sec. III, the φ fields in λ do not couple directly to the nucleon states, but only indirectly via the f_c fields (which do couple to the nucleons). Thus to evaluate the first matrix element, one must eliminate at least two of the φ fields by means of the g_1, g_2 couplings. However, since the φ couples only to an *even* number of f_c fields, there will result a nucleon matrix element of at least four f_c fields. Such a matrix element would be at least quartic in the ξ_5, ξ_6 couplings and hence of too high an order to be retained.[16] Thus only the terms linear in $\xi_1, \ldots, \xi_4$ in Eq. (4.8) need be retained in calculating $W^{\mu\nu}_{ab}$.

We illustrate the above discussion by sketching the calculation of the contribution of the ξ_1 interaction of Eq. (3.11) to $W^{\mu\nu}_{ab}$. For this case the source function of Eq. (4.3) is

$$D^{\mu\alpha}_a=\xi_1 d_{abc}\rho^{[\alpha}{}_{\beta b\,\rm in}\partial^{\mu]}\partial^\beta f_c(x). \qquad (4.12)$$

Inserting into Eq. (3.2), the last two commutators of Eq. (4.8) give rise to two terms related by the interchange $\mu\leftrightarrow\nu$, $a\leftrightarrow b$, $x\leftrightarrow y$. Thus the contribution of Eq. (4.12) to $\overline{W}^{\mu\nu}_{ab}$ reads

$$\overline{W}^{\mu\nu}_{ab} = +g_\rho{}^2\xi_1 d_{abc}(2\pi)^{-1}\int d^4z d^4x'\, e^{-iqz}\partial_\alpha\Delta_{\rm ret}(x-x')$$
$$\times\langle p|[\{\lambda(x)\rho^{[\alpha}{}_\beta(x')_{\rm in}\partial^{\mu]}\partial^\beta f_c(x')\},\{\lambda(y),\rho^\nu_b(y)_{\rm in}\}]|p\rangle - \{q^\mu\to -q^\mu\}, \tag{4.13}$$

where $\Delta_{\rm ret}$ is the scalar retarded Green's function of mass m_ρ. The $\rho_{\rm in}$ part of the matrix element now factors, so that Eq. (4.13) reduces to

$$\overline{W}^{\mu\nu}_{ab} = ig_\rho{}^2\xi_1 d_{abc}(2\pi)^{-1}\int d\alpha d\beta\,\Delta_{\rm ret}(q-\beta p)\Delta(q+\alpha p)$$
$$\times[f_c(\beta,\alpha)+g_c(\beta,\alpha)](\beta+\alpha)^2[\eta^{\mu\nu}(p\cdot q)^2 - p^{(\mu}q^{\nu)}p\cdot q + p^\mu p^\nu] - \{q\to -q\}, \tag{4.14}$$

where $\Delta_{\rm ret}(q)$ and $\Delta(q)$ are the Fourier transforms of the retarded and commutator functions[17] and $f_c(\beta, \alpha)$ and $g_c(\beta, \alpha)$ are the Fourier transforms of the remaining nuclear matrix element. Thus, from Eq. (4.13) there arise the two structures

$$M_c \equiv \tfrac{1}{4}\langle p|\{\lambda(y),\{\lambda(x),f_c(x')\}\}|p\rangle,$$
$$N_c \equiv \tfrac{1}{4}\langle p|[\lambda(y),\{\lambda(x),f_c(x')\}]|p\rangle. \tag{4.15}$$

Since $f_c(x)$ has scale dimension -2, both these matrix elements have scale dimension zero. Hence assuming these matrix elements are smooth on the light cone (i.e., the basic light-cone singularity arises only from the ρ commutator function Δ) they are functions only of $p\cdot(x-x')$ and $p\cdot(y-x')$. Their Fourier transforms are the functions $f_c(\beta, \alpha)$ and $g_c(\beta, \alpha)$ appearing in Eq. (4.14):

$$M_c = \int d\alpha d\beta\, e^{i\alpha p\cdot(y-x')}e^{i\beta p\cdot(x-x')}f_c(\beta,\alpha),$$
$$N_c = \int d\alpha d\beta\, e^{i\alpha p\cdot(y-x')}e^{i\beta p\cdot(x-x')}\epsilon(\alpha)g_c(\beta,\alpha). \tag{4.16}$$

In Eq. (4.14) we have already gone to the deep-inelastic limit where $q^2, p\cdot q \gg p^2 = -m^2$. Thus

$$\Delta_{\rm ret}(q-\beta p)\cong(q^2-2p\cdot q\beta)^{-1}$$

and

$$\Delta(q+\alpha p) = -2\pi i\delta(q^2+2p\cdot q\alpha).$$

Performing the α integration then yields

$$W^{\mu\nu}_{ab} = \tfrac{1}{4}\xi_1 g_\rho{}^2 d_{abc}h_{cA}(\omega)[\eta^{\mu\nu} - q^{(\mu}p^{\nu)}/q\cdot p + p^\mu p^\nu q^2/(q\cdot p)^2], \tag{4.17}$$

where $\omega \equiv -2p\cdot q/q^2$ and

$$h_{cA}(\omega) \equiv \int d\beta\,(\beta+1/\omega)[f_c(\beta,1/\omega)+g_c(\beta,1/\omega)] - (\omega\to -\omega). \tag{4.18}$$

We note that $h_c(\omega)$ is an odd function of ω and also real [since f_c and g_c obey the reality conditions $f_c^*(\beta, \alpha) = f_c(-\beta, -\alpha)$, $g_c^*(\beta, \alpha) = g_c(-\beta, -\alpha)$].

Before continuing on, it is interesting to note several features of the result. First, the $\xi_1 d_{abc}$ interaction is the one that governs the experimentally observed electroproduction process. The tensor structure of Eq. (4.17) implies that there is no $q^2\sigma_L$ contribution to the deep-inelastic cross section. Thus the *interaction of Eq. (3.11) combined with the current of Eq. (1.4) automatically produces a transverse electroproduction cross section in the deep-inelastic region.* Second, we see that it is the matrix elements of λ and f_c in Eq. (4.15) which determine the structure function. It is the φ-f-f and f-$\overline{N}$-N interactions that determine the precise functional dependence of the structure functions on ω. Thus one need not specify the detailed nature of these interactions to determine the general tensor and isotopic dependence on $W^{\mu\nu}_{ab}$.

The quantities in Eq. (4.15) play the role of the matrix elements of the bilocal currents in the quark model, though here they have a trilocal form. Actually, however, only *bilocal* structures contribute to the structure functions. For example, the term in Eq. (4.18) proportional to $\int d\beta f_c(\beta, \alpha)$ (where $\alpha = 1/\omega$) is equal to

$$2\pi\int d(p\cdot y)e^{-i\alpha p\cdot y}\langle p|\{\lambda(y),\{\lambda(0),f_c(0)\}\}|p\rangle \tag{4.19}$$

by Eq. (4.16), and similarly for the other structures of Eq. (4.18).[18]

The contribution of the $\xi_3 f_{abc}$ terms in Eq. (3.11) to $W_{ab}^{\mu\nu}$ can be calculated by the same technique used for the $\xi_1 d_{abc}$ interaction. One finds

$$W_{ab}^{\mu\nu} = \tfrac{1}{2}\xi_3 g_\rho{}^2 i f_{abc} \bar{h}_{cS}(\omega)[\eta^{\mu\nu} - q^{(\mu}p^{\nu)}/q\cdot p + p^\mu p^\nu q^2/(q\cdot p)^2], \tag{4.20a}$$

where $\bar{h}_{cS}(\omega)$ is given by

$$i\bar{h}_{cS}(\omega) \equiv \int d\beta\,(\beta + 1/\omega)[\bar{f}_c(\beta, 1/\omega) + \bar{g}_c(\beta, 1/\omega)] + (\omega \to -\omega), \tag{4.20b}$$

and $\bar{f}_c(\beta, \alpha)$, $\bar{g}_c(\beta, \alpha)$ are defined analogously to $f_c(\beta, \alpha)$ and $g_c(\beta, \alpha)$ [Eqs. (4.15) and (4.16)] but with $f_c(x')$ replaced by $\bar{f}_c(x')$. [Note that $\bar{h}_{cS}(\omega)$ is even in ω and is real.] In addition $\bar{f}_c$ and $\bar{g}_c$ obey one extra constraint. Thus defining $\tilde{h}_c(\omega)$ by

$$\tilde{h}_c(\omega) \equiv \int d\beta\,(\beta + 2/\omega)[\bar{f}_c(\beta, 1/\omega) + \bar{g}_c(\beta, 1/\omega)] + (\omega \to -\omega), \tag{4.21}$$

one must also require

$$\tilde{h}_c(\omega) = 0\,. \tag{4.22}$$

The tensor factor in Eq. (4.20a) also has the "transverse" nature that occurred in the electroproduction sector of Eq. (4.17). Equation (4.23) is imposed to guarantee this. That is, more generally, the calculation of $W_{ab}^{\mu\nu}$ yields an additional longitudinal piece proportional to $\xi_3 i f_{abc} \tilde{h}_c(\omega)(\eta^{\mu\nu} - q^\mu q^\nu/q^2)$. Thus Eq. (4.22) imposes the transverse property for the neutrino sector. One may view Eq. (4.22) as an extra condition to be satisfied by the φ-$\bar{f}$-f and $\bar{f}$-$\bar{N}$-N couplings.

The calculations of the contributions to $W_{ab}^{\mu\nu}$ of the $\xi_2 f_{abc}$ and $\xi_4 d_{abc}$ terms of Eq. (3.11) are straightforward. Adding these to the results of Eqs. (4.17) and (4.20) allows one to determine the structure functions of Eq. (3.4) for the total interaction:

$$\omega F_{2ab}(\omega; r) = d_{abc}[m g_\rho{}^2 \tfrac{1}{2}\xi_1 h_{cA}(\omega; r)] + i f_{abc}[m g_\rho{}^2 \xi_3 \bar{h}_{cS}(\omega; r)], \tag{4.23a}$$

$$F_{1ab}(\omega; r) = \tfrac{1}{2}\omega F_{2ab}(\omega; r), \tag{4.23b}$$

$$F_{3ab}(\omega; r) = r\{ i f_{abc}[2 m g_\rho{}^2 \xi_2 h_{cA}(\omega; r)] + d_{abc}[-2 m g_\rho{}^2 \xi_4 \bar{h}_{cS}(\omega; r)]\}. \tag{4.23c}$$

Now, the full content of the Fritzsch–Gell-Mann light commutators Eq. (1.2) for this process is Eq. (4.23b) plus the requirement that the coefficient of d_{abc} (and $i f_{abc}$) in Eq. (4.23a) equal the coefficient of $i f_{abc}$ (and d_{abc}) in Eq. (4.23c).[5] The latter conditions can be achieved by requiring

$$\xi_2 = \tfrac{1}{4}\xi_1\,, \qquad \xi_4 = -\tfrac{1}{2}\xi_3\,. \tag{4.24}$$

Thus by choosing relation (4.24) between the coupling constants, one satisfies completely the Fritzsch–Gell-Mann algebra for inelastic lepton scattering.

V. THE CALLAN-GROSS SUM RULE

In the previous section it was seen that the interaction of Eq. (3.11) produces a purely transverse cross section in the deep-inelastic region. This result appears to be in contradiction with previous ideas based on the Callan-Gross sum rule[19] that a field-current-identity current has vanishing $q^2\sigma_T$ in the limit $q^2 \to \infty$. We will show here that this is not necessarily the case and that the results of Sec. IV are in fact consistent with the Callan-Gross sum rule. The idea that the transverse cross section vanishes for a field-current-identity current arises in part from the application of the sum rule to the specific algebra of fields model of Lee, Weinberg, and Zumino.[20] Actually, if one assumes the validity of asymptotic scale invariance, one expects for this case that not only does $q^2\sigma_T$ vanish in the limit $q^2 \to \infty$, but also $q^2\sigma_L$ does. This is due to the fact that the currents appearing in the algebra of fields have scale dimension one. One needs currents of dimension three to obtain nonvanishing structure functions (as is indeed the case for both the quark model and the model of this paper). One can indeed obtain a nonvanishing $q^2\sigma_T$ (and a vanishing $q^2\sigma_L$) in a theory which uses the field-current identity provided that (i) the currents have the proper scale dimension three and (ii) a judicious choice of the coupling structure is made. The second condition is clearly vital, as the field-current identity itself is just an interpolating equation, and hence empty of dynamical content in itself until the interactions are chosen.

We begin by briefly reviewing the Callan-Gross sum rule and then examine it in the context of the interactions of Sec. III. The basic quantity considered is

$$R^{ij} = -[m/(p^0)^2] \int d^4x\, \delta(x^0) \langle p | [V^i(x), V^j(0)] | p \rangle_{p^0 \to \infty}, \tag{5.1}$$

where p^0 is the proton energy and $\vec{p}$ approaches infinity in a fixed direction. For a current of dimension 3, R^{ij} is dimensionless and can be decomposed into the form

$$R^{ij} = A\hat{p}_i\hat{p}_j + B(\hat{p}_i\hat{p}_j - \delta_{ij}), \tag{5.2}$$

where $\hat{p}_i \equiv p_i/|\vec{p}|$. Here A and B are also dimensionless. (On the other hand, if R^{ij} has scale dimension < 0, and if the interaction preserves scale invariance asymptotically, one would expect that A and B vanish.) Using dispersion relations for the virtual Compton amplitude, Callan and Gross relate A and B to the electroproduction structure functions

$$A = 2\int_0^1 \frac{d\alpha^2}{\alpha}[F_2(\alpha) - 2\alpha F_1(\alpha)], \tag{5.3a}$$

$$B = -\int_0^1 d\alpha^2 F_1(\alpha). \tag{5.3b}$$

Thus the vanishing of A implies the transverse condition $F_2 = 2\alpha F_1$ ($\alpha \equiv 1/\omega = -q^2/2q\cdot p$).

We now apply the sum rule to the interaction of Eq. (3.11). The electroproduction cross section is governed by the ξ_1 part of the interaction:

$$\mathcal{L}_{\rho H} = -\tfrac{1}{2}\xi_1 d_{abc}\rho_{\alpha\mu a}\rho^{\alpha}{}_{\nu b}\partial^\mu\partial^\nu f_c. \tag{5.4}$$

The canonical momentum of the ρ field π_{ia} is given by

$$\pi_{ia}(x) = \partial_0\rho_{ia} - \partial_i\rho_{0a} + 2\partial\mathcal{L}_{\rho H}/\partial\rho_{0ia}. \tag{5.5}$$

Using the ρ field equations and the canonical commutation relations, we find

$$\begin{aligned}\delta(x^0)[\dot{V}_a^i(x), V_b^j(0)] = ig_\rho{}^2\{&-m_\rho{}^{-2}\delta_{ab}\lambda(0)\partial^i\partial^j + \delta^{ij}\delta_{ab}\lambda(0)^2 \\ &+ m_\rho{}^{-2}\delta_{ab}\lambda(0)\lambda(x)^{-1}\partial^i\lambda(x)\partial^j + 4b^2\rho_a^i(0)\rho_b^j(0)\lambda(0)\}\,\delta^4(x) \\ &+ 2i\lambda(x)\lambda(0)[\partial^2\mathcal{L}_{\rho H}(x)/\partial\rho_{0ia}(x)\partial\pi_{kc}(0)](\partial\pi_{kc}/\partial\rho_{0jb}).\end{aligned} \tag{5.6}$$

The first term in Eq. (5.6) does not contribute to R^{ij}. In evaluating the next three terms, recall from the discussion at the beginning of Sec. IV that the effective Lagrangian is to be used only to second order in the ξ couplings. Thus, using the same analysis as the one following Eq. (4.11), the matrix element of the second term $\langle p|\lambda^2(0)|p\rangle$ is at least fourth order in the ξ coupling constants and hence is not to be retained. A similar argument holds for the third and fourth terms. Thus only the last term contributes to the Callan-Gross commutator, and there one may replace $\partial\pi_{kc}/\partial\rho_{0jb}$ by $\delta_j^k\delta_{ab}$. Since

$$\delta^2\mathcal{L}_{\rho H}/\delta\rho_{0jb}(0)\delta\rho_{0ia}(x) = \xi_1 d_{abc}(\partial^i\partial^j f_c - \delta^{ij}\partial^0\partial^0 f_c), \tag{5.7}$$

this term contributes a structure to R^{ij} proportional to

$$R^{ij} \sim [m/(p^0)^2]\,\xi_1 d_{abc}\langle p|[\lambda(0)]^2[\partial^i\partial^j f_c(0) - \delta^{ij}\partial^0\partial^0 f_c]|p\rangle. \tag{5.8}$$

Since f_c has scale dimension -2, and λ^2 has dimension $+4$, the matrix element has dimension $+2$. Lorentz covariance then implies

$$R^{ij} \sim [m/(p^0)^2]\,\xi_1 d_{abc}B_c[p^ip^j - \delta^{ij}(p^0)^2], \tag{5.9}$$

where B_c is a dimensionless constant. Thus R^{ij} is purely transverse in the limit $p^0 \to \infty$. The Callan-Gross sum rule then implies a purely transverse electroproduction cross section for the interactions and currents being considered here, in accord with the direct calculation of the light-cone commutators in Sec. IV.

VI. CONCLUSIONS

In this paper we have attempted to realize the formal structure of the Fritzsch–Gell-Mann light-cone algebra for deep-inelastic lepton scattering without the introduction of physical quark fields. To achieve this we have made use of a set of modified field-current-identity currents which arose naturally in the intermediate-energy region from the principles of scale invariance and scale breaking. The primary constituents of the 18-plet of chiral $\tilde{U}(3)\times U(3)$ currents are the nonets of vector and axial-vector mesons as in the low-energy vector-dominance currents. However, the scale-breaking conditions of the intermediate-energy domain produce an additional dilaton (ϵ-meson) "clothing" factor which automatically give the currents a scale dimension three. Thus the currents used here have the same scale dimension as the quark currents (though of very different physical nature) as is necessary if Bjorken scaling is to be achieved. Our procedure then was to set up an effective Lagrangian whose scale-invariant interactions were chosen to produce the same effects for deep-inelastic lepton scattering as the Fritzsch–Gell-Mann algebra does. The situation is similar here to the approach adopted in the low-energy domain. There too, the equal-time $SU(3) \times SU(3)$ algebra, first derived on the basis of the quark model, was adopted as a principle to determine interactions governing low-energy phenomenology without any further reference to physical quarks.

The Fritzsch–Gell-Mann algebra strongly restricts the type of interactions that can occur. The asymptotic chiral invariance which gives rise to the conservation of the currents is most directly achieved by assuming that the couplings involve only the vector-meson field strengths $\rho_{\mu\nu a} = \partial_\mu \rho_{\nu a} - \partial_\nu \rho_{\mu a}$. For inelastic lepton scattering, the coupling must be bilinear in the $\rho_{\mu\nu a}$. The simplest way of achieving the light-cone algebra then turns out to involve coupling the $\rho_{\mu\nu a}$ to a scalar field f_a, the latter then coupling directly to the nucleons. (The f_a are scalar functions of other hadron fields.) The light-cone algebra requires that the ρ-ρ-f interaction contain at least two derivatives of the fields. This is one derivative higher than the usual smoothness assumptions of low-energy current algebra. This higher momentum dependence distinguishes the interactions in the asymptotic high-energy region from those used in the low- and intermediate-energy region. The f_a may be thought of in some sense as representing "partons," for the nucleons "emit" the f_a fields and the ρ mesons (and hence the photons) are scattered by the f_a fields rather than directly by the nucleons.

In the quark model, the structure functions are determined from nucleon matrix elements of the bilocal currents. The detailed properties of the structure functions then depend on the detailed nature of the gluon coupling structure assumed. Similarly in the present model, the structure functions are determined by the nucleon matrix element involving the dilaton and "parton" field f_a. Again, the detailed properties of the structure function depend upon the interactions between the dilaton and f_a fields and their interactions with the nucleons. The light-cone algebra does, however, put constraints on these interactions. Thus only the f_a couples directly to the nucleons and the dilaton must couple to an even number of f_a fields. Aside from these conditions, we have purposely not specified the details of the dilaton and f_a interactions to illustrate the minimum number of assumptions necessary to achieve the light-cone algebra. However, the fact that experimentally[9] structure functions can be expressed as a low-order polynomial in $\alpha \equiv 1/\omega \equiv -q^2/2q\cdot p$ suggests that it should be possible to construct simple interactions that yield the experimental results.

In Sec. IV, it was seen that the effective Lagrangian approach can recover the full content of the Fritzsch–Gell-Mann light-cone algebra for the inelastic lepton scattering. Thus, as in the quark model, we also predict the vanishing of $q^2\sigma_L$ for deep-inelastic electroproduction, a result that appears to be in approximate agreement with present data. Further, we also recover the various parton formulas relating the neutrino and electron structure functions. One difference, however, does exist between the present model of this paper and the quark model. It is possible to relax the vanishing of the longitudinal part of the cross section for neutrino scattering processes (though *not* for electroproduction) by an appropriate choice of the f_a and dilaton couplings. Thus should future data indicate the existence of a longitudinal part to neutrino cross sections, the present model could accommodate it, though the quark model could not.

Finally, we mention that the formalism developed here can be extended to the calculation of higher multiple light-cone commutators (e.g., the analog of the bilocal algebra in the quark model). To obtain the higher commutators, one must add terms cubic and higher in the ρ fields to the effective Lagrangian, and require that the light-cone algebra be satisfied to higher order. The situation is similar to the treatment of the equal-time current algebra, where the effective Lagrangian for the matrix element of n currents is a polynomial of order n in the fields.

APPENDIX

In this Appendix we verify the theorem quoted in Eq. (4.9) that one may calculate the $W^{\mu\nu}_{ab}$ replacing the $\tilde{\Delta}^{\mu\nu}_{\text{ret}}$ and $\tilde{\rho}^{\mu}_{a}$ of Eqs. (4.5) by $\Delta^{\mu\nu}_{\text{ret}}$ and $\rho^{\mu}_{a\,\text{in}}$ provided one inserts the projection operators $P^{\mu\alpha}$ and $P_\beta{}^{\nu}$ (which guarantee the CVC conditions).

From the discussion at the beginning of Sec. IV, one must calculate the currents to first order in $\mu(\varphi) \equiv \lambda(\varphi) - 1$. The quantity depending on λ in $V^{\mu}_{a(1)}$ of Eq. (4.7b) is the factor[21] $\lambda(x)\tilde{\Delta}^{\mu\sigma}_{\text{ret}}(x-x')$. Using Eq. (4.5b) and expanding to first order in $\mu(\varphi)$ gives

$$\lambda(x)\tilde{\Delta}^{\mu\sigma}_{\text{ret}}(x-x')=\Delta^{\mu\sigma}_{\text{ret}}(x-x')+\int d^4x''\,[\eta^{\mu\alpha}\delta^4(x-x'')-m_\rho{}^2\Delta^{\mu\alpha}_{\text{ret}}(x-x'')]\mu(x'')\Delta_{\alpha}{}^{\sigma}_{\text{ret}}(x''-x')\,. \tag{A1}$$

The zeroth-order term, $\Delta^{\mu\sigma}_{\text{ret}}$, corresponds to using a current with no dilaton clothing, and hence produces no contribution in the deep-inelastic limit. When the second term of Eq. (A.1) is inserted into Eq. (4.7b), the commutator of Eq. (4.8) taken, and the matrix element $W^{\mu\nu}_{ab}$ of Eq. (3.2) computed, the bracket produces simply the factor

$$\eta^{\mu\alpha}-m_\rho{}^2\Delta^{\mu\alpha}_{\text{ret}}(q)=\eta^{\mu\alpha}-m_\rho{}^2(\eta^{\mu\alpha}+q^\mu q^\alpha/m_\rho{}^2)(q^2+m_\rho{}^2)^{-1}\,. \tag{A2}$$

In the $q^2\to\infty$ limit this term becomes $P^{\mu\alpha}(q)$.

The $P_\beta{}^\nu(q)$ factor of Eq. (4.9) arises similarly when one expands $V^\nu_{b(0)}(y)$ in Eq. (4.8) to first order in $\mu(\varphi)$. Thus using Eq. (4.5a) and the definition of $V^\nu_{b(0)}$ of Eq. (4.7a), one finds for the parts linear in $\mu(\varphi)$ the structure

$$\int d^4y'\,[\eta^\nu{}_\beta\delta^4(y-y')-m^2\Delta^\nu_{\beta\,\text{ret}}(y-y')]\mu(y')\rho^\beta_{\text{in}}(y')\,. \tag{A3}$$

Again, when this structure is inserted into the commutator and the Fourier transform taken, the extra factor in the bracket produces the $P_\beta{}^\nu(q)$ term of Eq. (4.9) in the scaling limit. The nucleon matrix elements that remain after the commutator is taken are of the form of Eq. (4.15) with $\lambda(\varphi)$ replaced now by $\mu(\varphi)$. However, in the scaling limit these structures become the same, as $\mu(\varphi)$ has scale dimension two and is the leading term of $\lambda(\varphi)$.

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[1]H. Fritzsch and M. Gell-Mann, Caltech Report No. CALT-68-297 (unpublished); and in *Broken Scale Invariance and the Light Cone,* 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, p. 1.

[2]Also see J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).

[3]K. G. Wilson, Phys. Rev. 179, 1499 (1969).

[4]Y. Frishman, Phys. Rev. Letters 25, 966 (1970); G. Altarelli, R. A. Brandt, and G. Preparata, *ibid.* 26, 42 (1971).

[5]D. J. Gross and S. Treiman, Phys. Rev. D 4, 1059 (1971).

[6]M. Gell-Mann, Physics 7, 63 (1964).

[7]R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters 19, 1085 (1967); H. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); S. G. Brown and G. B. West, Phys. Rev. Letters 19, 812 (1967); J. Schwinger, Phys. Letters 24B, 473 (1967); J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).

[8]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[9]M. Breidenbach *et al.*, Phys. Rev. Letters 23, 935 (1969); G. Miller *et al.*, Phys. Rev. D 5, 528 (1972).

[10]P. Nath, R. Arnowitt, and M. H. Friedman, Phys. Rev. D 6, 1572 (1972); M. H. Friedman, P. Nath, and R. Arnowitt, Phys. Letters 42B, 361 (1972).

[11]S. P. de Alwis and P. J. O'Donnell, Phys. Rev. D 2, 1023 (1970); P. Carruthers, Phys. Reports 1C, 1 (1970).

[12]The principle of scale breaking combined with the experimental f-meson decay rates determines $F_\sigma\simeq F_\pi$, where F_π is the pion decay constant. See R. Arnowitt, M. H. Friedman, and P. Nath, Northeastern Univ. Report No. NUB 2098 (unpublished); M. H. Friedman, P. Nath, and R. Arnowitt, Phys. Letters 42B, 361 (1972).

[13]In the chiral-symmetry limit all members of the 18-plet of spin-1 mesons are assumed to have the same mass. Also all the interpolating constants appearing in $V^\mu_a(x)$ have the same value g_ρ.

[14]Friedman, Nath, and Arnowitt, Ref. 10.

[15]If one wished to obtain the effective Lagrangian for a process such as electroproduction of μ pairs ($e+N\to e+\mu+X$) which involves higher commutators of currents (the bilocal algebra in the quark model) one would have to include terms cubic in $\mathcal{L}_{\rho H}$. The situation is analogous to the construction of the effective Lagrangian in the equal-time commutator case. There the effective Lagrangian was also a polynomial in the phenomenological fields, and terms up to order n were needed to calculate the vacuum matrix elements of n currents.

[16]A nucleon matrix element of the type $\langle p|f_af_bf_cf_d|p\rangle$ reduces to the form $\xi_5\langle 0|f_af_bf_c|0\rangle$ by the $\xi_5 f$-$\bar{N}$-N coupling. The three-point f_c function is a triangle diagram (again using the ξ_5 f-$\bar{N}$-N couplings). Such closed-loop diagrams are conventionally discarded in the effective-Lagrangian approach.

[17]We use functions as defined in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Appendix C.

[18]Thus the part of Eq. (4.18) which is the first moment of $f_c(\beta,\alpha)$, $\alpha=1/\omega$; i.e., $\int d\beta f_c(\beta,\alpha)\beta$; is determined by $[\partial^\mu_x M_c(p\cdot x, p\cdot y)]_{x=0}$. Thus only $M_c(p\cdot x, p\cdot y)$ and its first derivative at $x^\mu=0$ enter into the structure functions.

[19]C. G. Callan and D. Gross, Phys. Rev. Letters 22, 156 (1969).

[20]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[21]The additional term linear in $\mu(\phi)$ obtained by replacing $\lambda\tilde{\Delta}^{\mu\sigma}_{\text{ret}}$ by $\Delta^{\mu\sigma}_{\text{ret}}$ and expanding the $\tilde{\rho}^{\alpha\beta}_b$ in $J_{\sigma a}(x)$ to linear order in $\mu(\phi)$ can easily be seen to lead to a contribution to $V^\mu_{a(1)}$ of scale dimension one, and hence gives vanishing contribution to $W^{\mu\nu}_{ab}$ in the deep-inelastic limit.